Multi-objective open location-routing model for relief distribution networks with split delivery and multi-mode transportation under uncertainty

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Abstract

In this study, the response phase of the management of natural disasters is investigated. One of the important issues in this phase is determining the distribution areas and timely distribution of relief to affected areas in which transportation routing is of a critical matter. In the event of disasters, especially flood and earthquake, terrestrial transportation is not that much easy due to the damage to many infrastructures. For this reason, we propose that delivering relief from the distribution areas to disaster stricken places should be done simultaneously by terrestrial as well as aerial transportation modes to increase route reliability and reduce travel time. In this study, for relief allocation after earthquake, we offer a mixed-integer nonlinear open location-routing model in uncertainty condition. This model includes several contradictory objectives and variety of factors such as travel time, total costs, and reliability. In order to solve this model, a hybrid solution by combining robust optimization and fuzzy multi-objective programming has been used. The performance and effectiveness of the offered model and solution approach has been investigated through a case study on the earthquake in East Azerbaijan, Iran. Our computational results show the solution we have offered for real problems has been effective.

Keywords: Emergency logistics; Relief distribution; Location; Routing; Split delivery; Multi-objective programming; Robust optimization

1. Introduction

Given that natural catastrophe, earthquakes, torrents, and tornados are unpredictable, they can cause severe and lasting damage to countries. Of all events earthquake are the major reason for deaths. Examples of previous years include the deaths of 40 thousand people in 2003 in Bam earthquake in Iran [1], 70 thousand individuals in the 2007 Sichuan earthquake [2], Haiti 2010 earthquake that took the lives of 23,000 thousand ones [3].

As in the time of earthquake or any natural disasters, infrastructures would be ruined, therefore, the supplies and logistics services would be highly requested. Processes in emergency after earthquake should...
be planned in such a way that they can respond to the needs of causalities as fast as possible. The reason is that immediate distribution of emergency supplies can play a key role in reducing the damage and disastrous events. In the meantime, One of the required strategic decision in this regard is determining the place of distribution centers. The places which will be selected to serve as a warehouse need to meet some prerequisites such as physical and economic accessibility, and non-violation of local and state constraints. When the suitable, places of distributions are identified, a group of places which can address the limitations of the system and are in the best conditions are chosen. The objective of the problem may include items such as minimizing the cost, maximizing the demand covering, minimizing the maximum travelling time or a combination of these goals, and so on. In addition to the above-mentioned factors, the efficiency of the system depends on other factors.

In this regard, It is more important to know how the transport fleet are organized and managed as they positively contribute to efficacy of the distribution chain [4]. Therefore, the decision for selecting distribution centers (DC) depends on the number and means of transportation assigned to each DC, the delivery route to disaster areas, and allocation of disaster areas to the established centers [5].

The properties that make the problem closer to actual emergency condition are as follows: (1) at the time of post-earthquake, vehicles stay in last node of their routes and don not get back to DCs until another order is specified, for this reason, each region receiving the relief could be considered as a new distribution center [6]. Open location-routing problem (OLRP) is an emerging issue in the literature. The first study to investigate the issue is the research conducted by Qiu and Xumei [7], (2) In spite of a great demand for relief at the earthquake stricken areas after-earthquake, each of the area received service more than once because of the capacity of the vehicles was quite limited [8]. This method has been referred as split delivery. In their experimental research, Archeti et al. [9], investigated the routing of transportation vehicle and focused on the split-delivery of the demand and showed that when the demand means of the customers fell between fifty and seventy five percents of the capacity of the vehicles and the variance was smaller, better results in terms of profit and savings were achieved.

(3) One of main concern associated with earthquakes which differentiate it from terrorism attack or other natural catastrophe is that aftershocks can put the life of aid workers at jeopardy. Therefore, preparation for the aftermath of earthquake is necessary. One of the aftermath of earthquake is destruction of transport networks between disaster areas , highways, bridges and tunnels . In this case, we can conceptualize reliability as the likelihood of traveling through the network among disaster areas in the post-earthquake time. In this case, aid distribution in disaster increases with high reliability which not only can support the rescue team but also can guarantee timely delivery of necessary facilities to them [10]., (4) in the event of a disaster, especially earthquake, a lot of infrastructures for transportation become unusable or many disaster areas are remote and difficult to reach or are not available by land. In order to provide relief to these areas, air transportation network can be used which can increase the reliability of the route and timely delivery of relief supplies to these regions.

(5) a kind of uncertainty is associated with disorders. This type of uncertainty is resulted from unexpected disasters such as torrent, earthquakes, tornados, financial crises, and terrorist attacks. Dynamic and intricate nature of disaster relief chain imposes a high level of uncertainty in decision-making for logistics planning of relief and effectively affects the function of the chain. Important
parameters such as the demand rate of the damaged areas and also supply, the cost and time of transfer of goods in times of disaster because of incomplete or unavailable data required are inaccurate. As a result, addressing uncertainties can help to have a right strategic, tactical and operational decision [11].

Given the issues outlined, in this study, attempts were made to provide a multi-objective model under uncertainty to location of DCs and allocate disaster stricken regions and transportation vehicles to DCs and designing the routes from DCs to disaster stricken regions by taking into account the split delivery of the demand. Also, in this study, the relief guidance from DCs to disaster stricken regions has been considered by terrestrial and aerial transportation networks simultaneously and the route for all transportation vehicles has been considered open. The purpose of the model are (1) minimizing the fixed costs to establish DCs and the vehicle travelling cost, (2) Minimization of the maximum travel time of the vehicle route,(3) maximizing the minimum reliability of the route for all service vehicles in the process. In addition, a hybrid solution is proposed for better analysis of uncertainties by combining robust optimization and fuzzy multi-objective programming, and finally the efficiency of the model is shown through a case study of the earthquake in East Azarbaijan, Iran.

The paper is organized as follows: In section 2, we will review pertinent literature on relief logistics and LRP. Section 3 includes the statement of the problem and the proposed model. Also, the proposed solution is presented in Section 4. Computational results and the case study to validate the model are presented in Section 5. In section 6, we will make a conclusions and futuer recommendations.

2. Literature review

In general, the location of DCs and routing of transportation vehicles have so far been individually studied for logistics in the state of emergency; however, they heavily depend on each other. There is a dearth of research on the design of mathematical models and solution approaches for the integration of the location and routing in the state of earthquake.

Akkihal [12] proposed a model to location crisis management centers to manage the aid goods. His study was limited to only non-edible supplies such as water refining systems, medical resources, tents, and information tools because such goods are necessary when the disaster starts. The study determines the location of the optimal storage of non-consumable goods for the purpose of distance minimization. In other words, the average interval of any one who is at risk to the nearest facility is minimized by taking into account the limitations related to the lack of resources.

