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Dynamic analysis of thick plates on elastic foundations using Winkler foundation model

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Winkler model;
Shear locking problem;
Dynamic analysis.

Abstract. Dynamic analysis of rectangular thick plates on elastic foundations under partially distributed and centrally concentrated impulsive load is presented using Winkler foundation model. An 8-noded (PBQ8) Mindlin plate element are adopted for modeling the plate to account the transverse shear deformation effects neglected in the classical thin plate theory. Selective reduced integration technique is used to avoid shear locking problem which occurs as thickness/span ratio decreases. The results obtained from the study are compared with the solutions obtained by SAP2000 software to show the validity of the elements. The variation of the maximum displacement with various values of subgrade reaction modulus, aspect ratios, the ratio of plate thickness to shorter span of the plate and loaded area are investigated. Numerical examples show the applicability of the 8-noded element to dynamic analysis of thick plates on elastic foundation subjected to external loads using Winkler model.

1. Introduction

Vibration problems of plates on elastic foundation occupy significant place in many fields of structural and foundation engineering. Kirchhoff plate theory is used in most of the studies because of its simplicity. Although Kirchhoff plate theory works well for the thin plates, the result obtained by the Kirchhoff theory may not be accurate enough as the plate gets thicker since the shear deformation effect is ignored in the theory. Mindlin plate theory accounts for the angle changes within a cross section of the plate and assumes that transverse shear deformation occurs. Mindlin plate elements based on Mindlin plate theory can be used for the analysis of both thin and thick plates when reduced or selective reduced integration technique are used to avoid shear locking problem for calculation of the stiffness matrix. On the other hand, the real behavior of the foundation-plate system is quite complex. Various foundation models have been developed to simplify the complex phenomenon. The simplest and most widely used is Winkler model that the subsoil is represented by a set of discrete spring elements and the vertical displacement is assumed to be proportional to the contact pressure at an arbitrary point.

For the past twenty years, a lot of studies concerning dynamic analysis of plates on elastic foundation are performed. Omurtag et al. [1] developed a mixed finite element formulation based on Gateaux differential for free vibration analysis of Kirchhoff plates on elastic foundation. Daloglu et al. [2] examined dynamic analysis of plates on elastic foundation using modified Vlasov model and thin plate theory. Shen et al. [3] performed the free and forced vibration analysis of Reissner-Mindlin plates resting on Pasternak-type elastic foundation. Huang and Thambiratnam [4] discussed dynamic response of plates on elastic foundation subjected to moving loads and the investigated effects of velocity, subgrade reaction, moving path and

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distance between multiple moving loads on responses. Hasemi et al. [5] studied free vibration analysis of vertical rectangular Mindlin plates resting on Pasternak elastic foundation and fully or partially in contact with fluid on their one side for different combinations of boundary conditions. Hsu [6] presented the vibration responses of orthotropic plates on nonlinear elastic foundations using the differential quadrature method. Celep and Güler [7] studied the static behavior and forced vibration of a rigid circular plate supported by a tensionless Winkler elastic foundation by assuming that the plate is subjected to a uniformly distributed load and a vertical load having an eccentricity. Eröz and Yildiz [8] presented a finite element formulation of forced vibration problem of a prestretched plate resting on rigid foundation. Yu et al. [9] presented the dynamic response of Reissner-Mindlin plate resting on an elastic foundation of the Winkler-type and Pasternak-type using an analytical-numerical method. Tovstik [10] studied vibration and stability of a prestressed plate on elastic foundation. Ozgan and Daloglu [11] applied the modified Vlasov model to the free vibration analysis of thick plates resting on elastic foundation using PBQ4 Mindlin plate element with selective reduced integration technique. Wen and Aliabadi [12] used boundary element method for the analysis of Mindlin plates on elastic foundation subjected to dynamic load.

In this study, 8-noded (PBQ8) Mindlin plate elements are adopted for the dynamic analysis of thick plates resting on Winkler-type foundation under partially distributed impulsive load and centrally concentrated impulsive load. A computer program is coded in Fortran for the purpose. Newmark-β method is used for time integration. Selective reduced integration techniques are used to obtain stiffness matrix of plates. The results of the study using the presented elements are compared with the solutions obtained by SAP2000 software to show the validity of the elements. Later the maximum displacements of the plate are examined for various values of modulus of subgrade reaction, aspect ratios, loaded area and the ratio of plate thickness to shorter span of the plate.

