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Multi-objective optimization of orthogonally stiffened cylindrical shells using optimality criteria method

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Abstract. A multi-objective Optimality Criteria (OC) is used to obtain optimum design of metal cylindrical shells under combined external loading. The objectives are to maximize the axial and hoop stiffness and minimize the mass of stiffened cylinders subject to the constraints, including functions of weight and buckling load, in such a way that the stiffened shell has no increase in weight and no decrease in buckling load, with respect to the initial unstiffened shell. The optimization process contains six design variables, including shell thickness, number of circular ring stiffeners, number of longitudinal stringer stiffeners, height of ring stiffeners, width of ring stiffeners, and longitudinal stiffener eccentricity from the shell centerline. In the analytical solution, the Rayleigh-Ritz energy procedure is applied and the ring stiffeners are treated as discrete elements. The shapes of the ring and stringer stiffeners are assumed rectangular and Z, respectively. The shell is subjected to uniform axial and non-constant external pressure, simultaneously. The longitudinal stringers are placed in equal spacing, whereas the rings can be placed in an unequal space, due to the non-constant of external pressure over the cylinder length. The results show that the iteration numbers depend on the ring stiffener space states.

1. Introduction

Cylindrical shells are mostly used in structural applications, such as pipes conveying fluids or gases, aircraft and aerospace industries, and marine vessels. With the aim of having minimum weight, the load-carrying capacity of these structures is generally restricted by a loss of stability under axial compression and external pressure. Stiffening of the shell with longitudinal stringers and circumferential rings is one of a few ways to increase the shell buckling load and reduce its weight. Some researchers have treated the optimization of stiffened cylindrical shells. Damodar and Navin [1] examined the optimal design of composite grid-stiffened panels and shells with variable curvature subjected to global and local buckling constraints. Akl et al. [2] carried out the optimal design of underwater isotropic stiffened cylindrical shells using a Pareto/min-max multi-criteria optimization procedure. The adopted procedure aimed to, simultaneously, minimize either the shell vibration and associated sound radiation or the vibration, noise radiation, weight, and cost of the stiffened shell. Also, some researchers have treated the optimization of stiffened cylindrical shells. Topal [3] carried out the multi-objective optimization of laminated composite cylindrical shells for maximum frequency and buckling load without any constraint.

The present paper deals with the multi-objective optimization of ring and stringer stiffened cylindrical shells for minimum weight and maximum critical global buckling load, which includes axial compression and hydrostatic external pressure, using the OC method. The constraints involve functions of weight and critical

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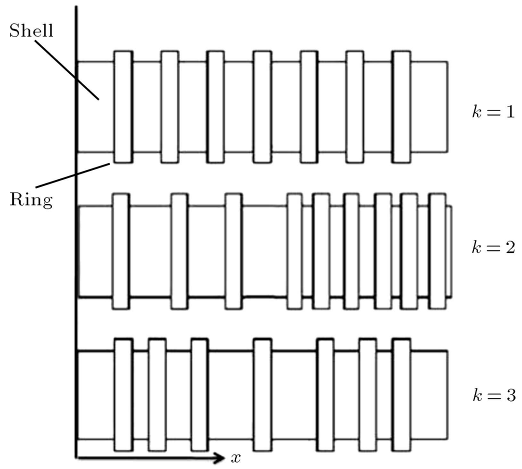

Figure 1. Three types of ring spacing on the cylindrical shells.

buckling load. The analysis is implemented based on Classical Shell Theory for shells, with simply supported boundary conditions. The optimization is carried out for stiffened shells with three types of ring spacing distribution, which is illustrated in Figure 1. The study provides the optimum values of the objective functions, corresponding to the optimal values of the design variables, including shell thickness, number of rings and stringers, height of rings, the eccentricity of stringers from the centerline of the shell, and the order of ring spacing distribution. Comparisons between the two single-objective weight and buckling optimizations, as well as the multi-objective weight and buckling optimization, are made, being useful for relevant research and applications.

2. OC method

In general, Optimality Criteria (OC) methods are optimization algorithms which seek the optimum through finding a solution that satisfies some pre-specified criteria, which are postulated to the corresponding optimal result for the problem. In these methods, the optimum is sought without explicit concern for an objective function. Design modifications in these methods are usually based on a recursive resizing formula derived from the optimality criterion [4].

The attractive features of OC methods caused further advances in modern structural optimization. For example, Sander and Fleury [5] proposed a mixture of optimality criteria and mathematical programming methods in a dual formulation of the design problem.

Since all sizes and numbers of stiffeners are defined in discrete value assembly, the OC method can be the appropriate option for optimal solution. The optimum value of the variable from Eq. (1) is found as:

$$\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\left(\frac{\partial g_k}{\partial y_i}\right)\left(\frac{\partial g_j}{\partial y_j}\right)}{x_i^3\left(\frac{\partial f}{\partial x_i}\right)}\times\lambda_j=\sum_{i=1}^{n}\frac{\frac{\partial g_k}{\partial y_i}}{x_i}+\eta g_k(x)y_i=\frac{1}{x_i},$$

$$k=1,2,...,m, \tag{1}$$

in which x_i is the variable, λ_j are the Lagrangian coefficients, η is the value of optimization steps, function (f) is the objective function and, (g_i) are condition functions that are defined, subsequently.

