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Optimality of network marketing integrated in a dual-channel distribution system

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Abstract. This paper provides a framework to study the integration of network marketing in a dual-channel distribution system. We develop an approach to optimize the main decision variables of this system simultaneously. These decision variables include the price paid by the customers of both channels, confidence level, the effort level of active distributors of network marketing, and wholesale price. Although both channels compete with each other, it is vital to have a balanced pricing system to keep them motivated. However, the prices in network marketing and traditional retailer system are not necessarily equal due to the differences in their nature. Furthermore, it is also required to develop an appropriate system of commissioning for the payoff of distributors at different levels of network marketing to keep them motivated. We also examine different scenarios of dual distribution systems, namely, centralized and decentralized operations of network marketing. Furthermore, in case of decentralized system, we investigate revenue or profit sharing for all parties involved (manufacturer, retailer, and network marketing distributors). To illustrate the proposed approach, we present some numerical studies and investigate the impact of customer loyalty degree to retail channel on decisions.

1. Introduction

Network Marketing (NM), as a strategic sales and marketing policy, has recently been gaining popularity in sales and marketing. Typically, it is a sales channel in which a manufacturer directly distributes its product or service to customers through a network of distributors. It is an agile distribution strategy through creating effective marketing relationship between distributors and customers [1]. The integration of traditional retailing with network marketing adds a new dimension of competition to the distribution channels of a product.

Network marketing seems to be a growing industry. Worldwide, there are more than 90 million independent sales people who sell nearly $154 billion of goods and services annually. In the United States, the direct sale in 2012 increased by 5.9% in comparison with 2011, from 29.87 billion $ to 31.63 billion [2]. Similarly, the number of people involved in direct selling in the United States increased to 15.9 million in 2012 from 15.6 million in 2011, at the rate of 1.9% [2].

Popularity of network marketing results from some of its natural characteristics, such as the ones discussed hereafter. One-to-one and personalized relationship marketing interaction takes place between distributor and customer, which is key to network marketing [3]. Many people prefer to buy from someone they know and trust. Network marketing satisfies this concept significantly well [4]. Providing cost advantage by reducing or eliminating the middlemen that yields greater margins is one of the major reasons for growth and popularity of network marketing [5]. Capital

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requirement and cash flow advantages are other aspects of attractiveness of network marketing. Network marketing provides better control and swift action in product introductions, sales, after-sales services, handling returns, and prompt delivery of goods and services [1]. It also paves the way for maximizing sales volume, market share, and market penetration by close relationship [6]. The cross-cultural characteristic of network marketing channel is an additional strong contributing factor for its widespread success [7]. Furthermore, network marketing is ideal for many people interested in entrepreneurs [7].

There is a relatively rich literature on different aspects of two areas related to our work, namely, network marketing [6,8,9] and the integration of internet channel with traditional cannel [10-13]. However, to our knowledge, not many have investigated different aspects of network marketing integrated in a distribution system.

To see the studies on different aspects of network marketing, such as developing, analyzing, and calibrating the growth of NM, see [6,8,9]. The advantages and disadvantages of a distribution system, which includes traditional and internet channel, were investigated in [10,11,14-16]. A hybrid channel, in which customers were classified into price-sensitive and service-sensitive segments, was studied by [17]. Cattani provided a survey of channel coordination of the internet-based supply chain with traditional supply chain through procurement, pricing, integration, fulfillment, and distribution [14]. Panda showed social responsibility effect on channel coordination and profit distribution [18]. Cárdenas-Barrón and Sana investigated a production-inventory model for a two-echelon supply chain when demand depended on initiatives of the sales teams [19,20]. Sana developed an inventory model to determine optimal order quantity of the retailer for homogeneous products based on initiatives of the sales teams [21,22].

Pricing and coordination are challenging issues between retailer and the newly added channel as network marketing. Chiang et al. [23] and many others employed a game theory model to study the price competition between direct channel of a manufacturer and its traditional-channel partner. They argued that vertically integrated direct channel allowed a manufacturer to constrain pricing behavior of the retailer partner [16]. Wang et al. investigated pricing and other service decisions of complementary products in a dual-channel supply chain [24]. Roy et al. studied a two-echelon supply chain when demand depended on sales price with random arrival of the customers [25]. Roy et al. showed that dual channels significantly influenced the pricing strategies as well as the effort level of the supply chain entities and they were always beneficial in an integrated system for the members of the chain [26]. Dual channel pricing and structure in a supply chain was also the focus of study of some researchers [10,13,27-30].