2. Mathematical model

The dynamics of elastic structures include the kinetic energy of the plate in addition to the strain energy and work of external forces. Applying the Hamilton's principle, the equation of motion for dynamic of plates on elastic foundation acted on by external loads is derived from the below equation (Figure 1).

$$\delta \int_{t_1}^{t_2} \left(\prod_k -U \right) dt = 0, \tag{1}$$

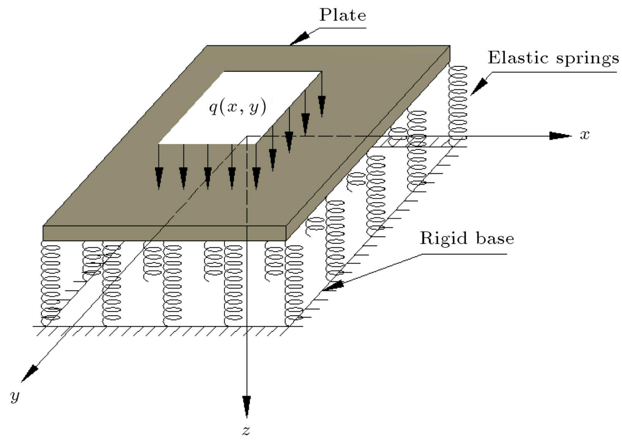


Figure 1. A plate resting on elastic foundation.

where Π_k is the kinetic energy and given as:

$$\prod_k = \frac{1}{2} \int_\Omega \{\dot{w}\}^T [\mu] \{\dot{w}\} d\Omega, \tag{2}$$

and $U = \Pi_p + \Pi_s + V$ in which Π_p is the strain energy stored in the plate, Π_s is the strain energy stored in the subsoil and V is the potential energy of external loads. These expressions can be written as:

$$\prod_p = \frac{1}{2} \int_\Omega [B]^T [D][B] d\Omega, \tag{3}$$

$$\prod_s = \frac{1}{2} \int_\Omega [w(x,y)]^T k [w(x,y)] d\Omega, \tag{4}$$

$$V = -\int_\Omega N^T q d\Omega, \tag{5}$$

where Ω is the domain of the plate, $[\mu]$ is the mass density matrix, $\{w\}$ the vector of generalized displacement components relevant to internal forces, the dot denotes the partial derivative of the vector of generalized displacement with respect to time variable, $[D]$ is the material matrix relating the stresses to strains, $[B]$ is the strain-displacement matrix of the plate, k is the subgrade modulus of the subsoil, $[N]$ is defined as displacement shape functions and q is external loads [13].

As mentioned before, in this study, rectangular finite elements based on Mindlin plate theory considering the transverse shear effects are used for finite element formulation. PBQ8 element has 8 nodes-24 degrees of freedom (Figure 2). At each node of the element, one displacement and two rotations are taken as unknowns.

$$w, \varphi_x, \varphi_y. \tag{6}$$

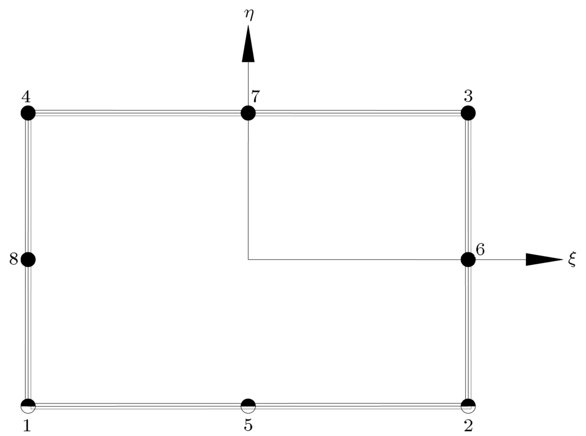


Figure 2. The finite elements used in this study.

The displacement function is:

$$w = [N]\{w_e\}. \tag{7}$$

The matrix $[N]$ contains the displacement shape functions found in explicit form in reference Weaver and Johnston [14]. The stiffness matrices, mass matrix and load vector can be derived by substituting Eq. (7) into Eqs. (2) to (5) and by using the standard procedure in the finite element methodology.