Tzeng et al. [13] proposed a multi-criteria decision model for the distribution of emergency supplies to affected regions. The model involved different factors such as the expenses, response time, customers’ satisfaction, and problem was solved by using fuzzy multi-objective programming. Rajagopalan et al. [14] suggested a multi-period location-covering model to send an ambulance in an emergency case. The purpose of the model was to enhance urgent medical care service to respond the needs at the time of the events. However the vehicle routing was not considered. In another study carried out by Balcik and Beamon [15], the radius of coverage for humanitarian relief was examined. The proposed model considered the financial limitations before and following the natural disaster. Moreover, they defined upper and lower limits for response time to the supply demands from each center. They also showed that aid time should be shorter than this time.

Vitoriano [16] proposed a multi-criteria optimization model founded on cost, time and priority for the delivery of humanitarian assistance. Ozdamar [17] provided an effective programming system for the use of helicopter in disaster relief operations which is a new model and a route management approach. The transfer of necessary items such as blood, medicine, etc. to damaged areas as well as the evacuation
of the damaged people from these locations is among the duties considered for the helicopter. The purpose of this model is to minimize the time needed to perfect the mission of transportation, including flight time, the time for unloading the equipment and boarding the injured. Ozdamar and Demir [18] proposed a hierarchical clustering method and routing to facilitate relief distribution in a large scale. To this end, they employed an algorithm to organize the demand nodes into smaller categories then offer an optimal solution for the vehicle routing. Bozorgi Amiri et al. [11] proposed a multi-objective robust stochastic programming for aid logistics under uncertainty. The study highlighted demand, supply and cost and transportations as uncertain factors and measured them.

Wang et al. [10] developed a multi-purpose relief distribution model for earthquake stricken zones. They solved open location-routing problem for relief after the earthquake. The disaster network in their research consisted of DCs and the disaster stricken regions. In the intended model, only the disaster areas that are accessible via ground transportation have been considered and those which need helicopter or other vehicles are not considered at all. In another study Talarico et al. [19] formulated an routing problem for ambulance at the time of natural catastrophes. In their study, ambulances were used not only to carry health care personnel but also patients. The patients fell into two groups: those who have fewer injuries and could directly get aid and people who are seriously injured and must be transferred to hospital. Because ambulances represent scarce resources in disaster conditions, an efficient use of them is very important. Two mathematical formula were used to obtain the route plans to minimize the total relief time. In continue, categorization in relation to the existing researches has been done in Table 1 in order to show the research gap and innovations of this article.

{Please insert Table 1 here.}

Regarding Table 1, it can be seen that the leading research has favorable innovations compared to the research background. Yet, it can be stated that the leading research is based on the study done by Wang et al. [10] to make the innovation of this research more clearly. In the mentioned research, the researchers provided the location-routing model for aid distribution post the crisis in a definite state. However, the changing and multi-faceted nature of disaster chain can cause a high level of uncertainties in decision associated with logistic planning. Also, only ground transportation has been used for rescue in their model. A point that should be noted here is that when occurring a disaster, transport infrastructures are often unreliable for rescue supplies. That’s why in the leading research, we have tried to remove this dependency by concentrating on the use of air carriers in addition to ground transportation. According to the descriptions provided, in this study, the open location-routing model is developed considering different transportation networks (ground and air) and also robust optimization method provided by Ben Tal et al. [29] is applied to deal with uncertainties in some parameters of the model. The fuzzy multi-objective programming proposed by Torabi and Hassini [30] is used for solving the multi-objective model.

3. Problem definition

In this study, a relief distribution network is intended after earthquakes. Distribution network structure is depicted in Fig 1. In this study, aid distribution includes the location of delivery centers, vehicle routing, and time programming after the earthquake. we offer a multi-purpose open location-routing problem with split delivery of demand. Because, in the event of disasters especially flood and earthquake, terrestrial transportation is not that much easy due to the damage to many infrastructures. In
this study, to aid from DCs to disaster stricken regions by terrestrial and aerial transportation networks is done simultaneously which in turn increases route reliability and reduces travel time. In the other words, in an earthquake, a lot of infrastructures are damaged and can be larger in aftershocks and also some of the disaster areas may not have good condition for helicopters to land and put the lives of relief personnel at risk. In this study, to avoid another causalities casualties, we offer the route reliability of each vehicle as the possibility of deliver relief to all the demand points on route of each vehicle has been successfully.

In the proposed problem, transferring aids from DCs to the disaster stricken regions is done in the way that, first, a subset of DCs are determined to be opened, the suppliers begin their tasks from the established DCs. crisis areas and vehicles will also be assigned to DCs. As soon as the volume of demand needed for each crisis area is larger than the capacity of the vehicle, split delivery is required and that any disaster stricken region can receive service more than once by distinct vehicles. Heterogeneous vehicles with different speeds and capacities have been taken into account and also vehicles in the last node without returning to where they started moving wait until next mission is specified. So each disaster area can be considered as a new distribution center.

3.1. Assumptions

- All transportations are considered in multi-mode (ground and air). In addition to the crisis areas that are available by ground vehicles, those that need helicopters due to the damage to the routs and roads and long-distance have been considered too.
- Fleet are not homogenous and are dissimilar in terms of speeds and capacity.
- The number of disaster stricken regions and DCs is known. The distance between the regions is available. The rout reliability can be detected with advanced technology and communication skills in the real-time.
- Any vehicles is permitted to transfer different kinds of relief in each assignment. This means that any vehicles is permitted to be loaded by different types of relief at the same time.
- Lack of awareness of information (demand, cost, etc): in actual humanitarian operations, it is often seen that in the first phase of disaster response is uncertain. Also, the complex and dynamic nature of various actors in a relief chain suggests the importance of uncertainty in the decision-makings of relief chain. Accordingly, in this model, the cost and amount of relief available is intended as uncertain parameters.

3.2. Notations and definitions

Sets and indices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Set of disaster stricken regions{1, ..., n}</td>
</tr>
<tr>
<td>M</td>
<td>Set of candidate DCs {n + 1, ..., n + m}</td>
</tr>
<tr>
<td>V</td>
<td>Set of node {1, ..., n + m}</td>
</tr>
<tr>
<td>H</td>
<td>Set of aerial vehicles{1, ..., h}</td>
</tr>
</tbody>
</table>
\[ G \] Set of terrestrial vehicles \( \{1, \ldots, g\} \)

\[ L \] Set of aid \( \{1, \ldots, l\} \)

\[ E \] Set of accessible traffic links \( \{(i, j), i, j \in V, i \neq j\} \)

\( i, j \) Indices to nodes \( i, j \in V \)

\( l \) Indices to aid

\( h \) Indices to aerial vehicles

\( g \) Indices to terrestrial vehicles

**Parameters**

\( f_j \) Fixed cost of establishing the DC \( j \), \( \forall j \in M \)

\( d_{ij} \) Distance of link \( (i,j) \), \( \forall (i,j) \in E \)

\( r_{ij} \) Probability of crossing arc \((i, j)\) successfully, \( \forall (i,j) \in E \)

\( D_l \) Quantity of aid \( l \) demanded by crisis area \( i \),

\( w_l \) Unit volume of relief \( l \), \( \forall l \in L \)

