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Estimation on bearing capacity of shallow foundations in heterogeneous deposits using analytical and numerical methods

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Abstract. In situ soil properties are spatially variable parameters causing soil deposits to be heterogeneous. Heterogeneity consists of two components: (i) A deterministic trend and (ii) The residual component. This paper presents the effect of different components of soil heterogeneity on the ultimate bearing capacity of a vertically loaded shallow foundation resting on clay deposits. The numerical model used in this study is based on finite difference simulations, employing FLAC 5.0. Results of numerical analysis are compared with other simple and analytical solutions. For heterogeneous soil deposits, considering both linear and bi-linear deterministic trends, finite difference tools were found to be able to reflect salient features of heterogeneity in bearing capacity estimation. An equivalent homogeneous analysis solution is introduced, in order to allow for heterogeneity, by adopting a representative depth for shear strength measurements. Stochastic variation of shear strength is shown to induce under conservatism by solely relying upon deterministic estimations.

1. Introduction

Soil is a naturally occurring material exhibiting noticeable changes in its engineering properties, due to spatial variability in physical and chemical environments, which brings about various formations. This will introduce uncertainty and variability in the estimation of engineering parameters from geotechnical engineering perspectives, defining the strength and stiffness characteristics of in situ soil, and also induces uncertainty in the safety limits required for assessing the safety and performance characteristics of the structures [1].

The inherent variability of soil properties is usually separated into a depth-dependent spatial trend and fluctuations around this mean value trend. The spatial trend is identified by regression analysis employing a sufficient number of in-situ test data, and is usually removed from the subsequent stochastic analyses.

Noticeable attempts have been made to study the effect of heterogeneity on the bearing capacity of shallow foundations on clays under undrained conditions, $\varphi = 0$ [2-5]. The majority of these studies found that heterogeneity has a paramount effect on the bearing capacity of clays.

Raymond [2] also studied the bearing capacity of footings and embankments on heterogeneous clay using the slip circle method. He presented a dimensionless plot of failure criteria for footings. For some simple cases, where the strength bears linear and bi-linear variations with depth, Davis and Booker [3] have provided approximate analytical solutions. However,

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for more general cases, such solutions do not exist. A general variation of undrained shear strength with depth is relatively easy to simulate in numerical analysis. The undrained shear strength can be realized using random field theory adopted in finite element or finite difference formulations.

The current study investigates the capability of different analytical and numerical methods in bearing capacity estimation for shallow foundations. It focuses on finite difference methods using the commercial software, FLAC 5.0 [6], taking into account different forms of heterogeneity. It will be finally compared with other conventional methods and a new translation of conservatism will be introduced.

2. Shear strength variability

Soil properties are well-known for variability from point to point in, so-called, heterogeneous natural alluvial deposits. Variability in measured properties in these layers emanates from different sources. The most important component of variability is the inherent spatial variability originating from the natural geological process that produces and gradually alters soil mass. The results of various in-situ tests on the variation of undrained shear strength of natural deposits with depth reveals that the ratio ($\frac{c_u}{\sigma'}$), where σ' is the effective overburden stress, is substantially a constant for normally consolidated clays [7]. Therefore, a linear increasing of overburden stress with depth introduces a linear increase in shear strength with depth.

However, numerous studies show that the deterministic trend of undrained shear strength decreases with depth in near-surface soils, and then, increases as the depth increases [7-10].

This is due mainly to surface desiccation. The depth at which the trend of undrained shear strength transforms is called the transformation depth (Z_t). Further discussion on the mentioned issue is provided elsewhere [11].

Other studies support the fact that the undrained shear strength of natural deposits can be detrended to extract the deterministic trend, which is a bilinear trend as discussed earlier, and the stochastic or residual component, which has its own effect. The current study considers the effect of both deterministic and stochastic heterogeneity on the bearing capacity of shallow foundations. The conducted analyses lend support to the contention that for nonlinear problems, like bearing capacity calculations and slope stability, the mutual contributions of different components of heterogeneity should be taken into account simultaneously. Figure 1 schematically demonstrates different components of shear strength heterogeneity.

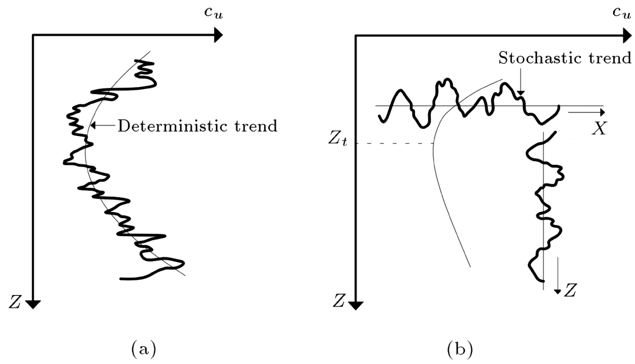

Figure 1. Deterministic and stochastic trend of undrained shear strength: (a) General variation; and (b) stochastic and deterministic components.

3. Bearing capacity

Limit equilibrium and slip line solution are two major analytical methods widely employed over the past few decades for considering the effect of "strength density" on bearing capacity problems. Strength density is the so-called increasing rate of the shear strength with depth. Numerical analysis includes methods which satisfy all theoretical requirements, including equilibrium, compatibility, constitutive behavior and boundary conditions. The Finite Element Method (FEM) and the Finite Difference Method (FDM) are two more widely-used numerical analysis schemes adopted in geotechnical engineering.

