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Prediction of unsaturated soils effective stress parameter using gene expression programming

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Abstract. Unsaturated soil shear strength can be determined using effective stress relation that depends on the effective stress parameter. Several models have been developed in the past few years to estimate this parameter. In this research, the Gene Expression Programming (GEP) is used as an Artificial Intelligence (AI) method for developing a model to predict the effective stress parameter, using efficient parameters. The principal advantage of the GEP approach is its ability to generate powerful prediction equations without any prior assumption on the possible form of the functional relationship. The input terminal set consists of net confining pressure, suction, Soil Water Characteristic Curve (SWCC) fitting parameter, bubbling pressure, residual and saturated volumetric water content. The output terminal set has one member, which is the effective stress parameter. An experimental database obtained from the literature is employed to develop the model. Comparison of the model prediction with the actual data, as well as other investigators, indicates a very good performance and ability of model. Sensitivity and parametric analyses are conducted to verify the results. It is also shown that soil suction is the most influential parameter in the effective stress parameter of unsaturated soils.

1. Introduction

An optimized design of many geotechnical problems above water table such as foundations, earth retaining structures and slopes are based on shear strength of unsaturated soil. Unsaturated soil shear strength may be determined directly in the laboratory [1-5] or indirectly using the developed models. The fundamental goal of the experimental methods is to establish the shear strength characteristics of unsaturated soils in terms of net normal stress and matric suction. The main challenges of the experimental determination of the shear strength of unsaturated soils are generally more complicated, more time consuming and more expensive when compared to conventional test methods for saturated soils. In the indirect method of determining the unsaturated shear strength, two major categories are available that are described below:

1. The models developed by considering two independent state variables, namely suction, S, and mean net stress, P. The model proposed by Fredlund et al. [1] as given below falls in this category.

$$\tau = c' + S\tan\phi^b + P\tan\phi', \tag{1}$$

where:

$\tau =$ Shear strength;

$S =$ $u_a - u_w$;

$P =$ $\sigma - u_a$;

$u_a =$ Pore air pressure;

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$u_w =$ Pore water pressure;

$\phi^b =$ Angle of shearing resistance with respect to matric suction;

$\phi' =$ Effective friction angle;

$c' =$ Effective cohesion.

Several models introduced recently fall in this group [6-8].

2. Effective stress-based method.

In this method, the shear strength is simply expressed in terms of the effective stress by:

$$\tau = c' + \sigma' \tan \varphi', \tag{2}$$

where σ' is the effective normal stress that is expressed by the effective stress equation proposed by Bishop [9] given by:

$$\sigma' = \sigma - u_a + \chi(s), \tag{3}$$

where $\chi(s)$ is the effective stress parameters. Substituting Eq. (2) in Eq. (1) yields:

$$\tau = c' + [P + \chi S] \tan \phi', \tag{4}$$

where χ is Effective stress parameter. This parameter is related to the matric suction with a value of $\chi = 0$ for dry soils and $\chi = 1$ for saturated soils.

Although the value of χ is known to be affected by the soil structure, stress changes and cycles of wetting and drying, this parameter has been expressed in different forms as listed in the next section. There has been a considerable work in the literature to come up with a suitable closed form relationship for the effective stress parameter [8,10,11]. However, the approaches employed so far make certain assumptions in order to arrive at the desired equation.

The main objective of this paper is to employ a powerful approach called Gene Expression Programming (GEP), a branch of artificial intelligence method, to propose a suitable relationship for the effective stress parameter. The main advantage of the GEP approaches over the regression and other soft computing techniques is their ability to generate prediction equations without assuming prior form of the existing relationship. In this study, soil water retention parameters such as bubbling pressure (h_b), residual volumetric water content (θ_r), saturated volumetric water content (θ_s), as well as soil suction (S) and net confining pressure (P), are considered independent variables.

2. Available methods for determining effective stress parameter

There are several methods available for obtaining the effective stress parameter for a particular soil. These methods can be classified into four major groups described below.

2.1. Experimental

The first group contains relationships that are obtained based on experimental result. In this group, Bishop et al. [12] were the first who measured χ for several soils, using volume change and shear strength processes, the results of which are indicated in Figure 1. Bishop and Donald [13] arranged several experiments and plotted the relationship between degree of saturation and χ as shown in Figure 1. Jennings [14] determined the χ by comparing the behavior of a soil specimen under changes in applied suction with the behavior of an identical saturated sample under changes in external pressure. Zerhouni [15] updated Figure 1 initially taken from Jennings and Burland [16].

2.2. Fitting

The second group includes methods that correlate effective stress parameters with unsaturated soil parameters such as matric suction, air entry value, saturated water content and residual volumetric water content. Aitchison [17] gives a fitted expression for the effective parameter, written as follows:

$$\chi = \begin{cases} 1 & \text{if } S_r = 1 \\ \left(\frac{\alpha}{S}\right) S_e & \text{if } S_r < 1 \end{cases} \tag{5}$$

where S is the matric suction, S_e is the air entry suction and α is a coefficient varying from 0.3 to 0.35.