Finally the equation for the plate-soil system to be solved is:

$$[M]\{\ddot{w}\} + [K]\{w\} = \{F\}, \tag{8}$$

where $[K]$ is the stiffness matrix of the plate-soil system, $[M]$ is the mass matrix of the plate-soil system, $\{F\}$ is the applied load vector, w and $\ddot{w}$ are the displacement and acceleration vector of the plate, respectively [13].

In this study the Newmark-β method is used for the time integration of Eq. (8) by using the average acceleration method [15]. Value of β is considered as $\frac{1}{4}$. Evaluation of the element stiffness and mass matrices is performed using the displacement function given in Eq. (7) [11].

It is well known that when Mindlin plate elements are used, very stiff results may be obtained for the solution of thin plates. This phenomenon is called shear locking problem. The selective reduced integration rule is used to obtain the element stiffness matrix of the plate to avoid shear locking. In the selective reduce integration, the bending terms of plates is integrated with the normal rule and the shear terms of plate with a lower-order rule.

3. Numerical examples

For the numerical example, a rectangular plate freely resting on elastic foundation subjected to partially distributed and centrally concentrated impulsive loads is considered. At first the results obtained in this study are compared with the solutions obtained by SAP2000 software to show the validity of the elements. Then the maximum displacements of the plate are examined for various values of subgrade reaction modulus, aspect ratios, loaded area and the ratio of plate thickness to shorter span of the plate since the maximum displacements are the most important for the design. The results are presented in graphical and tabular form.

The properties of the plate-soil system are as follows. The modulus of elasticity of the plate is $E_p = 27000000$ kN/m^2, Poisson's ratio of the plate is $\nu_p = 0.20$. The mass densities of the plate is taken to be $\rho = 2500$ kg/m^3. Longer span length is considered as 10, 15 and 20 m for $l_y/l_x = 1.0$, 1.5 and 2.0 respectively while the shorter span length of the plate is kept constant at 10 m. The thickness of the plate is considered as 0.5 m, 1.0 m and 2.0 m. The analysis is performed for three different values of subgrade reaction modulus of subsoil, $k = 500$, 5000 and 50000 kN/m^3. The impulsive pressure $q(\bar{t}) = q_0.F(\bar{t})$ is applied on the top surface of the plate, in which q_0 is the maximum amplitude which is 30 kN/m^2 for partially distributed load and is 10 kN for concentrated load at the center of the plate, $F(\bar{t})$ is a unit function in time domain which has the type in Figure 3. Partially distributed load is applied centrally to the plate and loaded area is $a/l_x = b/l_y = 0.5$ as shown in Figure 4. The values for initial displacement, velocity and acceleration are assumed to be equal to zero.

For the sake of accuracy in results, a convergence study is performed before analysis. First the time increment is fixed and the mesh density is increased until the maximum displacements converge satisfactorily. Then the time increment is increased for the constant value of mesh density determined in the previous step until the maximum displacements converge satisfactorily. It is concluded that the result is acceptable when equally spaced 6 elements for 10 m length for concentrated load and 8 element for 10 m length for partially distributed load if a 0.000025 s time increment are used.

The maximum displacements for $l_y/l_x = 1.0$ and 2.0, $h = 0.5$ m and $k = 500$ kN/m^3 have been compared first with the results obtained from SAP2000 software for verification of the present formulation as

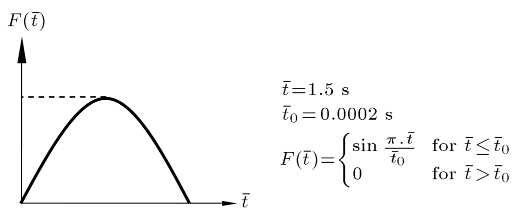


Figure 3. Shape of transverse impulsive load.

Table 1. Comparison of maximum displacements of plate on elastic foundation.