Raymond [2] studied the effect of the nonhomogeneity of strength on bearing capacity with the limit equilibrium method. Figure 2 shows a typical slip circle drawn through a uniform surface strip load of intensity, q; the distance from the edge of the slip circle to the edge of load is B.

The undrained shearing strength at any depth is given by Eq. (1):

$$c_u = cu_0 + \lambda Z, \tag{1}$$

where λ is the constant strength density obtained experimentally, Z is the depth, and c_{u_0} is the shearing strength at the surface. The results for the least load producing collapse are shown in Figure 3.

If a footing is analyzed according to the aforesaid assumption and the result plotted on the left side of the critical line, the footing will be theoretically unsafe, and if the result falls on the right side, it will be theoretically safe. As a numerical method solution, the commercially available finite difference code, FLAC 5.0, was used for the numerical modeling of the foundation model. For simplicity, the model was assumed weightless and the soil behavior was sought in an undrained condition. A plane-strain analysis was performed to model strip-footing on a heterogeneous stratum. Figure 4 provides the discretized model,

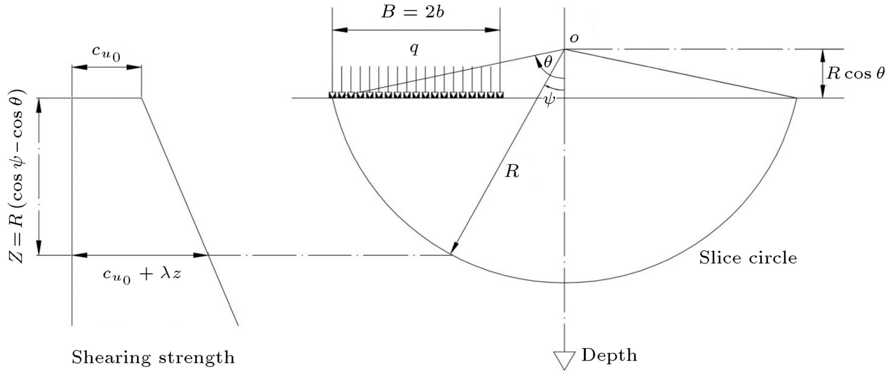

Figure 2. Geometrical layout of a typical slip circle [2].

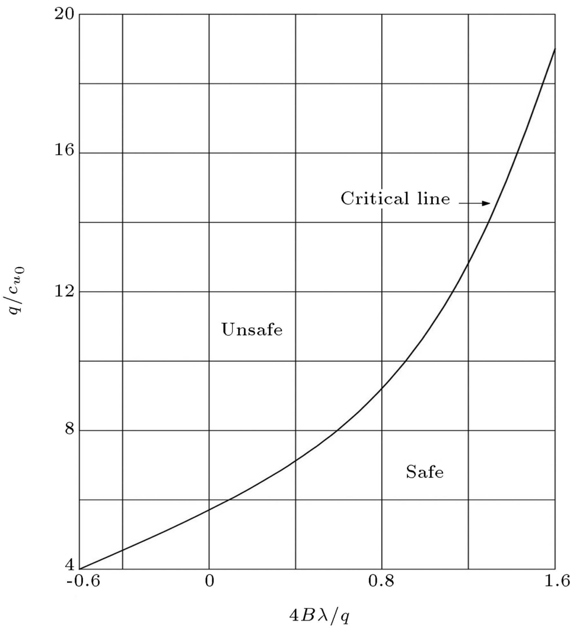

Figure 3. Dimensionless plot of failure criterion [2].

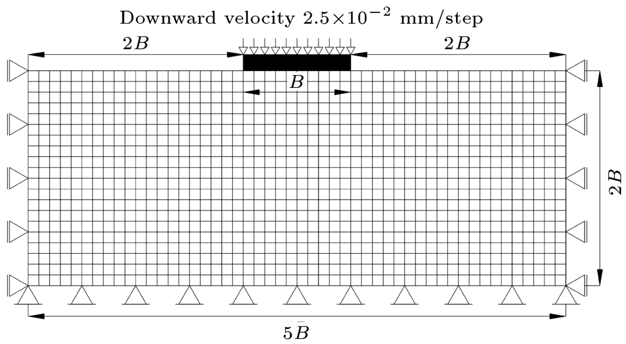

Figure 4. Model geometry and boundary conditions in finite difference model, $B = 10$ m.

along with the boundary conditions at the sides and bottom. A downward velocity field was applied to the area representing the footing. The value of the velocity applied to the footing area was 2.5×10^{-2} mm/step for analyses. This value is sufficiently small to minimize any inertial effects. A rough strip footing was simulated by fixing the x-velocity to zero for the grid points representing the footing.

4. Deterministic heterogeneity

Due to the inherent variability of soil properties, the failure surface under the footing will follow the weakest path through the soil, which is not necessarily the logarithmic spiral shape assumed by Terzaghi [12] and researchers afterwards. Deterministic trends, as shown in Figure 5, are considered for undrained shear strength to investigate the effects of deterministic heterogeneity on the bearing capacity of shallow foundations.

Undrained shear strength in Figure 5(a) increases linearly with depth and λ is the strength density. Figure 5(b) represents variations of shear strength when the undrained shear strength inherits a bi-linear trend. The undrained shear strength starts decreasing from the surface cohesion value of 30 kPa with a negative strength density, λ_1, but turns to increasing with a positive strength density, λ_2, as the transformation depth, Z_t, is passed. Undrained shear strength in Figure 5(c) is constant, spatially, over the entire layer.

