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Multi-objective optimization of structures using charged system search

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Abstract. Many industrial problems are concerned with optimization of large and complex systems involving many criteria. Indeed, optimization problems encountered in practice are seldom mono-objective. In general, there are many conflicting objectives to handle. This study introduces a new method for the solution of multi-objective optimization problems. Multi-objective optimization is utilized to find the most suitable solution, which covers the requirements and demands of decision makers. The main goal of the resolution of a multi-objective problem is to obtain a Pareto optimal set and, consequently, the Pareto front. This method is based on the Charged System Search (CSS) algorithm, which is inspired by the Coulomb and Gauss laws of electrostatics in physics. In order to illustrate the efficiency of the proposed method, numerical examples are solved and results are compared to show the ability of the CSS in finding optimal solutions.

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1. Introduction

In the last two decades, many efficient mono-objective optimization algorithms have been developed [1-7]. These algorithms search through possible feasible solutions, and ultimately identify the best results. Multi-objective optimization techniques play an important role in engineering design, resource optimization, and many other fields. Their main purpose is to find a set of best solutions from which a designer or decision maker can choose a solution to derive maximum benefit from available resources. The various objectives of a multi-objective optimization problem often conflict and/or compete with one another. In multi-criterion Decision Making (DM), no single solution can be termed as the optimum solution to the multiple conflicting objectives, as a multi-objective optimization problem is amenable to a number of trade-off optimal solutions. For this purpose, multi-objective optimization generates a Pareto front, which is a set of non-dominated solutions for problems with more than one objective. The major goal of a multi-objective optimization algorithm is to generate a well-distributed true Pareto optimal front or surface.

Over the past decade, a number of Multi-Objective Evolutionary Algorithms (MOEAs) have been developed, such as the Non-dominated Sorting Genetic Algorithm (NSGA)-II [8], the Strength Pareto Evolutionary Algorithm (SPEA2) [9], the Pareto Archive Evolution Strategy (PAES) [10], Multi-Objective Particle Swarm Optimization (MOPSO) [11], and hybrid multi-objective optimization comprised of CSS and PSO [12].

In this paper, a new multi-objective optimization approach, based purely on the Charged System Search (CSS) algorithm, is introduced. The CSS is a population based meta-heuristic optimization algorithm proposed recently by Kaveh and Talatahari [5,13,14]. In the CSS, each solution candidate is considered a charged sphere, called a Charged Particle (CP). The

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electrical load of a CP is determined considering its fitness. Each CP exerts an electrical force on all the others, according to the Coulomb and Gauss laws from electrostatics. Then, the new positions of all the CPs are calculated utilizing Newtonian mechanics, based on the acceleration produced by the electrical force, the previous velocity and the previous position of each CP. Many different structural optimization problems have been successfully solved by CSS [13,14].

In the present work, after a brief description of multi-objective optimization (MOP), the main concepts of the Charged System Search algorithm are provided. For better understanding of the MOPs, readers can refer to [15]. Then, the multi-objective charged system search algorithm is presented. A simple multi-criteria decision making process is also presented. Numerical examples are prepared to show the efficiency and accuracy of the proposed method. Finally, the concluding remarks are provided.

2. Multi-objective optimization concepts

Definition 1. Multi-objective optimization problem. A multi-objective optimization problem can be defined as:

$$\mathrm{MOP} = \begin{cases} \min \quad F(x) = (f_1(x), f_2(x), ..., f_n(x)) \\ S.C. \quad x \in S \end{cases} \tag{1}$$

where $n \geq 2$ is the number of objectives, $x = (x_1, x_2, ..., x_k)$ is the vector representing the decision variables, and S represents the set of feasible solutions associated with equality and inequality constraints and explicit bounds. $F(x) = (f_1(x), f_2(x), ..., f_n(x))$ is the vector of objectives to be optimized.

Definition 2. Pareto dominance. An objective vector, $u = (u_1, u_2, ..., u_n)$, is said to dominate $v = (v_1, v_2, ..., v_n)$, denoted by $u \prec v$, if and only if no component of v is smaller than the corresponding component of u, and at least one component of u is strictly smaller, that is:

$$\forall i \in \{1, ..., n\} : u_i \leq v_i \wedge \exists i \in \{1, ..., n\} : u_i < v_i.$$

Definition 3. Pareto optimality. A solution, $x^* \in S$, is Pareto optimal if for every $x \in S$, $F(x)$ does not dominate $F(x^*)$, that is $F(x) \nprec F(x^*)$.

Graphically, solution x^* is Pareto optimal if there is no other solution x such that point $F(x)$ is in the dominance cone of $F(x^*)$, which is the box defined by $F(x)$ with its projections on the axes and origin (Figure 1).

Definition 4. Pareto optimal set. For a given MOP(F, S), the Pareto optimal set is defined as $P^* = \{x \in S / \nexists\ x' \in S, F(x') \prec F(x)\}$.

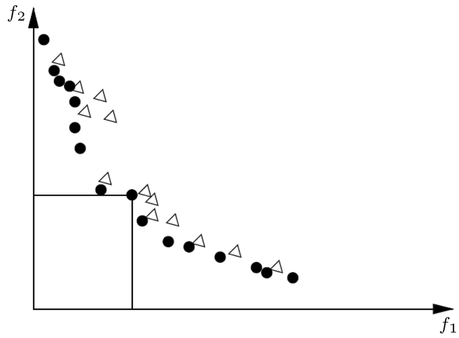


Figure 1. Pareto solution denoted by solid dots and dominate solution shown by triangles.

Definition 5. Pareto front. For a given MOP(F, S) and its Pareto optimal set, the Pareto front is defined as $PF^* = \{F(x), x \in P^*\}$.

The Pareto front is the image of the Pareto optimal set in the objective space. Obtaining the Pareto front of a MOP is the main goal of a multi-objective optimization. The Pareto front should have two desirable properties consisting of good convergence and diversity.

3. Charged system search algorithm

The charged system search contains a number of Charged Particles (CP), where each CP is treated as a charged sphere and can insert an electric force onto the others. The magnitude of this force for a CP located inside the sphere is proportional to the separation distance between the CPs, and, for a CP located outside the sphere, is inversely proportional to the square of the separation distance between the particles. The resultant forces persuade the CPs to move towards new locations, according to the motion laws of Newtonian mechanics. In the new positions, the magnitude and direction of the forces are reformed and this successive action is repeated until a terminating condition is satisfied. The pseudo-code for the CSS algorithm is summarized as follows:

Level 1: Initialization
Step 1. *Initialization.* The magnitude of charge for each CP is defined as:

$$q_i = \frac{\mathrm{fit}(i) - \mathrm{fitworst}}{\mathrm{fitbest} - \mathrm{fitworst}} \quad i = 1, 2, ..., N, \tag{2}$$

where fitbest and fitworst are the best and the worst fitness of all the particles, respectively, fit(i) represents the fitness of agent i, and N is the total number of CPs. The separation distance, r_{ij}, between two charged particles is defined as follows:

$$r_{ij} = \frac{\|X_i - X_j\|}{\|(X_i + X_j)/2 - X_{\mathrm{best}}\| + \varepsilon}, \tag{3}$$

where X_i and X_j are the positions of the ith and jth

CPs, respectively, X_{best} is the position of the best current CP, and ε is a small positive number. The initial positions of CPs are determined randomly.

Step 2. *CP ranking.* Considering the values of the fitness function, sort the CPs in an increasing order.

Step 3. *CM creation.* Store a number of the first CPs and the values of their corresponding fitness functions in the Charged Memory (CM).

Level 2: Search

Step 1. *The probability of moving determination.* Determine the probability of moving each CP towards the others using the following probability function:

$$p_{ij} = \begin{cases} 1 & \frac{\text{fit}(i)-\text{fitbest}}{\text{fit}(j)-\text{fit }(i)} > \text{rand} \ \vee \ or \ \text{fit}(j) > \text{fit}(i) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Step 2. *Forces determination.* Calculate the resultant force vector for each CP as:

$$F_j = q_j \sum_{i,i\neq j} \left(\frac{q_i}{a^3} r_{ij}.i_1 + \frac{q_i}{r_{ij}^2}.i_2\right) ar_{ij} p_{ij} (X_i - X_j),$$

$$\left\langle \begin{array}{l} j = 1, 2, \cdots, N \\ i_1 = 1, \quad i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, \quad i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{array} \right. \tag{5}$$

where F_j is the resultant force acting on the jth CP. ar_{ij} is a new parameter, so-called the kind of force, and determines the type of force, where $+1$ represents the attractive force and -1 denotes the repelling force, which is defined as:

$$ar_{ij} = \begin{cases} +1 & \text{w.p. } k_t \\ -1 & \text{w.p. } 1 - k_t \end{cases} \tag{6}$$

where "w.p." stands for "with the probability". In this algorithm, each CP is considered a charged sphere with radius a, which has a uniform volume charge density.