Khalili and Khabbaz [10] demonstrated that the effective stress parameter χ is unity at the suctions below bubbling pressure, and the relationship between

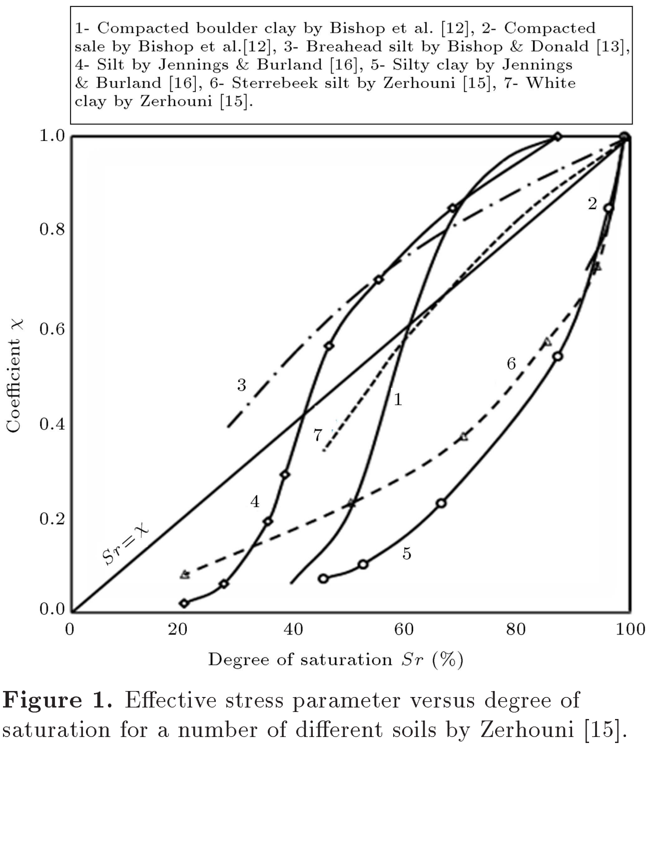

Figure 1. Effective stress parameter versus degree of saturation for a number of different soils by Zerhouni [15].

χ and logarithm of matric suction is linear:

$$\chi = \begin{cases} \left(\frac{S}{h_b}\right)^{\gamma} & \text{for } S > h_b \\ 1 & \text{for } S \le h_b \end{cases} \tag{6}$$

where

γ : Varies from -0.4 (lower bound of the equation) to -0.65 (upper bound), averaging -0.55;

S : Is the matric suction;

h_b : Is the air entry value in the drying process being equal to the air expulsion value in the wetting condition.

The validity of several forms of χ as a function of the degree of saturation was also examined by Vanapalli and Fredlund [18], using a series of shear strength test results for statically compacted mixtures of clay, silt and sand from Escario et al. [19]. For matric suction ranging between 0 and 1,500 kPa, the following two forms showed a good fit to the experimental results:

$$\chi = S_r^k = \left(\frac{\theta}{\theta_s}\right)^k, \tag{7}$$

where S_r is the degree of saturation, θ is volumetric water content, θ_s is the saturated water content, and k is a fitting parameter used to obtain a best-fit between measured and predicted values.

They also proposed an expression for χ in terms of effective saturation or effective volumetric water content as follows:

$$\chi = \frac{S_r - S_r^*}{1 - S_r^*} = \frac{\theta - \theta_r}{\theta_s - \theta_r}, \tag{8}$$

where S_r is the degree of saturation, θ is volumetric water content, θ_s is the saturated water content, θ_r is the residual volumetric water content and S_r^* is the residual degree of saturation. Russell and Khalili [20] developed the following equation for sand:

$$\chi = \begin{cases} 1 & \text{for } \left(\frac{S}{h_b}\right) < 1 \\ \left(\frac{S}{h_b}\right)^{-0.55} & \text{for } 1 < \left(\frac{S}{h_b}\right) < 25 \\ 25^{0.45}\left(\frac{S}{h_b}\right)^{-1} & \text{for } \left(\frac{S}{h_b}\right) > 25 \end{cases} \tag{9}$$

2.3. Theoretical

In this group, the relation for effective stress parameter developed mathematically. In these categories, Xu [11] defined the effective stress parameter, using surface fractal dimension of soil (D_s):

$$\chi = \left(\frac{S}{h_b}\right)^{3-D_s}, \tag{10}$$

where D_s is surface fractal dimension of soil pores.

2.4. Artificial intelligence

Artificial intelligence methods such as Neural Network (NN), Genetic Programming (GP), Gene Expression Programming (GEP), Evolutionary Polynomial Regression (EPR) and other machine learning methods have been used in various disciplines of civil engineering [21-26]. Prediction of effective stress parameter, using artificial intelligence, fall into the fourth group. Kayadelen [27] developed a neural network model with six neurons in the input layer representing the angle of shearing resistance, air entry value, sand fraction, silt+clay fraction, suction and plasticity index. The hidden layer includes: Three neurons and effective stress parameter as output layer. The model square correlation coefficient (R^2) for training data was 0.96.

Ajdari et al. [28] proposed a multilayer perceptron network with six neurons in the input layer representing the air entry value, the volumetric water content at residual and saturated conditions, the slope of soil water characteristic curve, the net confining stress, suction and bias. The hidden layer includes seven neurons and effective stress parameter as the output layer. The model square correlation coefficients for training and testing data were 0.96 and 0.75, respectively.

Gene expression programming by Ferreira [29] is a branch of artificial intelligence and recent extension to genetic programming that develops computer programs of different sizes and shapes encoded in linear chromosomes of fixed length. There have been some scientific efforts directed at applying GEP to the civil engineering tasks [30-34]. The objectives of this paper can be categorized as follows:

- Investigating the feasibility of gene expression programming in order to find dependence of effective stress parameter on soil suction, net stress and parameters defining the Soil Water Characteristic Curve;
- Assessing predictability of the model, using experimental data not exposed to the model during its development;
- Carrying out sensitivity analysis and parametric study, using the developed GEP model;
- Comparing the accuracy of the GEP model with a recent model.