A range for the Young's modulus for undrained loading can be considered between $300c_u$ and $1500c_u$ [13]. However, in this study, $E = 300c_u$ was considered. Therefore Young's modulus would also imitate the same deterministic trend as adopted for undrained shear strength. A value of 0.49, appropriate for undrained conditions, was considered for the Poisson ratio.

By adopting the trend shown in Figure 5(a) for undrained shear strength, the ultimate bearing capacity of heterogeneous soil, $q_{\mathrm{ult(N.H.)}}$, increases as the strength density, λ, increases (Figure 6).

From Figure 6, it is evident that surface cohesion contributes most to the undrained bearing capacity of shallow foundations, explaining why full heterogeneity with zero surface cohesion renders a trivial bearing capacity and it is thought unrealistic to make a zero surface cohesion assumption.

Bi-linear heterogeneity, which is usually observed

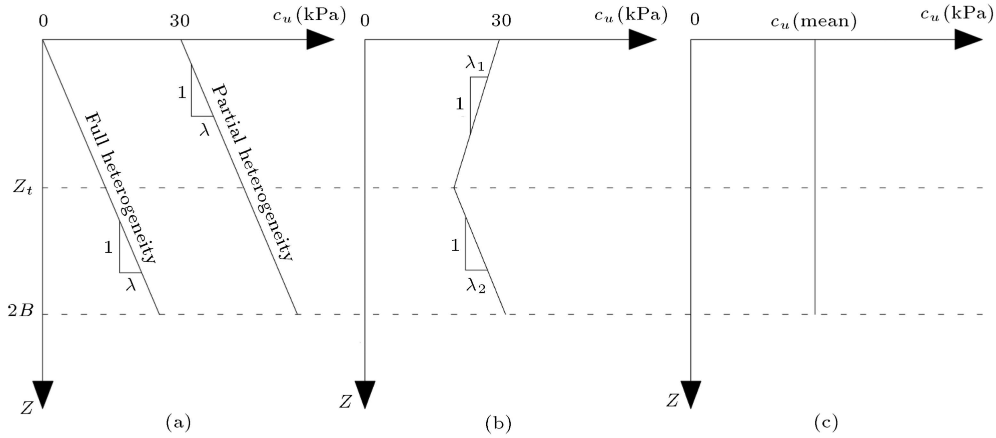

Figure 5. Assumed deterministic trends for undrained shear strength: (a) Linear heterogeneity (Gibson soil); (b) bi-linear heterogeneity; and (c) homogeneous field.

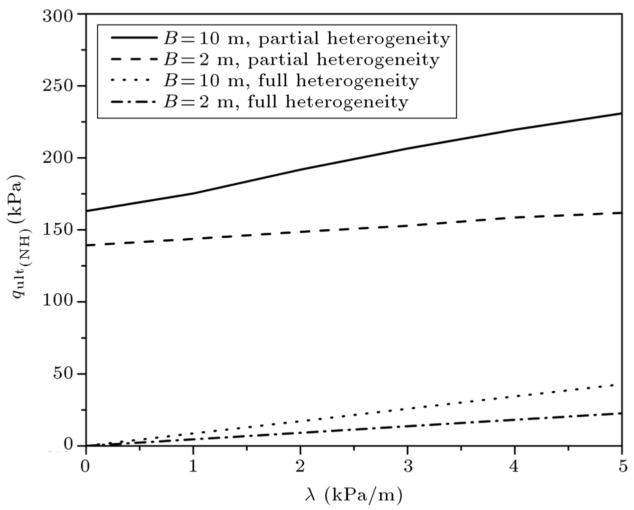

Figure 6. Ultimate bearing capacity of shallow foundation on heterogeneous Gibson-soil, $q_{\text{ult}_{N.H}}$, vs. strength density, λ.

for soft soils due to surface desiccation, adopts two inversely varying linear trends which intersect in the so-called transformation depth, Z_t. According to the general trend introduced in Figure 5(b), finite difference analyses were conducted for different strength density values, λ_1 and λ_2, and different footing widths.

The results illustrated in Figure 7(a) show that with increasing the rate at which the undrained cohesion, c_u, decreases (λ_1), the ultimate bearing capacity decreases. This effect is justified by the fact that the slip failure zones extending downward depend on the footing width. The ultimate bearing capacity of the footing is indeed the accumulation of shear strength along the failure surface, which is masked by the negative strength density in shallow depths.

Another observation from Figure 7 is that the bearing capacity increases with the increase of positive strength density (λ_2), which happens in the zone following the surface crust zone. However, it is seen that the bearing capacity of the footing with small width (B = 2 m) is, in effect, uninfluenced by the strength density of the second zone (λ_2). This is explained by the fact that failure zone dimensions are controlled by the footing width, and when the transformation depth is deeper than the footing width, the increase of shear strength in lower depths has actually no influence on the bearing capacity.

The effect of transformation depth, Z_t, is studied by performing a bearing capacity analysis for different transformation depths, while other strength parameters are assumed constant between different analyses. Figure 8 demonstrates the variation in the load bearing capacity of footings with two different widths and with transformation depth, Z_t. It is derived that for both small and large footings, the bearing capacity decreases as the transformation depth moves downward. As expected, the increase of transformation depth pushes the failure zone to be laid more widely within the surface crust zone where the undrained cohesion decreases. However, this effect fades after a specific transformation depth, which is beyond the failure zone for each specific footing width, and the bearing capacity converges to a constant value (Figure 8).