This paper is distinguished from the other studies in the literature from several aspects.

- We develop a novel approach for both centralized and decentralized operations of network marketing in an integrated distribution system. Since the nature of network marketing is different from an internet sales channel, the previous results for dual channels are not suitable for analyzing an integrated distribution system which includes network marketing and traditional retailing. In network marketing, in addition to customer price, other parameters such as the number or layers, distributions commissions, and the number of subordinates for each distributor should also be determined;
- We investigate the impact of complementary coordination based agreements on the profit of manufacturer, networker marketing channel, and retailers. We develop an approach for creating a coordinated dual channel of network marketing and traditional retailing in order to make it profitable for all members.

From the viewpoint of managerial insight, our approach can assist the companies that are willing to sell their products through network marketing in addition to traditional retailers. Our model makes it possible to determine the best decision to utilize network marketing and traditional sale channels as well as to coordinate them. The coordination of different parties involved in a parallel distribution is vital. Otherwise, it may result in a competition which can demotivate some channels.

There are many different types of network marketing models in the literature; however, all of them consider some specified layers of distributors. In each layer, there are several distributors while each distributor serves certain customers, as shown in Figure 1. In this figure, for simplicity, only two levels of distributors

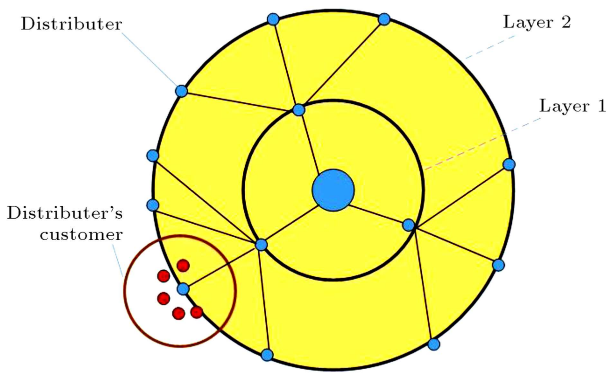

Figure 1. Network Marketing (NM) concept model.

are depicted, while in reality, a network can have multi levels of distributors.

The remainder of this paper is as follows. Section 2 introduces problem statement and some related variables such as *confidence level* and *effort level*. Moreover, distributors' commissions and demand function are defined in this section. In Sections 3, we introduce different scenarios for dual channel distribution systems without coordination mechanism. We also analyze the impacts of customer loyalty and commission coefficient on pricing decisions for different scenarios. In Section 4, two coordination-based contracts, namely, profit and revenue sharing contracts, are investigated. Furthermore, we determine the optimal range of coordination parameters. In Section 5, a numerical example is presented. We conclude the results and suggest topics for future research in Section 6.

2. Problem statement

We consider a manufacturer that distributes its single product through two parallel channels of retailers and network marketing. Customers may choose either channel to purchase the product. Three following distribution scenarios are investigated:

- Centralized dual-channel model: Network marketing and traditional channel centrally operated;
- Decentralized dual-channel model: The manufacturer is the owner of network marketing channel, but not of the traditional one;
- Fully decentralized dual-channel model: Network marketing and traditional channel decentrally operated.

2.1. Confidence level

The important factor that has impact on the sales in network marketing is called *confidence level*, which refers to customers' perception of the product (or brand). It is achieved through different means, such as advertising [16,27,31,32]. If $C(g_0)$ represents the cost of having confidence level of g_0, then $C(g_0) = \eta_1 \frac{g_0^2}{2}$, where η_1 measures the cost effectiveness of confidence level. The manufacturer (the owner of brand) pays this cost.

2.2. Effort level of distributors

The success of network marketing depends on the *effort level* of distributors. Let Ef be the average effort level of a distributor of network marketing to attract and convince customers. If $C(Ef)$ represents the cost of effort level, Ef, paid by a distributor, then $C(Ef) = \eta_2 \frac{(Ef)^2}{2}$, where η_2 measures the cost effectiveness of effort level. Let γ represent the probability of sales by a distributor, N_n be the number of distributors, and k be the adjustment factor. Accordingly, $N_n \gamma (Ef) k$ is the expected number of customers attracted by all distributors and $N_n \eta_2 \frac{(Ef)^2}{2}$ is the total cost of effort paid by the distributors.