3. Analytical formulation

Consider a thin uniform cylindrical shell with uniform thickness, t, radius, R, length, L, mass density, ρ, modulus of elasticity, E, Poisson ratio, v, and shear modulus, $G=E/2(1+v)$, as displayed in Figure 2. The shell is circumferentially stiffened by n_r rings of equal or unequal spacing and is also longitudinally stiffened by n_s stringers of equal spacing, which may be placed either internally or externally. The form of the ith ring stiffener cross section is rectangular with constant width, b_{ri}, and depth, d_{ri}, and is located at distance x_{ri} from one end of the shell, which can be arranged by one of the three types of ring spacing (Figure 1). The height of the rings may vary along the length of the shell but the width of the rings is equal along it. The stringer sections are Z-shaped and the same size. A coordinate system (x,y,z) is fixed on the middle surface of the shell at one of its two ends, as shown in Figure 2. In general, the stiffeners may be made of materials different from the parent shell material, with ρ_r, ρ_s, E_r, E_s, G_r, G_s, v_r and v_s denoting the ring and stringer mass density, Young modulus, shear modulus, and Poisson ratio, respectively. All rings and stringers are the same material.

The displacement field in the classical theory of thin shells can be expressed in the cylindrical coordinate system as follows:

$$u(x,\theta,z)=u_0(x,\theta)+z\left(-\frac{\partial w_0(x,\theta)}{\partial x}\right),$$

$$v(x,\theta,z)=v_0(x,\theta)+z\left(-\frac{\partial w_0(x,\theta)}{R\partial\theta}+\frac{v_0(x,\theta)}{R}\right),$$

$$w(x,\theta,z)=w_0(x,\theta), \tag{2}$$

where (u,v,w) are the orthogonal components of displacement of an arbitrary point (x,θ,z) in the shell along the coordinates (x,θ,z), respectively, and (u_0,v_0,w_0) are the displacements of the shell mid-surface at point (x,θ).

The strain-displacement relations from Love thin shell theory are defined as:

$$\varepsilon=\varepsilon^{(0)}+Z\kappa,$$

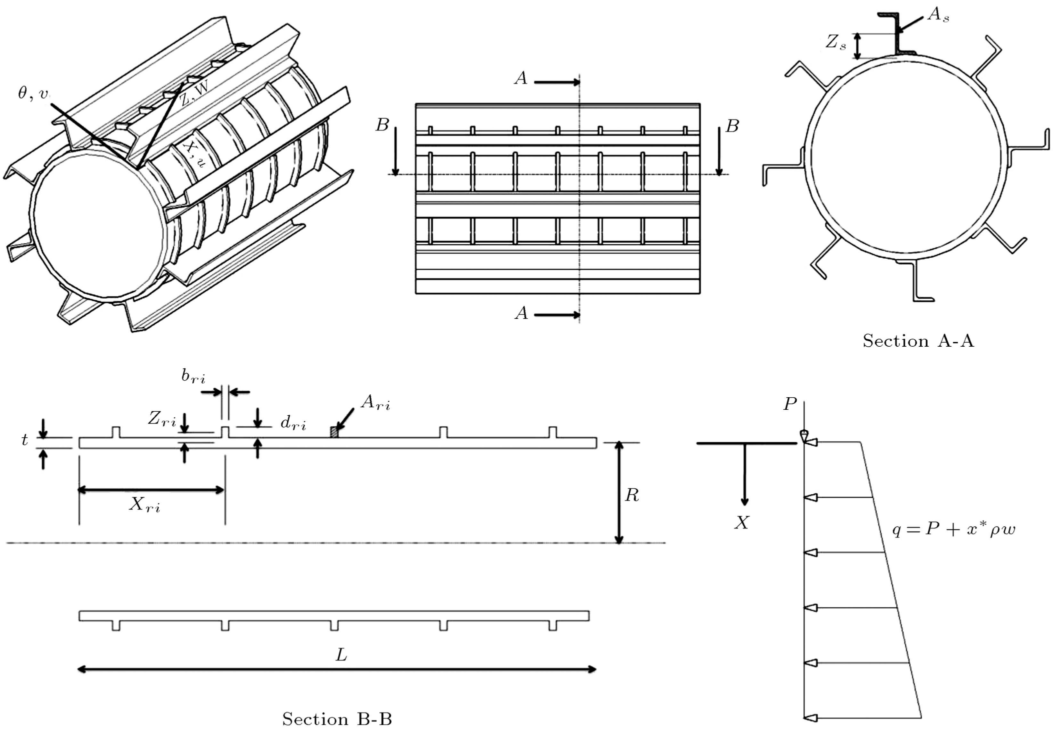

Figure 2. Stiffened shell configuration, geometrical properties, loading method and coordinate system.

$$\varepsilon^{(0)} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{\theta\theta}^{(0)} \\ \gamma_{x\theta}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{1}{R}\frac{\partial v_0}{\partial \theta} + \frac{w_0}{R} \\ \frac{1}{R}\frac{\partial u_0}{\partial \theta} + \frac{\partial v_0}{\partial x} \end{Bmatrix},$$

$$\kappa = \begin{Bmatrix} \kappa_{xx} \\ \kappa_{\theta\theta} \\ \kappa_{x\theta} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{1}{R^2}\left(\frac{\partial^2 w_0}{\partial \theta^2} - \frac{\partial v_0}{\partial \theta}\right) \\ -\frac{1}{R^2}\left(2\frac{\partial^2 w_0}{\partial x \partial \theta} - \frac{\partial v_0}{\partial x}\right) \end{Bmatrix}, \tag{3}$$

where $\varepsilon_{xx}^{(0)}$, $\varepsilon_{\theta\theta}^{(0)}$ and $\gamma_{x\theta}^{(0)}$ are the normal and shear strains at the mid-surface ($z = 0$), κ_{xx}, $\kappa_{\theta\theta}$ are the mid-surface changes in curvature and $\kappa_{x\theta}$ is the mid-surface twist.