Superimposed on Figure 8 is the result of bearing capacity calculations with Limit Equilibrium (LE) analysis, which shows how the bearing capacity of footing underlain by a bi-linear heterogeneous stratum varies with the transformation depth. Calculations were made according to Eq. (2) by assuming a circular failure surface:

$$q_u = 2\pi c_{u_0} - 4\lambda_1 B \qquad Z_t \geq B,$$

$$q_u = 2\pi c_{u_0} - 4\lambda_1 B - 4(\lambda_1+\lambda_2)\left(\frac{\pi}{2} - \theta_t - \sqrt{B^2 - Z_t^2}\right)$$

$$Z_t < B, \tag{2}$$

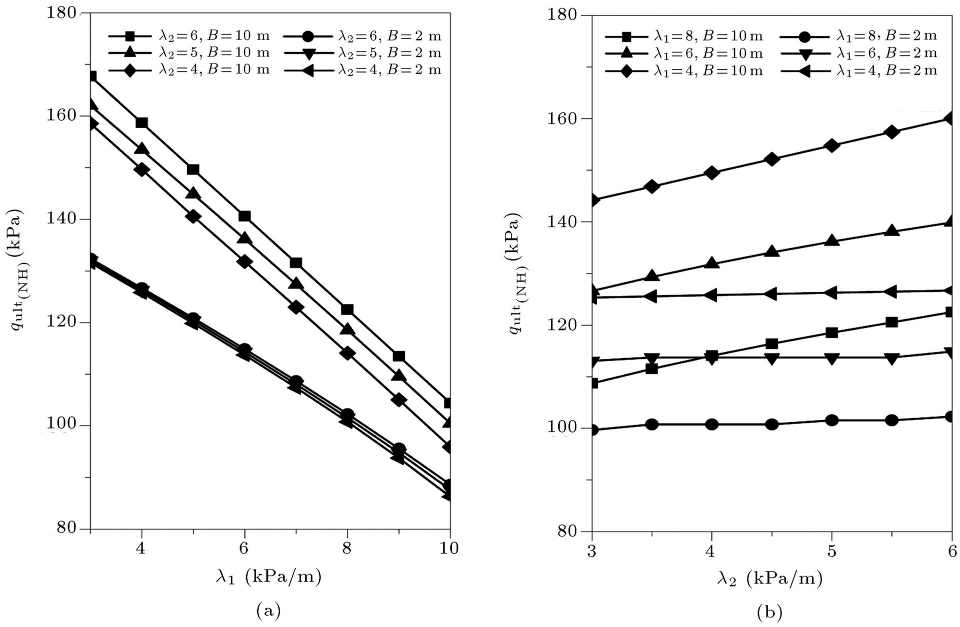

Figure 7. Ultimate bearing capacity of bi-linear heterogeneous soil, $Z_t = 2$ m: (a) λ_1 variation; and (b) λ_2 variation.

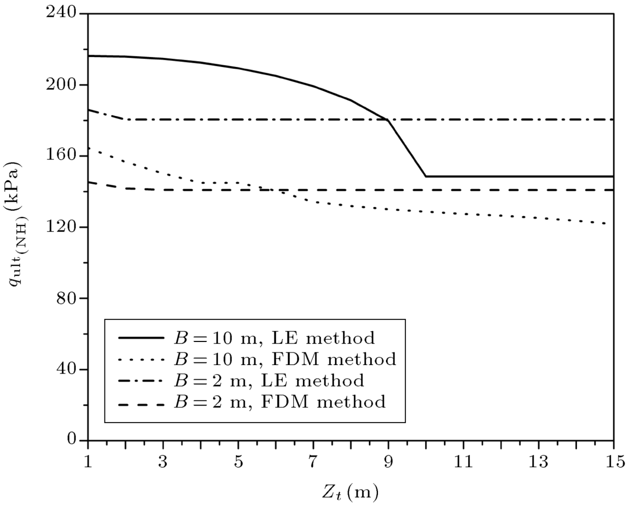

Figure 8. Effect of the transformation depth on bearing capacity (Z_t), $\lambda_2 = \lambda_1 = 1$ kPa/m and $c_{u_0} = 30$ kPa.

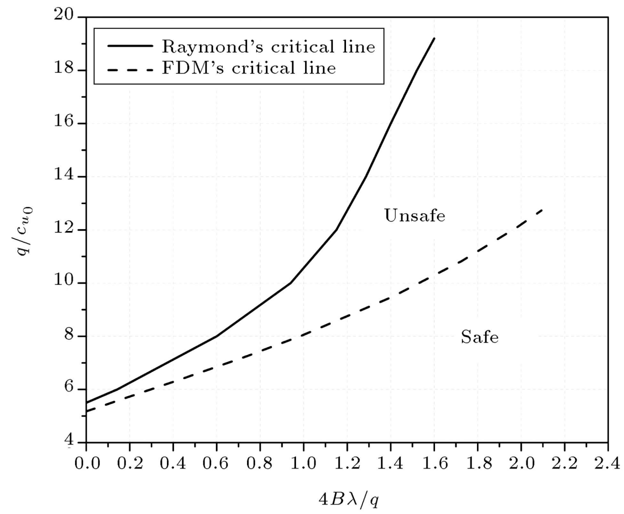

Figure 9. Finite difference results in comparison to Raymond's results [2].

where:

$$\theta_t = \sin^{-1}\left(\frac{Z_t}{B}\right).$$

A critical line based on the results of numerical FDM analyses is drawn in Figure 9 in a dimensionless fashion, along with the Raymond's line.