3. Gene expression programming

Gene expression programming is a method for learning the most fit computer programs by means of artificial evolution. It incorporates both the simple linear chromosomes of fixed length similar to Genetic Algorithms (GA) and the ramified structures of different sizes and shapes similar to the parse trees of genetic programming [29,35,36].

Behavior of GEP forms a metaphor of the processes of evolution in nature. GEP, similar to GA and GP, initializes a population that compounds the random members known as chromosomes. Afterwards, fitness of each chromosome is evaluated with respect to a target value. The principle of Darwinian natural selection is used to select and reproduce "fitter" programs. The process continues until a best solution for that problem is reached (Figure 2).

In brief, eight stages are employed in GEP to solve a problem. These stages are shown in Figure 2 and listed below:

1. Generation of a random population of chromosomes (genotype). Each chromosome is a symbolic expression that consists of variables (terminal) and several mathematical operators (function) in the Karva language;
2. Translating chromosomes into computer programs/models (phenotype);
3. Execution of the programs/models generated in the previous step;
4. Performance evaluation of the programs, using the selected fitness function;
5. Selection of the best performing programs;
6. Reproduction of chromosomes, using the best performing individuals programs through genetic operators replication, mutation, transposition and recombination;
7. Development of a new generation of programs as a result of the reproduction in the last step;
8. Re-execution of steps 1 to 7 until the chosen termination criteria are fulfilled.

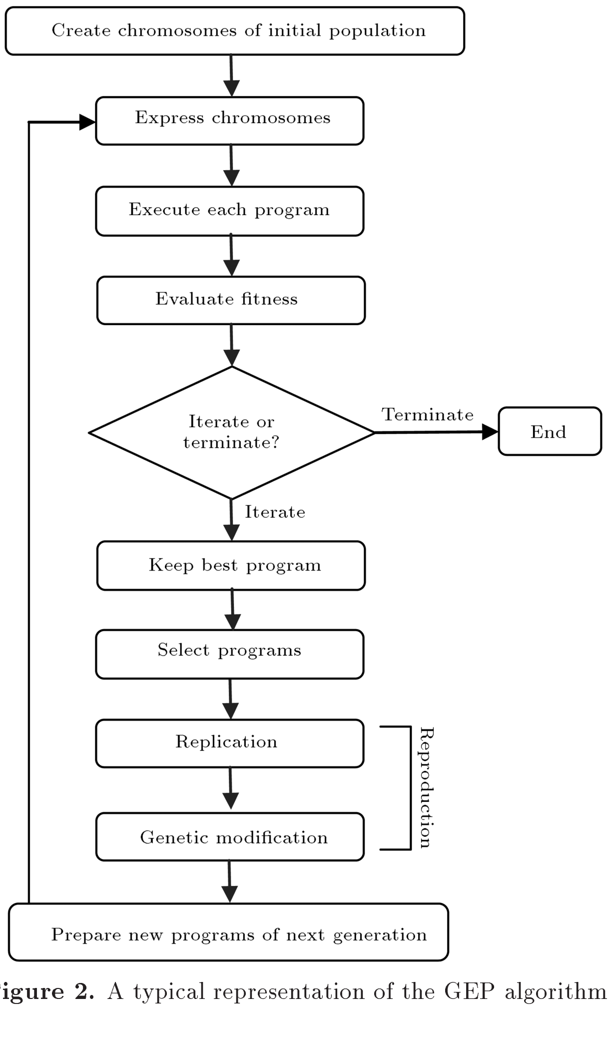

Figure 2. A typical representation of the GEP algorithm.

In GEP application, the chromosome can have one or more genes. The gene contains two types of information. The first type is stored in the head of the gene containing the information which is used in producing the overall GEP model. The head contains some of the functions from the pre-selected function set 'F', along with some terminals from the terminal set 'T'. The second type is stored in the tail, and contains only terminals. The tail contains information that can be used in generating future GEP models. The arrangement of functions and terminals in head and tail of a GEP gene is called its structural architecture. The fundamental structural and functional differences among GEP, GP and GA are summarized in Table 1. It is only in recent years that GEP has found its applications in geotechnical engineering [31-34].

4. GEP modeling of unsaturated soils effective stress parameter

A GEP software, GeneXproTools 4.0 [37] was used in this study to perform symbolic regression, using GEP, to find a formulation for the effective stress parameter. From previous works on the topic, it is well understood that the effective stress parameter is dependent on the SWCC of the soil considered. Hence, any of the SWCC parameters and/or any combination of these parameters may be considered appropriate candidates for inputs of the model. Therefore, six independent parameters, namely net confining pressure, suction, SWCC fitting parameter, bubbling pressure, residual volumetric water content, and saturated volumetric water content were considered as potential input variables. These variables were then converted to dimensionless quantities, listed in Table 2, to serve as input terminals. The output terminal was the effective stress parameter corresponding to the assigned input suction.

A large number of generations were needed to find a formula with minimum error. The formulation selection was based on simplicity and its relevance to the nature of the problem; thus, ensuring a simple and efficient final GEP model. Defining the chromosome structure requires the specification of the number of

Table 1. Comparison of GEP technique to GP and GAs, Ferriera [29].

Genetic programming	Genetic algorithms	Gene expression programming
Population individuals (chromosomes) are non-linear, varying in length as well as shape (also known as 'parse trees')	Population individuals (chromosomes) are linear and of fixed length	Population individuals (chromosomes) are linear and fixed length that are converted to non-linear with varying sizes and lengths (expression trees or computer programs) at a later stage
Uses a single entity working as genome (gene) and phenome (body) at the same time	Similar to GP	Has totally separated genomes and phenomes
Sometimes, invalid expressions can be obtained	Similar to GP	Always produces valid expressions
Not yet established beyond the replicator threshold	Similar to GP	Well established beyond the replicator threshold

Table 2. Range of basic properties adopted for developing the GEP model.

