Scientia Iranica A (2014) 21(5), 1471–1479

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions A: Civil Engineering
www.scientiairanica.com

Seat width requirement for skewed bridges under seismic loads

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Received 16 May 2012; received in revised form 31 October 2013; accepted 10 December 2013

KEYWORDS
Bridges;
Skew;
Seismic response;
Seat width;
Free vibration;
Bearing.

Abstract. In this paper, the dynamic characteristics of skewed bridges are explored analytically. Closed form solutions for translational and torsional periods of free vibration and mode shapes are given for slab-girder skewed bridges. Moreover, the seismic displacement of the deck of skewed bridges is calculated using the response spectrum method and its skew term is compared with the requirement of AASHTO. The effects of seismic force resisting elements, such as elastomeric bearings and end diaphragms are included. It is shown that the skew term in AASHTO's equation can underestimate the seat width requirement for some bridges. A new skew term for the bridge seat width requirement is suggested.

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1. Introduction

After the 1971 San Fernando earthquake in which a lot of bridges experienced unseating of the superstructure, many researchers investigated this phenomenon. It was observed that skewed bridges were more prone to unseating at least in the corners of the deck. Dicleli and Bruneau [1] investigated the seismic behavior of straight slab-girder bridges and presented the possible sliding displacements of bridges under several earthquake records. Saiidi et al. [2] have also investigated the seismic displacement of bridges and have presented some design requirements for seismic restrainers. In their investigation, the skew effect has been introduced as a simple expression, $1/\cos\alpha$, where α is the skew angle of the bridge. The seismic behavior of bridges considering the effects of skew angle, support stiffness, diaphragms and cross-frames has been investigated before by Zahrai and Bruneau [3] and Maleki [4-6]. However, an analytical investigation of the seismic displacement has not been investigated, using all these factors as it relates to the bridge seat requirement.

Some bridge design codes mandate a minimum seat width in seismic zones. AASHTO [7] has included a skew factor in the seat width requirement formula which is given as $(1 + 0.000125S^2)$. The seat width increases exponentially with skew angle S.

In this paper, the exact translational and torsional periods of free vibration for a skewed bridge are given. The maximum displacements of skewed slab-girder bridges are found using a response spectrum approach. The seat width requirement is compared with the AASHTO formula and the necessary changes are recommended.

2. Mathematical modeling

A typical slab-girder bridge is shown in Figures 1 and 2. The period of vibration and inertial forces induced in a bridge structure depend on the stiffness of supporting members in the two orthogonal directions. The end-diaphragms affect the lateral stiffness of the bridge in the direction perpendicular to the span,

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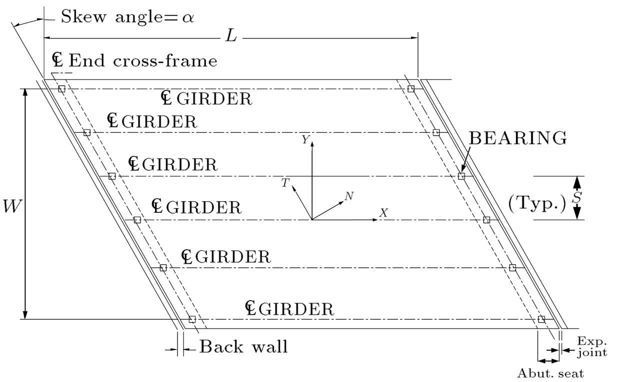

Figure 1. Plan view of a typical skewed slab-girder bridge.

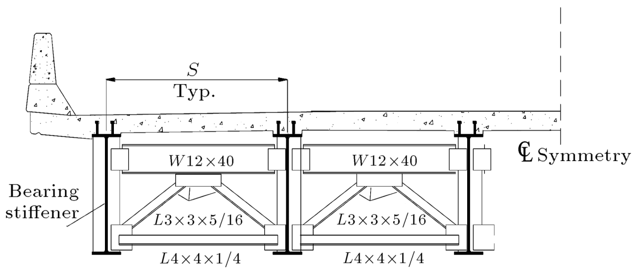

Figure 2. Cross-section of a typical skewed slab-girder bridge.