Raymond considered both positive and negative strength densities. However, the current study consists of two major sorts of heterogeneity, which are linear heterogeneity with positive strength density and bi-linear heterogeneity with mutual negative-positive strength densities. Therefore, the pure negative strength density as considered by Raymond is not realistic and it was shown in the previous section that the bearing capacity of footing on soft soils bears the mixed effects of crust and the layer underneath. The aforesaid discrepancy was resolved by solely choosing the positive side of Figure 3 and taking only the linear heterogeneity into consideration. From the lines provided in Figure 9, it is clear that the Raymond criterion is quite unconservative and overestimates the bearing capacity of footings on linear heterogeneous strata.

5. Stochastic bearing capacity

Random fields have been employed by many researchers to study a wide range of geotechnical issues. Among them, many adopt the finite difference method joined with random field theory. For instance, Srivastava and Babu [1] studied the effect of soil variability on two cases of bearing capacity and slope stability with the aid of the finite difference numerical code, FLAC 5.0. More recently, Zhalehjoo et al. [14] conducted a study, in which the so-called stochastic bearing capacity was compared to classic methods.

In this study, the stochastic variation of undrained shear strength is modeled by adopting a log-normal distribution with the aid of three recognized representative statistical parameters: mean value, standard deviation and correlation length. Also, there are other distributions, like Normal, Beta, etc., in practice. However, the use of log-normal distribution lies in the fact that shear strength is strictly non-negative and, in log-normal distribution, there is no possibility of the existence of negative values. For a detailed description of proper distributions in geomaterials, Harr [15] and Lee et al. [16] can be referred to. In practice, it is more common to use the dimensionless Coefficient Of Variation (COV) instead of standard deviation, which can be defined as the standard deviation divided by the mean. Typical values for the COV of the undrained shear strength have been suggested by several investigators [16,17]. The suggested values are based on in situ or laboratory tests and the recommended range is 0.1-0.5 for the COV of the undrained shear strength. The third important feature of a random field is its correlation structure. It is obvious that if two samples are close together, they will be usually more correlated compared to the case where they are widely separated. It is common in literature to use a correlation function in the following single exponential form, which is known as the Markovain spatial correlation function [18]:

$$\rho = \exp\left\{\frac{-2|\tau|}{\theta}\right\}, \tag{3}$$

where θ is called the correlation length or the scale of fluctuation and τ is the lag distance. The correlation length is the parameter which describes the degree of correlation of a soil property, and is defined as the distance beyond which the random values will be no longer correlated. We should note that in the case of a large correlation length, the random field tends to be smooth, and oppositely, when it is small, the random field tends to be rough [19]. The field is assumed to be characterized by a correlated log-normal distribution and is generated by the matrix decomposition algorithm, which is based on the Choleski method [20]. The technique is based on decomposing a symmetric, positive definite matrix into a lower triangular matrix.

Since the undrained shear strength field is log-normally distributed, taking its logarithm yields a normally distributed random field, meaning that $\ln c_u$ is normally distributed. The values of the undrained shear strength are estimated from:

$$\ln c_u = L.\varepsilon + \mu_{\ln c_u}, \tag{4}$$

where $\mu_{\ln c_u}$ is the mean of $\ln c_u$, ε is a Gaussian vector field (having zero mean and unit variance), and L is a lower triangular matrix defined by:

$$A = LL^T, \tag{5}$$

where A is the covariance matrix, which will be formed by using a specified form of the covariance function. The approach can allow consideration of anisotropy; however, in the present study, the isotropic field is adopted. The isotropic covariance matrix is given by [20]:

$$A = \sigma^2 e^{-\frac{2|\tau|}{\theta}}, \tag{6}$$

where σ^2 is the variance of $\ln c_u$, θ is the autocorrelation length and matrix τ is the lag distance, which tends to be the distance matrix. Figure 10 illustrates a sample realization of undrained shear strength with assumed stochastic properties.

As stated before, the bearing capacity will be calculated by a FISH program developed by the authors, which adopts the finite difference method merged with random field theory (RFDM). In this code, the Poisson ratio (v) is assumed to be constant, while the undrained shear strength (c_u) and the undrained Young's modulus (E_u) are randomized throughout the domain. The undrained Young's modulus is assumed to be fully correlated to the undrained shear strength by adopting a E_u/c_u ratio of 300.

Several runs are performed to investigate the effects of COV and λ (strength density) separately. The values of COV vary from 0.1 to 0.75. Adopted values of λ are 1, 3 and 5 (kPa/m), where the surface shear strength value is adopted as 30 (kPa). It should be noted that the correlation length is assumed constant, being equal to the footing width. For each set of adopted COV and λ values, Monte Carlo simulations have been conducted involving 500 realizations of the shear strength random field and the subsequent numerical analysis of the bearing capacity.

The results of the random finite difference method for each set of input parameters have been prepared by the explained method. Figure 11 shows the effect

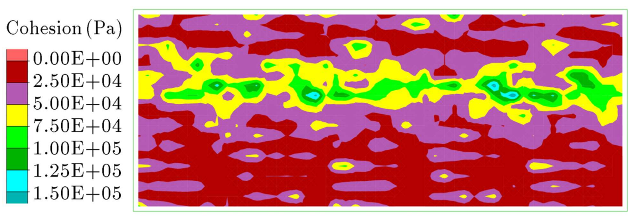

Figure 10. Realization of the undrained shear strength with COV = 0.75, c_{u_0} = 30 kPa and θ = 10 m.