\( Q_l \) Amount of aid \( l \) available in traffic network, \( \forall l \in L \)

\( c_g \) Transportation cost per kilometer of terrestrial vehicle \( g \), \( \forall g \in G \)

\( c_h \) Transportation cost per kilometer of aerial vehicle \( h \), \( \forall h \in H \)

\( v_g \) Normal speed of terrestrial vehicle \( g \), \( \forall g \in G \)

\( v_h \) Normal speed of aerial vehicle \( h \), \( \forall h \in H \)

\( l_h \) Loading capacity of aerial vehicle \( h \), \( \forall h \in H \)

\( l_g \) Loading capacity of terrestrial vehicle \( g \), \( \forall g \in G \)

**Decision variables**

\( x_i \) 1, if candidate DC \( j \) is opened, 0, else, \( \forall i \in j \)

\( y_{ijh} \) 1, if \( i \) precedes \( j \) in route of aerial vehicle \( h \), 0, else

\( y_{ijg} \) 1, if \( i \) precedes \( j \) in route of terrestrial vehicle \( g \), 0, else \( \forall g \in G, (i,j) \in E \)

\( z_{ih} \) 1, if \( i \) is on route of aerial vehicle \( h \), 0, else \( \forall h \in H, (i,j) \in E \)

\( z_{ig} \) 1, if \( i \) is on route of terrestrial vehicle \( g \), 0, else \( \forall g \in G, (i,j) \in E \)

\( VF_{ih} \) 1, if the last demand area serviced by aerial vehicle \( h \) is node \( i \in N; 0 \), else

\( VF_{ig} \) 1, if the last demand area serviced by terrestrial vehicle \( g \) is node \( i \in N; 0 \), else

\( d_{l\text{th}} \) Amount of unsatisfied demand aid type \( l \) at node \( i \) at the end of the operation

\( q_{lth} \) Quantity of aid \( l \) distributed by \( h \) to demand area \( i \), \( \forall h \in H, \forall l \in L, \forall i \in N \)

\( q_{ltg} \) Quantity of aid \( l \) distributed by \( g \) to demand area \( i \), \( \forall g \in G, \forall l \in L, \forall i \in N \)

### 3.3. Model

#### 3.3.1. Objective functions

Objective 1: minimizing relief distribution costs: OLRP simultaneously determines the number, location of DCs, assignment of earthquake stricken areas to DCs, and vehicle routes such that The total cost includes two parts: (1) the fixed cost to establish DCs \((j \in M)\) (2) the travel cost of air and ground vehicles. We can write the objective function (1) as below:
Min \( Z_1 = \sum_{j \in M} f_j x_j + \sum_{h \in H} \sum_{(i,j) \in E} c_h d_{ij} y_{ijh} + \sum_{g \in G} \sum_{(i,j) \in E} c_g d_{ij} y_{ijg} \)  

(1)

Objective 2: minimizing the maximum traveling time of vehicle route. Times \( t_{ijg} \) and \( t_{ijh} \) for vehicles \( h \) and \( g \) which are crossing the arc \( (i,j) \in E \), with \( d_{ij} \) and the normal speed of \( V_h \) and \( V_g \) are related to vehicles \( h \) and \( g \). If we do not consider service time (pickup and delivery time), we can assume that the departure time for serving vehicles from DCs is zero, the traveling time for aerial mode of transportation \( (t_h) \) is equal to the total traveling time needed to pass through all the connectors on the vehicle route \( h \).

\[
t_h = \sum_{(i,j) \in E} t_{ijh} = \sum_{(i,j) \in E} \frac{d_{ij} y_{ijh}}{(v_h)}
\]

(2)

And travel time for ground vehicle \( (t_g) \) is equivalent to the total travel time through all connectors on route of vehicle \( g \).

\[
t_g = \sum_{(i,j) \in E} t_{ijg} = \sum_{(i,j) \in E} \frac{d_{ij} y_{ijg}}{(v_g)}
\]

(3)

Objective 2 can be formulated as follows:

\[
Min \ Z_2 = \max \left\{ \sum_{(i,j) \in E} \frac{d_{ij} y_{ijh}}{v_h}, \sum_{(i,j) \in E} \frac{d_{ij} y_{ijg}}{v_g} \mid h \in H, g \in G \right\}
\]

(4)

Objective 3: the maximization of the minimum route reliability: earthquake can seriously damage different infrastructures (bridges, tunnels, etc.) and there is a great probability for more damages in aftershocks and also some of the disaster-stricken regions may not have good condition for helicopters to land and put the lives of relief personnel at risk. In this study, to avoid another casualties, we conceptualize the route reliability of each vehicle as the probability of rescuing the workers to deliver aid to all demand areas on the vehicle route. \( P_g \) and \( P_h \) indicates the reliability of the vehicle \( g \) and \( h \) to complete its relevant distribution activities successfully, assuming that the connections on each vehicle are independent of each other.

\[
P_g = r_{01} \times r_{12} \times \ldots \times r_{(n-1)n} = \prod_{(i,j) \in E_g} r_{ij}
\]

(5)

And

\[
P_h = r_{01} \times r_{12} \times \ldots \times r_{(n-1)n} = \prod_{(i,j) \in E_h} r_{ij}
\]

(6)
The objective 3 which shows the reliability for the entire distribution process in emergency chain is to be formulated as follows:

\[
\text{Max } Z_3 = \min \left\{ \prod_{(i,j) \in E, \gamma_{ijh} = 1} r_{ijh}, \prod_{(i,j) \in E, \gamma_{ijg} = 1} r_{ijg} \quad \forall \; h \in H, \forall \; g \in G \right\}
\]

(7)

3.3.2. Constraints of the model

\[s.t\]

\[x_i \geq y_{ijh}, \forall \; i \in M, (i,j) \in E, h \in H : i \neq j \quad (8)\]

\[x_i \geq y_{ijg}, \forall \; i \in M, (i,j) \in E, g \in G : i \neq j \quad (9)\]

\[x_i \geq z_{ih}, \forall \; i \in M, (i,j) \in E, h \in H \quad (10)\]

\[x_i \geq z_{ig}, \forall \; i \in M, (i,j) \in E, g \in G \quad (11)\]

\[z_{ih} \geq y_{ijh}, \forall \; i \in V, (i,j) \in E, h \in H : i \neq j \quad (12)\]

\[z_{ig} \geq y_{ijg}, \forall \; i \in V, (i,j) \in E, g \in G : i \neq j \quad (13)\]

\[z_{ih} \geq VF_{ih}, \forall \; i \in V, h \in H \quad (14)\]

\[z_{ig} \geq VF_{ig}, \forall \; i \in V, g \in G \quad (15)\]

\[\sum_{i \in V} VF_{ih} = 1, \forall h \in H \quad (16)\]

\[\sum_{i \in V} VF_{ig} = 1, \forall g \in G \quad (17)\]