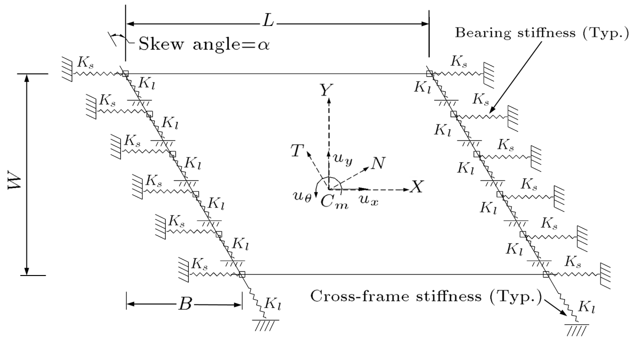

Figure 3. Two-dimensional model of the skewed bridge.

and elastomeric bearings affect the stiffness of the bridge in the longitudinal direction. It is assumed that the concrete deck is rigid in its plane [6]. Note that intermediate diaphragms do not affect the seismic behavior of bridges significantly [3], and are ignored in this study. Further, the substructure stiffness is excluded in this study, i.e. the bearings are assumed to be attached to a rigid support. However, the stiffness of substructure can easily be added by observing that they are, in effect, springs attached in series with the bearing springs. Hence, if desired, an equivalent spring can be substituted.

Based on the above assumptions, a two-dimensional model of the bridge is made, shown in Figure 3. Three degrees of freedom at the center of mass are also shown in the figure. The center of mass, C_m, is chosen as the origin and two sets of axes are defined at this point. The X-Y axes are in the longitudinal and transverse directions, and N-T axes are normal and parallel to the abutments. In general, due to asymmetry of support springs, the center of mass, C_m, and the center of stiffness, C_s, do not coincide. The spring k_s represents the stiffness of each elastomeric bearing in the X direction, and the spring k_l represents the lateral stiffness of each end-diaphragm panel in the T direction. In general, k_s and k_l represent the stiffness of equilibrium springs in X and T directions, respectively, and can model variety of support conditions. For example, in the case of a pin-supported bridge, respective longitudinal stiffness (k_s) should be chosen as infinite. Also, in the case of elastomeric bearings, without side retainers, k_l represents equivalent stiffness of end-diaphragm and elastomer attached in series; however in practice, side retainers exist and are relatively rigid. Therefore, in this case, k_l only represents the lateral stiffness of the end-diaphragm panel. Let M and J represent the total mass and the mass moment of inertia of the bridge superstructure. These are assumed to be concentrated at the center of mass. The equation of free vibration for the model bridge, shown in Figure 3, can be written as [8]:

$$\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J \end{bmatrix} \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} K_{xx} & K_{xy} & K_{x\theta} \\ K_{yx} & K_{yy} & K_{y\theta} \\ K_{\theta x} & K_{\theta y} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{1}$$

where J for a parallelogram is given as:

$$J = (M/12)(L^2 + W^2 + B^2) = (M/12)(L^2 + W^2 + W^2 \tan^2\alpha) = (M/12)(L^2 + W^2/\cos^2\alpha). \tag{2}$$

As shown in Figure 3, L and W are the length and width of the bridge, and B and α are the skew length and skew angle of the bridge. The stiffness sums for the supporting elements are:

$$K_s = \sum K_{si}, \tag{3}$$

$$K_l = \sum k_{li}. \tag{4}$$

Transforming the local coordinates along N and T axes of the K_l springs to the global X and Y coordinates and assembling stiffness terms:

$$K_{xx} = K_s + K_l \sin^2\alpha, \tag{5}$$

$$K_{xy} = -K_l \sin\alpha\cos\alpha = K_{yx}, \tag{6}$$

$$K_{yy} = K_l \cos^2\alpha, \tag{7}$$

$$K_{\theta x} = -\sum k_{si}.y_i - \sin\alpha\cos\alpha \sum k_{li}.x_i - \sin^2\alpha \sum k_{li}.y_i = K_{x\theta}, \tag{8}$$

$$K_{\theta y} = \cos^2\alpha \sum k_{li}.x_i + \sin\alpha\cos\alpha \sum k_{li}.y_i = K_{y\theta}, \tag{9}$$

$$K_{\theta\theta} = \sum k_{si}.y_i^2 + \sin 2\alpha \sum k_{li}.y_i.x_i + \cos^2\alpha \sum k_{li}.x_i^2 + \sin^2\alpha \sum k_{li}.y_i^2, \tag{10}$$

gives rise to the eigenvalue problem:

$$\begin{bmatrix} K_{xx} - \omega^2 M & K_{xy} \\ K_{yx} & K_{yy} - \omega^2 M \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{11}$$

$$\omega_t = \sqrt{\frac{K_{\theta\theta}}{J}}, \tag{12}$$

where ω is the translational and ω_t is the torsional natural frequency of free vibration. In the above equations, x_i and y_i are the coordinates of the springs attachment points.

In general, the eigen-solution is coupled in terms of translational and torsional modes. However, in the case of having symmetry, as shown in Figure 3, then $K_{\theta x} = K_{\theta y} = 0$ and translational modes are uncoupled from torsional mode.