Property	Range
P/P_0^*	0 - 3.95
S/h_b	0 - 300
SWCC fitting parameter, λ	0.19 - 11.82
θ_r/θ_s	0 - 0.714
Effective stress parameter, χ	0.091 - 1

* P, P_0: Net confining stress and atmospheric pressure, respectively.

genes per chromosome, as well as the size of the gene. The head size of gene is the maximum number of functions and terminals that can be stored in the head and the tail of the gene. The size of the gene is normally controlled by its head size and the complexity of the problem.

The stopping criterion for the GEP model evolution process was achieved when the coefficient of determination (R^2) reached 0.8 or more. The process was executed several times, and stopped when no significant changes were noticed in the model statistics (fittness value and R^2).

5. Database

A database consisting of 121 literature's triaxial shear and pressure plate/filter paper test results were used to train and test the GEP model. Frequently, 80 to 85 percentage of database are used for training while the remaining 15 to 20 percentage of data are used for testing the model. In this study, the results from 100 Consolidated Drained (CD) triaxial shear tests (83% of total data) performed on 14 different soil types were collected from the literature and employed to train the GEP model to determine the effective stress parameter under different suctions [19,38-46]. The model was further tested using another database containing 21 Constant Water (CW) data sets (17% of total data) obtained from triaxial shear tests on different soil by Thu et al. [44]. Table 2 indicates the range of basic soil properties adopted for this study. It should be noted that, like all empirical models, GEP performs best in interpretation rather than extrapolation; thus, the extreme values of the data used are included in the training set. For the testing data bubbling pressure, residual volumetric water content, saturated volumetric water content and SWCC fitting parameter were constant and equal to 27kPa, 8.75, 52 and 0.94, respectively.

Due to hydraulic hysteresis, the SWCC has two different branches, one corresponds to adsorption and another to desorption. The soil tends to dilate and absorb water during the softening phenomenon. However, the soil behavior in the strain hardening condition is characterized by a reduction in volume accompanied by flow of water out of the soil specimen. Hence, the drying branch of the SWCC should be employed for CW tests in the strain hardening condition and the wetting branch of the SWCC should be used if the strain softening and strength drop occur.

6. Performance

To set the model parameters a performance analysis was done. In GEP, values of setting parameters have significant influence on the fitness of the output model. These include the number of chromosomes, number of genes, gene's head size and the rate of genetic operators. This approach involved using different settings and conducting runs in steps. During each step, runs were carried out and the values of one of the above mentioned parameters were varied, whereas the values of the other parameters were kept constant [34]. The runs were stopped after one hundred thousand generations, which were found sufficient to evaluate the fitness of the output. At the end of each run, the Mean of Squared Errors (MSE) for both training and testing sets was recorded in order to identify the values that give the least MSE. The results are shown in Figures 3 to 7. Selection of the optimum chromosome was based on the following rules:

- When the output had several generations with similar MSE around the minimum value in the training set, the generation with the lower error for testing data was selected.
- When the difference in MSE between two or three cases was negligible, as in Figure 4, the one which leads to a model with a smaller length (smaller head size or gene number) was selected.

More details are given below:

In the first step, the number of chromosomes was determined. Figure 3 shows that the model has the best performance when the number of chromosomes was 25. This value corresponds to the least MSE for the training sets.

For selecting the number of genes and head size it was tried to find the values with minimum length and acceptable error measure. From Figures 4 and 5, it could then be concluded that the optimum chromosome structure consists of 4 genes of head size=5.

Figures 6 and 7 present the influence of the rates of genetic operators, mutation and gene recombination on the performance of the GEP model. It can be seen that the GEP model performs best when mutation and gene recombination rates are 0.1 and 0.3, respectively.

After obtaining the optimum GEP model, the influence of model operators on the model performance may be investigated. Figure 8 shows the effect of

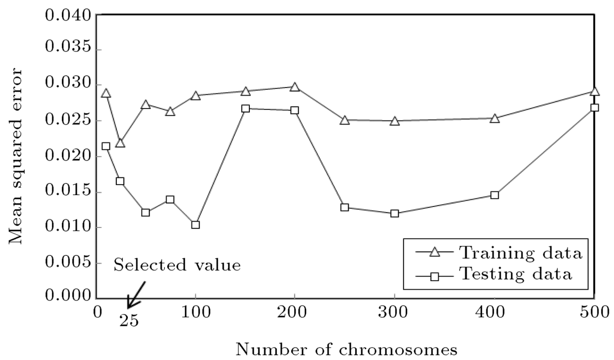

Figure 3. Effect of number of chromosomes on the performance of the GEP model.

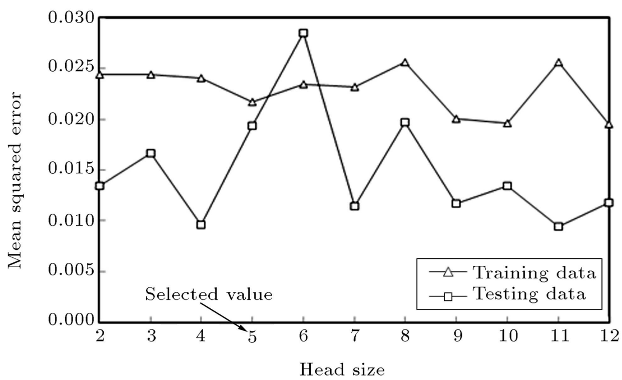

Figure 4. Effect of gene head size on the performance of the GEP model.