2.3. Demand functions

We adopt the following demand function, which has been used by many researchers, e.g., see [28,29,32]:

$$D_r = \alpha_r - \alpha_1 p_r + \alpha_2 p_{\text{net}} + \beta_r g_0, \tag{1}$$

$$D_N = \alpha_N - \alpha_1 p_N + \alpha_2 p_r + \beta_{\text{net}} g_0 + N_n \gamma (Ef) k, \tag{2}$$

where:

D_r and D_N	Are retailer and network marketing demand functions, respectively;
p_r and p_N	Are retailer and network marketing channel prices, respectively;
α_r and α_N	Are the base levels of demand, i.e., potential demands if the goods are sold at the lowest possible price in retailer and network marketing channels, respectively;
$\theta = \dfrac{\alpha_r}{\alpha_r + \alpha_N}$	Degree of customer loyalty to the retail channel;
α_1	Coefficient of price elasticity;
α_2	Cross-price sensitivity, which reflects the substitution degree by the other channel;
β_r, β_N	Demands sensitivity of confidence level in retail channel and network marketing channel, respectively;
g_0	Customers' perception of the product (or brand);
Ef	Average effort level of a distributor of network marketing to attract and convince customers;
η_1	Cost effectiveness of confidence level;
η_2	Cost effectiveness of effort level;
N_n	Number of distributors;
γ	Probability of sales by a distributor;
c	Production cost.

2.4. Distributions commissions

Let $c\lambda_N$ be the commission allocated to the network marketing channel, for each sale, where c is the product cost. Then, the total commissions allocated to the distributors are $D_N c \lambda_N$.

To see the distribution of the commissions allocated to the distributors of different layers, see [33].

2.5. The optimal effort of distributors

The total profit of network marketing is as follows:

$$\pi_{\text{networkers}} = D_N c \lambda_N - N_n \eta_2 \frac{(Ef)^2}{2}. \tag{3}$$

The first term of Eq. (3) is the total commissions received for selling D_N items, while the second term represents the equivalent cost of efforts by distributors.

Proposition 1. For any given g_0, the optimal average effort level in network marketing is as follows:

$$Ef = \frac{\gamma c \lambda_N}{\eta_2}. \tag{4}$$

Proof. Since Eq. (3) is a concave function, we set $\frac{\partial \pi_{\text{networkers}}}{\partial Ef} = 0$ and substitute D_N from Eq. (2).■

By updating network marketing demand, based on the optimal effort (Eq. (4)), it is concluded that the effort of network marketing distributors raises initial demand as follows:

$$D_n = \alpha_{N1} - \alpha_1 p_N + \alpha_2 p_r + \beta_N g_0, \tag{5}$$

where $\alpha_{N1} = \alpha_N + \frac{N_n \gamma^2 c \lambda_N}{\eta_2}$.

3. Dual channel with no coordination

In this section, we study three different scenarios of distribution systems consisting of network marketing and traditional retailers.

3.1. Scenario 1, centralized dual-channel model

In this scenario, there are two distribution channels, namely, network marketing and traditional channel, while the manufacturer is the owner of both. The profit function of the manufacturer in this model is as follows:

$$\pi_m = (p_n - c)D_N + (p_r - c)D_r - D_N c\lambda_N - \eta \frac{g_0^2}{2} - hD_r, \tag{6}$$

where h is handling cost, which includes multiple logistic and storage costs.