The translational periods can be written as [5]:

$$T = \sqrt{\frac{8\pi^2 M}{\left[(K_s+K_l) \pm \sqrt{(K_s+K_l)^2 - 4K_sK_l\cos^2\alpha}\right]}}. \tag{13}$$

By introducing the non-dimensional parameter $\beta = K_l/K_s$, Eq. (13) can be rewritten as:

$$T = \sqrt{\frac{8\pi^2 M}{K_s\left[(1+\beta) \pm \sqrt{(1+\beta)^2 - 4\beta\cos^2\alpha}\right]}}, \tag{14}$$

in Eqs. (13) and (14), the fundamental mode of vibration occurs when considering the negative sign under the square-root. The closed form solution for the torsional period of a skewed bridge with n spring attachments is therefore given by Eq. (15) shown in Box I.

Note that the number of end-diaphragm panels is one less than the number of girders and bearings at each end of the bridge. However, in Eq. (15) (also see Figure 3), it is assumed that the total lateral stiffness of the end-diaphragm panels (K_l) is distributed equally among the nodes at bearing locations. This assumption is not necessary in Eq. (14) because total stiffness is used in the latter.

The above equations and the following procedure in this paper are valid for single span and Multi Span Simply Supported (MSSS) bridges.

3. Numerical example and verification

Consider the bridge in Figure 1 with 20 m span and 10 m width and a skew of 30 degrees. Assuming the deck to be 0.19 m thick, then the total mass of the bridge superstructure is estimated as 130,500 kg. The lateral stiffness of each cross-frame panel (Figure 2) with 2 m girder spacing, is equal to 160×10^6 N/m. Typical shear stiffness for elastomeric bearing for this bridge is 1×10^6 N/m. Hence, for 12 bearings and 10 cross-frame panels, total stiffness of $K_s = 12 \times 10^6$ N/m and $K_l = 1600 \times 10^6$ N/m is assumed [5-6]. Using these values, one obtains $\beta = 133.3$, and using Eq. (14) results in $T_1 = 0.757$ s and $T_2 = 0.057$ s, which are the two translational periods of vibration. To obtain the torsional period, Eq. (15) is used which yields $T_t = 0.044$ s. In this case, the torsional mode is the third mode of vibration of the skewed bridge.

The eigenvectors for the free vibration are $\boldsymbol{\varphi}_1 =< 1,\ 0.582,\ 0 >^T$, $\boldsymbol{\varphi}_2 =< -0.582,\ 1,\ 0 >^T$, $\boldsymbol{\varphi}_3 =< 0,\ 0,\ 1 >^T$. Denoting the angle of motion in the first mode with the X axis as θ, then we have $\theta = \tan^{-1}(0.582/1) = 30.2°$. This is very close to the skew angle of 30°. In general, if the cross-frames are much stiffer than the elastomers ($\beta >> 1$), the first mode shape is directed in the perpendicular direction to the abutments (N direction). On the contrary, if the elastomers are much stiffer than the cross-frames ($\beta << 1$), the first mode shape is in the Y direction.

To compare the values derived above with an actual finite element model, a two-dimensional model of the same bridge is made with SAP2000 finite elements program by Computers and Structures Inc. [9], as shown in Figure 4. The total mass of 130,500 kg is assumed to be distributed uniformly over the concrete deck. The deck is modeled with shell elements of

$$T_t = \sqrt{\frac{n\pi^2 M(L^2 + W^2/\cos^2\alpha)}{3K_s(\sum y_i^2 + \beta\sin 2\alpha \sum x_i y_i + \beta\cos^2\alpha \sum x_i^2 + \beta\sin^2\alpha \sum y_i^2)}}. \tag{15}$$

Box I

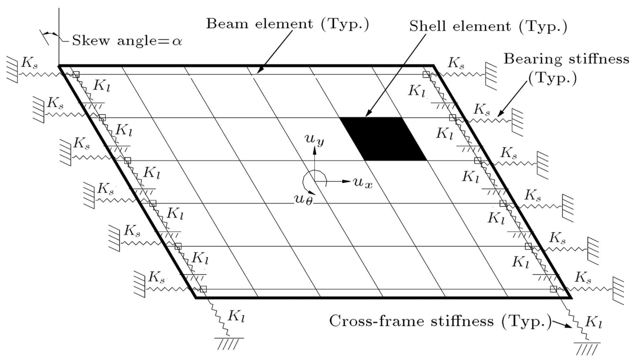

Figure 4. Bridge finite elements analysis model.

0.19 m thickness. The girders are $W36 \times 230$ and are rigidly attached to the deck slab. All joints are constrained in the deck slab plane to perform as a rigid diaphragm. The ends of the girders are attached to longitudinal springs that represent elastomeric bearing's shear stiffness. There are also springs in the transverse (T) direction that simulate the stiffness of cross-frames. The results from this model is as follows: $T_1 = 0.757$ s, $T_2 = 0.057$ s and $T_3 = 0.044$ s. It is seen that the values are in exact agreement with the analytical results derived earlier.