The boundary conditions of the shell are standard simply supported $v = w = 0$ at $x = 0, L$. The displacement components (u_0, v_0, w_0) may be defined as:

$$u_0(x, y) = u_1 \cos\left(\frac{m\pi x}{L}\right)\cos(n\theta),$$

$$v_0(x, y) = v_1 \sin\left(\frac{m\pi x}{L}\right)\sin(n\theta),$$

$$w_0(x, y) = w_1 \sin\left(\frac{m\pi x}{L}\right)\cos(n\theta), \tag{4}$$

where m and n are the longitudinal and circumferential wave numbers, respectively, and u_1, v_1 and w_1 are the displacement coefficients. The strain energy of the shell in terms of strains is expressed by:

$$U_{\text{shell}} = \frac{E}{2(1-v^2)} \int_0^L \int_0^{2\pi R} \int_{-\frac{t}{2}}^{\frac{t}{2}} \left(\varepsilon_{xx}^2 + \varepsilon_{\theta\theta}^2 + 2v\varepsilon_{xx}\varepsilon_{\theta\theta} + \frac{1-v}{2}\gamma_{x\theta}^2\right) dz d\theta dx, \tag{5}$$

in which ε_{xx} and $\varepsilon_{\theta\theta}$ are the normal strains in the x and θ directions of the shell, and $\gamma_{x\theta}$ is the shear strain in the x-θ plane.

The strain energy of all stringers is expressed as:

$$U_{\text{string}} = \frac{1}{d} \int_0^L \int_0^{2\pi R} \left\{ \frac{E_s}{2}\left[A_s\varepsilon_{xx}^2 - 2Z_s A_s \varepsilon_x \frac{\partial^2 w}{\partial x^2} + (I_{xs} + A_s Z_s^2)\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right] + \frac{G_s(I_{\theta s} + I_{zs})}{2}\left(\frac{\partial^2 w}{\partial x \partial \theta}\right)^2 \right\} d\theta dx, \tag{6}$$

where I_{xs}, $I_{\theta s}$ and I_{zs} are the second moment of areas about the x, θ and z axes, A_s is the cross-sectional area, Z_s is the eccentricity of the stringers, d is the equal space between the stringers, and ε_{xx} and w are the axial strain and out of plane deformations, respectively, as defined in Eqs. (1) and (2). In this formula, the stringers are considered a continuous shell above the main shell.

Table 1. Comparison of critical buckling loads, P_{xcr} (N/mm), and the number of iterations for finding optimum values of ring-stiffened cylindrical shells.

Stiffened type	P_{cr} in [8]	P_{cr} in the present study	Difference in P_{cr}	Iteration number in [8]	Iteration number in the present study	Difference in iteration number
Ring stiffened	9179.3	9181.26	2%	10	5	5

The displacement functions of the ring stiffeners in three directions are:

$$u_r = u_0 + Z_r\left(-\frac{\partial w}{\partial x}\right),$$

$$v_r = v_0\left(1+\frac{Z_r}{R}\right) + Z_r\left(\frac{1}{R}\frac{\partial w}{\partial \theta}+\frac{v_0}{R}\right),$$

$$w_r = w_0. \tag{7}$$

Now, the strain energy for the ith ring is expressed as:

$$\begin{aligned}U_{\text{ringi}} =& \frac{1}{2}\int_0^{2\pi R}\left[\frac{E_r A_r}{R+Z_r}\left(\frac{\partial v_r}{\partial \theta}+w_r\right)^2\right.\\ &+\frac{E_r I_{\theta r}}{(R+Z_r)^3}\left(\frac{\partial^2 w_r}{\partial \theta^2}+w_r\right)^2\\ &+\frac{E_r I_{zr}}{R+Z_r}\left(-\frac{1}{R+Z_r}\frac{\partial^2 u_r}{\partial \theta^2}+\frac{\partial w_r}{\partial x}\right)^2\\ &\left.+\frac{G_r(I_{\theta r}+I_{zr})}{R+Z_r}\left(\frac{\partial^2 w_r}{\partial x \partial \theta}+\frac{1}{R+Z_r}\frac{\partial u_r}{\partial \theta}\right)^2\right]d\theta,\end{aligned} \tag{8}$$

where $I_{\theta r}$ and I_{zr} are the second moment of areas about θ and z axes, A_r is the cross-sectional area, and Z_r is the eccentricity of the ith rings.