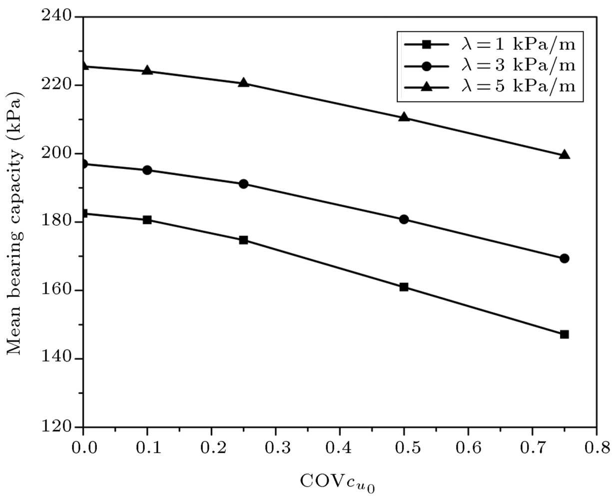

Figure 11. Bearing capacity variation with COV and λ.

of COV on the bearing capacity for different levels of strength densities, λ. It is evident from Figure 11 that the COV has a decremental effect on the bearing capacity of shallow foundations. It means that by an increase in the variation of strength parameters, the possibility of weak zone formation in underlying stratum increases, and the bearing capacity of the overlying foundation decreases. This finding is in conformity with other research [1,18].

6. Comparison with "equivalent" homogeneous soil

According to most modern building codes [21], the bearing capacity calculation for shallow foundations is based on uniform shear strength parameters acquired from laboratory tests. Regulations essentially refer to a homogenous or inhomogeneous profile, without strong gradients in shear strength parameters with depth.

On the other hand, in the case of inhomogeneous layers, such as those described in Eq. (1), the choice of pertinent "representative" shear strength has not yet been established. A simple approach to consider heterogeneity in the bearing capacity calculation is to pick up the shear strength parameters from a specific depth below the ground surface and then to follow the conventional homogeneous formulations. Towards this aim, the proposed numerical solution for a heterogeneous system is compared to the bearing capacity of an equivalent homogenous soil possessing shear strength parameters taken from a representative depth. Three homogenous layers are examined to this end, using the following alternative representative depths:

- $c_{u_{\text{hom1}}}$, equal to the undrained shear strength at the depth of $B/3$ beneath the foundation base.

$$c_{u_{\text{hom1}}} = c_u\left(\frac{B}{3}\right). \tag{7}$$

  Raymond [2] was the first to make such an assumption by evaluating the stability of surface footings, assuming a uniform strength equal to the strength at a depth of one third of the footing ($B/3$).

- $c_{u_{\text{hom2}}}$, calculated if a linear variation for shear strength is assumed and a circular failure mechanism is adopted. By choosing a circular failure mechanism, as shown in Figure 2, such that the radius of the failure is B, the representative depth for shear strength will be equal to $\frac{2B}{\pi}$.

$$c_{u_{\text{hom2}}} = c_u(\frac{2B}{\pi}). \tag{8}$$

  This means that the bearing capacity calculation for a linearly varying soil stratum with surface cohesion, c_{u_0}, and strength density, λ, holds in analogy with the homogeneous condition of the bearing capacity calculation when the shear strength at the representative depth introduced in Eq. (8) is picked.

- $c_{u_{\text{hom3}}}$, equal to the undrained shear strength at the depth of B beneath the foundation base.

$$c_{u_{\text{hom3}}} = c_u(B). \tag{9}$$

This is indeed believed to be the farther depth influenced by the shallow foundation stress bulbs. This is further confirmed if someone assumes a slip circle with radius B.

If someone takes the equivalent uniform shear strength value from a representative depth, Z_{rep}, then, the ultimate bearing capacity of the heterogeneous soil stratum calculated by the finite difference method can be compared to the equivalent homogenous solution by assuming the above mentioned representative depths, $Z_{\text{rep}} = B/3$, $2B/\pi$ and B, where B is the width of the footing.

The first adopted representative depth is to evaluate the Raymond assumption. The second one is reflecting the depth calculated by the limit equilibrium method, as discussed earlier, and the last is only to cover the maximum depth at which the slip surface for shallow foundation extends.

In Figure 12, the inhomogeneous system is compared to the above equivalent homogeneous soil in terms of its bearing capacity. NH analysis in this figure refers to non-homogeneous analysis.

Very good agreement between the FDM heterogeneous solution and the equivalent homogeneous solution at the representative depth of $B/3$ is found, confirming Raymond's assumption. The same level of conformity is observed when the soil layer beneath the foundation is soft and probably bears a bi-linear shear strength variation trend, as assumed in Figure 5(b). Figure 12(b) clearly shows this agreement. The numerical analyses results, illustrated in Figure 12, also