4. Variation of periods and mode shapes for skewed bridges

In this section, the exact variation of three periods of free vibration for a general case of a skewed bridge is given. From Eqs. (14) and (15), by introducing the dimensionless parameter $\beta = K_l/K_s$, one can obtain the period as a dimensionless quantity $T.(K_s/M)^{0.5}$. In Figure 5(a) and (b), the translational dimensionless periods are shown versus β for different skew angles. It can be seen that the first translational period increases while the second period decreases for increasing skew angles. Also, for $\beta > 1$, the first translational period is independent of K_l, while for $\beta < 1$ the same is true for the second translational period. For a complete discussion of the translational periods, the reader is referred to the work by Maleki [5].

Figure 5(c) shows the variation of the torsional period for a skewed bridge with 20 m span and 10 m width. These geometric properties must be known for the determination of torsional periods as Eq. (15) demonstrates. It is seen that the torsional period increases with increasing skew angles.

In Figure 5(d), the direction of motion in the first mode of vibration for skewed bridges is shown as derived from the eigenvector solution of Eq. (11). It is seen that for a straight bridge ($\alpha = 0$), the direction of motion in the first mode is along Y axis for $\beta < 1$, and along X axis for $\beta > 1$, while it is indeterminate for $\beta = 1$. For skewed bridges, when $\beta << 1$, the direction of motion in the first mode tends toward the Y axis, while for $\beta >> 1$, it tends toward the N axis. It is obvious that the second translational mode is directed in the perpendicular direction from the first mode.

5. Minimum seat width of skewed bridges

5.1. *Provisions of design codes*

This research is only concerned with the seat width requirement for skewed bridges due to seismic displacements of the deck. Other factors such as creep, shrinkage, temperature, prestressing and girder deflection can also cause the deck and girders to displace.

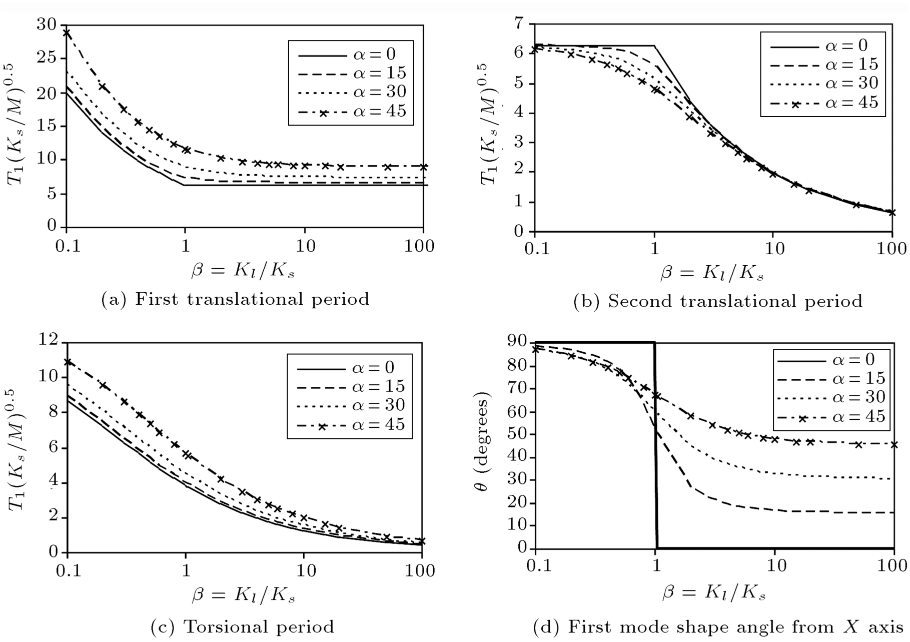

Figure 5. Dimensionless diagrams for periods and first mode shape angle.

However, these displacements are much less than the seismic displacements imposed on a structure in a severe earthquake. Some design codes, such as Eurocode 8 [10] prescribe a minimum seat width based on calculated displacement of superstructure under seismic and other loads. AASHTO [7], in addition to calculated displacement, prescribes a minimum seat width in the perpendicular direction to the abutments for all bridges in seismic zones based on an empirical equation. According to AASHTO (Article 4.7.4.4), the seat width of a bridge should satisfy:

$$N_A \geq (200 + 0.0017L + 0.0067H)(1 + 0.000125S^2), \tag{16}$$

in which N_A is support width normal to the centerline of bearing (mm), L is the length of the bridge deck to the adjacent expansion joint, or the end of the bridge (mm), H is the average height of the columns supporting the bridge deck to the next expansion joint (mm), and S is the skew angle of the abutments in degrees. For seismic zones 3 and 4, 150% of the above limit should be considered.