The work done by the compressive load (P) and external hydrostatic pressure ($q = P+L*\rho_w$) (Figure 2) during buckling is defined as [6]:

$$w_p = -\int_0^L\int_0^{2\pi}\left\{\frac{pR}{2}\left[\left(\frac{\partial u_0}{\partial x}\right)^2+\left(\frac{\partial v_0}{\partial x}\right)^2 + \left(\frac{\partial w_0}{\partial x}\right)^2\right]\right\}d\theta dx,$$

$$w_p = \frac{-(p+q)R}{2}\int_0^L\int_0^{2\pi}\left(2w\frac{\partial u_0}{\partial x}+\frac{2w\frac{\partial v_0}{\partial \theta}}{R} - \frac{\partial u_0}{\partial x}\frac{\partial v_0}{\partial \theta}-\frac{w^2}{R}\right)d\theta dx. \tag{9}$$

The total potential energy, U_{tot}, of the system is obtained by:

$$U_{\text{tot}} = U_{\text{shell}} + U_{\text{string}} + \sum_{i=1}^{N} U_{\text{ringi}} + w_p + w_q. \tag{10}$$

For starting the optimization process, we use the exact formula for critical load that is obtained according to [7].

Also, for the optimization process, the weights of the shell, rings, and stringers are given by:

$$W_{sh} = 2\pi R t L \rho_{sh} g,$$

$$W_r = 2\pi\left(R+\frac{t}{2}+Z_r\right)A_r n_r \rho_r g$$

$$\rightarrow W_{\text{tot}} = W_{sh} + W_r + W_s,$$

$$W_s = L A_s n_s \rho_s g. \tag{11}$$

4. Validation of buckling formulation

To examine the validity of the derived solution, the critical optimum values of variables obtained from the presented analysis are compared with those existing in well-known sources for the case of ring-stiffened cylindrical shells.

For a ring-stiffened cylindrical shell, the results of the current study are compared with those of Ref. [8], as presented in Table 1. The geometric and material properties of the simply supported, externally equal spaced, ring-stiffened cylindrical shell are $L = 247.5$ mm, $R = 82.5$ mm, $t = 2.5$ mm, $v = 0.29$ and $E = 200$ GPa. In Table 2, we describe our optimization procedure for this case ($k = 1$).

As seen in Table 2, our optimization procedure reached the same result as in Ref [8] in 5 steps less than in Ref [8].

5. OC optimization procedure

A multi-objective, weight and buckling optimization of orthogonally stiffened cylindrical shells is proposed for three types of circumferentially stiffened ring. Emphasis is placed on selecting the optimal design parameters of stiffened shells using the OC method. Consider a shell with length $L = 2$ m, radius $R = 0.75$ m, and initial thickness $t = 0.01$ m with simply supported boundary conditions. The shell and stiffeners are made of steel with $\rho = 7800$ kg/m^3, $E = 200$ GPa and $v = 0.3$. The unstiffened shell has a weight of W_0 and critical buckling load of P_{cr0}. The optimization aims to maximize shell buckling load and minimize shell

Table 2. Optimization path that program passed to reach the result of Ref. [8] for case $k = 1$.

Iteration	t (m)	Z_s- ($t/2$)	n_s	n_r	W (KN)	P_{cr} (KN/mm)		**Ring's no.=n_r** $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$
1	0.0025	-	-	-	24.61	9.17		-	-	-	-	-	-
2	0.002	-	-	3	21.6	9.18	$b_r =$	0.004	0.004	0.004	-	-	-
							$d_r =$	0.004	0.004	0.004	-	-	-
3	0.00165	-	-	5	19.8	9.1808	$b_r =$	0.0035	0.004	0.004	0.004	0.0035	-
							$d_r =$	0.0045	0.0045	0.0055	0.0045	0.0045	-
4	0.00155	-	-	6	19.66	9.1811	$b_r =$	0.0035	0.0035	0.004	0.004	0.0035	0.0035
							$d_r =$	0.0045	0.005	0.0055	0.0055	0.005	0.0045
5	0.0015	-	-	6	**19.6**	**9.1812**	$b_r =$	0.0035	0.0035	0.004	0. 004	0.0035	0.0035
							$d_r =$	0.005	0.0055	0.006	0.006	0.0055	0.005

weight, instantaneously. Throughout the optimization process, the thickness of the shell is reduced and the number of stiffeners is increased at each step. On this basis, the total objective function (f) is defined as:

$$f = \frac{W}{W_0} - \frac{P_{cr}}{P_{cr0}}. \tag{12}$$

In the OC method, the condition functions, (g_i), are defined as:

$$\begin{aligned} &g_1 = 0.005 < d_r, \quad g_2 = d_r < 0.05, \\ &g_3 = 0.01 < b_r, \quad g_4 = b_r < 0.1, \\ &g_5 = 0.02 < Z_s - t/2, \quad g_6 = Z_s - t/2 < 0.1, \\ &g_7 = 4 < n_s, \quad g_8 = n_s < 22, \\ &g_9 = 3 < n_r, \quad g_{10} = n_r < 21. \end{aligned} \tag{13}$$

Therefore, in Eq. (1), "m" is equal to number 10 and "n" is equal to number 5. Moreover, the identical sized rings can be spaced in different arrangements. A ring spacing distribution function (x_{ri}) representing the location of the ith ring along the shell length, is defined as:

$$\begin{aligned} x_{r1} &= 0.2m, \\ x_{ri} &= x_{r(i-1)} + \frac{3L}{2n_r} \\ &+ \left\{ \left[(2k-5) - \frac{(k-2)}{2} \right] (k-1) \right\} \\ &\left(\sum_{j=2}^{i \leq \frac{n_r}{2}} \frac{1}{2(j-1)} \frac{L}{n_r} \right) \\ &+ \left\{ \left[(k-4) - (k-3) + \frac{(k-2)}{2} \right] (k-1) \right\} \\ &\left(\sum_{j=\frac{n_r}{2}+1}^{i} \frac{1}{2(j-1)} \frac{L}{n_r} \right), \quad i > 1. \end{aligned} \tag{14}$$