Proposition 2. For any given g_0, the optimal network price and retail price are as follows:

$$p_r = A_{2r} g_0 + B_{2r}, \tag{7}$$

$$p_N = A_{2N} g_0 + B_{2N}, \tag{8}$$

where:

$$A_{2r} = \frac{(\alpha_1 \beta_r + \alpha_2 \beta_N)}{2\alpha_1^2 - 2\alpha_2^2},$$

$$B_{2r} = \frac{(c+h)}{2} + \frac{\alpha_1 \alpha_r + \alpha_2 \alpha_{N1}}{2\alpha_1^2 - 2\alpha_2^2},$$

$$A_{2N} = \frac{(\alpha_1 \beta_N + \alpha_2 \beta_r)}{2\alpha_1^2 - 2\alpha_2^2} \quad \text{and}$$

$$B_{2N} = \frac{c(1+\lambda_N)}{2} + \frac{\alpha_1 \alpha_{N1} + \alpha_2 \alpha_r}{2\alpha_1^2 - 2\alpha_2^2}.$$

Proof. It can be shown that for any g_0, Eq. (6) is a concave function of p_r and p_N, since the Hessian matrix of Eq. (6) is negative definite, see Appendix A. Therefore, the optimal price for both channels is obtained by setting the gradient of Eq. (6) equal to zero. ■

Proposition 3. The prices of both channels are increasing functions of confidence level. However, the price of retailer channel is more sensitive than the other one.

Proof. In Appendix B, we show that $\frac{\partial P_r}{\partial g_0} > 0$, $\frac{\partial P_N}{\partial g_0} > 0$, and $\frac{\partial P_r}{\partial g_0} > \frac{\partial P_N}{\partial g_0} > 0$.■

3.2. Optimal value of confidence level

To find an optimal g_0 that maximizes π_m, the manufacturer profit function (Eq. (6)) is rewritten with respect to g_0, while p_r and p_N are substituted by Eqs. (7) and (8), respectively. Then, the optimal value of g_0 is as follows:

$$g_0 = \frac{M_{H-C}}{M_{L-C}}, \tag{9}$$

where:

$$\begin{aligned} M_{H-C} =& A_{2N}\left(\alpha_{N1} - 2\alpha_1 B_{2N} + 2\alpha_2 B_{2r}\right) \\ &+ A_{2r}\left(\alpha_r - 2\alpha_1 B_{2r} + 2\alpha_2 B_{2N}\right) \\ &- c(1+\lambda_N)\left(\alpha_1 A_{2N} + \alpha_2 A_{2r} + \beta_N\right) \\ &- (c+h)\left(-\alpha_1 A_{2r} + \alpha_2 A_{2N} + \beta_r\right) \\ &+ B_{2N}\beta_N + B_{2r}\beta_r, \end{aligned}$$

$$\begin{aligned} M_{L-C} =& -2\Bigg(-\alpha_1 A_{2N}^2 - \alpha_1 A_{2r}^2 + 2\alpha_2 A_{2r} \\ &+ A_{2N}\beta_N + A_{2r}\beta_r - \frac{\eta}{2}\Bigg). \end{aligned}$$

Proposition 4. The confidence level (g_0) and retailer price (P_r) are increasing functions of customer loyalty to retailer channel (θ). Furthermore, the confidence level (g_0), the retailer price (p_r), and the network marketing price (p_N) are increasing functions of the commission coefficient (λ_N) and the probability of sale (γ).

Proof. The following relations hold:

$$\frac{\partial g_0}{\partial \theta} > 0 \text{ and } \frac{\partial p_r}{\partial \theta} > 0$$

$$\frac{\partial g_0}{\partial \lambda_N} > 0 \text{ and } \frac{\partial p_r}{\partial \lambda_N} > 0, \quad \frac{\partial p_N}{\partial \lambda_N} > 0$$

$$\frac{\partial g_0}{\partial \gamma} > 0 \text{ and } \frac{\partial p_r}{\partial \gamma} > 0, \quad \frac{\partial p_N}{\partial \gamma} > 0.\blacksquare$$

In Appendix C, we show the details of proving $\frac{\partial g_0}{\partial \theta} > 0$. The other relations can also be proved similarly.

3.3. Scenario 2, decentralized dual-channel model

In this scenario, the distribution system consists of traditional and network marketing channels. The manufacturer is the owner of network marketing channel, but not of the traditional one. Similar to the previous scenario, let $c\lambda_N$ be the commission allocated to the network marketing channel from each sale. The retailer and manufacturer profit functions are as follows:

$$\pi_r = (p_r - \omega - h)D_r, \tag{10}$$

$$\pi_m = (p_N - c)D_N + (\omega - c)D_r - D_N c\lambda_N - \eta\frac{g_0^2}{2}, \tag{11}$$

where ω is the wholesale price of each item. From Proposition 1, an optimal effort level (Eq. (4)) and updated network marketing demand function (Eq. (5)) are determined.