Finite element	Partially distributed load		Centrally concentrated load	
	10 m ×10 m plate	10 m ×20 m plate	10 m ×10 m plate	10 m ×20 m plate
	w_{max} (mm)	w_{max} (mm)	w_{max} (mm)	w_{max} (mm)
SAP2000	0.05202	0.06519	0.00107	0.00072
PBQ8(SRI)	0.05624	0.06828	0.00120	0.00080

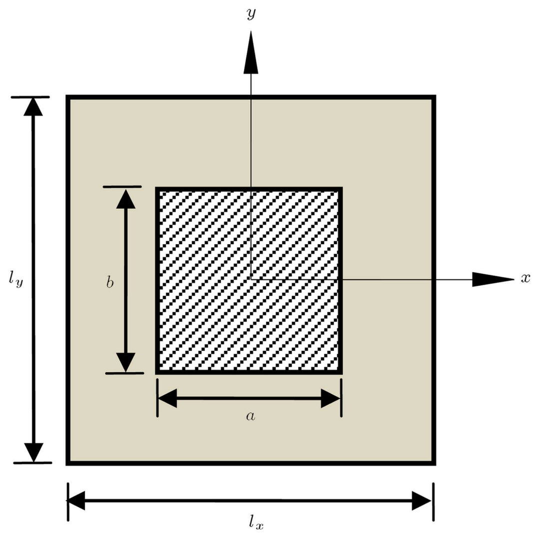


Figure 4. Plate subjected to partially distributed load.

presented in Table 1. It should be noted that Sap2000 software uses only 4 noded thick plate element while 8 noded Mindlin plate element is used in the computer program coded in this study. Nevertheless, all results are in good agreement with SAP2000 solutions for all load cases.

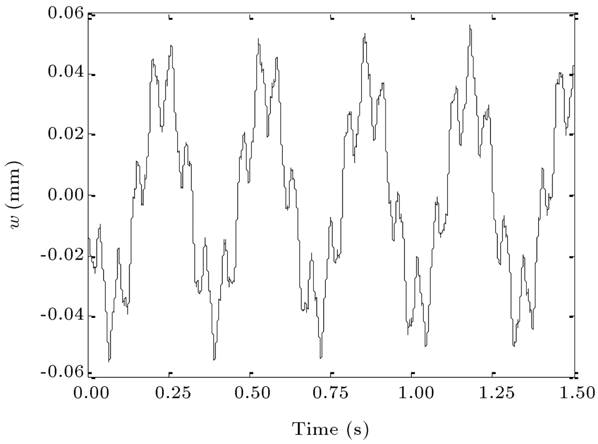
The results obtained for various values of l_y/l_x, h/l_x and k are given in Table 2. The time histories of the maximum displacements for only two cases are presented in Figure 5 for partially distributed load and in Figure 6 for centrally concentrated load because presentation of all of the time histories of the displacements at different points on the plate would take up excessive space. The time histories of the center displacement of the plate differ from each other depending on the dynamic characteristics of the system.

In the case of partially distributed load, the variation of the maximum displacement of the plate with various values of aspect ratio (l_y/l_x) for different values of the ratio of plate thickness to shorter span length of the plate (h/l_x) and subgrade reaction modulus (k) is plotted in Figures 7-9 in order to show the effects of the changes in these parameters better. The maximum displacement of the plate decreases as the subgrade reaction modulus (k) increases.

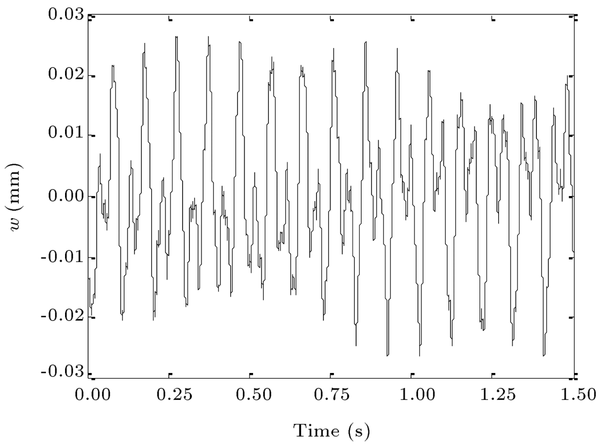
Table 2. Maximum displacements of plates on elastic foundation for PBQ8(SRI) element.

