A Global Method for Structural Damage Detection
Part I- Theory and Computational Aspects

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Abstract

Using the concept of strain energy change, due to damage at the element level, a new structural damage detection method is developed. The proposed method employs either static or dynamic response of a structure and simultaneously localizes and quantifies multiple damages. It requires only the stiffness and mass matrices of the baseline structure and a few measured responses of the current structure to find the exact location and severity of damage. A numerical example was used to investigate the behavior of the algorithm. It is shown that those mode shapes and static loadings which yield a uniform distribution of strain energy in elements predict the location and magnitude of damage with more accuracy. Some higher mode shapes which might induce significant levels of strain energy in some elements are not always reliable. Finally, it is shown that in order to obtain reliable results, the number of equations (mode shapes or static loadings) must be greater than the number of predicted damaged elements.

Keywords: Damage Detection, Strain Energy, Static Response, Dynamic Response.

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INTRODUCTION

Structural systems are susceptible and vulnerable to unrecoverable damages due to unexpected loadings such as sever earthquakes or other natural disasters. Besides, these systems and more specifically infrastructures including life lines and bridges might lose their strength during their service life due to some other causes. Some of these parameters which harm the health of a structure are lack of repair, inappropriate maintenance, and not following the most recent design codes. Concerning the safety and functionality of structures, Structural Damage Detection (SDD) and Structural Health Monitoring (SHM) has become an attractive and essential field of research in Civil Engineering.

Any change in the properties of the baseline structure is known as a structural damage. Basically, there are two categories of damages; linear and nonlinear. If an originally linear-elastic structure preserves its linear-elastic behavior after damage occurs, it bears a linear damage. A nonlinear damage is developed when an initially linear-elastic structure shows a nonlinear behavior. Cracks, fatigue, corrosion and broken welds and/or nuts could be some sources of damages.

There are many different linear and nonlinear damage detection procedures. These methods cover a wide range of techniques from visual inspection to very sophisticated nondestructive damage detection (NDD) methods. The NDD techniques classified as local and global methods. Local methods such as x-ray inspection, eddy current scanning, acoustic emission, magnet field, thermal field and ultrasonic methods require that the vicinity of the damage be known a priori and the structural components be readily accessible. The global methods are based on the measured response of the structures which could be either static or dynamic response.

In general, the global methods may yield model-based algorithms or non-model-based algorithms. Model-based algorithms need a finite element model of the damaged structure for the process of damage detection. Non-model-based algorithms identify the damage by comparing the responses of the structure before and after the damage occurrence. These algorithms do not explicitly use the structural parameters (stiffness, mass, and damping) and do not require the analytical model of the structure.

Rytter [1] defined four levels of damage identification as follows.

Level 1: determining the presence of a damage in the structure (damage detection),

Level 2: determining the location of the damage (damage localization),

Level 3: quantifying the severity of the damage,

Level 4: predicting the remaining life time of the structure.

The field of structural damage identification is a very broad field of study. Different methods based on different concepts and definitions of damage have already been developed. The concept of strain energy which is very suitable and promising for identification structural damages has been frequently used in literature. Stubbs et al. [2] are the first researchers who used the concept of strain energy to detect a structural damage. They presented a method based on the decrease in modal strain energy between two structural degrees of freedom, as defined by the curvature of the measured mode shapes. This method was applied to a damaged steel bridge. Their algorithm localized the damage by using three Eigen modes [3]. Later, Topole and Stubbs [4] investigated the feasibility of this method by using a limited set of modal parameters.

In 1996, Farrar and Jauregui [5] employed five different damage detection algorithms, including; the damage index method [3], the mode shape curvature method

[6], the flexibility change method [7], a method combining mode shape curvature and flexibility change [8], and the stiffness change method [9], to detect some cuts intentionally provided in the I-40 bridge over the Rio Grande in New Mexico. They concluded that in general, all methods could correctly identify the damage location for the most severe cut, one from the mid-web completely through the bottom flange. All these methods could not clearly identify the damage location when they were applied to three less severe damage cases. Finally, based on the detection results they concluded that the damage index method [3], which is based on the strain energy concept, performed the best.

In 2002, Park and Kim [10] examined the feasibility of damage index method for large, complex structures and found that the method can successfully be applied to this type of structures. The authors observed that the results might be improved if data from several modes are simultaneously used. Their method was suitable for structures that behave globally in a beam-like manner (one-dimensional). Cornwell et al. [11, 12] extended the method for plate-like structures characterized by two-dimensional curvature. The method employs only a few modes and has a non-model based algorithm, i.e. the method only requires the mode shapes of the structure before and after damage and these modes do not need to be mass normalized. This feature makes it very advantageous when using ambient excitation. The disadvantage of this method is that no appropriate data is usually available for undamaged structures. Basically, the method is a level-two method which can only identify the location of damage but not quantifying the magnitude of damage.

Another structural damage detection method based on modal strain energy change was presented by Shi et al. [13]. Later, in 2000 these authors improved their algorithm

[14]. The new algorithm is able to locate single as well as multiple damages and compute corresponding magnitude of the damage (a level-three method). In 2002, Shi et al. [15] modified their algorithm so that it could quantify the damage using fewer lower modes compare to their original algorithm. The improved algorithm reduces the truncation error in computation, avoids the finite element modeling error in higher modes, and improves the rate of convergence. This method has a model based algorithm, i.e. it requires the stiffness matrix of the structures to compute modal strain energy. Since the damaged elements are not known, the undamaged elemental stiffness matrix is used instead of the damaged one as an approximation in elemental modal strain energy for the damaged state. Also, it requires both damaged and undamaged mode shapes of the structure.

In last decades, some researchers developed various methods to determine location and magnitude of damage in different structural elements and bridges using strain energy changes [16-21].

Sharifi and Banan [17] developed a structural damage detection method based on the change of strain energy in each element before and after damage, which requires only the stiffness and mass matrices of the baseline structure and a few measured mode shapes of the current structure to find the location and severity of damage. The method has model based algorithm and it is a level-three method. The method has the capability of simultaneously localizing and quantifying multiple damages. The authors applied their method which is called Energy index method to the benchmark study sponsored by the IASC-ASCE Task Group on Structural Health Monitoring (details of the benchmark problem is presented in [22]) and the method successfully detected the damages of the benchmark.