Proposition 5. For a given network price (p_N), wholesale price (ω), and confidence level (g_0), the optimal retailer price is as follows:

$$p_r = \frac{\alpha_1(\omega + h) + \alpha_r + \alpha_2 p_N + \beta_r g_0}{2\alpha_1}. \tag{12}$$

Proof. Retailer profit function (Eq. (10)) is concave with respect to retailer price, since the second derivative is negative. Therefore, p_r is obtained by setting the gradient of profit function equal to zero.■

Proposition 6. For any g_0, the optimal network marketing price and the wholesale price are as follows:

$$p_N = A_{3N} g_0 + B_{3N}, \tag{13}$$

$$\omega = A_{3\omega} g_0 + B_{3\omega}, \tag{14}$$

where:

$$A_{3N} = \frac{\alpha_1\beta_N + \alpha_2\beta_r}{2\alpha_1^2 - 2\alpha_2^2},$$

$$B_{3N} = \frac{c(1+\lambda_N)}{2} + \frac{\alpha_1\alpha_{N1} + \alpha_2\alpha_r}{2\alpha_1^2 - 2\alpha_2^2},$$

$$A_{3\omega} = \frac{\alpha_1\beta_r + \alpha_2\beta_N}{2\alpha_1^2 - 2\alpha_2^2},$$

$$B_{3\omega} = \frac{(c-h)}{2} + \frac{\alpha_1\alpha_r + \alpha_2\alpha_{N1}}{2\alpha_1^2 - 2\alpha_2^2}.$$

Proof. It can be shown that for any g_0, Eq. (11) is a concave function of p_N and ω, since the Hessian matrix of Eq. (11) is negative definite. Therefore, the optimal price for network marketing channel and wholesale price are obtained by setting the gradient of Eq. (11) equal to zero. ■

Proposition 7. The wholesale and network marketing prices are increasing functions of confidence level, while the wholesale price is more sensitive.

Proof. In Appendix D, we show that $\frac{\partial p_N}{\partial g_0}, \frac{\partial \omega}{\partial g_0} > 0$, and $\frac{\partial \omega}{\partial g_0} > \frac{\partial P_N}{\partial g_0}$.■

3.4. Optimal value of confidence level

To find the optimal g_0 that maximizes π_m, we set $\frac{\partial \pi_m}{\partial g_0} = 0$, while p_r, p_N, and ω are substituted from Eqs. (12)-(14), respectively. Then, the optimal value of g_0 is as follows:

$$g_0 = \frac{M_{H-D}}{M_{L-D}}, \tag{15}$$

where:

$$\begin{aligned} M_{H-D} = \Big(& A_{3N}(F_0 + F_1 B_{3N} + F_3 B_{3\omega}) \\ & + \frac{1}{2}A_{3\omega}(\alpha_r - \alpha_{1r}hc + \alpha_2 B_{3\text{net}} - \alpha_1 B_{3\omega}) \\ & + (B_{3N} - c(1+\lambda_N))(F_1 A_{3N} + F_3 A_{3\omega} + F_2) \\ & + (B_{3\omega} - c)\frac{1}{2}(\alpha_2 A_{3N} - \alpha_{1r}A_{3\omega} - \beta_r)\Big), \end{aligned}$$

$$\begin{aligned} M_{L-D} = & \left(-\alpha_{11} + \frac{\alpha_2^2}{2\alpha_1}\right) A_{3N}^2 + \alpha_2 A_{3\omega} A_{3N} \\ & + A_{3N}\left(\beta_N + \frac{\alpha_2\beta_r}{2\alpha_1}\right) + \frac{-\alpha_1}{2}A_{3\omega}^2 \\ & + \frac{1}{2}A_{3\omega}\beta_r - \frac{\eta}{2'}, \end{aligned}$$

where:

$$F_0 = \alpha_{N1} + \frac{\alpha_2 h}{2} + \frac{\alpha_r\alpha_2}{2\alpha_1},$$

$$F_1 = \left(-\alpha_1 + \frac{\alpha_2^2}{2\alpha_1}\right),$$

$$F_2 = \left(\beta_N + \frac{\alpha_2\beta_r}{2\alpha_1}\right),$$

$$F_3 = \frac{\alpha_2}{2}.$$

Proposition 8. The confidence level and network marketing price are decreasing functions of customer loyalty if $\beta_N > \frac{\beta_r(\alpha_1 - \alpha_2)}{2\alpha_1}$. Moreover, the confidence level and network marketing price as well as wholesale price are increasing functions of λ_N and probability of sales (γ).