\[\sum_{i \in H} y_{ijh} + \sum_{g \in G} y_{ijg} \leq 1, \forall (i,j) \in E : i \neq j \quad (18)\]

\[\sum_{i \in H} y_{ijh} \leq 1, \forall (i,j) \in E : i \neq j \quad (19)\]

\[\sum_{g \in G} y_{ijg} \leq 1, \forall (i,j) \in E : i \neq j \quad (20)\]

\[\sum_{j \in V} y_{jih} \leq 1, \forall i \in V, h \in H : i \neq j \quad (21)\]

\[\sum_{j \in V} y_{jig} \leq 1, \forall i \in V, g \in G : i \neq j \quad (22)\]

\[\sum_{i \in M} \sum_{j \in N} y_{ijh} \leq 1, \forall h \in H \quad (23)\]

\[\sum_{i \in M} \sum_{j \in N} y_{ijg} \leq 1, \forall g \in G \quad (24)\]

\[\sum_{i \in M} \sum_{j \in N} \sum_{h \in H} q_{jih} z_{ih} + \sum_{i \in M} \sum_{j \in N} \sum_{g \in G} q_{jig} z_{ig} \leq Q_l \quad \forall l \in L \quad (25)\]
Constraints (8)-(11) ensure that only established distribution centers can provide service. Constraints (12) and (13) guarantee that any vehicle (aerial or terrestrial) can be travel via arc \((i,j)\) if and only if the node \(i\) is in the path of any vehicle (aerial or terrestrial). Constraints (14) and (15) indicate that the node located at the end of the route of any vehicle should be served by the same vehicle. Equations (16) and (17) show that any vehicle (aerial or terrestrial) must remain only in one node finally. Constraints (18)-(20) ensures that only one of the aerial or terrestrial ambulance is selected for each path. Constraints (21) and (22) ensure that any vehicle (aerial or terrestrial) serves once at most for any disaster area. Constraints (23) and (24) ensure that any vehicle (aerial or terrestrial) is dispatched from one DC at most. Constraint (25) ensures that the amount of aid distributed to disaster areas from all DCs does not exceed available amount of relief. Constraint (26) indicates that the relief which is \(l\) allocated to each node is less or equal to the demands of the node. Constraints (27) and (28) ensure that amount of all the aid distributed to earthquake stricken areas by vehicles (aerial or terrestrial) does not exceed their capacity. Equations (29) and (30) express the consecutive movement, and it ensures the assumption of openness of path. Equation (31) shows that each disaster stricken zone can be visited minimum once. The assumption of
split delivery in this constraint has been well illustrated. Constraints (32) and (33) ensure that distribution centers are not related with each other. Equations (34) and (35) are constraints of elimination sub-tours. Constraints (36)-(42) are the limitations on the decision variables.

The presented model was provided in section 3.3 assuming that parameters are certain. In the real world, there is uncertainty in many of these parameters. To get closer to the real conditions in the next section, model has been developed in uncertainty conditions. To develop model, robust optimization approach has been used.

3.4. Linearization of the model

In dealing with the real world decision problems, we face with problems that each of the objective functions or constraints can be considered as a non-linear function. In the model presented in section 3.3, objectives 2 and 3 as well as constraint (25) are nonlinear. In this section, the linearization of the equation is addressed. Non-linear equation (4) which has been defined for the objective function of time in Section 3.3 becomes linear as follows:

\[
\min t
\]

\[
t \geq \sum_{(i,j) \in E} \frac{d_{ij}y_{ijh}}{v_h}, \forall h \in H
\]  

\[
t \geq \sum_{(i,j) \in E} \frac{d_{ij}y_{ijg}}{v_g}, \forall g \in G
\]

Also, non-linear equation (7) becomes linear for the objective function of the reliability as follows:

\[
\max p
\]

\[
p \leq \prod_{\substack{(i,j) \in E, \\ y_{ijh}=1}} r_{ijh} \quad \forall h \in H
\]

\[
p \leq \prod_{\substack{(i,j) \in E, \\ y_{ijg}=1}} r_{ijg} \quad \forall g \in G
\]

In this equation, the probability of \( r_{ijh} \) and \( r_{ijg} \) is between 0 and 1, the variables \( y_{ijh} \) and \( y_{ijg} \) are binary variables if one of the variables \( y \) becomes zero, the objective function value will be zero. So the reliability of the routes must be calculated that vehicles pass through them, so rather than maximizing \( p \), we can maximize \( \dot{p} = a + f(x) \) that can be defined as follows:

\[
\max \dot{p}
\]
\[
\hat{p} \leq \left\{ \prod_{(i,j) \in E} (r_{ijh} y_{ijh} + 1 - y_{ijh}) \right\} \quad \forall h \in H \tag{50}
\]

\[
\hat{p} \leq \left\{ \prod_{(i,j) \in E} (r_{ijg} y_{ijg} + 1 - y_{ijg}) \right\} \quad \forall g \in G \tag{51}
\]

With this change, if one of the variables \( y \) is 0, the target function will not be 0.

\[
(r_{ijh} y_{ijh} + 1 - y_{ijh}) = \begin{cases} 1 & \text{, } y_{ijh} = 0 \\ r_{ijh} & \text{, } y_{ijh} = 1 \end{cases}
\tag{52}
\]

\[
(r_{ijg} y_{ijg} + 1 - y_{ijg}) = \begin{cases} 1 & \text{, } y_{ijg} = 0 \\ r_{ijg} & \text{, } y_{ijg} = 1 \end{cases}
\tag{53}
\]

Now, the nonlinear constraint (25) is converted to linear constraint as follows:

First, we define two positive variables: \( qz_{ijh} \) and \( qz_{ijg} \)

\[
\sum_{i \in M} \sum_{j \in N} qz_{ijh} + \sum_{i \in M} \sum_{j \in N} qz_{ijg} \leq Q_l \quad \forall l \in L \tag{54}
\]

\[
qz_{ijh} \geq q_{ijh} - (1 - z_{ih}) \times bigm
\tag{55}
\]

\[
qz_{ijh} \leq z_{ih} \times bigm
\tag{56}
\]

\[
qz_{ijh} \leq q_{ijh}
\tag{57}
\]

\[
qz_{ijg} \geq q_{ijg} - (1 - z_{ig}) \times bigm
\tag{58}
\]

\[
qz_{ijg} \leq z_{ig} \times bigm
\tag{59}
\]

\[
qz_{ijg} \leq q_{ijg}
\tag{60}
\]