Step 3. *Solution construction.* Move each CP to the new position and find the velocities as:

$$X_{j,\text{new}} = \text{rand}_{j1}.k_a.\frac{F_j}{m_j}.\Delta t^2 + \text{rand}_{j2}.k_v.V_{j,\text{old}}.\Delta t + X_{j,\text{old}}, \tag{7}$$

$$V_{j,\text{new}} = \frac{X_{j,\text{new}} - X_{j,\text{old}}}{\Delta t}, \tag{8}$$

where k_a and k_v are the acceleration and velocity coefficients, respectively. These can be obtained as follows: If rand_{j1} and rand_{j2} are two random numbers uniformly distributed in the range [0,1], then:

$$k_a = 0.5\,(1 + \text{iter}/\text{iter}_{\max}),$$

$$k_v = 0.5\,(1 - \text{iter}/\text{iter}_{\max}). \tag{9}$$

Step 4. *CP position correction.* If each CP swerves off the predefined bounds, correct its position using the harmony search-based handling approach, as described in [16].

Step 5. *CP ranking.* Considering the values of the fitness function, sort the CPs in an ascending order.

Step 6. *CM updating.* Include the better new vectors in the CM and exclude the worst ones from the CM. The number of substitutions is not constant. In primary iterations, many CM vectors may be excluded, but in later iterations (when the particles are converged to the optimal answer), this number is decreased.

Level 3: Terminating criterion controlling

Repeat the search level steps until a terminating criterion is satisfied.

Figure 2 shows the flowchart of the CSS algorithm.

4. Multi-objective charged system search optimization algorithm

This algorithm is based on a pure Charged System Search (CSS) algorithm. For using this algorithm in a multi-objective optimization procedure, some changes are made and some additional steps are considered.

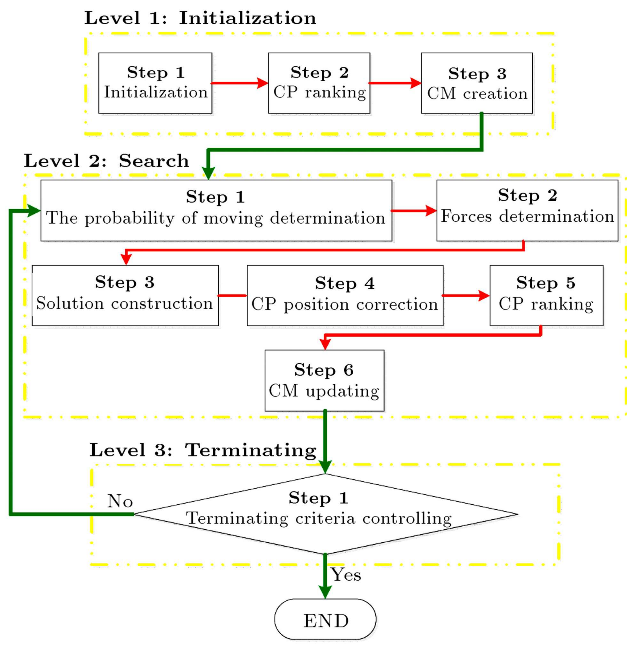


Figure 2. Summarized flowchart of the CSS.

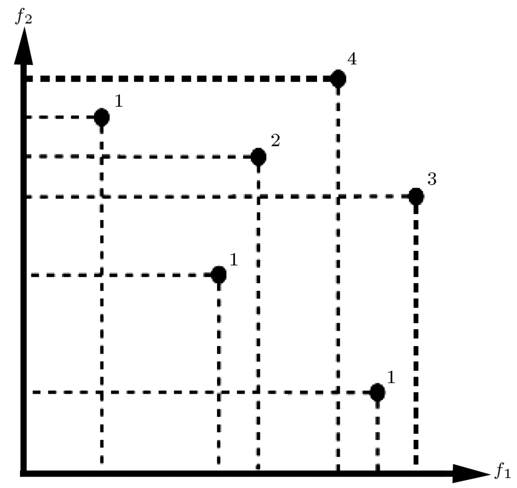


Figure 3. Dominance rank determination.

4.1. Algorithm

This algorithm consists of the following steps:

Step 1. Initialize the Charged Particles (CPs) magnitudes randomly. The initial speed of each particle is considered zero.

Step 2. Determine the magnitude of charge for each CP. For this purpose, the vector of objectives for each CP is calculated. Then, dominance rank of each CP is obtained. The dominance rank of a solution is related to the number of solutions in the population that dominates the considered solution. Figure 3 represents the procedure for determining the dominance rank of some solutions.

Diversity loss is observable in many meta-heuristics. To face the drawback related to the stagnation of a population, diversity must be maintained in the population. In general, the diversification method deteriorates solutions that have a high density in their neighborhoods. For solution i, distances d_{ij} between i and other solutions of population j, are computed.

The magnitude of charge for solution i, $q(i)$, is calculated as:

$$q(i) = \frac{1}{\mathrm{DR}_i \times m_i} \qquad i \in [1, 2, ..., N], \tag{10}$$

where DR_i is the dominance rank of solution i and $m_i = \sum_{j \in \mathrm{pop}} \mathrm{sh}(d_{ij})$.

Sharing function, $\mathrm{sh}(d_{ij})$, is defined as follows:

$$\mathrm{sh}(d_{ij}) = \begin{cases} 1 - \frac{d_{ij}}{\sigma} & \text{if } d_{ij} < \sigma \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The constant σ represents the non-similarity threshold. The effectiveness of the sharing principle depends mainly on these two parameters that must be set carefully. Indeed, diversification becomes inefficient with a low value of σ, but, the convergence speed of the front becomes too small when this value is too high.

Step 3. Now, CM should be created. For this purpose, the particles with dominance rank equal to 1 are selected as CM.

Step 4. While iter <= itermax, in other words, since a terminating criterion is not satisfied, repeat the following steps:

a) Determine the CMpart and CPpart. This means that the location of all the particles in the population and archive should be determined. It should be noted that the objective space is divided into z parts.

The space division method employed here is the same as the formulation introduced in [16]. According to this method, to each particle with $F(x) = (f_1(x), f_2(x))$, a value, σ_i is defined as:

$$\sigma = \frac{f_1^2 - f_2^2}{f_1^2 + f_2^2}. \tag{12}$$

In case the objectives are not in the same range, for a two-objective optimization problem, σ can be calculated as below:

$$\sigma = \frac{m_1^2 - m_2^2}{m_1^2 + m_2^2},$$

$$m_1 = \frac{f_1 - f_{\min 1}}{f_{\max 1} + f_{\min 1}},$$

$$m_2 = \frac{f_2 - f_{\min 2}}{f_{\max 2} - f_{\min 2}}, \tag{13}$$

where $f_{\max 1}(f_{\min 1})$ and $f_{\max 2}(f_{\min 2})$ are the maximum (minimum) values of the first and second objective of the particles in the population or archive, respectively. The schematic demonstration of different parts is shown in Figure 4.

b) Calculate the resultant force vector for each CP or CM particles as:

$$F_j = q_j \sum_{i, i \neq j} \left(\frac{q_i}{a^3} r_{ij}.i_1 + \frac{q_i}{r_{ij}^2}.i_2 \right) ar_{ij} p_{ij} (X_i - X_j),$$

$$\begin{cases} i_1 = 1, \ i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, \ i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{14}$$

where the probability of moving, p_{ij}, can be calculated according to Eq. (4). F_j is the resultant force acting on the jth particle, and ar_{ij} is the kind of force, and determines the type of force explained in the previous sections. This parameter can be

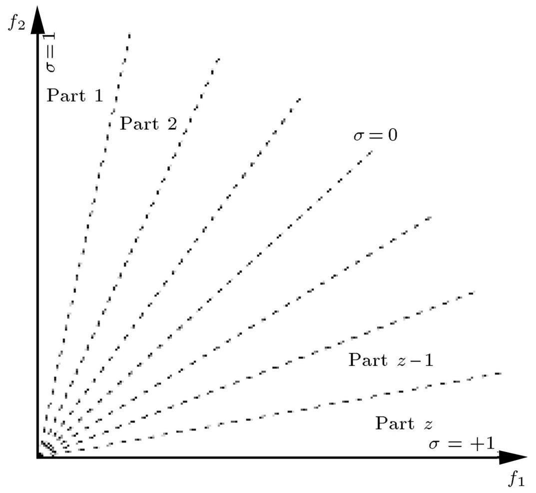


Figure 4. Division of the objective space by assigning parameter σ to each particle.