6. Optimization results and discussion

The discrete values of the design variables used in the OC method are listed below. At each step, the value of the variables will be compared with those available in the list, and by considering the constraints, the nearest value will be selected. The OC procedure will be continued until the convergence in the results is achieved. In this work, the control of convergence was undertaken on variables b_{ri} and d_{ri} in which, in two consecutive iterations, the value of b_{ri} or d_{ri} was almost the same; then, the OC procedure would be stopped. Also, the eccentricity of the stringer stiffeners, as a variable, was considered in this study. The Z-shaped cross sections are selected from available commercial products, listed as below:

$$\begin{aligned} t = \{&0.005, 0.0045, 0.004, 0.0035, 0.003, 0.0025, \\ &0.002, 0.0015, 0.001, 0.0005\}, \\ d_r = \{&0.05, 0.045, 0.04, 0.035, 0.03, 0.025, \\ &0.02, 0.015, 0.01, 0.005\}, \\ b_r = \{&0.1, 0.09, 0.08, 0.07, 0.06, 0.05, 0.04, \\ &0.03, 0.02, 0.01\}, \\ (Z_s - t/2) = \{&0.1, 0.09, 0.08, 0.07, 0.06, 0.05, \\ &0.04, 0.03, 0.025, 0.02\}, \\ n_s = \{&22, 20, 18, 16, 14, 12, 10, 8, 6, 4\}, \\ n_r = \{&21, 19, 17, 15, 13, 11, 9, 7, 5, 3\} \end{aligned}$$

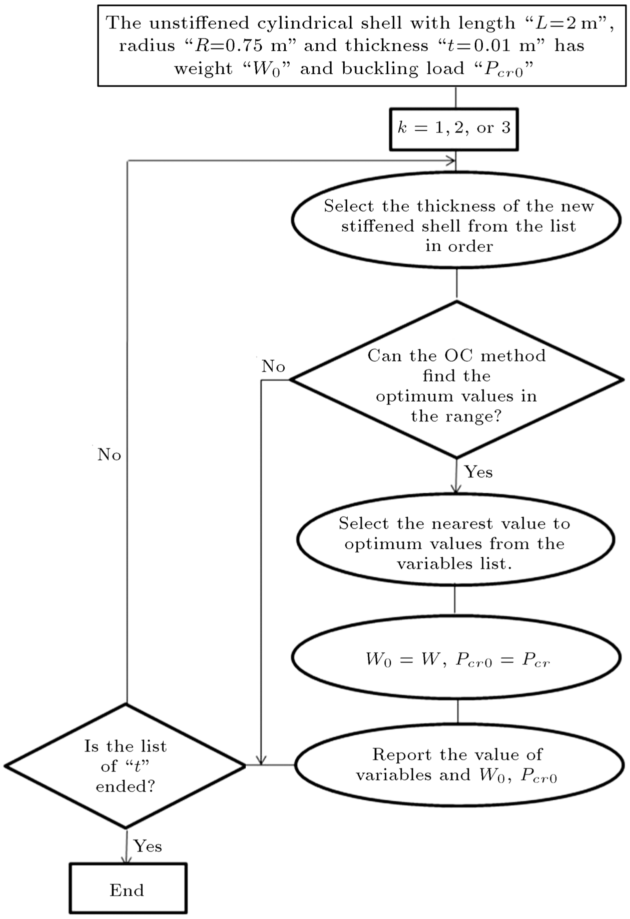

Figure 3. Flowchart of the optimization procedure.

The following three examples are presented; In each example, the optimization procedure is illustrated in Figure 3.

I. The stiffened shell is subjected only to uniform axial pressure;

II. The stiffened shell is subjected only to external pressure;

III. The stiffened shell is subjected to both uniform axial and external pressure.

Tables 3 to 5 show the optimization process and convergence study for a stiffened cylindrical shell under uniform axial pressure only (example I), for different values of "k", denoting different ring spacing. The obtained results show that for axial pressure only, the uniform distribution of ring spacing provides the highest buckling load with minimum weight. The convergence process yields to 4 iterations, and is one less than that for $k = 2$ and 3.

Tables 6 to 8 illustrate the optimization progress and convergence study of a stiffened cylindrical shell under external pressure only (example II), for different values of k. It is seen that for the state of non-uniform external pressure (example II), the optimum design, based on maximum buckling load and minimum weight, has been achieved for the case when $k = 2$, with the lowest iteration in the optimization process. The next optimum design belongs to $k = 3$ and the worst case is when $k = 1$ was chosen.

In the last example, the optimization process has been practiced for stiffened cylindrical shells under combined axial and external pressure. The results of the convergence study have been tabulated in Tables 9 to 11. It can be observed that for this case, the optimum design was found for $k = 3$, with the highest buckling strength, and minimum weight and optimization iteration of all cases, including $k = 1$ and 2. The second optimum design was obtained for $k = 1$ and the worst for $k = 2$.