3.5 Robust counterpart mathematical model-box uncertainty

In order to develop the robust counterpart of the proposed model, the fixed cost of opening distribution centers, transportation costs between the points and the amount of first aid available in network are the uncertain parameters. In robust optimization approach, linear programming model changes to its robust counterpart model by putting any constraint with uncertain coefficients with constraint that explanatory the set of uncertain. In our model, we changed each factors of uncertain in a closed bounded box [29, 31]. The overview of the box is as follows:

\[
u_{Box} = \left\{ \xi \in \mathbb{R}^n : \| \xi_t - \bar{\xi}_t \| \leq \rho G_t, \quad t = 1, 2, ..., n \right\}
\tag{61}
\]
Where $\bar{\xi}_t$ is the $t^{th}$ normal value or the $t^{th}$ parameter of vector $\xi$ and the positive value of $G_t$ represents a "scale of uncertainty" and $\rho > 0$ is "level of uncertainty". A particular case in which $G_t = \bar{\xi}_t$ is related to the state in which the relative deviation $\xi_t$ of nominal data is as large as $\rho$. We suggest the following sources [29, 32] for further information on robust optimization approach. It should be noted that in order to simplify the robust optimization approach in the objective function first, $(c_h \ d_{ij})$ then $(A_{ijh})$ and $(c_g \ d_{ij})$ and $(B_{ijg})$ are considered. Therefore, we have:

\[ \begin{align*}
\text{Min } Z_1 &= \sum_{i,j \in E} d_{ij} y_{ijh} v_h, \quad \sum_{i,j \in E} d_{ij} y_{ijg} v_g \quad h \in H, g \in G \\
\text{Min } Z_2 &= \max \left\{ \sum_{i,j \in E} d_{ij} y_{ijh} v_h, \quad \sum_{i,j \in E} d_{ij} y_{ijg} v_g \quad h \in H, g \in G \right\} \\
\text{Max } Z_3 &= \min \left\{ \prod_{i,j \in E, y_{ijh}=1} r_{ijh}, \quad \prod_{i,j \in E, y_{ijg}=1} r_{ijg} \quad \forall h \in H, \forall g \in G \right\} \\
\text{s.t.} \quad \sum_{j \in M} (f_j x_j + \eta_j) + \sum_{h \in H} \sum_{i,j \in E} (A_{ijh} y_{ijh} + \eta_{ijh}^d) + \sum_{g \in G} \sum_{i,j \in E} (B_{ijg} y_{ijg} + \eta_{ijg}^b) \\ &\leq Z_1 \\
\rho_f G_j^f &\leq \eta_j^f \quad \forall j \\
\rho_f G_j^f &\geq -\eta_j^f \quad \forall j \\
\rho_A G_{ijh}^d &\leq \eta_{ijh}^d \quad \forall i, j, h \\
\rho_A G_{ijh}^d &\geq -\eta_{ijh}^d \quad \forall i, j, h \\
\rho_B G_{ijg}^b &\leq \eta_{ijg}^b \quad \forall i, j, g \\
\rho_B G_{ijg}^b &\geq -\eta_{ijg}^b \quad \forall i, j, g \\
x_i &\geq y_{ijh}, \forall i \in M, (i,j) \in E, h \in H \quad i \neq j \\
x_i &\geq y_{ijg}, \forall i \in M, (i,j) \in E, g \in G \quad i \neq j \\
x_i &\geq z_{ih}, \forall i \in M, (i,j) \in E, h \in H \\
x_i &\geq z_{ig}, \forall i \in M, (i,j) \in E, g \in G
\end{align*} \]
\[ z_{ih} \geq y_{ijh}, \forall i \in V, (i, j) \in E, h \in H : i \neq j \]  
(76)

\[ z_{ig} \geq y_{ijg}, \forall i \in V, (i, j) \in E, g \in G : i \neq j \]  
(77)

\[ z_{ih} \geq VF_{ih}, \forall i \in V, h \in H \]  
(78)

\[ z_{ig} \geq VF_{ig}, \forall i \in V, g \in G \]  
(79)

\[ \sum_{i \in V} VF_{ih} = 1, \forall h \in H \]  
(80)

\[ \sum_{i \in V} VF_{ig} = 1, \forall g \in G \]  
(81)

\[ \sum_{h \in H} y_{ijh} + \sum_{g \in G} y_{ijg} \leq 1, \forall (i, j) \in E : i \neq j \]  
(82)

\[ \sum_{h \in H} y_{ijh} \leq 1, \forall (i, j) \in E : i \neq j \]  
(83)

\[ \sum_{g \in G} y_{ijg} \leq 1, \forall (i, j) \in E : i \neq j \]  
(84)

\[ \sum_{j \in V} y_{jih} \leq 1, \forall i \in V, h \in H : i \neq j \]  
(85)

\[ \sum_{j \in V} y_{jig} \leq 1, \forall i \in V, g \in G : i \neq j \]  
(86)

\[ \sum_{i \in M} \sum_{j \in N} y_{ijh} \leq 1, \forall h \in H \]  
(87)

\[ \sum_{i \in M} \sum_{j \in N} y_{ijg} \leq 1, \forall g \in G \]  
(88)

\[ \sum_{i \in M} \sum_{j \in N} \sum_{h \in H} q_{jih} z_{ih} + \sum_{i \in M} \sum_{j \in N} \sum_{g \in G} q_{jig} z_{ig} \leq \overline{Q}_l - \rho Q G^G_Q \quad \forall l \in L \]  
(89)

\[ d_{eV_{jl}} = D_R - \left( \sum_{h \in H} q_{jih} + \sum_{g \in G} q_{jig} \right) \geq 0, \forall j \in N, l \in L \]  
(90)

\[ \sum_{j \in N} \sum_{l \in L} u_{l} q_{jih} \leq L_h, \forall h \in H \]  
(91)

\[ \sum_{j \in N} \sum_{l \in L} u_{l} q_{jig} \leq L_g, \forall g \in G \]  
(92)
4. Solution approach

In order to solve multi-objective mathematical programming models (MOLP), different methods have been proposed in previous studies. Among these methods, fuzzy programming approaches have wide application. The first fuzzy solution for MOLP problems developed by Zimmermann [33] is min-max method; however, Solutions offered by this method were far from inefficient and ineffective [34]. So then, several methods were proposed to overcome this defect. Mahaptra et al. [35] to solve their multi-objective problem, they improved Max-Min Method. In this improved approach, the decision maker can achieve the optimal results due to his expectations. Also, Islam and Roys [36] presented a new fuzzy multi-objective planning method called PGP. To develop the weakness of min-max approach, Lai and Hwang [34] provided an interactive fuzzy approach called LH method to solve MOLP problems. Also,
Selim and Ozkarah [37] presented a new fuzzy method called MW for solving multi-objective problems. In this method, they took advantage of a modified merged function based on Werner [38] method. Li et al. [39] proposed a two-stage fuzzy model called LZL. Finally, Torabi and Hassini [30] performed these models for solving their multi-objective problem in a series of elementary numerical experiments and found some defects. Single-step models such as LH and MW solve the main model straight by an adjunctive certain model. Sometimes LH model provides ineffective solutions dominated by the solution of LZL model. MW model usually gives a useful solution which is poorly compromised solution. Thus, satisfaction degree of objectives would have significant differences and this wouldn’t be acceptable for decision-maker. However, LZL always produces an efficacious solution; but this is a two-stage a that requires more calculations compared with single-stage models. Therefore, Torabi & Hassini proposed a new single-stage fuzzy solution called TH for solving multi-objective problems which eliminates the disadvantages of previous methods. Their TH method is actually a combination of LH and MW methods. A solution method by combining the method provided in antecedent section and a fuzzy solution approach derived from Torabi and Hassini [30] are used in this research to solve the offered model.