calculated as follows:

if $i, j \in$ CP or $i, j \in$ CM

$$\Rightarrow ar_{ij} = \begin{cases} +1 & \text{if rand} \leq k_t \\ -1 & \text{if rand} < 1 - k_t \end{cases}$$

if $i \in$ CP and $j \in$ CM $\Rightarrow ar_{ij} = -1$

This means: A CM particle is repeled by all CP particles

if $i \in$ CM and $j \in$ CP $\Rightarrow ar_{ij} = +1$

This means: A CM particle attracts all CP particles

c) Compute the new position and velocity of each particle using Eqs. (7) and (8). When the current position of a particle is obtained, the following control should be performed:

if $j \in$ CP $\Rightarrow \text{Part}(X_{\text{new}j})$

should be the same as $\text{Part}(X_{\text{old}j})$,

otherwise $X_{\text{new}j} = X_{\text{old}j}$.

This means that each particle of CP should remain in its initial part up to the end of the optimization procedure, but CM particles can be moved to other parts.

d) Update the magnitude of each particle of CP and CM. Calculate their dominance ranks and select the new members of CM. This means that all particles which have a dominance rank equal to one should be selected as the new CM.

e) In this step, each particle of CM is compared with other particles of CM. In other words, the Euclidean distance between the objective vectors of all the particles in the CM is calculated, and, if this value is smaller than a positive predefined value, one of them is eliminated. Using this approach, a crowding region cannot be generated in the objective space.

5. Multi-criteria decision making

The aim of solving multi-objective optimization problems is to help a Decision Maker (DM) find a Pareto solution that copes with his preferences. One of the fundamental questions in MOPs resolution is related to interaction between the problem solver and the decision maker. Indeed, the Pareto optimal solutions cannot be ranked globally. The role of the decision maker is to specify some extra information to select his favorite solution.

Many different approaches can be used for the decision making process [17]. A simple method for the multi-criteria decision making problem, so-called the multi-criteria tournament decision making method (MTDM), is described in [18]. This method provides the ranking of alternatives from best to worst, according to the preferences of a human decision maker. It has another positive aspect, involving few input parameters, just the importance weight of each criterion. This method introduces a function, R, capable of reflecting the DM global interests. In order to find this function, first, each possible solution is compared to the others, considering only the ith-criterion. The pairwise comparisons are performed through the tournament function, $T_i(a, A)$, which counts the ratio of times alternative a wins the tournament against each other b solution from A. Hence, considering that a is a non-dominated point in the objective space, $T_i(a, A)$ can be stated as:

$$T_i(a, A) = \sum_{\forall b \in A, a \neq b} \frac{t_i(a, b)}{(|A| - 1)}, \tag{15}$$

where:

$$t_i(a, b) = \begin{cases} 1 & \text{if } f_i(b) - f_i(a) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

The tournament function, $T_i(a, A)$, assigns a score to each solution in the Pareto front. The assigned score works as a performance measure, which provides a distinct ordering of the elements of A for each criterion. In order to generate the global ranking, taking into account all criteria and their respective weights, w_i (priority factors), the scores are aggregated into the global ranking function, R. The weighted geometric mean, which is utilized by many different researchers, is considered the aggregation function in this study, as follows:

$$R(a) = (\Pi_{i=1}^{n} T_i(a, A)^{w_i})^{\frac{1}{n}}, \qquad (17)$$

where n is the number of objective functions. The priority weights must be specified by the DM in accordance with the following conditions:

$$w_i > 0 \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1. \qquad (18)$$

The ranking index, $R(a)$, gives an idea of how much each alternative is preferred to the others. In other words: if $R(a) > R(b)$, then, a is preferred to b, and when $R(a) = R(b)$, then a is indifferent to b.

6. Numerical examples

In this section, some numerical results are presented in order to show the performance of the pure CSS algorithm in multi-objective optimization problems. The algorithms are coded in MATLAB and, in order to handle the constraints, a penalty approach is utilized. When the constraints are in the range of allowable limits, the penalty is zero. Otherwise, the amount of penalty is obtained by dividing the violation of allowable limit by the limit itself. For the examples presented in this paper, the CSS algorithm parameters are set as follows: $k_a = 2$, $k_v = 2$, the number of agents is taken as 100, the maximum number of iterations is set to 100, $a = 1$, $\Delta T = 1$ and $k_t = 0.5$. The algorithm is run with an archive size of 100. In this paper, a real coded NSGA-II is utilized with a population size of 100, a crossover probability of 0.9 ($p_c = 0.9$), tournament selection, a mutation rate of $1/u$ (where u is the number of decision variables), and distribution indexes for crossover and mutation operators are taken as $\eta_c = 20$ and $\eta_m = 20$, respectively (as recommended in [8]). MOPSO used a population of 100 particles, an archive size of 100 particles, a mutation rate of 0.5, and 30 divisions for the adaptive grid [11]. Also, s-MOPSO is run with a population of 100 particles, an archive size of 100 particles, and a mutation probability of 0.05 [16]. The parameters considered for CSS-MOPSO consist of $C1 = 1$, $C2 = 2$, $R = 15$, $rld = 0.01$, $rud = 0.05$, mutation probability = 0.1, archive size of 100 and a population of 50 particles [12]. For all examples presented in this paper, the number of fitness function evaluations (structural analysis) in the multi-objective optimization phase is restricted to 30,000.

The results obtained by CSS is compared to the original MOPSO [11], s-MOPSO [19], NSGA [8] and MOCHS [20].

Example 1. *A 2-bar truss design.* This problem was originally studied using the ϵ-constraint method [21].

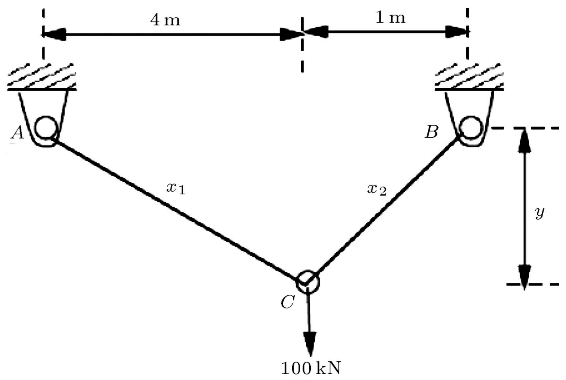


Figure 5. The two-bar truss problem.

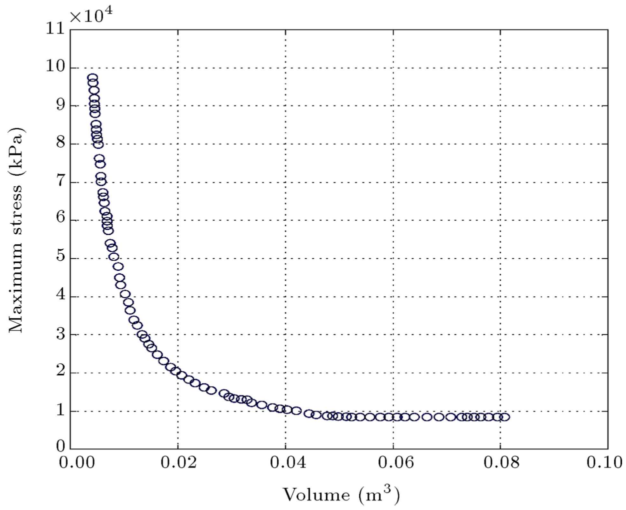


Figure 6. Pareto optimal front obtained using CSS method for two-bar truss design problem.