Although, there are many methods which can be used to determine the mode shapes of a damaged structure by using either ambient vibration or transient dynamic response of the structure [23], which is very useful for structural damage detection of complex structures such as high-rise building (damage detection from dynamic response of structures), but in some structures such as bridges, it is simpler and more accurate to measure the static response of structures subjected to some static load cases (damage detection from static response of structures).

In this paper, we have extended the energy index method developed by Sharifi and Banan [17], which works only for modal data, to cover static cases, as well. The proposed method is now capable of detecting damage based on availability of either static or dynamic response of structures (measurements from either static tests or dynamic tests). Using the static test results, the algorithm requires only the stiffness matrix of the baseline structure and a few number of statically loading cases of the current structure. For a dynamic case, a few number of measured mode shapes of the current structure and the stiffness and mass matrices of the baseline structure are required. The computational aspects of the advanced energy index method are also investigated in this paper.

THEORY AND FORMULATION DEVELOPMENT

The total stored strain energy, U, in an undamaged structure with n elements due to a virtual deformation is equal to the sum of the element strain energies, u_e, which is

$$\sum_{e=1}^{n} u_e = U \tag{1}$$

Suppose an element of the structure is damage. If the damaged structure is subjected to the same virtual deformation which was induced on the undamaged structures, then the strain energy of the damaged element, $\tilde{u}_e$, and thereby the total stored strain energy in the damaged structure, $\tilde{U}$, are reduced. The reduction of the element strain energy is as follows

$$\Delta u_e = u_e - \tilde{u}_e = u_e\left(\frac{u_e - \tilde{u}_e}{u_e}\right) = \delta_e u_e \quad ; \quad \delta_e \in [0,1] \tag{2}$$

In this relation δ_e is defined as the energy index of the *e*th element which varies between zero and one. If the *e*th element is undamaged then δ_e is zero, and δ_e is equal to one if the *e*th element is completely lost.

The decreased strain energy in the damaged structure is the sum of the Δu_e of each element which is

$$\Delta U = U - \tilde{U} = \sum_{e=1}^{n} (u_e - \tilde{u}_e) \tag{3}$$

Substituting Eqn. (2) into Eqn. (3) yields

$$\sum_{e=1}^{n} \delta_e u_e = U - \tilde{U} \tag{4}$$

Now, let us assume the structure is subjected to *m* different virtual deformations. It supplies a system of *m* different equations like Eqn. (4) as follows

$$\sum_{e=1}^{n} \delta_e u_{ei} = U_i - \tilde{U}_i \quad \text{for } i = 1, \ldots, m \tag{5}$$

Solving this system of equations δ_e is found for each element, and then the location and damage severity of the damaged elements are determined. Depending on the number of virtual deformations, the above system of equations may be determined ($m = n$), over determined ($m > n$), or underdetermined ($m < n$).

To solve over determined and under determined system of equations, the Non-Negative Least Square (NNLS) method proposed by Lawson and Hanson [24] is used. This method minimizes the objective function $\| \boldsymbol{S}\boldsymbol{\delta}_e - \boldsymbol{R} \|$ subjected to constraints $\delta_e \geq 0$. The performance of other methods such as Least Square, Moor-Penrose Pseudo Inverse and Singular Value Decomposition are also investigated. But comparing results leads to employing the NNLS method.

Static Response

For an elastic system subjected to conservative static forces, the work done by the external forces on the system is stored as strain energy in the system. The term "conservative forces" refers to those forces whose potential energy depends only on the final values of deflections, not the specific paths to reach these final values.

If we assume that the virtual deformation is the same as the deformation of the damaged structure due to a certain static loading, the stored strain energy in the damaged structure ($\tilde{U}$) is equal to the work down by the external forces on the damaged structure ($\tilde{W}$) which is

$$\tilde{U} = \tilde{W} \tag{6}$$

In the finite element form, by denoting the displacement vector of the structure and displacement vector of the *e*th element for damaged structure as $\tilde{\boldsymbol{Q}}$ and $\tilde{q}_e$, respectively, we will have

$$u_e = \frac{1}{2}\tilde{\boldsymbol{q}}_e^T \boldsymbol{k}_e \tilde{\boldsymbol{q}}_e \ , \ U = \frac{1}{2}\tilde{\boldsymbol{Q}}^T \boldsymbol{K} \tilde{\boldsymbol{Q}} \ , \ \tilde{U} = \tilde{W} = \frac{1}{2}\sum_{j=1}^{L} P_j \tilde{Q}_j \tag{7}$$

where $\boldsymbol{k}_e$ and $\boldsymbol{K}$ are the *e*th element stiffness and the global stiffness matrices of the undamaged structure, respectively. $\tilde{Q}_i$ is the generalized displacement of the damaged structure and P_i is the generalized external force acting in the direction of $\tilde{Q}_i$. The word generalized is used here to emphasize that the displacement can be either transitional or rotational and that the forces can be concentrated forces and/or moments. Substituting Eqns. (7) into Eqn. (4), one can get

$$\sum_{e=1}^{n} \tilde{\boldsymbol{q}}_e^T \boldsymbol{k}_e \tilde{\boldsymbol{q}}_e \delta_e = \tilde{\boldsymbol{Q}}^T \boldsymbol{K} \tilde{\boldsymbol{Q}} - \sum_{j=1}^{L} P_j \tilde{Q}_j \tag{8}$$

In this equation the only unknown is δ_e. If the damaged structure is subjected to *m* different static load cases, we will have *m* different equations like Eqn. (8), as follows

$$\sum_{e=1}^{n} \tilde{\boldsymbol{q}}_{ie}^T \boldsymbol{k}_e \tilde{\boldsymbol{q}}_{ie} \delta_e = \tilde{\boldsymbol{Q}}_i^T \boldsymbol{K} \tilde{\boldsymbol{Q}}_i - \sum_{i=1}^{L} P_{ij} \tilde{Q}_{ij} \quad \text{for } i = 1, \ldots, m \tag{9}$$

In compact matrix form, Eqn. (9) has the following form

$$\boldsymbol{S}\boldsymbol{\delta}_e = \boldsymbol{r} \ , \ s_{ie} = \tilde{\boldsymbol{q}}_{ie}^T \boldsymbol{k}_e \tilde{\boldsymbol{q}}_{ie} \ , \ r_i = \tilde{\boldsymbol{Q}}_i^T \boldsymbol{K} \tilde{\boldsymbol{Q}}_i - \sum_{j=1}^{L} P_{ij} \tilde{Q}_{ij} \tag{10}$$

where s_{ie}, the members of the system matrix ***S***, is the element strain energy of the *e*th element due to the deformation of the *i*th static loading of the damaged structure and r_i, the elements of the residual vector ***r***, is the difference between the total strain energies of the undamaged structure and the damaged structure due to the deformation of the ith static loading of the damaged structure.