Steps in the offered hybrid solutions approach are summarized as follows:

**Step 1:** determining the parameters and variables of uncertainty and considering the distribution functions needed to use in the model.

**Step 2:** formulating the proposed model with the parameters defined in the previous step.

**Step 3:** converting the constraints of mixed-integer programming model to constraints of the certain counterpart by applying the approach outlined in the antecedent section.

**Step 4:** converting the robust model to the equivalent certainty model by applying the approach outlined in the previous section.

**Step 5:** determining the positive ideal and negative ideal solution for $\alpha$ for every objective function in which $\alpha$ is possible level. To calculate the positive and negative ideal solution i.e. $(W_1^{PIS}, x_1^{PIS})$ and $(W_2^{PIS}, x_2^{PIS})$ of each certainty model is separately solved for each of the objective functions and the positive ideal solution is obtained, and then the negative ideal solution is estimated as follows:

$$W_1^{NIS} = W_1(x_2^{PIS}), W_2^{NIS} = W_2(x_1^{PIS}).$$

**Step 6:** calculating a linear membership function using the formulation. below. This is used for each objective function.

$$\mu_1(x) = \begin{cases} 
1 & \text{if } W_1 < W_1^{PIS} \\
\frac{W_1^{NIS} - W_1}{W_1^{NIS} - W_1^{PIS}} & \text{if } W_1^{PIS} \leq W_1 \leq W_1^{NIS} \\
0 & \text{if } W_1 > W_1^{NIS}
\end{cases} \quad (107)$$

$$\mu_2(x) = \begin{cases} 
1 & \text{if } W_2^{NIS} \leq W_2 \leq W_2^{PIS} \\
\frac{W_2^{PIS} - W_2}{W_2^{PIS} - W_2^{NIS}} & \text{if } W_2 < W_2^{NIS} \\
0 & \text{if } W_2 > W_2^{PIS}
\end{cases} \quad (108)$$

In fact, $\mu_h(x)$ represents the satisfaction degree of the $h^{th}$ objective function. It should be noted that $\mu_1(x)$ has been used for minimization objective functions and $\mu_2(x)$ for maximization objective function.
Step 7: converting the certainty mixed integer programming model to a certainty single-objective mixed integer programming model using the integrated function which is calculated as follows:

$$\max \lambda(x) = \psi \lambda_0 + (1 - \psi) \sum_h \theta_h \mu_h(x)$$

s.t

$$\lambda_0 \leq \mu_h(x), \quad h = 1,2$$

$$x \in F(x), \quad \lambda_0 \text{and } \lambda \in [0,1]$$

Which $\mu_h(x)$ and $\lambda_0 = \min \{\mu_h(x)\}$ show respectively the satisfaction degree of $h^{th}$ objective function and minimum degree of objectives satisfaction. This formulation is determined as a convex combination from the lower bound of satisfaction degree of functions ($\lambda_0$) and total weighted of these degrees to achieve ($\mu_h(x)$) which guarantees to obtain a balanced solution. In addition, $\theta_h$ and $\psi$ indicate respectively relative importance of $h^{th}$ objective function and coefficient of restitution. Parameter $\theta_h$ is specified by decision-maker based on $\sum_h \theta_h = 1$ and $\theta_h > 0$. Also, the parameter $\psi$ controls the minimum level of objectives satisfaction and the importance degree between the objectives. TH method is capable to obtain balanced and unbalanced solutions based on preferences of decision-maker through setting the parameter $\psi$. To have more value for $\psi$ parameter means that there has been high consideration for obtaining a big low bound for satisfaction degree of objectives ($\lambda_0$) and more objective solution are achieved. In contrast, having low value for $\psi$ means that there has been high consideration for obtaining an optimal solution showing a high level of satisfaction for same relative important objectives regardless of satisfaction degree of other objectives [30].

Step 8: determining the parameters $\theta_h$, $\rho$ and $\psi$ and solving single-objective models created in the previous step. If the answer is satisfactory for decision makers, it stops; otherwise, in order to achieve new answers, change the values of parameters $\psi$ and $\rho$ and if needed, change the value of $\theta_h$.

5. Computational results

In this section, in order to evaluate the validity of the offered model and show the efficiency of the offered solution methodology, first, a case study for relief distribution in the post-earthquake is presented, and then to further investigate the performance of the model, a series of additional tests are randomly generated.

5.1. The case study

The earthquake of East Azarbaijan province in Iran, the southern region of Arasbaran (towns of Ahar, Varzaghan and Harris) with a magnitude of 6.2 on the Richter scale caused the death of about 320 people, destroyed more than 160 villages and 250 villages were damaged. Ahar-Varzaghan earthquake during the last century along with 1930 Salmas earthquake and 1996 Golestan Ardebil earthquake are among the most devastating earthquakes in northwest provinces of Iran. The earthquake was felt in most Northern provinces of Iran and neighboring countries (Azerbaijan, Armenia and Turkey) in a radius of over 300 km. The events of landslides and rock fall because the area was mountainous, cutting off the communications; breakdowns on the roads and damaged bridges faced the rescue groups with additional
problems and in the event of the main earthquake and its aftershocks disturbed the traffic of the rescue teams.

In this study, three cities (kharvana, Tabriz, Duzduzan) where the intensity of earthquake was less and without serious injury were considered as candidates for establish distribution centers. Also, 11 areas among Ahar, Haris, and Varzeghan where the damage was so severe were targeted for relief. Relief items intended for distribution that were needed in the early hours after the event, including water and tents are for the temporary housing of people. Also, one helicopter and two trucks were considered to transport goods to these areas. Reliefs from other provinces and towns of the country were collected that is not within the scope of this research.