Proof. If $\beta_N > \frac{\beta_r(\alpha_1-\alpha_2)}{2\alpha_1}$, then:

$$\frac{\partial g_0}{\partial \theta} < 0 \text{ and } \frac{\partial P_N}{\partial \theta} < 0,$$

$$\frac{\partial g_0}{\partial \lambda_N} > 0 \text{ and } \frac{\partial P_N}{\partial \lambda_N} > 0, \quad \frac{\partial \omega}{\partial \lambda_N} > 0,$$

$$\frac{\partial g_0}{\partial \gamma} > 0 \text{ and } \frac{\partial P_r}{\partial \gamma} > 0, \quad \frac{\partial P_N}{\partial \gamma} > 0.$$

In Appendix E, we show the details for proving $\frac{\partial g_0}{\partial \theta} < 0$ and $\frac{\partial P_N}{\partial \theta} < 0$. The other relations can also be proved similarly. ■

By substituting g_0 from Eq. (15) in Eqs. (12)-(14), the optimal p_r, ω, and p_N are obtained.

3.5. Scenario 3, fully decentralized dual-channel model

In this scenario, there are two parallel channels, namely, traditional retailer and network marketing, for distribution. The manufacturer is the owner of neither network marketing channel nor traditional channel.

We apply Stackelberg game model to optimize the system. The manufacturer acts as leader and determines wholesale price as well as confidence level. Moreover, the retailer and network marketing channels act as followers and determine their channel prices at the same time. In addition, the distributors as followers of network marketing channel determine their effort level. The profit functions of network marketing channel and manufacturer are as follows:

$$\pi_m = (\omega - c)(D_r + D_N) - \eta \frac{g_0^2}{2}, \tag{16}$$

$$\pi_{NM} = (p_n - \omega - \lambda_N c) D_N, \tag{17}$$

where π_{NM} is network marketing channel profit.

The optimal retailer price is determined as in the previous scenario by Eq. (12). In order to maximize the profit of manufacturer and network marketing channel, we propose the following propositions.

Proposition 9. For any given retailer price (p_r), wholesale price (ω), and confidence level (g_0), the optimal network marketing price is as follows:

$$p_N = \frac{\omega + \lambda_N c}{2} + \frac{(\alpha_{N1} + \alpha_2 p_r + \beta_N g_0)}{2\alpha_1}. \tag{18}$$

Proof. Network channel profit function (Eq. (17)) is concave with respect to its price. Therefore, the network marketing price (p_N) is obtained from $\frac{\partial \pi_{NM}}{\partial p_N} = 0$.■

Proposition 10. For any g_0, wholesale price is as follows:

$$\omega = A_{4\omega} g_0 + B_{4\omega}, \tag{19}$$

where $A_{4\omega}$ and $B_{4\omega}$ are defined in Box I.

Proof. It can be shown that for any g_0, profit function (Eq. (16)) is strictly concave with respect to ω. Accordingly, the optimal wholesale price is obtained by setting $\frac{\partial \pi_m}{\partial g_0} = 0$.■

Proposition 10 shows that wholesale price is a linear function of confidence level. Furthermore, by increasing confidence level, wholesale price increases, since $\frac{\partial \omega}{\partial g_0} > 0$. To find the optimal g_0 that maximizes π_m, we set $\frac{\partial \pi_m}{\partial g_0} = 0$, while p_r, p_N, and ω are substituted by Eqs. (12), (18), and (19), respectively. Then, the optimal value of g_0 is as follows:

$$g_0 = \frac{M_{H-DN}}{M_{L-DN}}, \tag{20}$$

where:

$$M_{H-DN} = A_{4\omega}\Big(\alpha'_r + \alpha'_N + (\alpha'_{2r} + \alpha'_{2N})B_{4\omega}\Big) + (B_{4\omega} - c)\Big(\alpha'_{1r} + \alpha'_{1N} + (\alpha'_{2r} + \alpha'_{2N})A_{4\omega}\Big)$$

$$M_{L-DN} = -2\Big(A_{4\omega}(\alpha'_{1r} + \alpha'_{1N} + A_{4\omega}(\alpha'_{2r} + \alpha'_{2N}))\Big) - \frac{\eta}{2},$$

$$A_{4\omega} = \frac{1}{2}\left(\frac{(\alpha_2 + 2\alpha_1)\beta_N \alpha_1 + (2\alpha_1 + \alpha_2)\alpha_1 \beta_r}{2(2\alpha_1 - \alpha_2)\alpha_1^2 - 2\alpha_1 \alpha_2^2}\right),$$

$$B_{4\omega} = \frac{1}{2}\left(\frac{(2\alpha_1 + \alpha_2)\alpha_1 \alpha_r + (2\alpha_1 + \alpha_2)\alpha_1 \alpha_{N1} + (\alpha_1 \alpha_2 + \alpha_2^2 - 2\alpha_1^2)\alpha_1(\lambda_N c + h)}{2(\alpha_{1N} + \alpha_1 - \alpha_2)\alpha_{1N}\alpha_1 - (\alpha_1 + \alpha_{1N})\alpha_2^2}\right) + \frac{c}{2}.$$

Box I

$$\alpha_N' = \left(\frac{2\alpha_1^2 + 2\alpha_1^2\lambda_N c + \alpha_1\alpha_2 h + \alpha_r\alpha_2}{4\alpha_1^2 - \alpha_2^2}\right)\alpha_1 - \alpha_1\lambda_N c,$$

$$\alpha_{1N}' = \left(\frac{\beta_r\alpha_2 + 2\alpha_1\beta_N}{4\alpha_1^2 - \alpha_2^2}\right)\alpha_1,$$

$$\alpha_{2N}' = \left(\frac{\alpha_1\alpha_2 - 2\alpha_1^2 + \alpha_2^2}{4\alpha_1^2 - \alpha_2^2}\right)\alpha_1,$$

$$\alpha_r' = \left(\frac{2\alpha_1\alpha_r + 2\alpha_1^2 h + \alpha_2\alpha_1\lambda_N c + \alpha_{N1}\alpha_2}{4\alpha_1^2 - \alpha_2^2}\right)\alpha_1 - \alpha_1 h,$$

$$\alpha_{1r}' = \left(\frac{\alpha_2\beta_N + 2\alpha_1\beta_r}{4\alpha_1^2 - \alpha_2^2}\right)\alpha_1,$$

$$\alpha_{2r}' = \left(\frac{\alpha_2\alpha_1 - 2\alpha_1^2 + \alpha_2^2}{4\alpha_1^2 - \alpha_2^2}\right)\alpha_1.$$

By substituting g_0 from Eq. (20) in Eqs. (12), (18), and (19), the optimal p_r, ω, and p_N are obtained.

4. Coordination of decentralized dual-channel system

In this section, decentralized scenario model is reinforced by coordination concepts. We develop revenue (profit) sharing for the case of decentralized scenario, in which the manufacturer is the owner of network marketing channel, but not of the traditional one. The main idea is that although the retailer and manufacturer share their revenue (profit), it is profitable for them if their revenue (profit) is increased. To achieve this goal, it is necessary to enhance the performance of the system.

Since the maximal performance of the system is achieved if the system is operated centrally, we set the price of decentralized scenario equal to that of the centralized one. Furthermore, an appropriate whole price is determined in order to maximize the total profit of the system. It should be noted that in the centralized scenario, the performance of the system is independent of whole price, because the manufacturer is the owner of both channels.