k (kN/m^3)	l_y/l_x	h/l_x	Distributed load w_{max} (mm)	Concentrated load w_{max} (mm)
500	1.0	0.05	0.05624	0.00120
		0.10	0.03103	0.00058
		0.20	0.02001	0.00035
	1.5	0.05	0.06031	0.00094
		0.10	0.03337	0.00039
		0.20	0.02055	0.00021
	2.0	0.05	0.06828	0.00080
		0.10	0.03618	0.00039
		0.20	0.02112	0.00019
5000	1.0	0.05	0.02659	0.00089
		0.10	0.01270	0.00036
		0.20	0.00724	0.00018
	1.5	0.05	0.03207	0.00065
		0.10	0.01443	0.00022
		0.20	0.00763	0.00010
	2.0	0.05	0.03574	0.00065
		0.10	0.01676	0.00028
		0.20	0.00823	0.00014
50000	1.0	0.05	0.01338	0.00072
		0.10	0.00651	0.00030
		0.20	0.00303	0.00014
	1.5	0.05	0.01557	0.00047
		0.10	0.00688	0.00017
		0.20	0.00363	0.00006
	2.0	0.05	0.01592	0.00061
		0.10	0.00817	0.00028
		0.20	0.00420	0.00014

As expected, the maximum displacement of the plate increases as the value of aspect ratio (l_y/l_x) increases. The increase in the value of maximum displacement of the plate with increasing the aspect ratio (l_y/l_x) is less for the larger value of subgrade

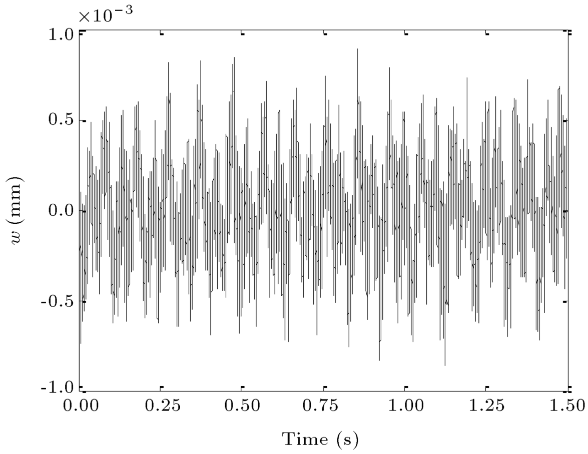


(a) $k = 500$ kN/m^3, $l_y/l_x = 1.0$ and $h/l_x = 0.05$

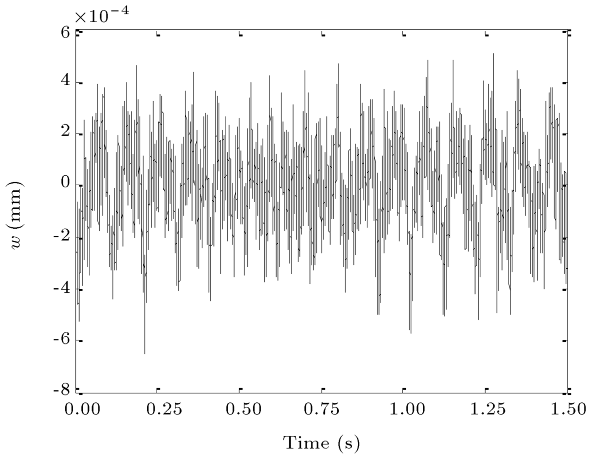


(b) $k = 5000$ kN/m^3, $l_y/l_x = 1.0$ and $h/l_x = 0.05$

Figure 5. The time histories of the center displacements of the plate subjected to uniformly distributed load.



(a) $k = 5000$ kN/m^3, $l_y/l_x = 1.0$ and $h/l_x = 0.05$



(b) $k = 5000$ kN/m^3, $l_y/l_x = 2.0$ and $h/l_x = 0.05$

Figure 6. The time histories of the center displacement of the plate subjected to concentrated load.