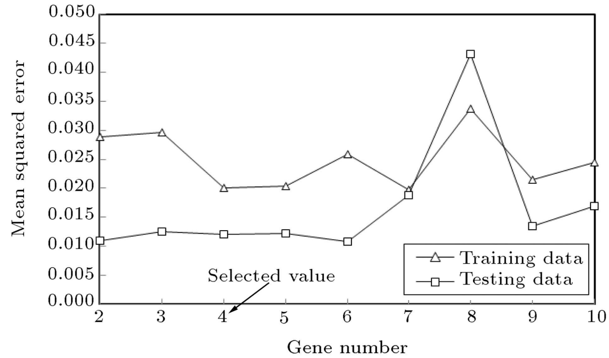

Figure 5. Effect of gene number on the performance of the GEP model.

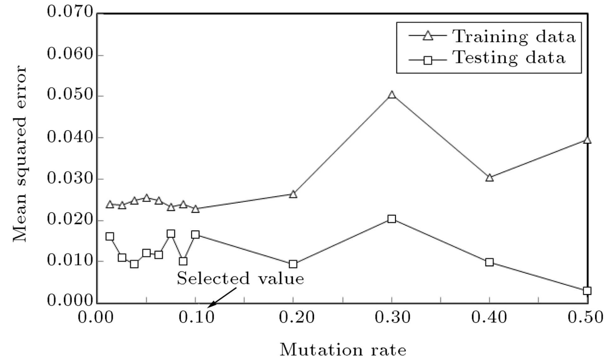

Figure 6. Effect of mutation rate on the performance of the GEP model.

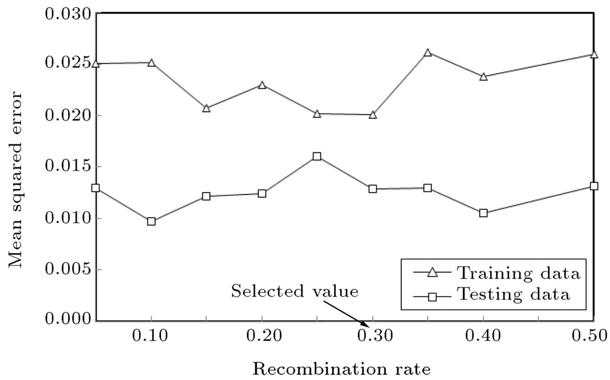

Figure 7. Effect of recombination rate on the performance of the GEP model.

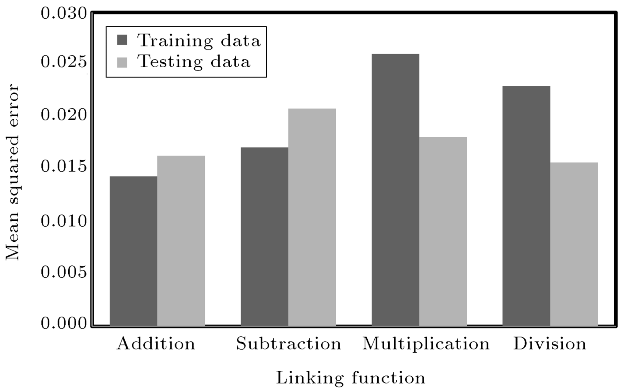

Figure 8. Effect of linking function on the performance of the GEP model.

Table 3. Optimum GEP setting based on performance operation.

Parameters	Achieved functions, values and rates
Linking function	Addition (+)
Number of chromosomes	25
Number of genes	4
Gene head size	5
Recombination rate	0.3
Mutation rate	0.1

the linking function on the performance of the GEP model. It can be seen that the GEP model performs best when linking function is addition. The optimum GEP settings according to performance operation are presented in Table 3.

7. Model development

The optimum GEP program (optimum formulation) was obtained by evolving the programs toward the formulation with minimum error, compared with the actual test results. In this process, performance was also checked using the sum of absolute differences between the predicted and actual values of the effective stress parameter. The average relative error is defined as:

$$\text{Relative Absolute Error (RAE)} = \sum_{i=1}^{n} \left| \frac{A_i - P_i}{A_i} \right| \times 100, \qquad (11)$$

where A_i and P_i are, respectively, the actual and predicted output values for the ith output and n is the number of data.

Iterations continued until this error measure did not decrease appreciably for training and testing data. Figure 9 indicates the variation of error (relative absolute error) measure during model development. The model training error dropped from 2.69 in the first generation to about 0.375 after 100,000 generations and, in testing, the error dropped from 3.37 to about 0.633 in the same generation.

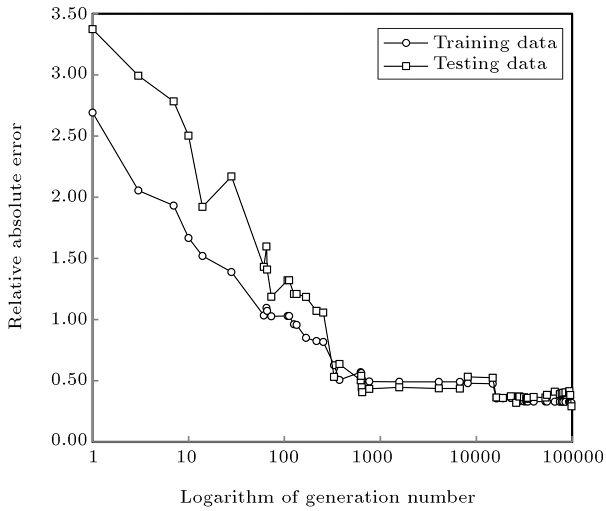

Figure 9. Variation of error measured during training and testing generations.