Table 12 compares the optimum design of three different types of ring stiffened cylinder. The non-dimensional parameter, $\eta = P_{cr}/W$, is the ratio of the critical buckling load over the weight.

Conclusions

This study considers an optimization study of double stiffened cylindrical shells under uniform axial loading and non-uniform external pressure. The objective is to obtain the optimum design of the ring arrangement, in order to achieve maximum buckling strength and minimum weight. Being a powerful discrete constraint optimization, optimality criteria was implemented for this research.

Table 3. Optimization path that program passed to reach the result of example I and for $k = 1$.

Iteration	t (m)	$Z_s - (t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)		Ring's no.=n_r $i = 1$	$i = 2$	$i = 3$
1	0.01	-	-	-	7.212	16.14		-	-	-
2	0.004	0.07	4	3	7.204	20.45	$b_r =$	0.06	0.06	0.06
							$d_r =$	0.045	0.04	0.045
3	0.0035	0.05	8	3	6.238	23.72	$b_r =$	0.06	0.05	0.06
							$d_r =$	0.04	0.035	0.04
4	0.003	0.04	12	3	6.124	**24.14**	$b_r =$	0.05	0.04	0.05
							$d_r =$	0.04	0.03	0.04

Table 4. Optimization path that program passed to reach the result of example I and for $k = 2$.

Iteration	t (m)	$Z_s-(t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)		Ring's no.$=n_r$ $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$
1	0.01	-	-	-	7.212	16.14		-	-	-	-	-	-	-
2	0.004	0.07	4	3	7.204	20.45	$b_r=$	0.06	0.06	0.06	-	-	-	-
							$d_r=$	0.045	0.04	0.045	-	-	-	-
3	0.003	0.06	6	5	7.088	21.56	$b_r=$	0.06	0.05	0.03	0.04	0.05	-	-
							$d_r=$	0.045	0.04	0.03	0.03	0.04	-	-
4	0.0025	0.05	8	7	6.884	21.97	$b_r=$	0.06	0.05	0.04	0.03	0.03	0.03	0.04
							$d_r=$	0.04	0.035	0.03	0.025	0.025	0.025	0.035
5	0.0025	0.05	10	7	**6.807**	**22.23**	$b_r=$	0.06	0.05	0.04	0.03	0.03	0.03	0.04
							$d_r=$	0.035	0.03	0.025	0.02	0.02	0.02	0.03

Table 5. Optimization path that program passed to reach the result of example I and for $k = 3$.

Iteration	t (m)	$Z_s-(t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)		Ring's no.$=n_r$ $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$
1	0.01	-	-	-	7.212	16.14		-	-	-	-	-	-	-
2	0.004	0.07	4	3	7.204	20.45	$b_r=$	0.06	0.06	0.06	-	-	-	-
							$d_r=$	0.045	0.04	0.045	-	-	-	-
3	0.003	0.06	6	5	7.051	21.98	$b_r=$	0.06	0.05	0.03	0.05	0.06	-	-
							$d_r=$	0.04	0.03	0.03	0.03	0.04	-	-
4	0.0025	0.05	8	7	6.94	22.17	$b_r=$	0.05	0.04	0.04	0.03	0.04	0.04	0.05
							$d_r=$	0.04	0.03	0.025	0.025	0.025	0.03	0.04
5	0.0025	0.05	10	7	**6.862**	**22.53**	$b_r=$	0.05	0.04	0.03	0.03	0.03	0.04	0.05
							$d_r=$	0.035	0.025	0.025	0.02	0.025	0.025	0.035

Table 6. Optimization path that program passed to reach the result of example II and for $k = 1$.

Iteration	t (m)	$Z_s-(t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)
1	0.01	-	-	-	7.212	52.26
2	0.004	0.05	4	5	7.028	64.37
3	0.0035	0.04	6	9	6.951	66.84
4	0.003	0.03	6	11	6.843	67.65
5	0.0025	0.025	6	13	**6.756**	**68.12**

Iteration		Ring's no.$=n_r$ $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	$i=12$	$i=13$
1		-	-	-	-	-	-	-	-	-	-	-	-	-
2	$b_r=$	0.04	0.04	0.04	0.04	0.04	-	-	-	-	-	-	-	-
	$d_r=$	0.04	0.04	0.045	0.045	0.05	-	-	-	-	-	-	-	-
3	$b_r=$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-	-	-
	$d_r=$	0.03	0.03	0.03	0.035	0.035	0.035	0.035	0.04	0.04	-	-	-	-
4	$b_r=$	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-
	$d_r=$	0.03	0.03	0.03	0.035	0.035	0.035	0.035	0.04	0.04	0.04	0.04	-	-
5	$b_r=$	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.03	0.03	0.03
	$d_r=$	0.03	0.03	0.03	0.035	0.035	0.035	0.035	0.035	0.035	0.04	0.04	0.04	0.04

Table 7. Optimization path that program passed to reach the result of example II and for $k = 2$.