As shown in Figure 5, the truss has to carry a certain load without elastic failure. Thus, in addition to the objective of designing the truss for minimum volume, there are additional objectives of minimizing stresses in each of the two members, AC and BC. The two-objective optimization problem for three variables y (vertical distance between B and C in m), x_1 (cross sectional area of AC in m^2), and x_2 (cross sectional area of BC in m^2) is constructed as follows:

$$\text{Minimize } f_1(x) = x_1\sqrt{16 + y^2} + x_2\sqrt{1 + y^2}$$

$$\text{Minimize } f_2(x) = \max(\sigma_{\mathrm{AC}}, \sigma_{\mathrm{BC}})$$

$$\text{s.t.} \begin{cases} \max(\sigma_{\mathrm{AC}}, \sigma_{\mathrm{BC}}) \leq 10^5 \\ 1 \leq y \leq 3 \\ x \geq 0 \end{cases}$$

where $\sigma_{\mathrm{AC}} = \frac{20\sqrt{16+y^2}}{yx_1}$ and $\sigma_{\mathrm{BC}} = \frac{80\sqrt{1+y^2}}{yx_2}$.

Figure 6 shows the Pareto front obtained using

Table 1. Comparison of the results for two-bar truss design problem.

Optimization method	EM-MOPSO [22]	NSGA-II [8]	MOCHS [20]	CSS (present work)
Obtained extreme values (m^3, kN)	$[0.004026, \ 99996]$ $[0.05273, \ 8434.493]$	$[0.00407, \ 99755]$ $[0.05304, \ 8439]$	$[0.00375, \ 99847]$ $[0.0537, \ 7685]$	$[0.00412, \ 99457]$ $[0.08078, \ 8434.23]$

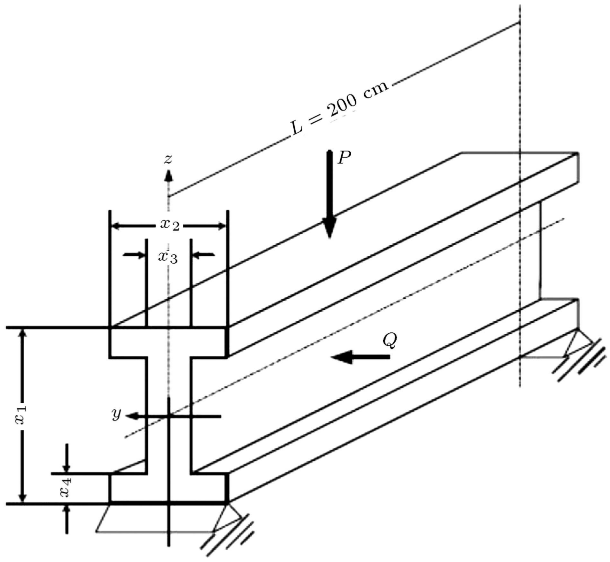


Figure 7. The I-beam design problem.

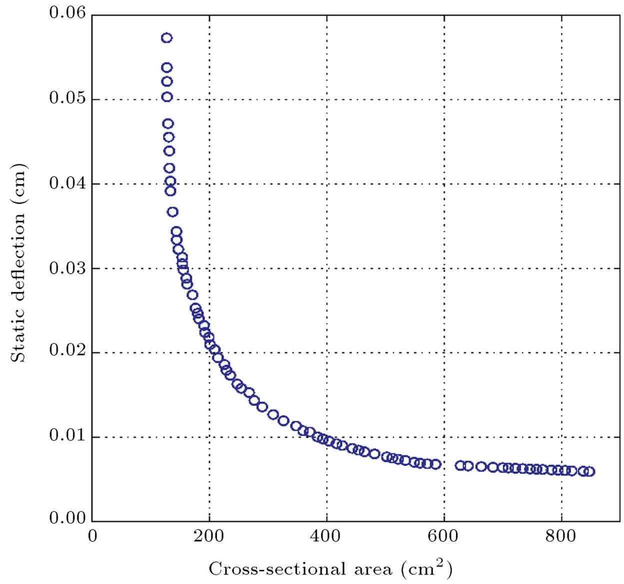


Figure 8. Pareto optimal front obtained using the CSS method for the I-beam design.

the CSS method. Also, the two extreme objective values obtained by various algorithms are compared in Table 1.

Example 2. *An I-beam design.* The second design problem is taken from [21]. The problem is to find the dimension of the beam shown in Figure 7. In this design problem, the dimensions of the geometric and strength constraints should be satisfied, and, at the same time, the cross-sectional area of the beam and the static deflection of the beam should be minimized under a force, P. The mathematical formulation of the problem is as follows:

Minimize cross-sectional area (cm^2):

$$f_1 = 2x_2x_4 + x_3(x_1 - 2x_4),$$

Minimize displacement (cm^2):

$$f_2 = \frac{\text{PL}^3}{48\text{EI}},$$

where:

$$I = \frac{1}{12}\left\{x_3(x_1-2x_4)^3+2x_2x_4\left[4x_4^2+3x_1(x_1-2x_4)\right]\right\}.$$

Find $x_i, \quad i = 1, 2, 3, 4$

Subject to:

$$g(x) = \sigma_a - \left(\frac{M_y}{Z_y} + \frac{M_z}{Z_z}\right) \geq 0 \text{ and } \begin{cases} 10 \leq x_1 \leq 80 \\ 10 \leq x_2 \leq 50 \\ 0.9 \leq x_3 \leq 5 \\ 0.9 \leq x_4 \leq 5 \end{cases}$$

where:

$$M_y = \frac{P}{2} \times \frac{L}{2}, \quad M_z = \frac{Q}{2} \times \frac{L}{2},$$

$$Z_y = \frac{1}{6x_1}\left\{x_3(x_1-x_4)^3+2x_2x_4\left[4x_4^2+3x_1(x_1-2x_4)\right]\right\},$$

$$Z_z = \frac{1}{6x_1}\left\{x_3^3(x_1-x_4)+2x_2^3x_4\right\},$$

$$E = 2 \times 10^4 \text{kNcm}^{-2}, \quad \sigma_a = 16 \text{kNcm}^{-2},$$

$$P = 600 \text{kN}, \quad Q = 50 \text{kN}, \text{ and } L = 200 \text{cm}.$$

Figure 8 shows the Pareto front obtained after 100 iterations. The CSS obtained the minimal cross-sectional area of 127.8201 units for a deflection of

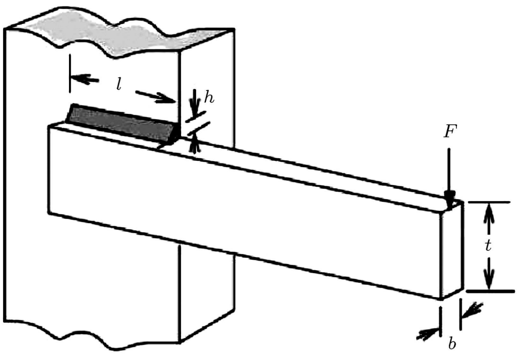


Figure 9. The welded beam design.

0.0573, and for the minimal deflection of 0.0059 units, the cross-sectional area is 847.5709 units. EM-MOPSO obtained the minimal cross-sectional area of 127.9508 units for a deflection of 0.05368 units, and for the minimal deflection of 0.005961 units, the cross-sectional area was 829.5748 units. NSGA-II obtained a minimal cross-sectional area of 127.2341 units with a deflection of 0.0654 units, and a minimal deflection of 0.0060 units with a cross-sectional area of 829.8684 units.

Example 3. *Welded beam design.* The third design problem was studied by [22]. A beam needs to be welded onto another beam and must carry a certain load (Figure 9). The overhang has a length of 14 inches, and a force, F, of 6000 lb is applied at the end of the beam. The objective of the design is to minimize the cost of fabrication and the end deflection. The mathematical formulation of the two-objective optimization problem is as follows:

$$\text{Minimize}\begin{cases} f_1(x) = 1.10471h^2l + 0.04811tb(14+l) \\ f_2(x) = \delta(x) = \frac{2.1952}{t^3b} \end{cases}$$

$$\text{Subject to}\begin{cases} g_1(x) = 13.600 - \tau(x) \geq 0 \\ g_2(x) = 30.000 - \sigma(x) \geq 0 \\ g_3(x) = b - h \geq 0 \\ g_4(x) = P_c(x) - 6000 \geq 0 \end{cases}$$

The first constraint ensures that the shear stress developed at the support location of the beam is less than the allowable shear strength of the material (13,600 psi). The second one ensures that the normal stress developed at the support location of the beam is less that the allowable yield strength of the material (30,000 psi). The third ensures that the thickness of the beam is not less than weld thickness, from a practical standpoint. The fourth one ensures that the allowable buckling load of the beam (along the t direction) is greater than the applied load, F. The stress and buckling terms are as follows:

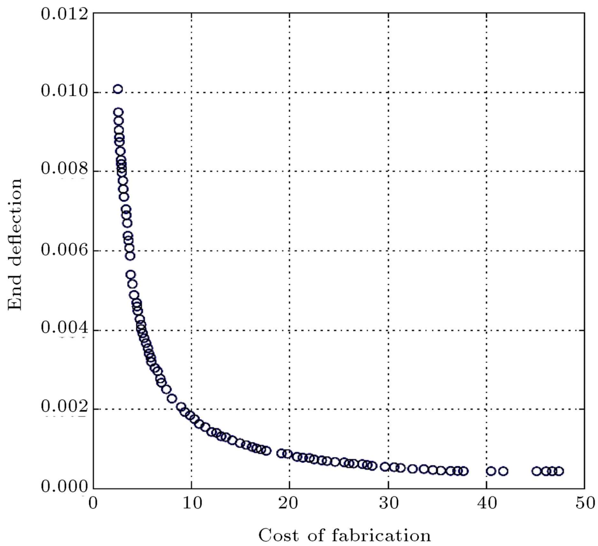


Figure 10. Pareto optimal front obtained using the CSS method for the welded beam design.

$$\tau(x) = \sqrt{(\tau')^2 + (\tau'')^2 + \frac{l\tau'\tau''}{\sqrt{0.25(l^2+(h+t)^2)}}},$$

$$\tau' = \frac{6,000}{\sqrt{2}hl},$$

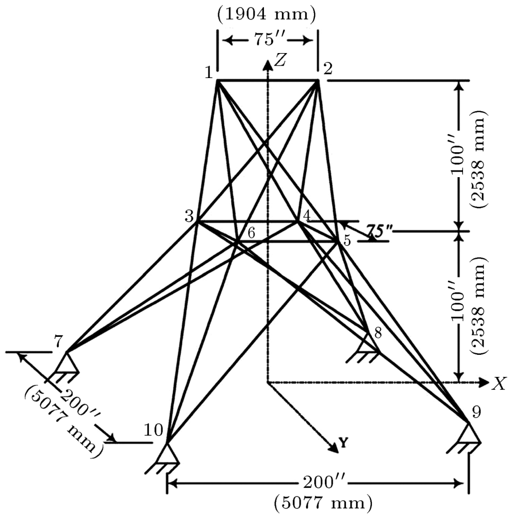
$$\tau'' = \frac{6,000(14+0.5l)\sqrt{0.25(l^2+(h+t)^2)}}{2\{0.707hl(\frac{l^2}{12}+0.25(h+t)^2)\}},$$

$$\sigma(x) = \frac{504,000}{t^2b},$$

$$P_c(x) = 64,746.022\,(1-0.0282346t)\,tb^3.$$

Figure 10 shows the optimized non-dominated solutions obtained using the CSS algorithm. EM-MOPSO found the minimal cost solution as 2.382 units with a deflection of 0.0157 inches, and the minimal deflection as 0.000439 with a cost of 36.4836 units. For NSGA-II, the minimal cost was 3.443 units for a deflection of 0.0101 units, and the minimal deflection was 0.004 with a cost of 36.9121 units. For the CSS, the minimal cost is 2.5112 units for a deflection of 0.000439 units, and the minimal deflection is 0.0108 with a cost of 47.3722 units.

Example 4. *A 25-bar truss structures.* Another famous 25-bar truss is considered, as shown in Figure 11 [12]. Again, the problem is to find the cross-sectional area of members, such that the total structural weight and the displacement in the Y-direction at node 1 are minimized concurrently. The structure includes 25 members, which are divided into eight



Group number	Members
1	1-2
2	1-4, 2-3, 1-5, 2-6
3	2-5, 2-4, 1-3, 1-6
4	3-6, 4-5
5	3-4, 5-6
6	3-10, 6-7, 4-9, 5-8
7	3-8, 4-7, 6-9, 5-10
8	3-7, 4-8, 5-9, 6-10

Figure 11. A 25-bar space truss structures and its member grouping.

groups, as follows: (1) A_1, (2) $A_2 - A_5$, (3) $A_6 - A_9$, (4) $A_{10} - A_{11}$, (5) $A_{12} - A_{13}$, (6) $A_{14} - A_{17}$, (7) $A_{18} - A_{21}$ and (8) $A_{22} - A_{25}$.

The applied load to this structure is:

$$F_{X(1)} = 4.45(\text{ kN}), \quad F_{Y(1)} = -44.5(\text{ kN}),$$

$$F_{Z(1)} = -44.5(\text{ kN}), \quad F_{Y(2)} = -44.5(\text{ kN}),$$

$$F_{Z(2)} = -44.5(\text{ kN}), \quad F_{X(3)} = 2,25(\text{ kN}),$$

$$F_{X(6)} = 2.67(\text{ kN}).$$

The upper and lower bounds for the cross sections of each truss element are 64.45 mm^2 (0.1 in^2) and 2191.47 mm^2 (3.4 in^2), respectively. The modulus of elasticity is taken as $E = 68.97$ kN/mm^2 (1×10^4 ksi) and the weight density as $\rho = 2.714E - 8$ kN/mm^2 (0.1 lb/in^2). Constraints on the truss limit the principal stress, σ_j, in each element to a maximum allowable stress value of $\sigma_j = \pm 0.27584$ kN/mm^2 ($\pm$40 ksi).

The Pareto front obtained by the CSS algorithm is shown in Figure 12. Also, the two extreme objective values obtained in 10 runs of algorithms are shown in Table 2.

In this example, after finding the Pareto front, the next step is to ask DMs to notify their preferences by considering all the information integrated in the Pareto front. Many different scenarios are possible for a considered problem. For example, these scenarios can be as follows:

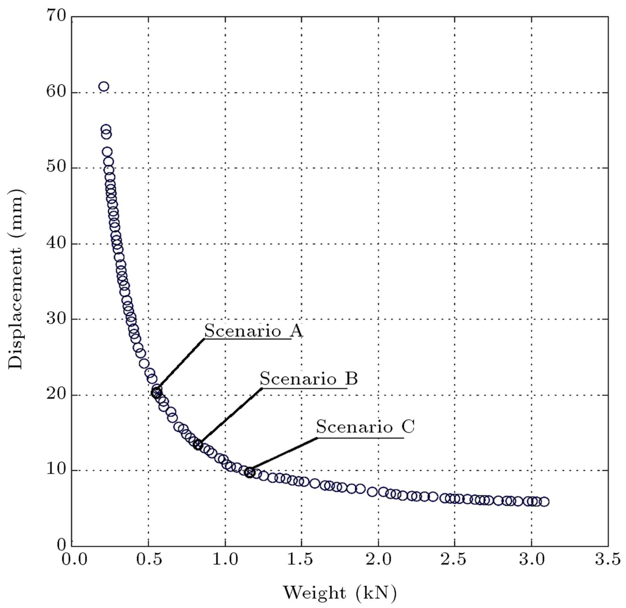


Figure 12. The Pareto front of 25-bar truss structure and the best solutions according to three different scenarios.

Table 2. Comparison of the extreme values obtained by different methods for two-bar truss design problem.

Optimization method	Obtained extreme values (mm, kN)
CSS-MOPSO [12]	[5.8437, 4.8111] [62.9807, 0.3440]
s-MOPSO [19]	[5.8437, 4.8917] [62.7832, 0.3239]
MOPSO [11]	[5.8791, 4.4836] [60.3942, 0.3642]
NSGA-II [8]	[5.8437, 4.8297] [64.5579, 0.3141]
CSS (present work)	[5.8697, 4.7989] [63.6643, 0.2176]

Scenatio A. The first criterion (objective) is more important: e.g. $(w_1, w_2) = (0.6, 0.4)$

Scenario B. The first criterion (objective) is as important as the second criterion: e.g. $(w_1, w_2) = (0.5, 0.5)$.

Table 3. Best selected solutions for two-bar truss design problem.

Algorithm	Scenario A			Scenario B			Scenario C		
	f_1 (kN)	f_2 (mm)	R_i	f_1 (kN)	f_2 (mm)	R_i	f_1 (kN)	f_2 (mm)	R_i
	$w_1 = 0.6$	$w_2 = 0.4$		$w_1 = 0.5$	$w_2 = 0.5$		$w_1 = 0.4$	$w_2 = 0.6$	
CSS (present work)	0.558	20.2810	**1.5325**	0.823	13.4183	**1.8229**	1.159	9.6868	**2.0355**
CSS-MOPSO [12]	1.189	16.7307	**1.8504**	1.548	12.7422	**2.0962**	2.036	9.6144	**2.2732**

Scenario C. The second criterion (objective) is more important: e.g. $(w_1, w_2) = (0.4, 0.6)$.