As mentioned earlier, one of the advantages of the GEP technique is that the relationship between the inputs and corresponding output is automatically constructed in the Expression Trees (ET). In this research the appropriate ETs (ET1 to ET4) that are linked to each other with addition to produce the final model are presented in Figure 10. The trees are easily formulated into a mathematical equation given by:

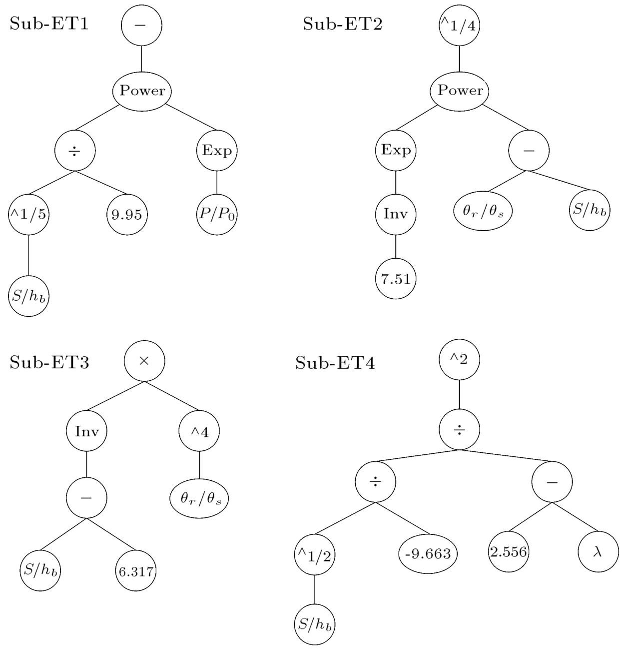

Figure 10. Expression Tree (ET) of the developed GEP mode.

$$\chi = \frac{\left(\frac{S}{h_b}\right)}{93.373(2.556-\lambda)^2} + \frac{\left(\frac{\theta_r}{\theta_s}\right)^4}{\left(\frac{S}{h_b}\right) - 6.317} + \left(\exp(0.133)^{\left(\frac{\theta_r}{\theta_s} - \frac{S}{h_b}\right)}\right)^{0.25} - \left(0.1\left(\frac{S}{h_b}\right)^{0.2}\right)^{\exp\left(\frac{P}{P_0}\right)}, \quad (12)$$

where:

$P =$ Net confining pressure;

$P_0 =$ Atmospheric air pressure (101.325 kPa);

$S =$ Suction;

$\theta_s =$ Saturated water content;

$\theta_r =$ Residual volumetric water content;

$h_b =$ Bubbling pressure;

$\lambda =$ Soil water characteristic curve fitting parameter.

Several GEP models were developed using different arrange of input variables. The performance of four GEP models and effect of input parameter on error of training and testing data-set are shown in Table 4. It can be seen that model 1 has a significantly superior performance. Therefore, the effective stress parameter of unsaturated soils strongly depends on the whole selected input parameters (P/P_0), (S/h_b), (λ), (θ_r/θ_s).

8. Results and discussion

Eq. (12) was used to predict all 100 effective stress parameters of the training set and 21 in the testing set. Figures 11 and 12 compare the predicted effective stress parameter with the actual data for training and testing, respectively. These figures show a good correlation between the predictions made, using GEP formulation, and the actual data, both for training and testing data. Furthermore, the proposed model is compared with the model presented by Russell and Khalili [20]. Figures 13 and 14 show the effective stress parameters predicted the model proposed by Russell and Khalili [20] for training and testing set data, respectively. In these figures, square correlation coefficient, R^2, is used to compare the results given by:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (A_i - P_i)^2}{\sum_{i=1}^{n} \left(A_i - \bar{A}_i\right)^2}, \quad (13)$$

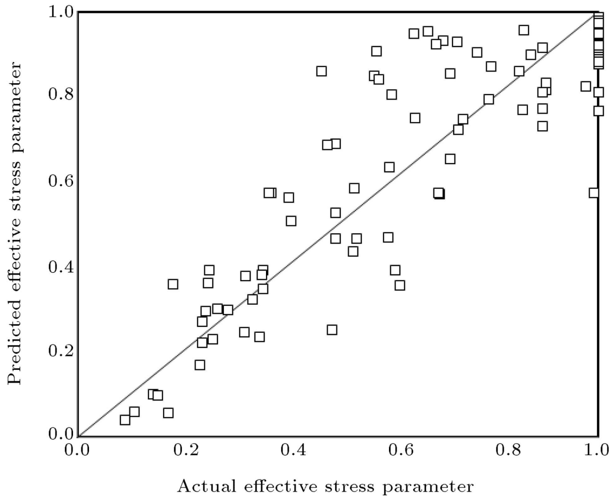

Figure 11. Actual versus predicted effective stress parameter values for training data, $R^2 = 0.81$.

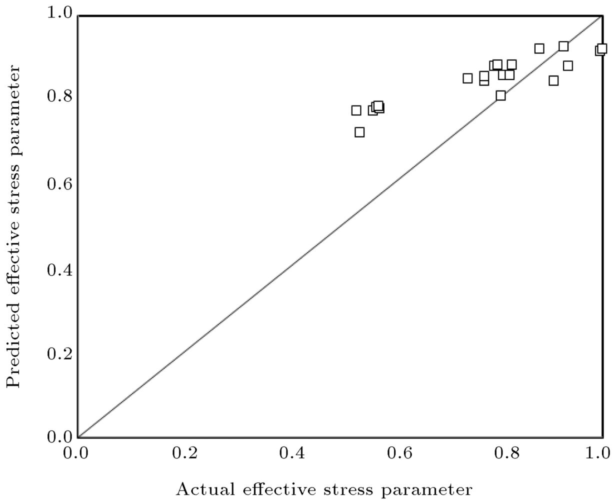

Figure 12. Actual versus predicted effective stress parameter values for testing data, $R^2 = 0.83$.