The second parenthesis in the above equation considers the skew effect on the seat width requirement. In other words, a skewed bridge should have a seat width that is wider than a straight bridge by a factor of $(1 + 0.000125S^2)$. It is the objective of this paper to examine the skew term analytically by using the theoretical derivation of seismic response of skewed bridges and to propose a more accurate replacement.

5.2. Modal analysis of skewed bridges

For a skewed bridge, as shown in Figure 3, with a skew angle of α, assuming the stiffness K_s and K_l to vary (variable β), the first modal displacement makes an angle θ with the positive X axis direction. According to Figure 5(d), with the axes defined as such, angle θ varies between zero and $\pi/2$. As mentioned before, for the symmetrical case, as shown in Figure 3, the torsional mode of vibration decouples from translational modes. Since the second translational mode of free vibration is always perpendicular to the first mode, one can write the equations for mode shape vectors as:

$$\boldsymbol{\varphi_1} = \begin{Bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{Bmatrix}, \quad \boldsymbol{\varphi_2} = \begin{Bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{Bmatrix}, \quad \boldsymbol{\varphi_3} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}. \tag{17}$$

It is our objective to maximize the displacement along the N axis. Hence, the input motion is assumed to be applied in the N direction. For more information on the input motion direction in the seismic analysis of skewed bridges the reader is referred to the work by Maleki and Bisadi [11]. Therefore, the influence vector ($\mathbf{L}$) for the ground motion is:

$$\mathbf{L} = \begin{Bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{Bmatrix}.$$

The differential equation of a structure subjected to ground motion acceleration is given as:

$$\mathbf{m\ddot{u}} + \mathbf{c\dot{u}} + \mathbf{ku} = -\mathbf{mL}\ddot{u}_g(t), \tag{18}$$

in the above equation, $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$ are mass, damping and stiffness matrices of the structure, respectively, and the vector $\mathbf{u} =< u_x, u_y, u_0 >^T$ is the displacement vector for the assumed degrees of freedom of the structure. Response of the structure in the first mode can be obtained using the response spectrum analysis, giving rise to the equations:

$$M_1 = \boldsymbol{\varphi_1}^T \mathbf{m} \boldsymbol{\varphi_1} = (\cos\theta, \sin\theta, 0) \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J \end{bmatrix} \begin{Bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{Bmatrix} = M, \tag{19}$$

$$L_1 = \boldsymbol{\varphi_1}^T \mathbf{mL} = (\cos\theta, \sin\theta, 0) \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J \end{bmatrix} \begin{Bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{Bmatrix} = M\cos(\alpha - \theta), \tag{20}$$

$$H_1 = \frac{L_1}{M_1} = \cos(\alpha - \theta), \tag{21}$$

$$\mathbf{u_1} = \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix}_1 = Sd_1 H_1 \boldsymbol{\varphi_1} = Sd_1 \cos(\alpha - \theta) \begin{Bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{Bmatrix} = \frac{1}{2} Sd_1 \begin{Bmatrix} \cos\alpha + \cos(2\theta - \alpha) \\ \sin\alpha + \sin(2\theta - \alpha) \\ 0 \end{Bmatrix}, \tag{22}$$

in which M_1 is the generalized mass in the first mode, H_1 is the modal participation factor in the first mode, $\mathbf{u_1}$ is the displacement vector and Sd_1 is the spectral displacement in the first mode. The latter is a function of the period and damping of the structure. Similarly, the second mode response can be found as:

$$M_2 = \boldsymbol{\varphi_2}^T \mathbf{m} \boldsymbol{\varphi_2} = (-\sin\theta, \cos\theta, 0) \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J \end{bmatrix} \begin{Bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{Bmatrix} = M, \tag{23}$$

$$L_2 = \boldsymbol{\varphi_2}^T \mathbf{mL} = (-\sin\theta, \cos\theta, 0) \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J \end{bmatrix} \begin{Bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{Bmatrix} = M\sin(\alpha-\theta), \quad (24)$$

$$H_2 = \frac{L_2}{M_2} = \sin(\alpha-\theta), \quad (25)$$

$$\mathbf{u_2} = \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix}_2 = Sd_2 H_2 \boldsymbol{\varphi_2} = Sd_2\cos(\alpha-\theta) \begin{Bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{Bmatrix} = \frac{1}{2} Sd_2 \begin{Bmatrix} \cos\alpha - \cos(2\theta-\alpha) \\ \sin\alpha - \sin(2\theta-\alpha) \\ 0 \end{Bmatrix}. \quad (26)$$