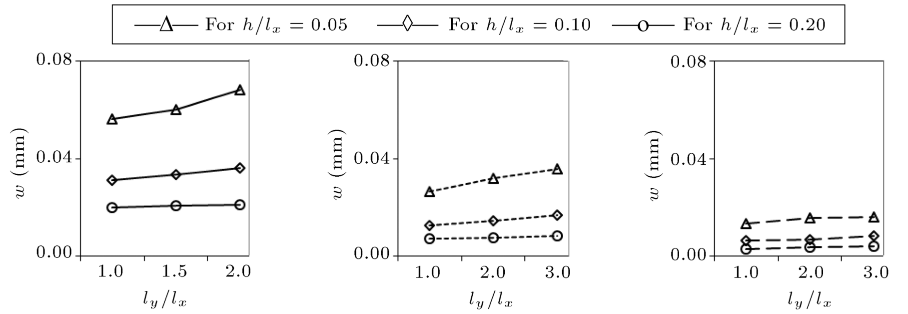
reaction modulus (k). It should be said that aspect ratio (l_y/l_x) does not affect the maximum displacement of the plate after a certain value of subgrade reaction modulus (k).

The maximum displacement of the plate decreases as the ratio of plate thickness to shorter span length of the plate (h/l_x) increases. This behavior is understandable in that a plate with a larger thickness becomes more rigid. But the decrease in the value of maximum displacement of the plate with increasing the ratio of plate thickness to shorter span length of the plate (h/l_x) is less for the larger value of subgrade reaction modulus (k). It should be said that the ratio of plate thickness to shorter span length of the plate (h/l_x) does not affect the maximum displacement of the plate after a certain value of subgrade reaction modulus (k).

Figure 10 shows the effect of loaded area on the time histories of the displacement for thick plate on Winkler elastic foundation subjected to partially distributed impulsive load. As expected, results show that the central displacements increase when the area of the loaded region increases.

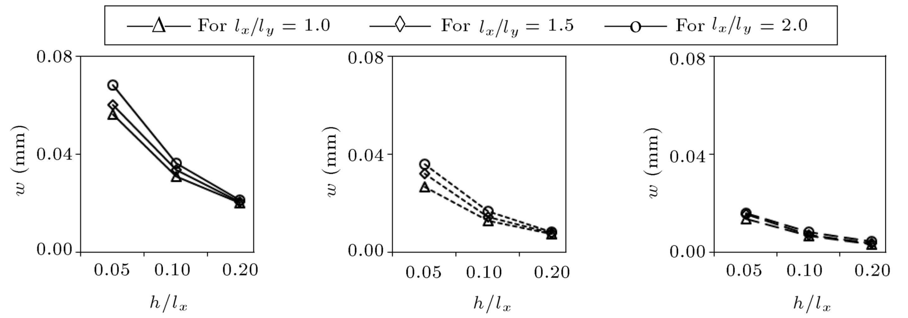
In the case of concentrated load, the variation of the maximum displacement of the plate with the aspect ratio (l_y/l_x), the ratio of plate thickness to shorter span length of the plate (h/l_x) and the subgrade reaction modulus (k) is plotted in Figures 11-13 in order to show the effect of the changes in these parameters better. The maximum displacement of the plate decreases as the value of aspect ratio (l_y/l_x) increases. An increase may be expected as the span of the plate gets larger but it should be noted that the load on per unit area gets smaller because the concentrated load is kept constant. The decrease in the value of maximum displacement of the plate with increasing aspect ratio (l_y/l_x) is less for the larger value of subgrade reaction modulus (k). It should be said that aspect ratio (l_y/l_x) does not affect the maximum displacement of the plate after a certain value of subgrade reaction modulus (k).

The maximum displacement of the plate decreases as the ratio of plate thickness to shorter span length of



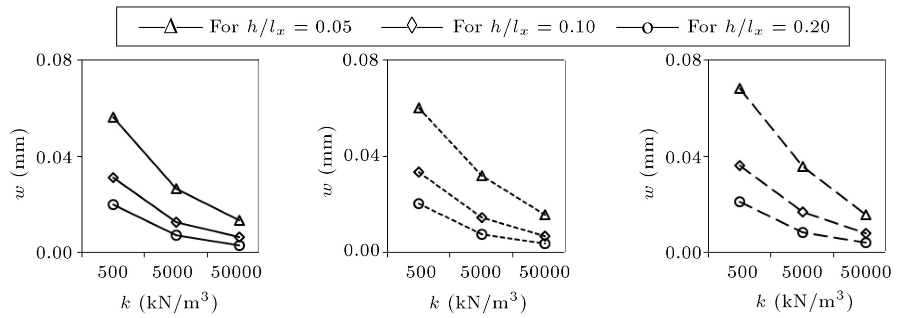
(a) $k = 500$ kN/m^3 (b) $k = 5000$ kN/m^3 (c) $k = 50000$ kN/m^3

Figure 7. The variation of the maximum displacement with l_y/l_x for partially distributed load for $h/l_x = 0.05$, $h/l_x = 0.10$ and $h/l_x = 0.20$.