Table 4. Performance of different GEP models.

	Set type	Training set		Testing set	
Model	**Used variables**	R_2	**RAE**	R_2	**RAE**
1	(P/P_0), (S/h_b), (λ), (θ_r/θ_s)	0.81	0.375	0.83	0.633
2	(P/P_0), (S/h_b), (θ_r/θ_s)	0.78	0.399	0.84	0.613
3	(S/h_b), (λ), (θ_r/θ_s)	0.73	0.434	0.59	0.929
4	(P/P_0), (S/h_b), (λ)	0.74	0.424	0.67	0.685

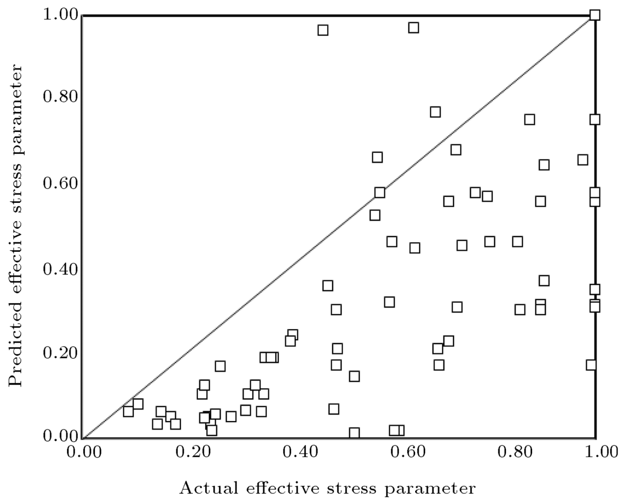

Figure 13. Actual versus predicted effective stress parameter by Russell and Khalili [20] for training data, $R^2 = 0.52$.

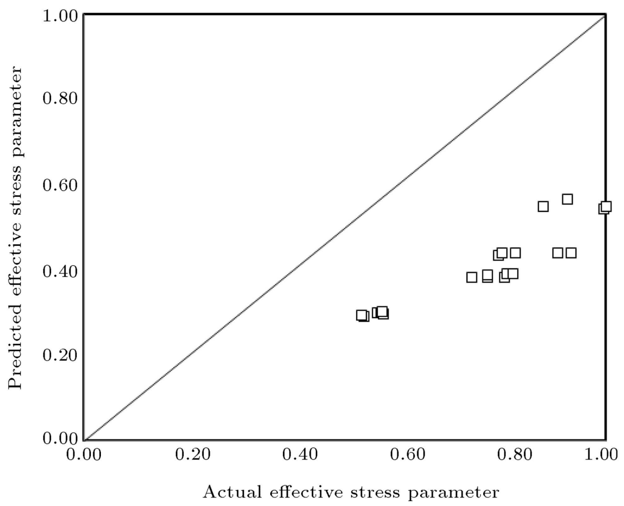

Figure 14. Actual versus predicted effective stress parameter by Russell and Khalili [20] for testing data, $R^2 = 0.848$.

where A_i and P_i are, respectively, the actual and predicted output values for the ith output; $\bar{A}_i$ is the average of the actual outputs; and n is the number of data.

9. Sensitivity analysis

To evaluate the model response to changes in input parameters, a sensitivity analysis was carried out. For this purpose, all input parameters, the normalized net confining pressure (P/P_0), SWCC fitting parameter (λ), and normalized suction by bubbling pressure (S/h_b), ratio of residual volumetric water content to the saturated volumetric water content (θ_r/θ_s) were considered. To evaluate the influence of each parameter on the effective stress parameter, the mean value of the input parameter was increased approximately 20%, while the ranges of the other input parameters were kept constant. The results are given in Table 5. In this table, negative change means reduction and positive means increasing effect on the effective stress parameter. It is shown that, with an increase in P/P_0 and λ, an increase occur in the effective stress parameter. Furthermore, Table 5 shows that with an increase in S/h_b and θ_r/θ_s, the effective stress parameter decreases. This table shows that the applied mean net stress (P/P_0) plays an important role in the effective stress parameter.

10. Parametric analysis

For further verification of the proposed GEP models, a parametric analysis was performed. The main goal was to find that how each parameter affects the effective stress parameter. Figures 15 to 18 present the predicted values of the effective stress parameter as a function of each parameter where others were constant. For this purpose, several arbitrary data sets from training and testing data set were considered for the parametric analysis. Response from typical data set, given in Table 6, was selected to investigate influence of various parameters.

The results of the parametric analysis indicate that as expected, the effective stress parameter continuously increases due to increasing mean net stress and SWCC fitting parameter. The effective stress

Table 5. The change in effective stress parameter corresponding to 20% increase in the mean value of the input parameters.

Parameter	P/P_0	S/h_b	λ	θ_r/θ_s
Change (%)	0.47	-0.174	0.00025	-0.218

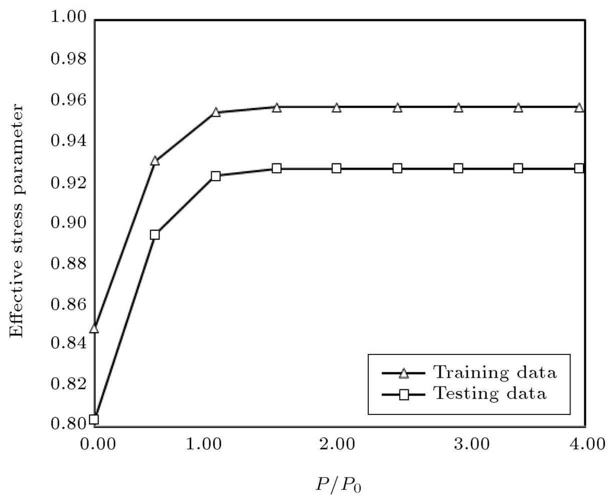

Figure 15. Parametric analysis of output model with respect to P/P_0.