Iteration	t (m)	$Z_s - (t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)
1	0.01	-	-	-	7.212	52.26
2	0.004	0.05	4	5	7.104	64.37
3	0.003	0.03	6	9	6.575	75.96
4	0.003	0.025	6	11	**6.460**	**77.23**

Iteration		Ring's no.=n_r										
		$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$
1		-	-	-	-	-	-	-	-	-	-	-
2	$b_r =$	0.04	0.04	0.04	0.04	0.04	-	-	-	-	-	-
	$d_r =$	0.04	0.045	0.05	0.045	0.05	-	-	-	-	-	-
3	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-
	$d_r =$	0.035	0.035	0.035	0.04	0.04	0.04	0.04	0.035	0.035	-	-
4	$b_r =$	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	$d_r =$	0.03	0.03	0.03	0.03	0.035	0.035	0.035	0.035	0.035	0.03	0.03

Table 8. Optimization path that program passed to reach the result of example II and for $k = 3$.

Iteration	t (m)	$Z_s - (t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)
1	0.01	-	-	-	7.212	52.26
2	0.004	0.05	4	5	7.18	64.37
3	0.0035	0.04	6	7	6.852	68.24
4	0.003	0.03	6	9	6.798	69.55
5	0.0025	0.025	6	11	**6.747**	**69.67**

Iteration		Ring's no.=n_r										
		$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$
1		-	-	-	-	-	-	-	-	-	-	-
2	$b_r =$	0.04	0.04	0.04	0.04	0.04	-	-	-	-	-	-
	$d_r =$	0.04	0.04	0.05	0.05	0.05	-	-	-	-	-	-
3	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-	-	-
	$d_r =$	0.035	0.035	0.04	0.05	0.05	0.045	0.045	-	-	-	-
4	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-
	$d_r =$	0.03	0.03	0.035	0.04	0.05	0.045	0.045	0.04	0.04	-	-
5	$b_r =$	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	$d_r =$	0.03	0.03	0.035	0.035	0.04	0.045	0.045	0.045	0.04	0.04	0.04

It is found that the cylinder stiffened with uniform distributed and equal space rings, $k = 1$, under axial pressure only, is the optimum design, with maximum η. When the stiffened shell was subjected to external pressure only, the second type of proposed ring space ($k = 2$) was found to be more efficient. For the double stiffened cylinder under combined axial and non-uniform external pressure, the third type ring space ($k = 3$) was obtained as the optimum design. The OC method was found to be a powerful constraint optimization method for such complex non-linear engineering problems, and the convergence speed was found to be considerably fast. The results also show that the sensitivity of the design variables to loading is

Table 9. Optimization path that program passed to reach the result of example III and for $k = 1$.

Iteration	t **(m)**	$Z_s - (t/2)$	n_s	n_r	W **(KN)**	P_{cr} **(KN/mm)**
1	0.01	-	-	-	7.212	50.14
2	0.004	0.06	4	5	7.179	61.13
3	0.0035	0.05	6	7	7.127	64.97
4	0.003	0.05	8	9	7.122	66.22
5	0.0025	0.04	10	11	7.065	66.68
6	0.0025	0.04	12	11	**7.017**	**66.85**

Iteration		**Ring's no.=n_r**										
		$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$
1		-	-	-	-	-	-	-	-	-	-	-
2	$b_r =$	0.04	0.04	0.03	0.03	0.04	-	-	-	-	-	-
	$d_r =$	0.045	0.045	0.05	0.05	0.05	-	-	-	-	-	-
3	$b_r =$	0.04	0.04	0.03	0.03	0.03	0.03	0.04	-	-	-	-
	$d_r =$	0.035	0.035	0.035	0.035	0.04	0.04	0.04	-	-	-	-
4	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-
	$d_r =$	0.025	0.025	0.03	0.03	0.03	0.035	0.035	0.04	0.04	-	-
5	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	$d_r =$	0.02	0.02	0.025	0.025	0.03	0.03	0.03	0.035	0.035	0.035	0.04
6	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	$d_r =$	0.02	0.02	0.02	0.02	0.025	0.025	0.03	0.03	0.03	0.035	0.035

Table 10. Optimization path that program passed to reach the result for example III and for $k = 2$.

Iteration	**t (m)**	$Z_s - (t/2)$	n_s	n_r	W **(KN)**	P_{cr} **(KN/mm)**
1	0.01	-	-	-	7.212	50.14
2	0.004	0.06	4	5	7.141	60.85
3	0.0035	0.05	6	7	7.071	63.55
4	0.003	0.05	8	9	7.03	64.31
5	0.0025	0.04	10	11	7.009	64.98
6	0.0025	0.04	12	13	**6.975**	**65.22**

Iteration		**Ring's no.=n_r**												
		$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	$i=12$	$i=13$
1		-	-	-	-	-	-	-	-	-	-	-		
2	$b_r =$	0.04	0.04	0.04	0.03	0.04	-	-	-	-	-	-	-	-
	$d_r =$	0.04	0.045	0.045	0.05	0.045	-	-	-	-	-	-	-	-
3	$b_r =$	0.04	0.04	0.03	0.03	0.03	0.03	0.04	-	-	-	-	-	-
	$d_r =$	0.035	0.035	0.035	0.035	0.035	0.04	0.04	-	-	-	-	-	-
4	$b_r =$	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-	-	-
	$d_r =$	0.025	0.025	0.025	0.025	0.03	0.03	0.03	0.035	0.04	-	-	-	-
5	$b_r =$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-
	$d_r =$	0.025	0.025	0.025	0.025	0.025	0.03	0.03	0.03	0.035	0.035	0.04	-	-
6	$b_r =$	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03
	$d_r =$	0.02	0.02	0.02	0.02	0.025	0.025	0.025	0.025	0.03	0.03	0.03	0.03	0.03

Table 11. Optimization path that program passed to reach the result for example III and for $k = 3$.