The third mode is the torsional mode of vibration and is decoupled from the translational modes due to symmetric conditions, hence:

$$L_3 = \boldsymbol{\varphi_3}^T \mathbf{mL} = (0,0,1) \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J \end{bmatrix} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{Bmatrix} = 0$$

$$\Rightarrow \quad H_3 = 0 \quad \Rightarrow \quad \mathbf{u_3} = 0.$$

Indeed, in the case of having symmetry, as shown in Figure 3, the torsional mode is unnecessary for future calculations. The projection of deck displacement on the N axis in the first and second modes ($\bar{u}_i$), knowing that $u_\theta = 0$, can be found as:

$$\bar{u}_i = u_{xi}.\cos\alpha + u_{yi}.\sin\alpha, \quad \text{for} \quad i = 1, 2 \quad \text{(mode number)}. \quad (27)$$

Substituting Eqs. (22) and (26) in Eq. (27) leads to:

$$\bar{u}_1 = Sd_1.\cos^2(\alpha-\theta), \quad (28)$$

$$\bar{u}_2 = Sd_2.\sin^2(\alpha-\theta). \quad (29)$$

Examining Eq. (28) reveals that the displacement of the deck along N axis is maximum when $\alpha = \theta$. In this case $\bar{u}_1 = Sd_1$ and the second mode effects vanish. Therefore one can obtain $\bar{u} = \bar{u}_1 = Sd_1$, where $\bar{u}$ is the deck displacement along N axis based on modal combinations. Referring to Figure 5(d), it is seen that, the case of ($\alpha = \theta$) occurs for different α's when $\beta >> 1$. As mentioned before, when the end diaphragm stiffness is much higher than the longitudinal stiffness of elastomeric bearings ($\beta >> 1$), then the first mode displacement is along the N axis ($\alpha \approx \theta$).

It can be shown that for all β's, one can assume $\bar{u} \leq Sd_1$. Using SRSS method to combine the modal responses, the deck displacement along N-axis is obtained as:

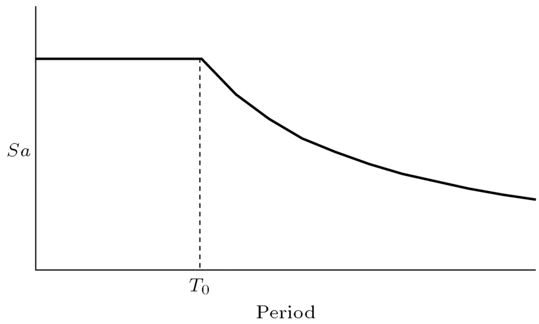

Figure 6. A typical design spectrum (Sa).

$$\bar{u} = \sqrt{\bar{u}_1^2 + \bar{u}_2^2} = \sqrt{Sd_1^2.\cos^4(\alpha-\theta) + Sd_2^2.\sin^4(\alpha-\theta)}. \quad (30)$$

Since the first mode period is higher than the second mode, assuming a smooth displacement response spectrum yields $Sd_1 \geq Sd_2$. This is also true based on a typical acceleration design spectrum. A typical design spectrum is shown in Figure 6. The acceleration response spectrum consists of a straight line segment up to T_0 and a curved part usually in the form of an inverse exponential function of period (T) with an exponent not higher than 1. Knowing that spectral displacement and acceleration are related as:

$$Sd = \frac{T^2}{4\pi^2} Sa, \quad (31)$$

then $Sd_1 \geq Sd_2$. So we can write down:

$$\bar{u} \leq Sd_1\sqrt{\cos^4(\alpha-\theta) + \sin^4(\alpha-\theta)} = Sd_1\sqrt{1 - 2\sin^2(\alpha-\theta)\cos^2(\alpha-\theta)} \leq Sd_1. \quad (32)$$

5.3. Seat width requirement for skewed bridges

Based on the response spectrum analysis of a skewed bridge and the derived equations, the ratio of seismic displacement along N axis of a skewed bridge to the displacement along X axis of a similar straight bridge for the case of maximum displacement ($\beta >> 1$) is:

$$\bar{u}_S/\bar{u}_R = \frac{Sd_{1S}}{Sd_{1R}}, \quad (33)$$

where subscripts S and R denote skewed and right bridges. The above equation depends on the period of the first mode of vibration of the skewed bridge (T_{1S}) to the similar right bridge (T_{1R}) and it is comparable to ($1 + 0.000125S^2$) as prescribed in the design codes. Although for practical cases we usually have $\beta >> 1$, for other values of β, the right-hand of Eq. (33) can be multiplied by a factor equal to $\sqrt{\cos^4(\alpha-\theta)+Sd_2^2/Sd_1^2\times\sin^4(\alpha-\theta)}$, which is smaller than 1.