Dynamic Response

In dynamic tests, the dynamic properties of structure such as natural frequencies and mode shapes are obtained via vibration measurements. Now, let us assume that the virtual deformation is the same as one of the mode shapes of the damaged structure. In the finite element form, we will have

$$u_e = \frac{1}{2} \tilde{\varphi}_e^T \boldsymbol{k}_e \tilde{\varphi}_e \ , \ U = \frac{1}{2} \tilde{\boldsymbol{\Phi}}^T \boldsymbol{K} \tilde{\boldsymbol{\Phi}} \ , \ \tilde{U} = \frac{1}{2} \tilde{\boldsymbol{\Phi}}^T \tilde{\boldsymbol{K}} \tilde{\boldsymbol{\Phi}} = \frac{1}{2} \tilde{\omega}^2 \tilde{\boldsymbol{\Phi}}^T \tilde{\boldsymbol{M}} \tilde{\boldsymbol{\Phi}} \tag{11}$$

where $\tilde{\boldsymbol{\Phi}}$ and $\tilde{\omega}$ are the mode shape and circular natural frequency of the damaged structure, respectively; $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{K}}$ are the mass and stiffness matrices of the damaged structure, respectively, and $\tilde{\varphi}_e$ is the displacement vector imposed on the *e*th element due to the mode shape $\tilde{\boldsymbol{\Phi}}$. It is reasonable to assume that the mass of the structure does not change due to damage, *i.e.* $\tilde{\boldsymbol{M}} = \boldsymbol{M}$. By substituting Eqns. (11) into Eqn. (4), one can get

$$\sum_{e=1}^{n} \tilde{\varphi}_e^T \boldsymbol{k}_e \tilde{\varphi}_e \delta_e = \tilde{\boldsymbol{\Phi}}^T \boldsymbol{K} \tilde{\boldsymbol{\Phi}} - \tilde{\omega}^2 \tilde{\boldsymbol{\Phi}}^T \boldsymbol{M} \tilde{\boldsymbol{\Phi}} \tag{12}$$

The only unknown in Eqn. (12) is δ_e. Using the other mode shapes of the damaged structure, a system of equations like Eqn. (12) is obtained. If *m* mode shapes of the damaged structure are available, we will have the following system of equations

$$\sum_{e=1}^{n} \tilde{\varphi}_{ie}^T \boldsymbol{k}_e \tilde{\varphi}_{ie} \delta_e = \tilde{\boldsymbol{\Phi}}_i^T \boldsymbol{K} \tilde{\boldsymbol{\Phi}}_i - \tilde{\omega}_i^2 \tilde{\boldsymbol{\Phi}}_i^T \boldsymbol{M} \tilde{\boldsymbol{\Phi}}_i \quad \text{for } i = 1, \ldots, m \tag{13}$$

If the mode shapes are normalized with respect to the mass matrix, we will have

$$\sum_{e=1}^{n} \tilde{\varphi}_{ie}^T \boldsymbol{k}_e \tilde{\varphi}_{ie} \delta_e = \tilde{\boldsymbol{\Phi}}_i^T \boldsymbol{K} \tilde{\boldsymbol{\Phi}}_i - \tilde{\omega}_i^2 \quad \text{for } i = 1, \ldots, m \tag{14}$$

This equation can be written in the compact matrix form as follows

$$\boldsymbol{S} \boldsymbol{\delta}_e = \boldsymbol{r} \ , \ s_{ie} = \tilde{\varphi}_{ie}^T \boldsymbol{k}_e \tilde{\varphi}_{ie} \ , \ r_i = \tilde{\boldsymbol{\Phi}}_i^T \boldsymbol{K} \tilde{\boldsymbol{\Phi}}_i - \tilde{\omega}_i^2 \tag{15}$$

where s_{ie}, the members of the system matrix ***S***, is the element strain energy of the *e*th element due to the *i*th mode shape and r_i, the elements of the residual vector ***r***, is the difference between the total strain energies of the undamaged structure and the damaged structure due to the *i*th mode.

SIMULATION STUDY AND COMPUTATIONAL ASPECTS

Using the developed formulation a finite element program named StruDD is developed. To investigate the behavior of the proposed method a two-span continues plane truss shown in Fig.1 is studied in a simulation environment. This truss is the same truss that Yeo et al. [25] used to study the performance of their algorithm. The cross-sectional properties are given in Fig.1. All members have the same Young's modulus of $19994.798\ {}^{KN}\!/_{cm^2}$ and mass density, ρ, equal to $7.827\text{E-}08\ {}^{KN\cdot S^2}\!/_{cm^4}$. The finite element model of the truss structure consists of 55 elements with 44 degrees of freedom. Different damage scenarios are simulated for this truss, which are summarized in Table1. For each case, damage is simulated by reduction in the sectional area of a truss member. The same truss structure is used to simulate required measurements for both static and dynamic tests.

Static Response

For the simulation of static tests, five load cases are selected as shown in Fig.2. The levels of elemental strain energies in the undamaged truss due to load case 1 are shown in Fig.3. As one can observe, the levels of strain energies in vertical and diagonal elements are very small compare to the strain energies in the top and bottom elements. Fig.4 presents the damage identification results for damage case 1. For this scenario of damage, all five load cases are used to identify the damage. In this case, different methods are used to solve Energy Equation System. As it is shown, the Non-Negative Least Square (NNLS) method provides the best solution. It can be seen that damage in member 22 is successfully located and quantified by using NNLS. The severity of damage in member 22 is predicted as 29.94% and all undamaged members are identified as undamaged.