(a) $k = 500$ kN/m^3 (b) $k = 5000$ kN/m^3 (c) $k = 50000$ kN/m^3

Figure 8. The variation of the maximum displacement with h/l_x for partially distributed load for $l_y/l_x = 1.0$, $l_y/l_x = 1.5$, and $l_y/l_x = 2.0$.



(a) $l_y/l_x = 1.0$ (b) $l_y/l_x = 1.5$ (c) $l_y/l_x = 2.0$

Figure 9. The variation of the maximum displacement with k for partially distributed load for $h/l_x = 0.05$, $h/l_x = 0.10$, and $h/l_x = 0.20$.

the plate (h/l_x) increases. This behavior is understandable in that a plate with a larger thickness becomes more rigid. But the decrease in the value of maximum displacement of the plate with increasing the ratio of plate thickness to shorter span length of the plate (h/l_x) is less for the larger value of subgrade reaction modulus (k). It should be said that the ratio of plate thickness to shorter span length of the plate (h/l_x) does not affect the maximum displacement of the plate after a certain value of subgrade reaction modulus (k).

The maximum displacement of the plate decreases as the value of subgrade reaction modulus (k) increases. The decrease in the maximum displacement decreases with increasing subgrade reaction modulus (k).

The effect of the subgrade reaction modulus (k) on the maximum displacement is larger than that of the other parameters.

4. Conclusions

The dynamic analysis of thick plates on Winkler-type elastic foundation subjected to impulsive load is carried out using an 8-noded Mindlin plate element in which transverse shear effect is taken into consideration and a parametric study is performed to show the effect of the thickness/shorter-span ratio, the aspect ratio, loaded area and the subgrade reaction modulus on the maximum displacement of the plate. The following

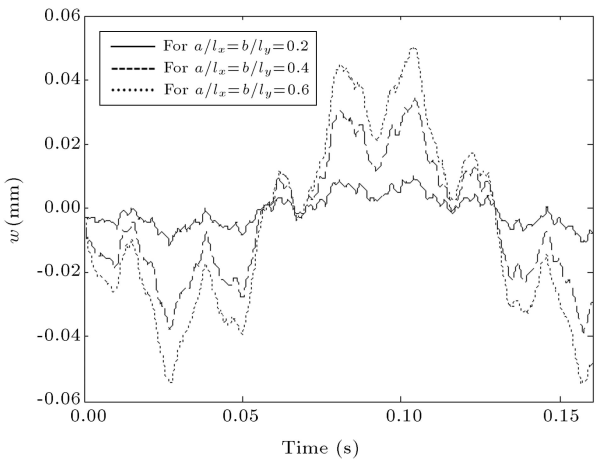
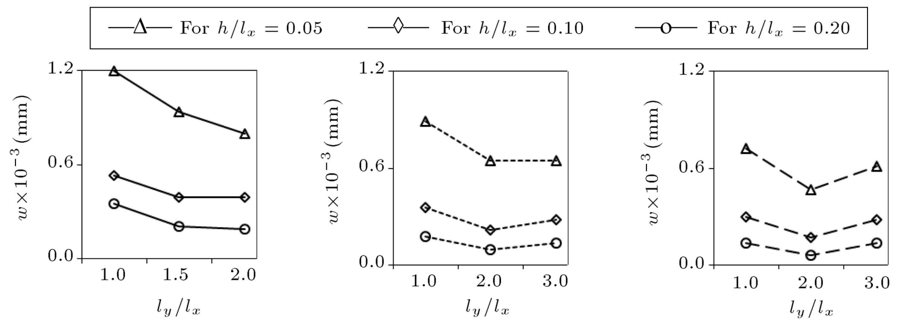


Figure 10. Effect of loaded area on dynamic behaviors of thick plate for $l_y/l_x = 1.0$, $h/l_x = 0.05$ and $k = 500$ kN/m^3, for $a/l_x = b/l_y = 0.2$, $a/l_x = b/l_y = 0.4$, and $a/l_x = b/l_y = 0.6$.