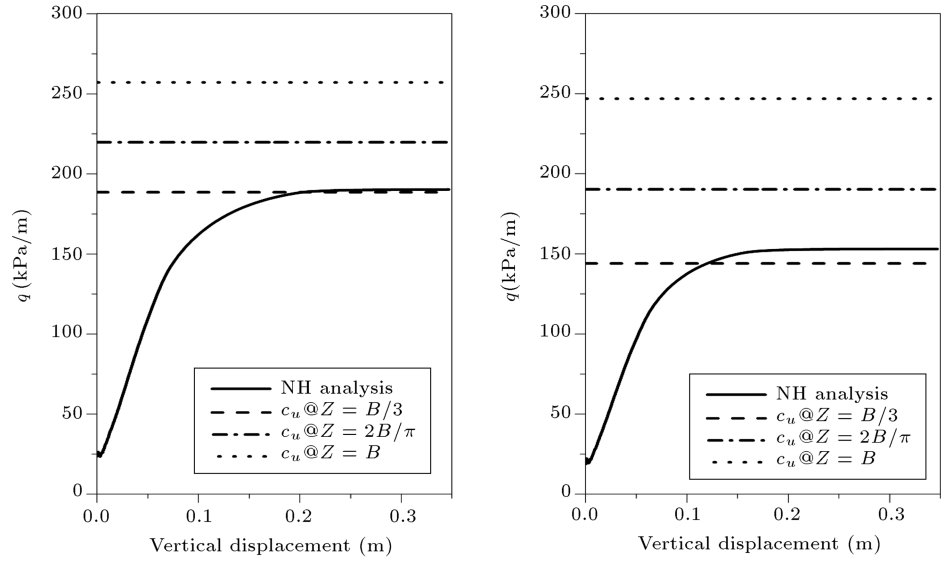

Figure 12. Load-displacement curves, $B = 10$: (a) Partial heterogeneous Gibson soil, $\lambda = 2$ kPa/m; and (b) bi-linear deterministic trend with $\lambda_1 = 3$ kPa/m, $\lambda_2 = 3$ kPa/m and $Z_t = 2$ m.

show that assuming a deeper representative depth for shear strength will lead to under-conservatism in the shallow foundation bearing capacity estimation. This becomes more important when paying attention to the fact that the adoption of a safety factor of 3 or even more does not necessarily guarantee safety when the representative depth for shear strength is not realistic.

The implication of the aforesaid consideration of representative depth for shear strength, in practice, is to properly select the sampling depth for conducting shear strength tests, and apply a consistent value of confining stress in laboratory test schemes that best represents the actual condition at that depth.

7. Implication to design

The current study examines different approaches leading to bearing capacity calculation for shallow footings. It was shown that finite difference analysis, among all, is an efficient tool for performing such calculations. It was used to investigate the effect of different components of heterogeneity on the ultimate bearing capacity of footings. The normal practice in a construction field is to conduct some limited numbers of laboratory tests and calculate the bearing capacity of foundations from conventional equations, which are mainly derived by adopting limit equilibrium analysis. Terzaghi [12], Meyerhof [22,23] and Hansen [24] are good examples of such equations. These equations have found general use in geotechnical practice. However, none of them consider heterogeneity as explained and analyzed here. We should now investigate the implication of heterogeneity on engineering design and the effect of its negligence on design conservatism. Short-term bearing capacity (undrained condition) was considered in the course of this research; however, it can be extended to a long-term (drained) condition in a similar manner. The equivalent homogeneous ultimate bearing capacity calculated from homogeneous FDM analyses, $q_{\mathrm{ult}_{(\mathrm{EH})}}$, was compared with heterogeneous FDM analysis results. A factor of conservatism, FOC, defined as the ratio of non-homogeneous to homogeneous bearing capacity (Eq. (10)), is employed herewith to reflect the level of conservatism.

$$\mathrm{FOC} = \frac{q_{\mathrm{ult}_{(\mathrm{NH})}}}{q_{\mathrm{ult}_{(\mathrm{EH})}}}. \tag{10}$$

The equivalent homogeneous bearing capacity calculation ($q_{\mathrm{ult}_{(\mathrm{EH})}}$) is based on shear strength at the representative depth, considered as $B/3$, where B is the width of footing.

Figure 13 shows the variation of FOC values with the strength density, λ, for footings of two different widths embedded on a Gibson-soil stratum with partial heterogeneity.

It is concluded that for medium to stiff clay possessing linear heterogeneity, performing homogeneous bearing capacity analysis, based on the strength data from the representative depth ($B/3$), will not lead to significant over conservatism, as the FOC value remains almost invariable and independent of strength density, λ, and is close to 1.

For soft soils, the variation of undrained cohesion was stated to be of a bi-linear nature. The results of FOC variation with different strength parameters, namely, densities and transformation depth, are plotted in Figure 14.

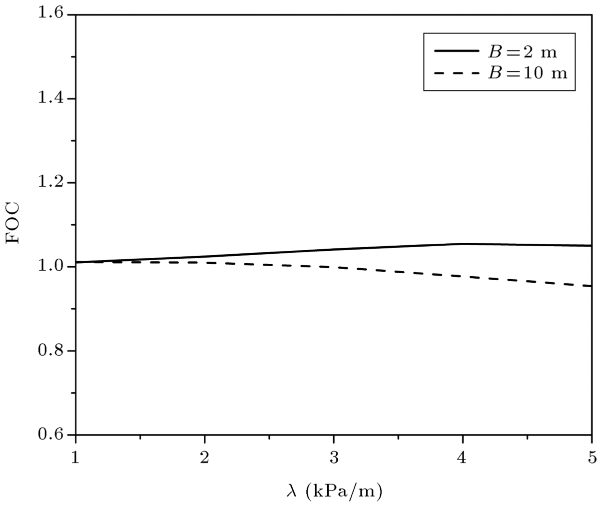

Figure 13. Variation of FOC values with strength density, λ, for partially heterogeneous Gibson-soil.

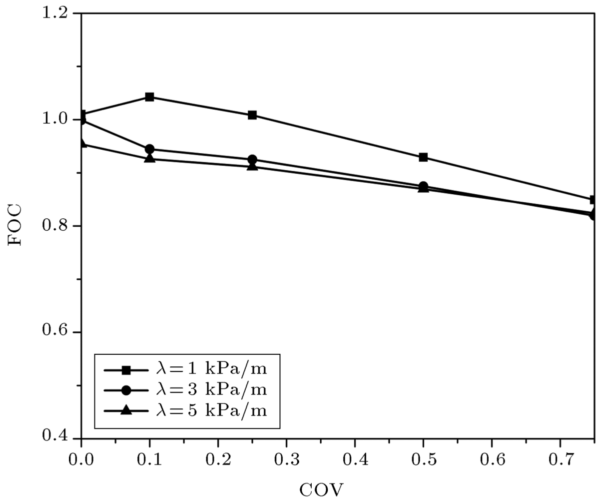

Figure 15. Variation of FOC values with COV for stochastic variation of undrained cohesion.