When we used load case 5 alone and NNLS method, the severity of damage in member 22 is successfully predicted as 30% and all undamaged members are identified as undamaged. But when only load case 1 is used, damage in member 22 is not detected, and the undamaged member 3 is identified as damaged member with very low damage severity, i.e. 1.79%. By speculating in Fig.3 and comparing the stored strain energy in element 3 with other elements due to load case 1, we can notice why the member 3 is identified as potential damaged member when only load case 1 is singly used. The level of developed strain energy in element 22 is very small in load case 1, and it causes that the energy index method be incapable to identify member 22 as a damaged member. But in load case 5, the stored strain energy in member 22 is relatively adequate for the algorithm. It is also observed that damage in member 22 is not detectable when either load case 2 or 3 is singly used, because in these load cases the stored strain energy in member 22 is very small. But for load case 4, which the level of developed strain energy in member 22 is relatively high, the severity of damage in this member is successfully predicted as 30% and all undamaged members are identified as undamaged.

For damage case 2, it is also observed that the NNLS method yields the best solution. The severity of damages in member 22 and 4 are successfully predicted as 30% and 40%, respectively and all undamaged members are identified as undamaged. When each load case 1 through 5 is singly used, we have only one equation and the algorithm can only identify one element as the damaged member. When load case 1 or 2 is used, the algorithm identifies member 4 as the damaged member. For load case 4 and 5, member 22 is identified as the damaged member. For load case 3 the algorithm falls into error. Fig.5 presents the damage identification results for the scenarios that two load cases are simultaneously used, i.e. load cases 1 and 5, and load cases 2 and 4. It can be seen that the algorithm performs well. It correctly identified the actual damaged member.

In damage case 3, by simultaneously using NNLS and all 5 load cases, all damaged members are successfully identified along with their exact damage severities. The damage detection and assessment results are illustrated in Fig.6 for the case that different combinations of load cases are used to identify damage. As one can notice, if only one load case is used the damage identification algorithm could identify one element as the damaged member. When two load cases are used the algorithm can identify two members as damaged members, if there exist any, and so forth. For example when three load cases 2, 3 and 4 are used, all three damaged member are correctly identified as damaged members, but when all five load cases are used, the algorithm does not identify any addition member as damaged member. In general, to get a reliable result the minimum number of required deformed shapes due to different load cases must be equal to the number of damaged elements. Those elements of the structure with high level of stored strain energies when subjected to damage can be easily detected. The loading case that develops the highest level of strain energy in the damaged element is the best load case to detect the damage of that element. But since the locations of damaged elements are unknown, one can not simply use this conclusion. An applicable suggestion might be using those load cases which develop almost a uniform level of strain energy in all elements.

Damage case 4 can be regarded as a very light damage. In this case, the algorithm fails to identify damage in member 16. The results of the damage identification process for element 16 when damage severity in this element increases, are shown in Fig.7. One can observed that the algorithm could not identify damage in member 16 with damage severity less than 20%. When damage severity in this element increases more than 20%, the element is successfully identified with its exact actual damage severity.

Dynamic Response

It is assumed that the natural frequencies and mode shapes of the first four modes of the structure are available as measured data. The levels of elemental strain energies in the undamaged truss due to first mode shape are shown in Fig.8. Any damage induced in diagonal and vertical members can be detected with more difficultly than detecting damages in top and bottom members. Because the strain energies developed in top and bottom elements are significantly larger than energy levels in other members. Fig.9 presents the elemental strain energies due to the first three mode shapes. It is observed that, higher mode shapes (e.g. mode shape 3) induce higher levels of strain energy in some elements, which may mislead the algorithm. Distribution of the strain energy due to the lower mode shapes (mode shapes 1 and 2) is relatively smooth. It means the lower mode shapes contain much more reliable information compare to information content of mode shape 3.

The results are summarized and shown in Fig.10 through Fig.13. For the damage case1, different methods are used to solve damage equations system. As one can notice, NNLS method gives the best solutions. When there is only one damaged element in structure (damage case1 and 4), only the first mode shape is sufficient to identify damage. Usually the first mode shape provides enough data to detect only damaged element, even with small magnitude (damage case4). But when the number of damaged elements is more than one, more mode shapes should be used to detect all damages.

For dynamic cases like the static ones the number of required mode shapes depends on the number and locations of damaged elements. The minimal number of required mode shapes must be equal to the number of damaged elements. But since some of the higher mode shapes develop higher levels of the strain energy in some elements, these mode shapes are not reliable. Thus, more mode shapes must be used to achieve the reliable results. By speculating in Fig.9 and comparing the stored strain energies due to different mode shapes in elements 4 and 46 together, one can easily understand the importance of this observation and fact. To support this conclusion, the damage case 3 is examined with different numbers of mode shapes and the results are summarized in Table 2.

Conclusion

A new global damage detection method based on strain energy change concept was developed. The method is referred as Energy Index Method. It is a model based and a level-three method, i.e. it not only flags damage but gives location and severity of damage. One of the advantages of the method is that the multiple damages can be simultaneously localized and quantified. Another feature of the method is that it can be used for both static and modal response of a structure (static tests and dynamic tests). The algorithm requires only the stiffness and mass matrices of the baseline structure and measured response of the current structure.

A numerical simulation study was employed to examine the capabilities of the algorithm in damage identification. It has been shown that the Non-Negative Least square is the best method to solve derived energy equations. Although the number of equations is much less than the number of unknowns, no methodology error was seen. Also it is demonstrated that, energy index method requires only a few number of mode shapes (in dynamic tests) or static loading cases (in static tests) to obtain reliable results. From a practical point of view, this is important because during a field test the number of measured mode shapes or static responses is limited.

From simulation study, it has been found that the strain energy distribution is very important. Those elements of the structure with high level of stored strain energies can be detected easier when subjected to damage. The mode shape or static loading that develops the highest level of strain energy in the damaged element is the best one for detecting the location and the magnitude of damage. But in reality the locations of damaged elements are unknown so one can not use this idea. Mode shapes and static loadings which develop almost a uniform distribution of strain energy in elements could be more suitable and desirable. Some higher mode shapes which might induce significant levels of strain energy in some elements are not always reliable.

Finally, it is shown that in order to obtain reliable results, the number of equations (mode shapes or static loadings) must be greater than the number of predicted damaged elements. It means that when the number of predicted damages is less than the number of equations, the results are reliable; otherwise more equations must be used to achieve a reasonable prediction.