From Eq. (14) and Figure 5(a), one can assume that $T_{1S} > T_{1R}$. Considering the design spectrum of AASHTO and Eq. (31), Eq. (33) can be rewritten as:

$$\bar{u}_S/\bar{u}_R=\begin{cases}(\frac{T_{1S}}{T_{1R}})^2 & \text{if} \quad T_{1R}<T_{1S}\leq T_0\\ T_0^{2/3}.\frac{T_{1S}^{4/3}}{T_{1R}^2} & \text{if} \quad T_{1R}<T_0<T_{1S}\\ (\frac{T_{1S}}{T_{1R}})^{4/3} & \text{if} \quad T_0<T_{1R}<T_{1S}\end{cases} \tag{34}$$

where T_0 depends on the site's soil condition. For maximum displacement along N axis (when $\beta >> 1$), one can obtain the ratio of T_{1S}/T_{1R} using Eq. (14) as:

$$\text{for} \quad \beta\geq 1: T_{1R}=2\pi\sqrt{\frac{M}{K_s}}, \tag{35}$$

$$\text{for} \quad \beta\to\infty: \frac{T_{1S}}{T_{1R}} = \sqrt{\frac{2}{\left[(1+\beta)-\sqrt{(1+\beta)^2-4\beta.\cos^2\alpha}\right]}} \approx \sqrt{\frac{2}{[(1+\beta)-(\beta+1-2\cos^2\alpha)]}}=\frac{1}{\cos\alpha}. \tag{36}$$

From Eqs. (34) and (36), one concludes that $(\bar{u}_S/\bar{u}_R)$ varies between $(1/\cos\alpha)^{4/3}$ to $(1/\cos\alpha)^2$ depending on the site's soil condition.

Table 1 summarizes the increase in seat width needed for skewed bridges based on AASHTO's equation and Eq. (34) for soil condition II ($T_0 = 0.44$ sec.) and soil condition III ($T_0 = 0.85$ sec.). Comparison is done for different skew angles (α) using the bridge properties of the previous numerical example in this paper. The last columns show the lower bound $(1/\cos\alpha)^{4/3}$ and the upper bound $(1/\cos\alpha)^2$ of the skew term of seat width demand for the same bridge. It is seen that AASHTO's equation underestimates the seat width requirement in all cases. However, $(1/\cos\alpha)^2$ provides a simple and conservative estimate for different sites and skew angles. It can also be concluded that for soft soil condition, the suggested formula is more accurate than AASHTO's equation.

5.4. Effect of bidirectional earthquakes

The nature of the three-dimensional seismic wave propagation is very complex. In the absence of near source effects, it is valid to assume that ground motions have one principal (strong) direction, and motions take place in the perpendicular direction with a smaller magnitude [12]. These motions are statistically independent of each other and act in two perpendicular directions simultaneously. To satisfy these criteria, design codes usually prescribe earthquake ground motions to be applied in two orthogonal directions independently and then the responses are combined by one of the accepted combinations rules such as the 100/30 percentage rule.

For seismic analysis of a skewed bridge under the effects of bidirectional earthquakes, we can follow the procedure of Sections 5.2 and 5.3 for two independent and perpendicular ground motions. Assume that the influence vector for the second component of ground motions is given by:

$$\mathbf{L}'=\begin{Bmatrix}\cos\alpha'\\ \sin\alpha'\\ 0\end{Bmatrix}.$$

Here, $\mathbf{L}'$ is perpendicular to $\mathbf{L}$, so $\alpha'=\alpha+90°$. From now, we use the prime sign ($'$) to express the parameters corresponding to the second component of the earthquake ground motion. We can write equations similar to Eqs. (18)-(29) for the second component of the earthquake. Therefore, corresponding to Eqs. (28), (29), we may obtain:

$$\bar{u}'_1=Sd_1.\cos^2(\alpha'-\theta)=Sd_1.\sin^2(\alpha-\theta), \tag{37}$$

$$\bar{u}'_2=Sd_2.\sin^2(\alpha'-\theta)=Sd_2.\cos^2(\alpha-\theta), \tag{38}$$

with $\bar{u}'_1$ and $\bar{u}'_2$ being the projections of deck displacements on the N axis under second component of ground motions in the first and second modes, respectively. The spectral displacements in the first and second

Table 1. Skew term of seat width needed for seismic requirement of skewed bridges.