The selected solutions corresponding to each considered scenario are indicated in Figure 11, and in Table 3, the best solutions for different scenarios are presented and compared to those of Ref. [12]. By calculating the index $R_i = \sqrt{f_1^{w_1} \times f_2^{w_2}}$ for the results obtained by CSS and Kaveh and Laknejadi [12], the efficiency of the proposed algorithm is clarified.

Example 5. *A 56-bar truss structure.* This example is a 56-bar space truss studied in [23], with members categorized in three groups, as shown in Figure 13. Joint 1 is loaded with 4 kN (899.24 lb) in the Y-direction and 30 kN (6744.267 lb) in the Z-direction, while the remaining free nodes are loaded with 4 kN (899.24 lb) in the Y-direction and 10 kN (2248.09 lb) in the Z-direction.

The vertical displacements of joints 4, 5, 6, 12, 13 and 14 are restricted to 40 mm (0.158 in), while the displacement of joint 8 in the Y-direction is limited to 20 mm (0.079 in). The modulus of elasticity and the minimum and maximum member-cross sectional areas are taken as 210 kN/mm^2 (3.05×10^4 ksi), 200 mm^2 (0.31 in^2) and 2000 mm^2 (3.1 in^2), respectively. The total structural volume, $F_1(x)$, and the displacement at node 1, $F_2(x)$, have to be minimized simultaneously. Objective functions are:

$$\text{Min} \begin{cases} F_1(x) = \sum_{i=1}^{56} A_i l_i \\ F_2(x) = \sqrt{\delta_{1X}^2 + \delta_{1Y}^2 + \delta_{1Z}^2} \end{cases} \tag{19}$$

The two extreme objective values, obtained in 10 runs of various algorithms and the proposed method, are compared in Table 4. In addition, the Pareto front obtained via the CSS algorithm is shown in Figure 14.

The process of decision making and finding the best solution is performed identical to the previous example. In this example, in order to show the wide range of possible solutions, five different scenarios are considered. The results are aggregated in Table 5. The selected solutions corresponding to each considered scenario are provided in Figure 14.

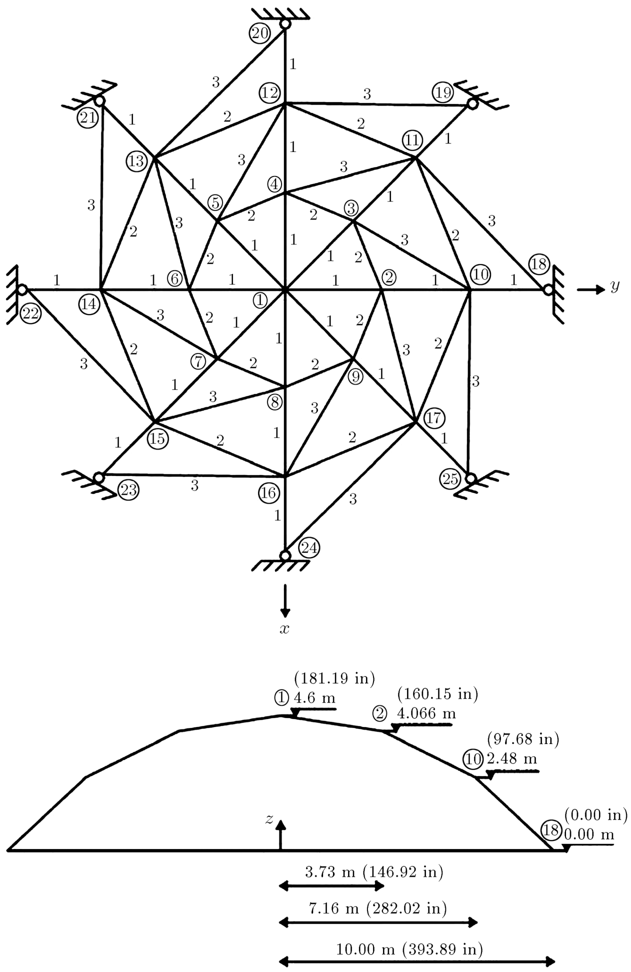


Figure 13. A 56-bar space truss structure.

Example 6. *A 272-bar transmission tower.* The fifth test example is the transmission tower, depicted in Figure 15, together with its geometric characteristics. This example is generated by the authors of this paper. The nodal coordinates and end nodes of each member are provided in Tables 6 and 7, respectively. The design variables considered are the cross-sectional area of the members, divided into twenty eight groups, as shown in Table 8.

Joints 1, 2, 11, 20 and 29 are loaded with 20 kN in the X- and Y-directions and -40 kN in the Z-direction, while the remaining free nodes are loaded with 5 kN in the X- and Y-directions. The vertical displacement of joints 2, 11, 20 and 29 is restricted to 20 mm, while the displacements in the X- and Y-directions are limited to 100 mm. The modulus of elasticity and the minimum and maximum member-cross sectional areas are taken as 2×10^8 kN/m^2 (3.05×10^4 ksi), 1000 mm^2 and 16000 mm^2, respectively. The principal stress, σ_j, in each element is restricted to the maximum allowable stress, $\sigma_j = \pm$ 275000 kN/m^2. The total structural volume, $F_1(x)$, and the displacement at node 1, $F_2(x)$, have to be minimized simultaneously.

Table 4. Comparison of the extreme values obtained by different methods for the 56-bar truss.

Optimization method	Obtained extreme values (mm, mm^3)
CSS-MOPSO [12]	$\begin{bmatrix}2.2148, & 402923368.6\end{bmatrix}$ $\begin{bmatrix}7.5495, & 120812690.1\end{bmatrix}$
s-MOPSO [19]	$\begin{bmatrix}2.2137, & 402417631.6\end{bmatrix}$ $\begin{bmatrix}7.4721, & 120151168.8\end{bmatrix}$
MOPSO [11]	$\begin{bmatrix}2.2154, & 403070300.4\end{bmatrix}$ $\begin{bmatrix}7.0825, & 123191518.1\end{bmatrix}$
NSGA-II [8]	$\begin{bmatrix}2.2137, & 402403612.4\end{bmatrix}$ $\begin{bmatrix}7.4883, & 119960278.7\end{bmatrix}$
CSS (present work)	$\begin{bmatrix}1.1061, & 478422670.1\end{bmatrix}$ $\begin{bmatrix}10.4342, & 50644453.1\end{bmatrix}$

Objective functions are:

$$\text{Min}\begin{cases} F_1(x) = \sum_{i=1}^{272} A_i l_i \\ F_2(x) = \sqrt{\delta_{1X}^2 + \delta_{1Y}^2 + \delta_{1Z}^2} \end{cases} \tag{20}$$

The Pareto front obtained via the CSS algorithm is shown in Figure 16. The process of decision making and finding the best solution is performed completely similar to those of previous examples. In this example, in order to show the wide range of possible solutions, nine different scenarios are considered. The results are aggregated in Table 9. The selected solutions corresponding to each considered scenario are provided in Figure 16.

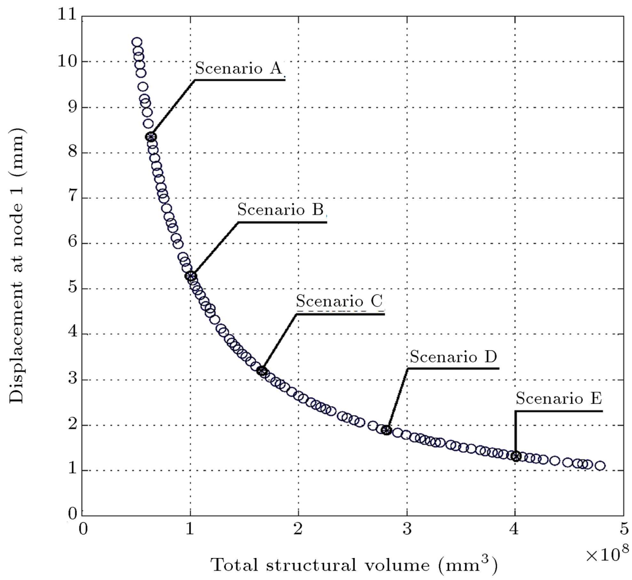


Figure 14. The Pareto front of 56-bar space truss structure and the best solutions according to five different scenarios.

Table 5. Different possible scenarios for the 56-bar truss with corresponding solutions.