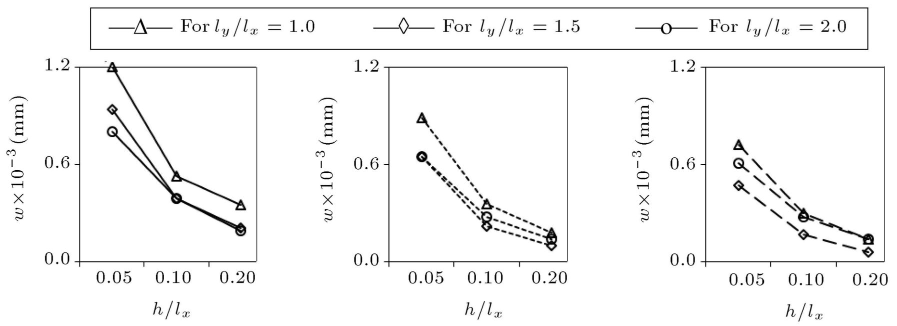
conclusions can be drawn from the results obtained in this study.

- The maximum displacement of the plate decreases with increasing the subgrade reaction modulus for both loading cases.
- The maximum displacement of the plate decreases with increasing thickness/span ratio for loading cases.
- The maximum displacement of the plate increases with increasing aspect ratio for partially distributed load.
- The maximum displacement of the plate decreases with increasing aspect ratio for centrally concentrated load while an increase is expected. But it should be noted that the load on per unit area gets smaller because the concentrated load is kept constant.
- The changes in the maximum displacement decreases for larger value of the thickness/span ratio and the subgrade reaction modulus while the changes in the maximum displacement increases for larger value of aspect ratio.
- The effects of thickness/span ratio and aspect ratio on the maximum displacement of the plate decrease after a certain value of subgrade reaction modulus of the subsoil.
- The subgrade reaction modulus has stronger effect on the maximum displacement of the plate compared to the other parameters.
- Results show that the central displacements increases with increasing the loaded area.



(a) $k = 500$ kN/m^3 (b) $k = 5000$ kN/m^3 (c) $k = 50000$ kN/m^3

Figure 11. The variation of the maximum displacement with l_y/l_x for centrally concentrated load for $h/l_x = 0.05$, $h/l_x = 0.10$, and $h/l_x = 0.20$.



(a) $k = 500$ kN/m^3 (b) $k = 5000$ kN/m^3 (c) $k = 50000$ kN/m^3

Figure 12. The variation of the maximum displacement with h/l_x for centrally concentrated load for $l_y/l_x = 1.0$, $l_y/l_x = 1.5$, and $l_y/l_x = 2.0$.

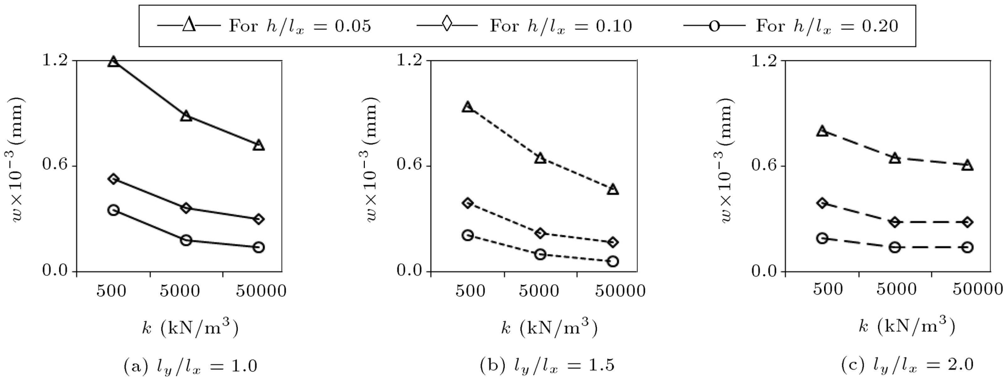


Figure 13. The variation of the maximum displacement with k for centrally concentrated load for $h/l_x = 0.05$, $h/l_x = 0.10$, and $h/l_x = 0.20$.

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Biographies

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