α (degree)	T_1 (sec.)	$\bar{u}_S/\bar{u}_R$: $1+0.000125S^2$	Eq. (34), $T_0=0.44$ s	Eq. (34), $T_0=0.85$ s	$(1/\cos\alpha)^{4/3}$	$(1/\cos\alpha)^2$
0	0.655	1	1	1	1	1
15	0.679	1.028	1.049	1.075	1.047	1.072
30	0.757	1.113	1.213	1.336	1.211	1.333
45	0.928	1.253	1.591	1.893	1.587	2

modes under the second component of an earthquake are the same as for the first component (Sd_1 and Sd_2). This is because the same design spectra are usually used for two perpendicular directions. Maximization of the deck displacement for combined responses of two components of an earthquake seems to be very complex and it depends on many different parameters. However, the condition of the maximum displacement of the deck along N axis under the effects of the first (principal) component of an earthquake is $\alpha = \theta$ and hence $\beta >> 1$, as it was noted in Section 5.2. In this case $\bar{u}'_2 = Sd_2$ and the first mode effects vanish. Therefore, one can obtain, $\bar{u}' = \bar{u}'_2 = Sd_2$, where $\bar{u}'$ is the deck displacement along N axis based on modal combinations under the effects of the second component of an earthquake. Most of the design codes (e.g., AASHTO) recommend 100/30 percentage rule for the combination of responses of two orthogonal components of an earthquake. Using this rule, $\bar{u}^t$, the deck displacement, along N axis, based on the bidirectional effects of an earthquake can be obtained as:

$$\bar{u}^t = \bar{u} + 0.3\bar{u}' = Sd_1 + 0.3Sd_2. \tag{39}$$

Therefore, instead of Eq. (33) we get the expression:

$$\bar{u}^t_S/\bar{u}^t_R = \frac{Sd_{1S} + 0.3Sd_{2S}}{Sd_{1R} + 0.3Sd_{2R}}, \tag{40}$$

where subscripts S and R denote skewed and right bridges. We showed in Section 5.3 that Eq. (33) is at most equal to $(1/\cos\alpha)^2$. With similar reasoning, we can show that Eq. (40) is at most equal to $(1/\cos\alpha)^2$ as:

$$\text{for} \quad \beta \geq 1 : T_{2R} = 2\pi\sqrt{\frac{M}{K_l}} = 2\pi\sqrt{\frac{M}{\beta K_s}}, \tag{41}$$

$$\text{for} \quad \beta \to \infty : \frac{T_{2S}}{T_{2R}} = \sqrt{\frac{2\beta}{\left[(1+\beta) + \sqrt{(1+\beta)^2 - 4\beta.\cos^2\alpha}\right]}} \approx \sqrt{\frac{2\beta}{2\beta}} = 1 \quad \Rightarrow \quad \frac{Sd_{2S}}{Sd_{2R}} = 1. \tag{42}$$

Since Sd_{1S}/Sd_{1R} is greater than one for $\beta >> 1$ (it is at most equal to $1/\cos^2\alpha$), and considering Eq. (42), we can rewrite Eq. (40) as:

$$\bar{u}^t_S/\bar{u}^t_R = \frac{Sd_{1S} + 0.3Sd_{2S}}{Sd_{1R} + 0.3Sd_{2R}} \leq \frac{Sd_{1S}}{Sd_{2R}} \leq (\frac{1}{\cos\alpha})^2. \tag{43}$$

It can be seen that the result obtained in Section 5.3 for unidirectional earthquakes appears again when we consider bidirectional earthquakes.

6. Conclusions

For a single span slab-girder skewed bridge, supported on elastomeric bearings, and having cross-frames or diaphragms at the ends with a rigid deck on top, this study concluded that the exact periods for translational and torsional modes of vibration can be calculated by Eqs. (14) and (15), respectively. Close agreements between the derived periods and finite element analysis were obtained in the examples. By introducing the dimensionless parameters, variation of periods and mode shapes for skewed bridges was studied. It can be seen that the first translational period and torsional period increase, while the second period decreases for increasing skew angles. Also, for skewed bridges, when $\beta << 1$, the direction of motion in the first mode of free vibration tends toward the Y-axis, while for $\beta >> 1$, it tends toward the N axis.

In addition, using a modal response spectrum analysis, the maximum displacement of a single span skewed bridge along N axis was found to be equal to the spectral displacement of the bridge in the first mode. This displacement occurs when stiffness parameter β is much greater than 1. The increase in the required seat width for a skewed bridge, in comparison with a straight bridge, can be calculated using Eq. (34), when a design response spectrum is given. It was also shown that for the maximum displacement perpendicular to the abutment Eq. (34) yields $(1/\cos^2\alpha)$ for the worst type of soil. Some rather conservative assumptions were used for this conclusion. Similar result was obtained when we considered the effect of bidirectional earthquakes. Comparing the above procedure to the current AASHTO requirement shows that the latter can be nonconservative. It is recommended that the current skew factor for the seat width requirement, i.e., $(1 + 0.00125S^2)$, be replaced with $(1/\cos^2 S)$, where S is the skew angle of the bridge.

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Biographies

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