Scenario	Importance of criteria	Possible priority weights	Selected solution by MTDM (mm, mm^3)					
			CSS-MOPSO [12]			CSS (present work)		
			f_1 (mm)	f_2 (mm^3)	R_i	f_1 (mm)	f_2 (mm^3)	R_i
A	$c_1 >> c_2$	$\begin{bmatrix}0.9, & 0.1\end{bmatrix}$	6.1144	134900811.2	**5.7592**	8.3478	63593336.6	**6.3808**
B	$c_1 > c_2$	$\begin{bmatrix}0.7, & 0.3\end{bmatrix}$	4.4740	166100766.2	**28.8934**	5.2836	100114327.7	**28.3857**
C	$c_1 \sim c_2$	$\begin{bmatrix}0.5, & 0.5\end{bmatrix}$	3.2959	208951138.7	**161.9961**	3.2022	165628602.5	**151.7560**
D	$c_1 < c_2$	$\begin{bmatrix}0.3, & 0.7\end{bmatrix}$	2.5679	267415709.3	**1025.5389**	1.8809	281480915.0	**996.4649**
E	$c_1 << c_2$	$\begin{bmatrix}0.1, & 0.9\end{bmatrix}$	2.2709	353607163.4	**7322.2470**	1.3168	401089550.8	**7541.0982**

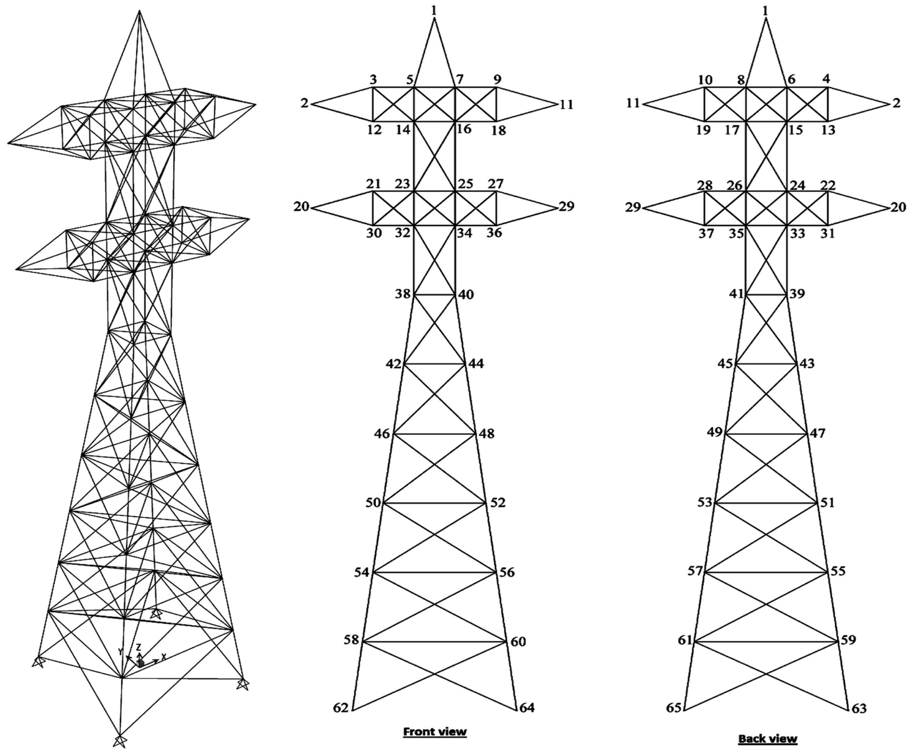


Figure 15. A 272-bar transmission tower.

Table 6. Nodal coordinates of the transmission tower.

Node	X (m)	Y (m)	Z (m)	Node	X (m)	Y (m)	Z (m)	Node	X (m)	Y (m)	Z (m)	Node	X (m)	Y (m)	Z (m)	Node	X (m)	Y (m)	Z (m)
1	0	0	20	**14**	-0.5	-0.5	17	**27**	1.5	-0.5	15	**40**	0.5	-0.5	12	**53**	1.25	1.25	6
2	-3	0	17.5	**15**	-0.5	0.5	17	**28**	1.5	0.5	15	**41**	0.5	0.5	12	**54**	-1.5	-1.5	4
3	-1.5	-0.5	18	**16**	0.5	-0.5	17	**29**	3	0	14.5	**42**	-0.75	-0.75	10	**55**	-1.5	1.5	4
4	-1.5	0.5	18	**17**	0.5	0.5	17	**30**	-1.5	-0.5	14	**43**	-0.75	0.75	10	**56**	1.5	-1.5	4
5	-0.5	-0.5	18	**18**	1.5	-0.5	17	**31**	-1.5	0.5	14	**44**	0.75	-0.75	10	**57**	1.5	1.5	4
6	-0.5	0.5	18	**19**	1.5	0.5	17	**32**	-0.5	-0.5	14	**45**	0.75	0.75	10	**58**	-1.75	-1.75	2
7	0.5	-0.5	18	**20**	-3	0	14.5	**33**	-0.5	0.5	14	**46**	-1	-1	8	**59**	-1.75	1.75	2
8	0.5	0.5	18	**21**	-1.5	-0.5	15	**34**	0.5	-0.5	14	**47**	-1	1	8	**60**	1.75	-1.75	2
9	1.5	-0.5	18	**22**	-1.5	0.5	15	**35**	0.5	0.5	14	**48**	1	-1	8	**61**	1.75	1.75	2
10	1.5	0.5	18	**23**	-0.5	-0.5	15	**36**	1.5	-0.5	14	**49**	1	1	8	**62**	-2	-2	0
11	3	0	17.5	**24**	-0.5	0.5	15	**37**	1.5	0.5	14	**50**	-1.25	-1.25	6	**63**	-2	2	0
12	-1.5	-0.5	17	**25**	0.5	-0.5	15	**38**	-0.5	-0.5	12	**51**	-1.25	1.25	6	**64**	2	-2	0
13	-1.5	0.5	17	**26**	0.5	0.5	15	**39**	-0.5	0.5	12	**52**	1.25	-1.25	6	**65**	2	2	0

7. Conclusions

Optimization problems encountered in practice are seldom mono-objective. In general, there are many conflicting objectives to handle. An efficient procedure for solving multi-objective optimization problems using the Charged System Search algorithm is presented in this study. The algorithm is also applied to six engineering design problems to demonstrate its applicability in practical problems. The results obtained amply demonstrate that the presented approach is efficient in converging to the true Pareto fronts and

Table 7. End nodes of the members of 272-bar transmission tower.