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Dr. Banan earned the Ph. D. in civil engineering at University of Illinois at Urbana-Champaign in 1993. He has been on the faculty of Department of Civil and Environmental Engineering at Shiraz University since 1993. Dr. Banan was a visiting Associate Professor in the Civil Engineering Department at the American University of Sharjah, UAE for three years. He has been an active researcher in the field of non–destructive damage detection, seismic behavior of steel structures and steel connections, nonlinear structural analysis, and earthquake engineering. Dr. Banan has been the principal or co-principal investigator on 25 industry-supported researches. Dr. Banan is a member of 5 professional societies and has served as a reviewer for 7 technical Journals. He is a technical consultant, and analyst. Dr. Banan has expertise in designing complex steel structural systems including tall and long span residential and commercial buildings, buried life lines and bridges. His prior project involvement includes seismic evaluation, design, and retrofit of steel structures (buildings) and bridges in high seismic zones, retrofit of historical buildings and design of shell structures.

Dr. Sharifi received his Ph.D. in Civil Engineering from Shiraz University in 2011. He joined Department of Civil Engineering at Science and Research branch of Islamic Azad University as a faculty member in 2011. Dr. Sharifi is currently an assistant professor.

Figure Captions

Fig. 1. Cross-sectional properties and the layout of bridge truss.

Fig. 2. Damaged members and load cases of bridge truss.

Fig. 3. Elemental strain energies of undamaged bridge truss due to load case 1.

Fig. 4. Predicted damages for case1 using the all load cases simultaneously.

Fig. 5. Predicted damages for case2, when two load cases are used simultaneously.

Fig. 6. Predicted damages for case3 using the different combination of load cases.

Fig. 7. Predicted damage for element 16 with different damage severity from static response.

Fig. 8. Elemental strain energies of the undamaged structure due to the first mode shape.

Fig. 9. Elemental strain energies of the undamaged structure due to the first three mode shapes.

Fig. 10. Predicted damages for case1 using the first mode shape.

Fig. 11. Predicted damages for case2 using the first four mode shapes.

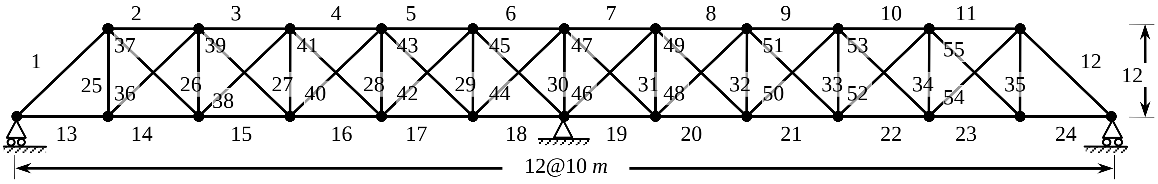
Fig. 12. Predicted damages for case3 using the first four mode shapes.

Fig. 13. Predicted damages for case4 using the first mode shape.

Table Captions

Table 1. Simulated damage for bridge truss structure.

Table 2. Predicted damages for case3 using different number of mode shapes.



Member	(cm^2) Area	Member	(cm^2) Area
Top	250	Vertical	200
Bottom	300	Diagonal	220

$\rho = 7.827\text{E}-08\ {}^{\text{KN}\cdot\text{S}^2}/_{cm^4}\ ;\ E = 19994.798\ {}^{KN}/_{cm^2}$

Fig. 1. Cross-sectional properties and the layout of bridge truss.

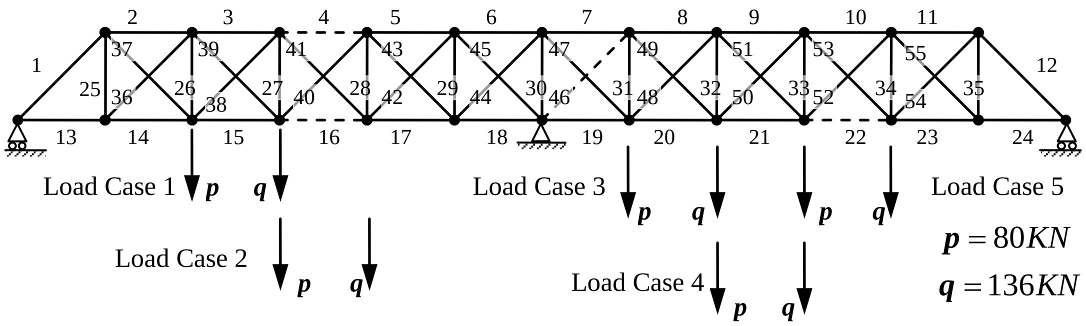


Fig. 2. Damaged members and load cases of bridge truss.

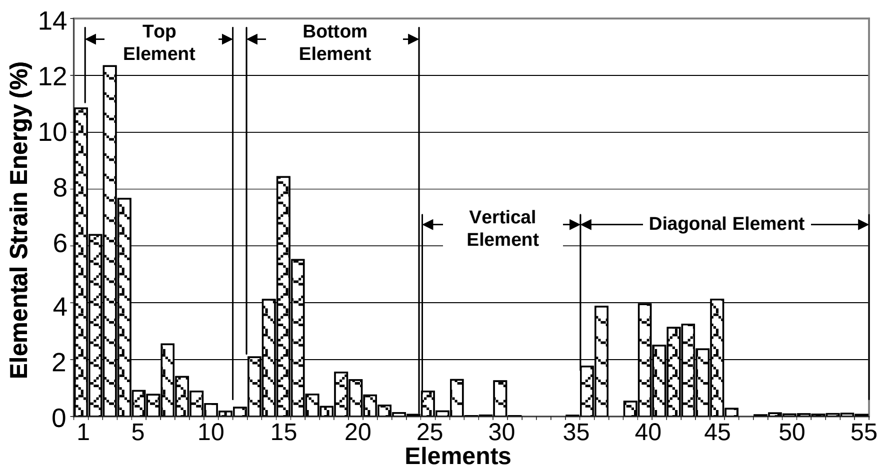


Fig. 3. Elemental strain energies of undamaged bridge truss due to load case 1.