It is evident that for small footing widths, the FOC values do not vary significantly. However, it is slightly less than 1, which implies under conservatism. As the footing width becomes large enough (10 m in this case), FOC values grow and homogeneous analysis will lead to over conservatism. However, the large footing width is not usually the case in engineering practice, as the strip footing assumed in this study is usually of small size and bears values even less than those adopted in this study (2 m).

Another observation is that the introduced representative depth for undrained cohesion proposed by Raymond [2] is a competent approximation, which does not lead to significant over or, more importantly, under conservatism.

Stochastic variation of soil properties will induce further uncertainty in bearing capacity estimation. Uncertainty is translated into unconservatism in practice when relying upon the homogeneous formulation for bearing capacity estimation, as found in classic foundation engineering literature. Figure 15 illustrates how the spatial variability of undrained shear strength represented by COV affects design conservatism.

As expected, the increase in COV values will lead to the probability of weak zone formations and, consequently, to reducing the mean bearing capacity. FOC values as defined earlier, will then show a decrease.

8. Conclusion

This study introduced and compared different techniques to compute the ultimate bearing capacity of

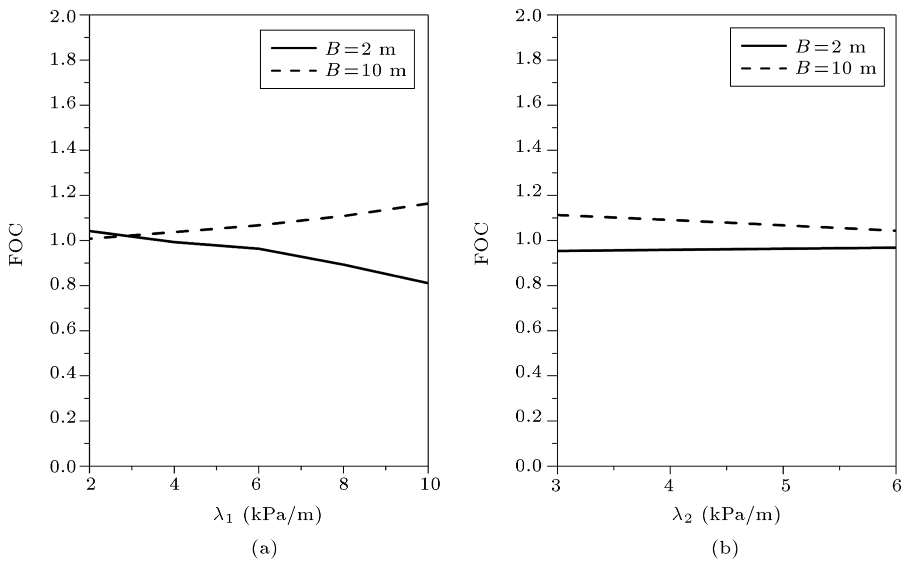

Figure 14. FOC values vs. different strength parameters: (a) λ_1 variation, λ_2=5 kPa/m and Z_t=2 m; and (b) λ_2 variation, λ_1=6 kPa/m and Z_t=2 m.

strip footings, inclusive of simple analyses and numerical methods representing complex analysis techniques. For heterogeneous soil deposits, the following results were obtained:

- In linear heterogeneous medium to stiff soil deposits, the bearing capacity increases as the strength density, λ, increases.
- For partial heterogeneous soil, which inherits non-zero surface cohesion and positive strength density, surface cohesion was found to contribute most to the undrained bearing capacity.
- For soft soils where the shear strength variation is of a bi-linear nature, by increasing the rate at which the undrained cohesion decreases, λ_1, the ultimate bearing capacity decreases. However, the positive strength density, λ_2, belonging to the region underlying the surface desiccation zones, has an incremental effect, especially for larger footing width, where the stress bulbs extend deeper and beyond the transformation depth.
- For small and large footings, the bearing capacity decreases with the increase in transformation depth. However, this effect disappears when a threshold transformation depth is reached. The threshold transformation depth is footing width dependent and is indeed beyond the failure zone.
- In order to avoid under or, most importantly, over estimation of the ultimate bearing capacity of shallow footings, it was found that performing an equivalent homogenous finite difference analysis by adopting shear strength parameters from a representative depth would be a competent alternative. The formerly introduced representative depth for undrained cohesion proposed by Raymond, $Z_t = B/3$, is still deemed to be valid. For both small and large footing widths, it was seen that the results of equivalent homogeneous bearing capacity analyses are close enough to actual heterogeneous calculations. This leads to the contention that FOC values remain near 1 for the practical ranges of strength density.
- Faced with common geotechnical issues, a practitioner usually refers to classic methods, such as limit equilibrium and slip line solution, available in literature. Homogenous conditions or in its most complex form of study, deterministic heterogeneity, are recognized usually by practitioners or even academicians. However, the results of the current study imply that in real world situations, the spatial variability of strength reduces the estimated bearing capacity, especially by increasing the COV of parameters. This means that neglecting the spatial variability of soil properties leads to an overestimation in bearing capacity prediction.

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