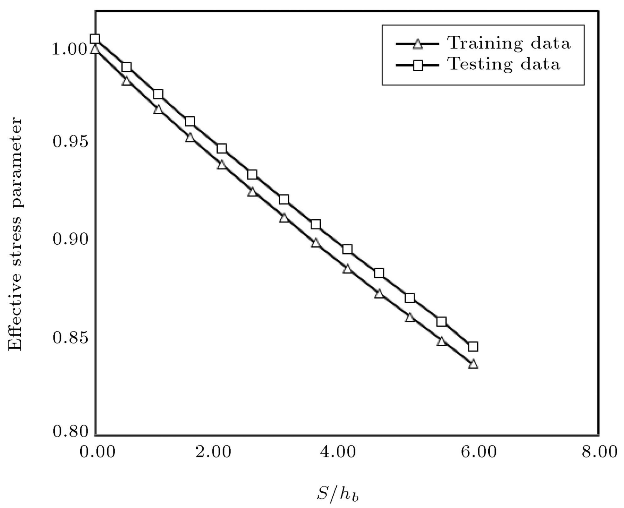

Figure 16. Parametric analysis of output model with respect to S/h_b.

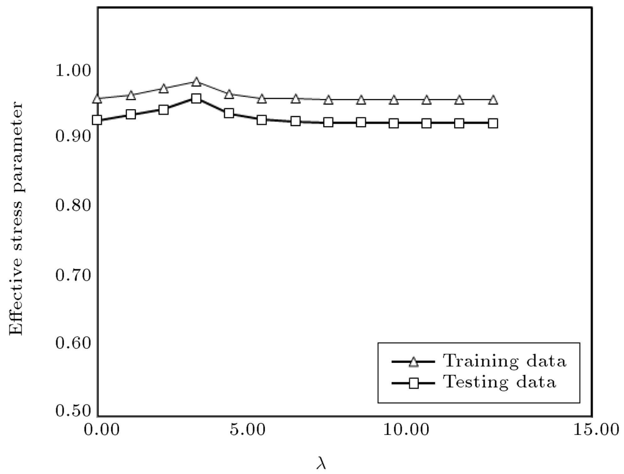

Figure 17. Parametric analysis of output model with respect to SWCC fitting parameter.

parameter decreases when the ratio of suction to bubbling pressure and the ratio of residual to saturated volumetric water content increases.

11. Conclusion

A model based on GEP as an artificial intelligence method was proposed to estimate the effective stress parameter for unsaturated soils. The input model consisted of net confining pressure, suction, soil water characteristic curve fitting parameter, bubbling pressure, residual volumetric water content, and saturated volumetric water content. A database containing the results of 121 literature's triaxial shear and pressure plate/filter paper test carried out was employed to develop the model. The results from 100 consolidated drained triaxial shear tests performed on 14 different soil types were employed to train the model to determine the effective stress parameter under different suctions. The model was further tested using another database containing 21 constant water data sets obtained from triaxial shear tests on different soils. The model prediction indicated a reasonable accuracy both for the results used in the training, as well as the results in the testing. The model prediction compared to the actual test data indicated its good performance for prediction of the effective stress parameter. The sensitivity analysis also showed that the soil suction is the most influential parameter in effective stress parameter. Furthermore, a parametric analysis showed an acceptable trend for the effective stress parameter with changing the input parameters of the model. These models have certain limitations in that they do not take into account the hysteresis phenomena and soil fabric effects.

Table 6. Selected data from training and testing data-set for parametric analysis.

Parameter	P/P_0	S/h_b	λ	θ_r/θ_s
Training	0.987	1.50	0.89	0.0051
Testing	2.960	2.815	0.94	0.1680

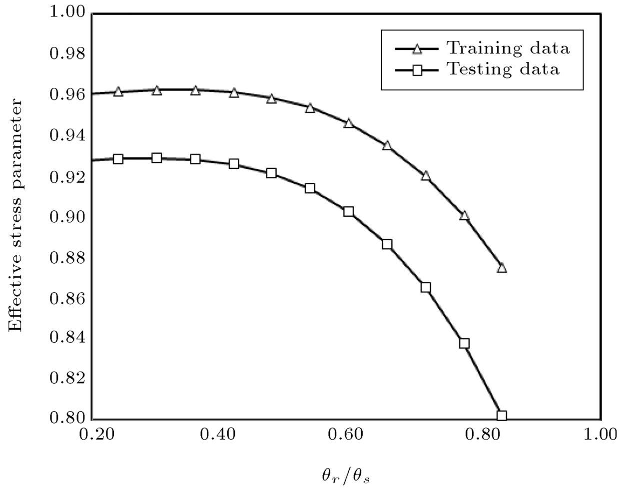

Figure 18. Parametric analysis of output model with respect to θ_r/θ_s.

The authors suggest the following future works for further improvements and extension on the topic:

- Studying other types of AI systems such as EPR;
- Validating the conclusions drawn in this paper as further data becomes available;
- Extending the AI systems to include hysteresis phenomena and soil fabric;
- Reliability assessment of the developed model.

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