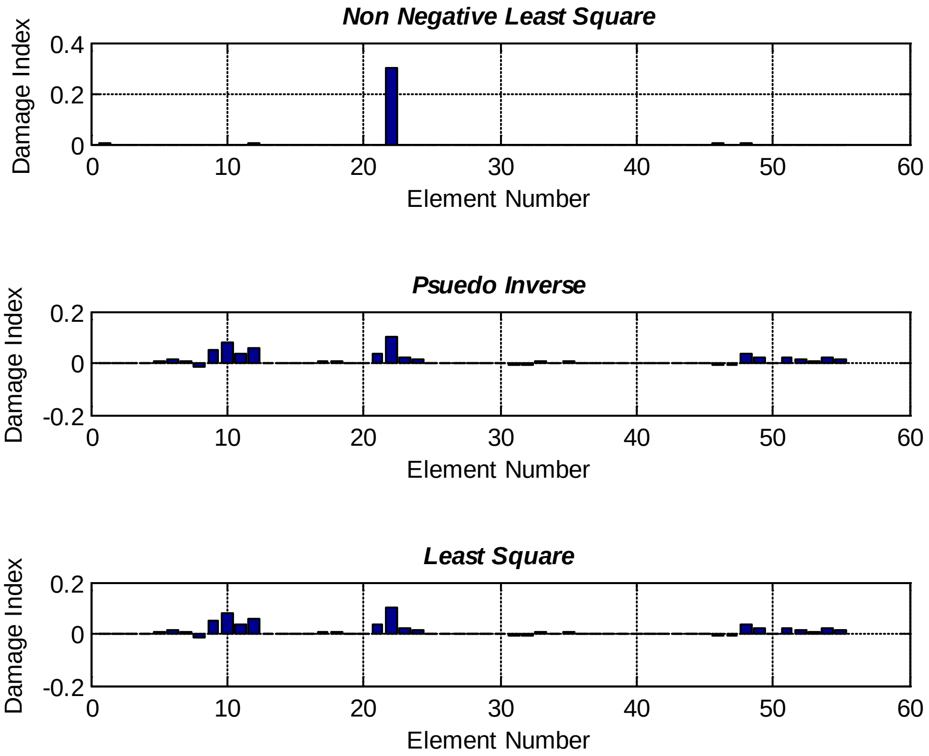


Fig. 4. Predicted damages for case1 using the all load cases simultaneously.

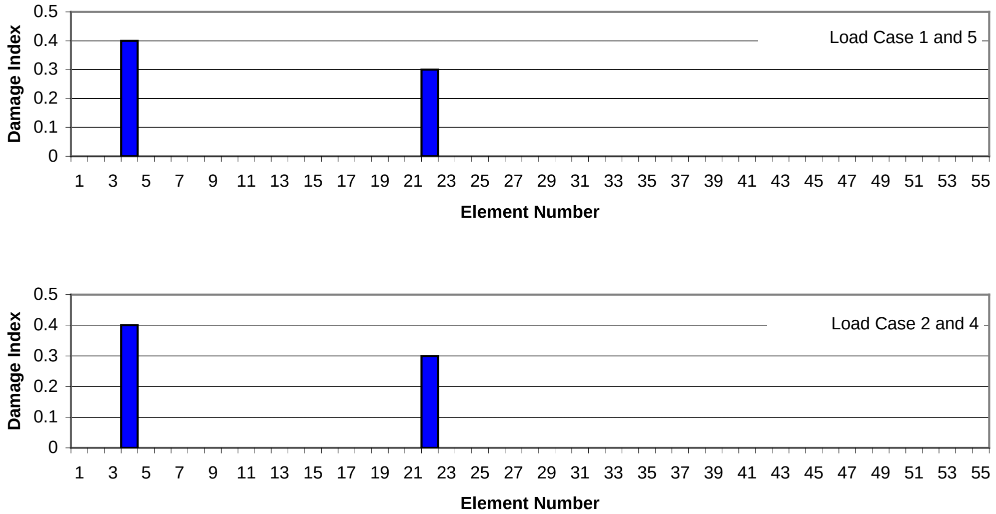


Fig. 5. Predicted damages for case2, when two load cases are used simultaneously.

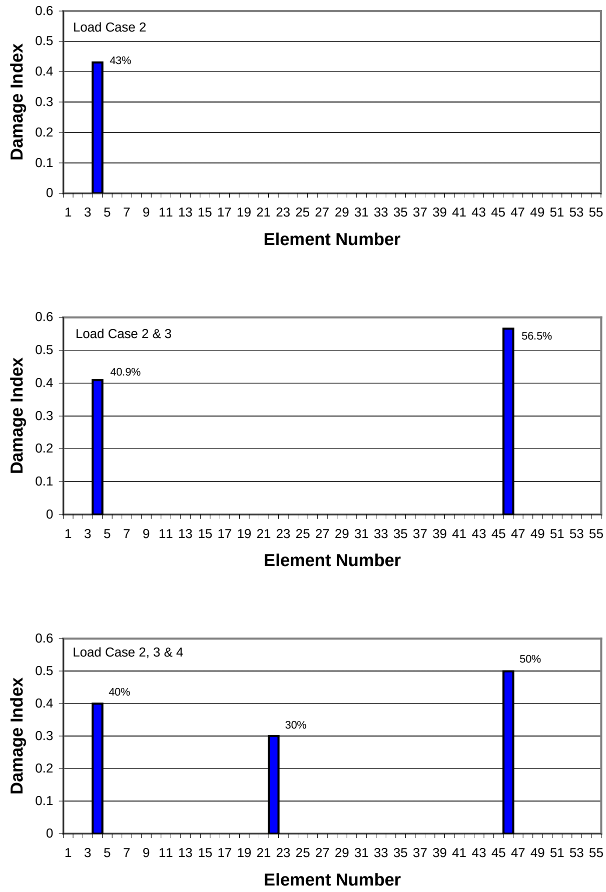


Fig. 6. Predicted damages for case3 using the different combination of load cases.

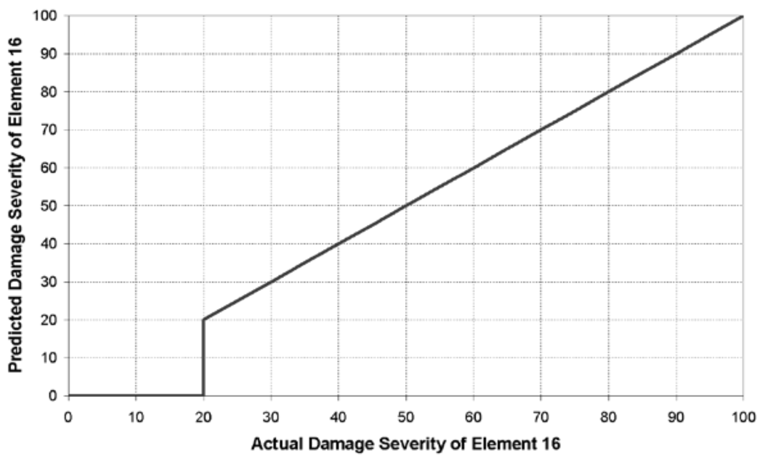


Fig. 7. Predicted damage for element 16 with different damage severity from static response.