Member	Start point	End point	Member	Start point	End point	Member	Start point	End point	Member	Start point	End point	Member	Start point	End point	Member	Start point	End point	Member	Start point	End point	Member	Start point	End point
1	1	5	**35**	10	19	**69**	34	32	**103**	4	15	**137**	9	19	**171**	38	42	**205**	47	48	**239**	57	56
2	1	6	**36**	8	17	**70**	35	37	**104**	6	13	**138**	10	18	**172**	40	44	**206**	46	49	**240**	56	54
3	1	7	**37**	6	15	**71**	37	36	**105**	21	24	**139**	21	31	**173**	41	45	**207**	46	50	**241**	54	57
4	1	8	**38**	4	13	**72**	36	34	**106**	22	23	**140**	22	30	**174**	39	43	**208**	48	52	**242**	55	56
5	2	3	**39**	12	13	**73**	21	30	**107**	24	25	**14 1**	23	33	**175**	38	43	**209**	49	53	**243**	54	58
6	2	4	**40**	13	15	**74**	23	32	**108**	23	26	**14 2**	32	24	**176**	39	42	**210**	47	51	**244**	56	60
7	2	12	**41**	15	14	**75**	25	34	**109**	26	27	**1 43**	25	35	**177**	40	45	**211**	46	52	**245**	57	61
8	2	13	**42**	14	12	**76**	27	36	**110**	25	28	**1 44**	26	34	**178**	41	44	**212**	48	50	**246**	55	59
9	11	9	**43**	15	17	**77**	28	37	**111**	31	32	**1 45**	27	37	**179**	38	44	**213**	49	51	**247**	54	60
10	11	10	**44**	17	16	**78**	26	35	**112**	30	33	**1 46**	28	36	**180**	40	42	**214**	47	53	**248**	56	58
11	11	18	**45**	16	14	**79**	24	33	**113**	33	34	**1 47**	14	23	**181**	41	43	**215**	46	51	**249**	57	59
12	11	19	**46**	17	19	**80**	22	31	**114**	32	35	**1 48**	16	25	**182**	39	45	**216**	47	50	**250**	55	61
13	20	21	**47**	19	18	**81**	4	5	**115**	35	36	**1 49**	17	26	**183**	42	43	**217**	48	53	**251**	54	59
14	20	22	**48**	18	16	**82**	3	6	**116**	34	37	**1 50**	15	24	**184**	43	45	**218**	49	52	**252**	55	58
15	20	30	**49**	38	39	**83**	6	7	**117**	21	32	**1 51**	32	38	**185**	45	44	**219**	50	51	**253**	56	61
16	20	31	**50**	39	41	**84**	5	8	**118**	30	23	**15 2**	34	40	**186**	42	44	**220**	51	53	**254**	57	60
17	29	27	**51**	41	40	**85**	8	9	**119**	23	34	**1 53**	35	41	**187**	43	44	**221**	53	52	**255**	58	59
18	29	28	**52**	40	38	**86**	7	10	**120**	32	25	**1 54**	33	39	**188**	42	45	**222**	52	50	**256**	59	61
19	29	36	**53**	21	22	**87**	12	15	**121**	25	36	**1 55**	14	25	**189**	42	46	**223**	51	52	**257**	61	60
20	29	37	**54**	22	24	**88**	13	14	**122**	34	27	**1 56**	16	23	**190**	44	48	**224**	50	53	**258**	60	58
21	3	4	**55**	24	23	**89**	15	16	**123**	28	35	**157**	17	24	**191**	4 5	49	**225**	50	54	**259**	59	60
22	4	6	**56**	23	21	**90**	14	17	**124**	26	37	**158**	15	26	**192**	4 3	47	**226**	52	56	**260**	58	61
23	6	5	**57**	24	26	**91**	17	18	**125**	26	33	**159**	14	24	**193**	4 2	48	**227**	53	57	**261**	58	62
24	5	3	**58**	26	25	**92**	16	19	**126**	24	35	**160**	15	23	**194**	4 4	46	**228**	51	55	**262**	60	64
25	6	8	**59**	25	23	**93**	3	14	**127**	24	31	**161**	16	26	**195**	45	47	**229**	50	56	**263**	61	65
26	8	7	**60**	26	28	**94**	5	12	**128**	22	33	**162**	17	25	**196**	43	49	**230**	52	54	**264**	59	63
27	7	5	**61**	28	27	**95**	5	16	**129**	39	40	**163**	32	40	**197**	42	47	**231**	53	55	**265**	58	64
28	8	10	**62**	27	25	**96**	7	14	**130**	38	41	**164**	38	34	**198**	4 3	46	**232**	51	57	**266**	60	62
29	10	9	**63**	30	31	**97**	7	18	**131**	3	13	**165**	35	39	**199**	44	49	**233**	50	55	**267**	61	63
30	9	7	**64**	31	33	**98**	9	16	**132**	12	4	**166**	33	41	**200**	45	48	**234**	51	54	**268**	59	65
31	3	12	**65**	33	32	**99**	10	17	**133**	5	15	**167**	32	39	**201**	4 6	47	**235**	52	57	**269**	58	63
32	5	14	**66**	32	30	**100**	8	19	**134**	14	6	**168**	33	38	**202**	4 7	49	**236**	53	56	**270**	59	62
33	7	16	**67**	33	35	**101**	8	15	**135**	7	17	**169**	34	41	**203**	4 9	48	**237**	54	55	**271**	60	65
34	9	18	**68**	35	34	**102**	6	17	**136**	8	16	**170**	35	40	**204**	4 8	46	**238**	55	57	**272**	61	64

Table 8. Member grouping of the 272-bar transmission tower.

Group number	Members	Group number	Members	Group number	Members	Group number	Members
1	$[M1-M4]$	**8**	$[M175-M182]$	**15**	$[M207-M210]$	**22**	$[M241-M242]$
2	$[M5-M20]$	**9**	$[M183-M186]$	**16**	$[M211-M218]$	**23**	$[M243-M246]$
3	$[M21-M80]$	**10**	$[M187-M188]$	**17**	$[M219-M222]$	**24**	$[M247-M254]$
4	$[M81-146M]$	**11**	$[M189-M192]$	**18**	$[M223-M224]$	**25**	$[M255-M258]$
5	$[M147-M154]$	**12**	$[M193-M200]$	**19**	$[M225-M228]$	**26**	$[M259-M260]$
6	$[M155-M170]$	**13**	$[M201-M204]$	**20**	$[M229-M236]$	**27**	$[M261-M264]$
7	$[M171-M174]$	**14**	$[M205-M206]$	**21**	$[M237-M240]$	**28**	$[M265-M272]$

Table 9. Different possible scenarios for the transmission tower with corresponding solutions.

Scenario	Possible priority weights	Selected solutions by MTDM (m, m^3)	$R_i=\sqrt{f_1^{w_1}\times f_2^{w_2}}$
A	$[0.9, 0.1]$	$[0.4571, 0.1102]$	0.6296
B	$[0.8, 0.2]$	$[0.4793, 0.0947]$	0.5887
C	$[0.7, 0.3]$	$[0.5067, 0.0763]$	0.5359
D	$[0.6, 0.4]$	$[0.5646, 0.0606]$	0.4809
E	$[0.5, 0.5]$	$[0.8278, 0.0504]$	0.4520
F	$[0.4, 0.6]$	$[1.4683, 0.0439]$	0.4228
G	$[0.3, 0.7]$	$[2.1702, 0.0398]$	0.3633
H	$[0.2, 0.8]$	$[3.0858, 0.0384]$	0.3039
I	$[0.1, 0.9]$	$[4.0674, 0.0377]$	0.2453

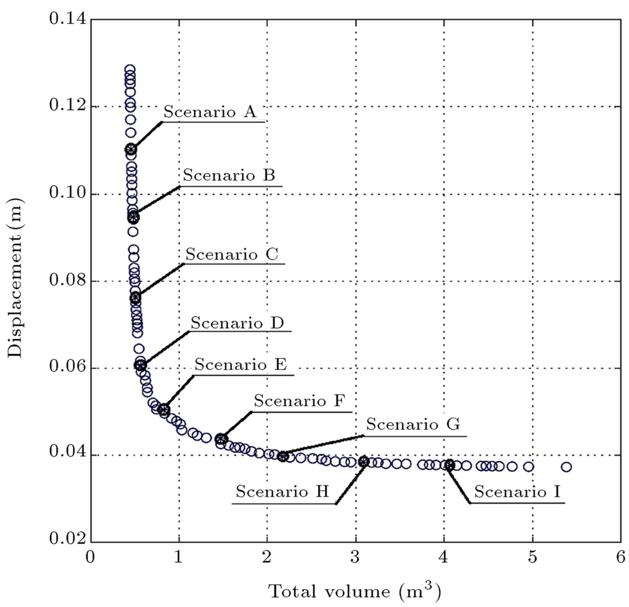


Figure 16. The Pareto front of transmission tower and the best solutions according to nine different scenarios.

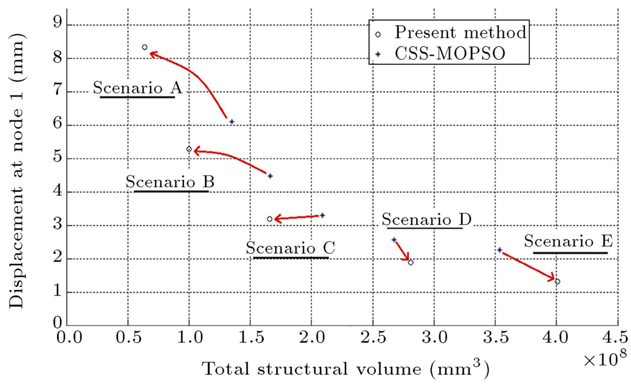


Figure 17. Comparison between best solutions according to five different scenarios obtained by the present work and CSS-MOPSO for Example 5.

in finding a diverse set of solutions along the Pareto front. Considering Tables 1, 2 and 4, it is obvious that the Pareto front obtained by the present method is more diverse than other methods. After computing the Pareto front, the engineers involved in making design decisions, express their preferences about different criteria (objectives or other independent criteria). By aggregating different ideas, the final solution is selected by an algorithm called MTDM. Comparison of the best solutions, corresponding to five different scenarios obtained by the present work and CSS-MOPSO, is shown in Figure 17 for Example 5.

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