Information on candidate distribution centers is shown in Table 2. The parameters of the reliefs is given in Table 3 in which the size of a tent and boxes of mineral water have been used as a benchmark for measuring the equivalent volume. The capacity and speed of each vehicle is shown in Table 4. Relief needs and demand for each earthquake stricken area on the first day after the earthquake is shown in Table 5. It should be noted that for the information on the deployment of vehicles for transport, network traffic should be determined which can be obtained through aerial photos taken instantly post the earthquake. In addition, based on the human resources and related equipment, the plan, length, and the reliability of the roads will be known and estimable. (See Tables 2-5 in The Appendix A)

The study has been solved for two uncertainty and certainty conditions using the approach proposed in section 4 by GAMS Software. In this solution, the importance of the objective functions has been considered as \((\theta =0.3, 0.3, \text{and} 0.4)\), respectively. Also, the penalty factor is taken into account as \((\varphi = 0.4)\). To solve the problem in uncertainty condition, uncertainty has been considered at three levels \((\rho = 0.3, 0.5, 0.7)\). Fig. 2 shows the solution under certainty. As it is observed, distribution center No. 13 is opened and vehicles appropriate to the route and the demand of the disaster area have been allocated. It can be seen that in order to give service to the demand point No.6, helicopter has been used that could decrease not only the frequency of distribution but also the travel time. It can also increase the route reliability. The resulting total cost is 8583.5 and the latest travel time is 16 hours, and the reliability achieved is 0.81. Fig.3 shows the solution under uncertainty condition in which the distribution center No. 14 has been opened and the route for the vehicles have been recovered as shown in the Fig. In uncertainty condition, the value obtained for the total cost is 11710.79, the latest travel time is 18.08 and the reliability is 0.805. The solution time under certainty is 14 minutes and 37 seconds, whereas in uncertainty, the solution time increased and lasted for 16-minute and 40 seconds. In the following, in Table 6 the percentage of the estimated demand in disaster locations has been determined under uncertainty and certainty conditions.

5.2. Generating testing problems
To demonstrate the credit of the proposed model and the efficiency of the proposed solution, in addition to the case study offered in the antecedent section, several tests were run and the numerical results are reported in this section. To this end, five problems with different aspects have been considered and the results have been shown under both certainty and uncertainty conditions in Tables 6 and 7 at different levels and different degrees of importance for the objective functions. The tests are conducted under three levels of uncertainty ($\rho = 0.3, 0.5, \text{ and } 0.7$). Also, the level of uncertainty in each test problem has been considered the same for all parameters. In the problems, the parameters and required information have been generated based on the data and information the studied earthquake. As shown in Table 7, all uncertainty problems have answers worse that certainty ones because in robust optimization approach, we consider the worst situation until in action, we face to the lowest harm and risk. In addition, with regard to of Table 8, it can be concluded that TH method obtains exclusive solutions for every degree of importance for objective functions. In total, it can be concluded that TH method is an appropriate and eligible way for multi-objective programming problems because they are able to obtain more efficient and effective results.

{Please insert Table 7 here.}

{Please insert Table 8 here.}

In problem No. 1, the impact of the penalty rate ($\psi$) on objective functions has been investigated under certainty and uncertainty conditions. For this purpose, 8 experiments with different penalty rate in the range of $(0.1, 0.8)$ were generated and analyzed. As can be seen in table 9 and figs 4-6, balanced and imbalanced solutions for a problem is obtained based on decision-maker preferences as $\theta = (0.3, 0.3, 0.4)$ via setting the parameter $\psi$. The results indicate that balanced solutions are produced for the values of $\psi$ between 0.4-0.6 for the problem.

{Please insert Table 9 here.}

In Table 10, given $\rho = 0.3$ and $\theta = (0.3, 0.3, 0.4)$ for each problem in certainty and robust cases, the established DC, the route of any vehicle and the value of functions are provided separately.

{Please insert Table 10 here.}

In Figs 7-9, sensitivity analysis of the level of uncertainty on the objective function is depicted in which using the data related to problem 1, different levels of uncertainty have been investigated. As it is shown, it cannot be said that the relationship between the level of uncertainty and the objective function is a direct relationship. A sensitivity analysis for more validity of the efficiency of the model and grade of the penetration of parameters in the model are investigated. As it can be seen in Figs 10 and 11, with an increase in the mean distance the value of the objective function increases too.

{Please insert Fig 4 here.}
Another effective parameter in the model is Reliability of any connection that is different for Air and ground transportation network. As can be seen in Figures 12-14, Cost increases by reducing the reliability, but travel times and reliability for the entire route decrease.

6. Conclusion and future research

In this study, a mixed-integer nonlinear open location-routing model for the delivery of aid after the earthquake with multiple objectives including the time travel, the total cost and reliability was presented. Then, using a robust optimization approach, the proposed model was developed in uncertainty conditions. Finally, the proposed model was solved using fuzzy multi-objective programming. For the performance of the model, a case study of East Azarbaijan earthquake was presented and the certainty and uncertainty results were compared with each other. According to the obtained results, it is believed that the proposed model can be effective and valid methodology to manage relief distribution in an uncertain condition. However, the time solution for proposed model in either a certain or uncertain state is reasonable for small sizes.

In the future research, The model can also be considered as multi-period as well as with regard to repair the damaged roads. Also, in addition to carrying essential goods to the earth quack-stricken areas, the discharge evacuation of the injured people from these locations and their transfer to emergency units can be considered in this model. Since location-routing problems are NP-Hard, a variety of solutions of meta-heuristic algorithms can be considered for future research.

Appendix A: information about parameters

{Please insert Table 2 here.}

{Please insert Table 3 here.}
References


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Fig1. Network for OLRP in emergency logistics
Fig2. Transportation network in certainty condition
Fig3. Transportation network in uncertainty condition

Fig4. Objective function (1) and penalty rate
Fig 5. Objective function (2) and penalty rate

Fig 6. Objective function (3) and penalty rate
Fig 7. Objective function (1) and uncertainty level

Fig 8. Objective function (2) and uncertainty level
Fig 9. Objective function (3) and uncertainty level

Fig 10. Objective function (1) and average distance
Fig11. Objective function (2) and average distance.

Fig12. Objective function (1) and average reliability
Fig 13. Objective function (2) and average reliability

Fig 14. Objective function (3) and average reliability
### Table 1
Categories research in the relief chain

<table>
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<th>Reference</th>
<th>location</th>
<th>Routing</th>
<th>Split delivery</th>
<th>Single-objective</th>
<th>Multi-objective</th>
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<th>uncertain</th>
<th>Single transport</th>
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<th>Case study</th>
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### Table 2
Candidate DCs parameters

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<td>Kharvana $j_{12}$</td>
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<td>Duzduzan $j_{13}$</td>
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<td>Tabriz $j_{14}$</td>
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### Table 3
Parameters of relief commodity

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<th>Item</th>
<th>Available quantity $Q_i$</th>
<th>(cm$^3$) cUnit volume</th>
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<tr>
<td>Mineral water</td>
<td>35000</td>
<td>45× 25 × 11</td>
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<tr>
<td>Tent</td>
<td>12000</td>
<td>36× 26 × 30</td>
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### Table 4
Parameters of the vehicles

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<th>Normal velocity (km/h)</th>
<th>Cost per unit of length ($)</th>
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<td>Helicopter 1</td>
<td>750 × 330 × 280</td>
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### Table 5
Parameters of the disaster areas

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<th>Demand points</th>
<th>Demands ($D_{i1}, D_{i2}$)</th>
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<td>Sarand $i_{10}$</td>
<td>(1000, 400)</td>
</tr>
<tr>
<td>Bashir $i_{11}$</td>
<td>(900, 300)</td>
</tr>
</tbody>
</table>