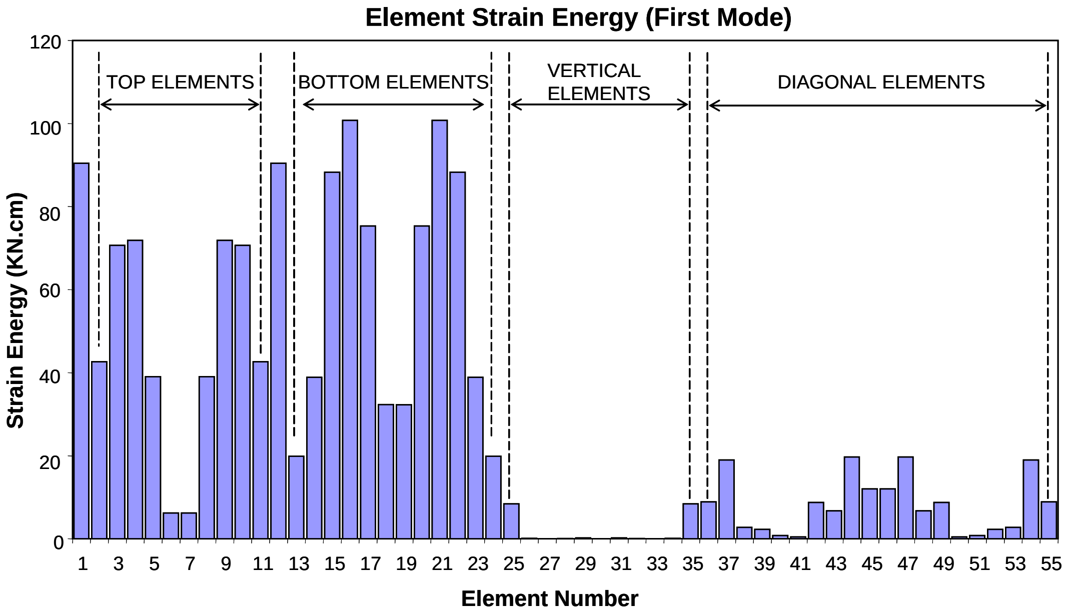


Fig. 8. Elemental strain energies of the undamaged structure due to the first mode shape.

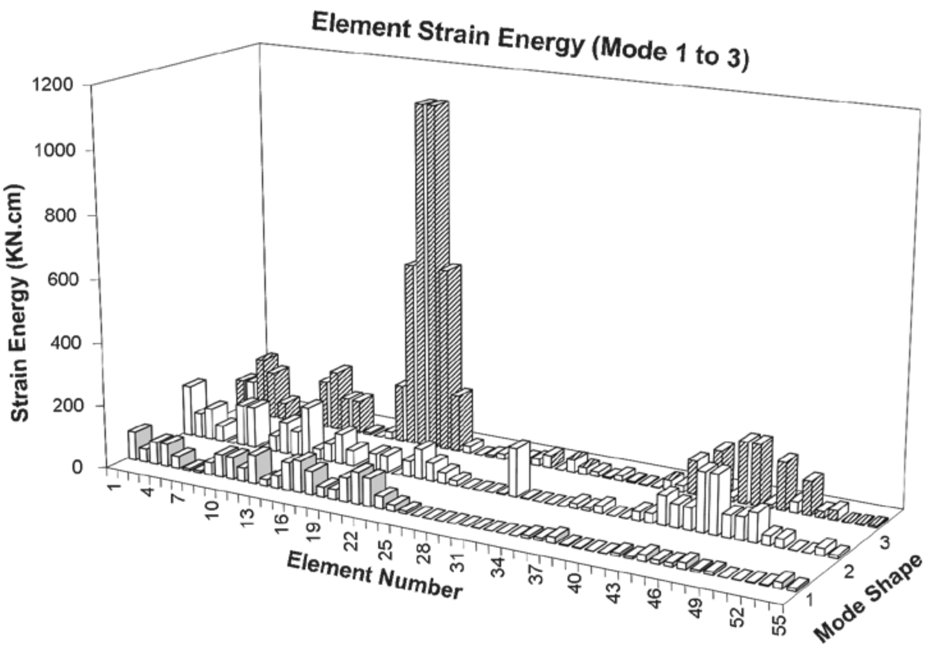


Fig. 9. Elemental strain energies of the undamaged structure due to the first three mode shapes.

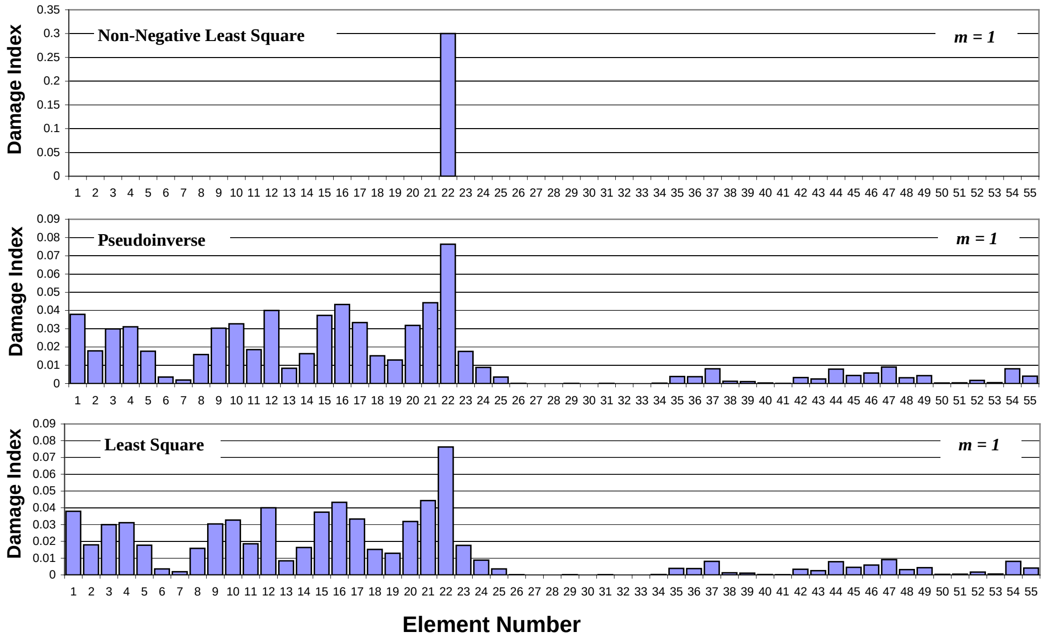


Fig. 10. Predicted damages for case1 using the first mode shape.