Iteration	t (m)	$Z_s - (t/2)$	n_s	n_r	W (KN)	P_{cr} (KN/mm)
1	0.01	-	-	-	7.212	50.14
2	0.0045	0.06	4	5	7.13	63.24
3	0.0035	0.05	8	9	6.948	72.69
4	0.003	0.04	10	11	6.799	73.95
5	0.003	0.04	12	11	**6.753**	**74.32**

Iteration		Ring's no.=n_r										
		$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$
1		-	-	-	-	-	-	-	-	-	-	-
2	$b_r =$	0.04	0.03	0.04	0.03	0.03	-	-	-	-	-	-
	$d_r =$	0.04	0.04	0.05	0.045	0.045	-	-	-	-	-	-
3	$b_r =$	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	-	-
	$d_r =$	0.02	0.02	0.025	0.025	0.025	0.03	0.03	0.03	0.03	-	-
4	$b_r =$	0.03	0.03	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	$d_r =$	0.02	0.02	0.025	0.025	0.025	0.025	0.025	0.03	0.03	0.03	0.03
5	$b_r =$	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03
	$d_r =$	0.02	0.02	0.02	0.02	0.02	0.025	0.025	0.03	0.03	0.03	0.03

Table 12. the optimum parameter η.

Examples	$k = 1$	$k = 2$	$k = 3$
I	3.942×10^3	3.266×10^3	3.283×10^3
II	10.083	11.955	10.326
III	9.527	9.35	11.005

dissimilar. For axial pressure, the existence of stringers is found to be more efficient than rings, and for lateral pressure, the existence of rings is more efficient than stringers. Also, for axial pressure increase, the width of the rings (b_r) is more efficient than increasing the height of the rings (d_r), but, for lateral pressure increase, the height of the rings (d_r) is more efficient than increasing the width of the rings (b_r). The results also show that the iteration numbers depend on the ring stiffener space states.

Nomenclature

u, v, w	The orthogonal components of displacement of an arbitrary point (x, θ, z) in the shell along the coordinates (x, θ, z), respectively
$\varepsilon_{xx}^{(0)}, \varepsilon_{\theta\theta}^{(0)}, \gamma_{x\theta}^{(0)}$	The normal and shear strains at the mid-surface ($z = 0$)
$\kappa_{xx}, \kappa_{\theta\theta}$	The mid-surface bending curvatures
$\kappa_{x\theta}$	The mid-surface twist curvature
m, n	The longitudinal and circumferential wave numbers, respectively
U_{shell}	Shell internal strain energy
U_{string}	Stringers internal strain energy
U_{ringi}	The strain energy for the ith ring
w_p	External work by the compressive load (P)
w_q	The work done by the external hydrostatic pressure ($q = P + L * \rho_w$)
W_{sh}, W_s, W_r	The weights of the shell, stringers, and rings, respectively
L	The length of the shell
R	The radius of the shell
t	The thickness of the shell
$I_{xs}, I_{\theta s}, I_{zs}$	The moment of inertia of stringers about x, θ and z axes
A_s	The cross-sectional area of stringers
Z_s	The eccentricity of stringers
n_s	The number of stringers
d	The equal space among stringers
$I_{xr}, I_{\theta r}, I_{zr}$	The second moment of areas of rings about x, θ and z axes
A_r	Rings cross section
Z_r	Ring eccentricity
x_{ri}	A ring spacing distribution function representing the location of the ith ring along the shell length
n_r	The number of rings

References

1. Damodar, R.A. and Navin, J. "Optimal design of grid-stiffened panels and shells with variable curvature", *Compos. Struct.*, **53**, pp. 173-80 (2001).
2. Akl, W., Ruzzene, M. and Baz, A. "Optimal design of underwater stiffened shells", *Structural and Multidisciplinary Optimization*, **23**, pp. 297-310 (2002).
3. Topal, U. "Multiobjective optimization of laminated composite cylindrical shells for maximum frequency and buckling load", *Mater Des.*, **30**, pp. 2584-94 (2009).
4. Moharrami, H. "Optimality criteria method AN efficient robust solution for nonlinear optimization problems", *7th International Congress on Civil Engineering*, Tehran (1385).
5. Sander, G. and Fleury, C. "Amixed method in structural optimization", *International Journal of Numerical Methods in Engineering*, **13**, pp. 385-404 (1978).
6. Lim, C.W., Ma, Y.F. "Computational p-element method on the effects of thickness and length on self-weight buckling of thin cylindrical shells via various shell theories", *Computational Mechanics*, **31**, pp. 400-8 (2003).
7. Eslami, G. and Kabir, M.Z. "Analytical solution of non-linear stability of double stiffened cylindrical shells under combined axial and variable external pressure", *The 10th Conference Shell Structures: Theory and Applications*, 16-18 October, Poland (2013).
8. Bagheri, M., Jafari, A.A. and Sadeghifar, M. "Multi-objective optimization of ring stiffened cylindrical shells using a genetic algorithm", *Journal of Sound and Vibration*, **330**, pp. 374-384 (2011).

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