### Table 6
Demand fill rate (%)

<table>
<thead>
<tr>
<th>Demand points</th>
<th>Deterministic</th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khormalou $i_1$</td>
<td>81</td>
<td>57</td>
</tr>
<tr>
<td>Varzeqan $i_2$</td>
<td>95</td>
<td>100</td>
</tr>
<tr>
<td>Bakhshayesh $i_3$</td>
<td>81</td>
<td>80</td>
</tr>
<tr>
<td>Mehraban $i_4$</td>
<td>92</td>
<td>75</td>
</tr>
<tr>
<td>Ahar $i_5$</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Heris $i_6$</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Khaje $i_7$</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 7
Sensitivity analysis of the level of uncertainty (\(\rho\)) given that \(\psi = 0.4\)

<table>
<thead>
<tr>
<th>Test problem</th>
<th>(\theta_1) Value</th>
<th>Deterministic</th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>(m)</td>
<td>(h)</td>
<td>(g)</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>&amp; 0.3</td>
<td>(16901.85, 0.90)</td>
<td>(17.96, 0.53)</td>
<td>(0.77, 0.59)</td>
</tr>
<tr>
<td>&amp; 0.7</td>
<td>(23002.36, 0.89)</td>
<td>(19.21, 0.55)</td>
<td>(0.652, 0.62)</td>
</tr>
<tr>
<td>&amp; 0.3</td>
<td>(18313.7, 0.28)</td>
<td>(10.70, 0.61)</td>
<td>(0.90, 0.85)</td>
</tr>
<tr>
<td>&amp; 0.5</td>
<td>(21713.40, 0.37)</td>
<td>(13.44, 0.46)</td>
<td>(0.80, 0.75)</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>&amp; 0.3</td>
<td>(4287.14, 0.66)</td>
<td>(4.87, 0.69)</td>
<td>(0.95, 0.89)</td>
</tr>
<tr>
<td>&amp; 0.5</td>
<td>(4528.65, 0.64)</td>
<td>(4.87, 0.69)</td>
<td>(0.85, 0.75)</td>
</tr>
<tr>
<td>&amp; 0.7</td>
<td>(5017.89, 0.62)</td>
<td>(5.11, 0.67)</td>
<td>(0.73, 0.58)</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>&amp; 0.5</td>
<td>(5017.5, 0.60)</td>
<td>(3.37, 0.70)</td>
<td>(0.89, 0.95)</td>
</tr>
<tr>
<td>&amp; 0.7</td>
<td>(5088.27, 0.67)</td>
<td>(4.10, 0.62)</td>
<td>(0.85, 0.88)</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>3</td>
<td>1</td>
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<tr>
<td>&amp; 0.5</td>
<td>(27489.41, 0.63)</td>
<td>(22.94, 0.58)</td>
<td>(0.76, 0.84)</td>
</tr>
<tr>
<td>&amp; 0.7</td>
<td>(32678.10, 0.53)</td>
<td>(24.54, 0.65)</td>
<td>(0.72, 0.78)</td>
</tr>
</tbody>
</table>

Table 8
The results of the sensitivity analysis on \(\theta\)-value for problems given that \(\rho = 0.3\) and \(\psi = 0.4\)

<table>
<thead>
<tr>
<th>Test problem</th>
<th>(\theta_1, \theta_2, \theta_3)</th>
<th>Deterministic</th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x_1, \mu_1))</td>
<td>((x_2, \mu_2))</td>
<td>((x_3, \mu_3))</td>
<td>((x_1, \mu_1))</td>
</tr>
<tr>
<td>1</td>
<td>(0.3,0.3,0.4)</td>
<td>(8583.5, 0.69)</td>
<td>(16.72, 0.81)</td>
</tr>
<tr>
<td>&amp; (0.3,0.4,0.3)</td>
<td>(9438.3, 0.83)</td>
<td>(15.16, 0.86)</td>
<td>(0.79, 0.71)</td>
</tr>
<tr>
<td>&amp; (0.4,0.3,0.3)</td>
<td>(8152.8, 0.94)</td>
<td>(16.44, 0.76)</td>
<td>(0.80, 0.72)</td>
</tr>
<tr>
<td>&amp; (0.2,0.4,0.4)</td>
<td>(10322.5, 0.68)</td>
<td>(15.08, 0.86)</td>
<td>(0.847, 0.87)</td>
</tr>
<tr>
<td>&amp; (0.3,0.3,0.4)</td>
<td>(12639.7, 0.64)</td>
<td>(9.744, 0.66)</td>
<td>(0.90, 0.85)</td>
</tr>
</tbody>
</table>
Table 9
Results of sensitivity analysis on $\varphi$-value for problems based on the $\rho = 0.3$ and $\theta = (0.3, 0.3, 0.4)$

<table>
<thead>
<tr>
<th>Test problem</th>
<th>$\psi$</th>
<th>Deterministic</th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varphi$</td>
<td>$(x_1, \mu_1)$</td>
<td>$(x_2, \mu_2)$</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>(10369.3,0.64)</td>
<td>(13.57,0.98)</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>(9309.2,0.57)</td>
<td>(15.04,0.86)</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>(9048.1,0.65)</td>
<td>(15.27,0.80)</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>(8583.5,0.69)</td>
<td>(16.07,0.72)</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>(7635.1,0.69)</td>
<td>(16.27,0.72)</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>(7517.0,0.69)</td>
<td>(16.61,0.72)</td>
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<tr>
<td>7</td>
<td>0.7</td>
<td>(7461.7,0.71)</td>
<td>(17.83,0.71)</td>
</tr>
<tr>
<td>Test problem</td>
<td>DCs</td>
<td>vehicle</td>
<td>Route</td>
</tr>
<tr>
<td>--------------</td>
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<td>---------</td>
<td>-------</td>
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<tr>
<td>Deterministic</td>
<td>1</td>
<td>13</td>
<td>h₁ : 13-6</td>
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<tr>
<td></td>
<td></td>
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<td>g₁ : 13-4-11-1-10-7</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>g₂ : 13-3-8-7-11-5-9-2</td>
</tr>
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<td>2</td>
<td>13</td>
<td>h₁ : 13-4-8-7</td>
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<td>h₂ : 13-10</td>
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<td>g₁ : 13-2-6-10</td>
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<td>g₂ : 13-3-1-9-5</td>
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<tr>
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<td>9</td>
<td>h₁ : 9-8-4</td>
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<td>g₁ : 9-6-2-5-7</td>
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<td>g₂ : 9-1-3</td>
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<td>4</td>
<td>9</td>
<td>h₁ : 9-4</td>
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<td>g₁ : 9-1-2-6-3</td>
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<td>g₂ : 9-2</td>
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<td>g₁ : 14-10-8</td>
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<td>g₂ : 16-6-1-9-2</td>
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<td>g₃ : 16-7-10-11</td>
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<td>h₁ : 7-6</td>
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<td>g₁ : 7-2-6-3</td>
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<td>g₂ : 7-5</td>
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<td>g₃ : 7-1-2-4</td>
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<td>g₄ : 17-12-11-10-7</td>
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