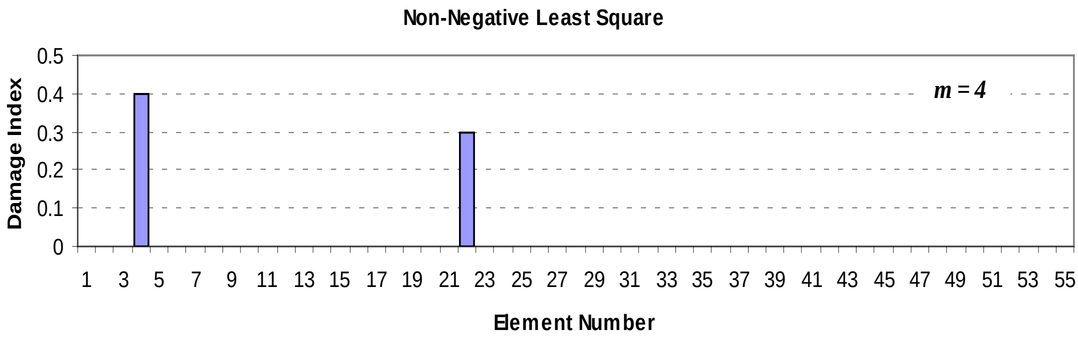


Fig. 11. Predicted damages for case2 using the first four mode shapes.

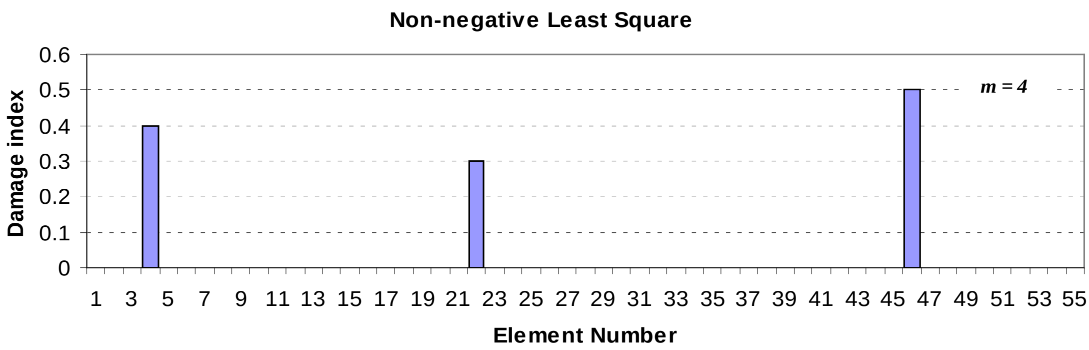


Fig. 12. Predicted damages for case3 using the first four mode shapes.

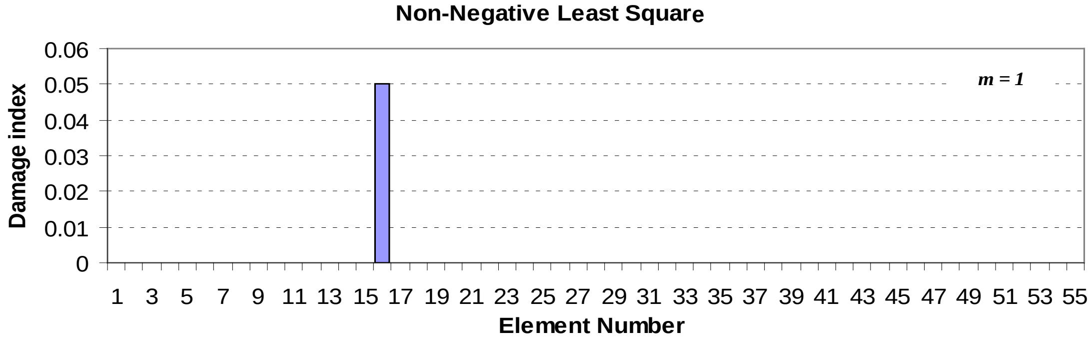


Fig. 13. Predicted damages for case4 using the first mode shape.

Table 1. Simulated damage for bridge truss structure.

Case	Damage scenarios	Damaged Members		Damage Severity (%)
		Elements	Area (cm^2)	
1	Single damaged member	E_{22}	210	30
2	Two damaged members different damage severity	E_{22} E_4	210 150	30 40
3	Three damaged members different damage severity	E_{22} E_4 E_{46}	210 150 110	30 40 50
4	Single light damaged member	E_{16}	285	5

Table 2. Predicted damages for case3 using different number of mode shapes.

Simulated damages	Number of Mode Shapes	Predicted Damages
$\delta_4 = 0.40$ $\delta_{46} = 0.50$ $\delta_{22} = 0.30$	$m = 1$	$\delta_{22} = 0.78$
	$m = 2$	$\delta_{22} = 0.70$, $\delta_{46} = 0.49$
	$m = 3$	$\delta_{46} = 0.48$, $\delta_{22} = 0.68$, $\delta_{18} = 0.11$
	$m = 4$	$\delta_{46} = 0.50$, $\delta_{22} = 0.30$, $\delta_4 = 0.40$
	$m = 5$	$\delta_{18} = 0.05$, $\delta_6 = 0.08$, $\delta_4 = 0.27$ $\delta_{47} = 0.13$, $\delta_{46} = 0.43$, $\delta_{22} = 0.40$
	$m = 6$	$\delta_{46} = 0.50$, $\delta_{22} = 0.30$, $\delta_4 = 0.40$
	$m = 7$	$\delta_{46} = 0.50$, $\delta_{22} = 0.30$, $\delta_4 = 0.40$