4.1. Revenue sharing coordination

In the revenue sharing model, one player lends a portion of their revenue to the other player to make more motivation for coordination. It is assumed that the retailer gives $(1-\varphi)$ percent of their revenue to the manufacturer. Therefore, the coordinated profit functions of retailer, π_r^{RS}, and manufacturer, π_m^{RS}, are updated as follows:

$$\pi_r^{RS} = \varphi(p_r - \omega - h)D_r, \tag{21}$$

$$\pi_m^{RS} = (p_N - c)D_N + (1-\varphi)(p_R - \omega - h)D_r + (\omega - c)D_r - D_N c\lambda_N - \eta\frac{g_0^2}{2}, \tag{22}$$

in which $(1-\varphi)(p_r-\omega-h)D_r$ is the transferred revenue that retailer pays to manufacturer; $0 \le \varphi \le 1$.

As mentioned before, to maximize the system performance, the manufacturer offers network marketing price and confidence level (g_0) equal to those in the centralized scenario. Furthermore, the manufacturer sets wholesale price in order to stimulate retailer to set their order equal to the optimal demand in the centralized scenario. As a result, the retailer price and wholesale price are achieved as follows:

$$p_r = \frac{\omega}{2} + \frac{(\alpha_1 h + \alpha_r + \alpha_2 B_{2N})}{2\alpha_{1r}} + \frac{(\alpha_2 A_{2N} + \beta_r)}{2\alpha_1} g_0, \tag{23}$$

$$\omega = 2\left(B_{2r} - \frac{h}{2} - \frac{\alpha_r + \alpha_2 B_{2N}}{2\alpha_1}\right) + 2\left(A_{2r} - \frac{\alpha_2 A_{2N} + \beta_r}{2\alpha_1}\right) g_0. \tag{24}$$

Let π_r^{CD*} and π_m^{CD*} represent the profit functions of manufacturer and retailer in decentralized scenario, while we set the retailer and wholesale prices as Eqs. (23) and (24), and network marketing price and confidence level equal to those in the centralized scenario.

Proposition 11. If φ is set within the range of $[\underline{\varphi} = \frac{\pi_r^{D*}}{\pi_r^{CD*}}, \overline{\varphi} = \frac{\pi_m^{CD*} - \pi_m^{D*}}{\pi_r^{CD*}} + 1]$, a revenue sharing contract results in increasing the profit of both parties. In other words, $\pi_r^{RS} \ge \pi_r^{D*}$ and $\pi_m^{RS} \ge \pi_m^{D*}$, where π_r^{D*} and π_m^{D*}, respectively, represent the optimal profits of retailer channel and manufacturer in the decentralized model when coordination is not applied.

Proof. From Eqs. (21), (23), and (24), we have $\pi_r^{RS} = \varphi\pi_r^{CD*}$. If $\varphi \ge \underline{\varphi}$, where $\underline{\varphi} = \frac{\pi_r^{RS}}{\pi_r^{CD*}}$, then $\pi_r^{RS} \ge \pi_r^{D*}$.

On the other hand, from Eqs. (22), (23), and (24), we have $\pi_m = (1-\varphi)\pi_r^{CD*} + \pi_m^{CD*}$. If $\varphi \le \overline{\varphi}$, where $\overline{\varphi} = \frac{\pi_m^{CD*} - \pi_m^{D*}}{\pi_r^{CD*}} + 1$, then $\pi_m^{RS} \ge \pi_m^{D*}$.■

Therefore, if φ is set within the range of $[\underline{\varphi}, \overline{\varphi}]$, both retailer and manufacturer benefit from the new contract.

4.2. A profit sharing coordination model

In profit sharing contract, the objective of both players is to agree upon profit sharing. The prices of the retail channel and network marketing channel, as well as the confidence level and wholesale price, are set equal to the corresponding values in the *revenue sharing* *contract*. Afterwards, the extra profit gained in this model, compared to the decentralized model, is shared among them. It is assumed that the shared ratios of retailer and manufacturer are δ and $(1-\delta)$, respectively, where $0 < \delta < 1$.

Therefore, profit functions of retailer and manufacturer in profit sharing contract are achieved as $\pi_r^{PS*} = \delta\pi^{C*}$ and $\pi_m^{PS*} = (1-\delta)\pi^{C*}$, where π^{C*} represents the centralized total profit of supply chain. The retailer prefers the profit sharing contract if $\pi_r^{PS*} - \pi_r^{D*} \geq 0$ and the manufacturer prefers it if $\pi_m^{PS*} - \pi_m^{D*} \geq 0$.

Proposition 12. If δ is within the range of $[\underline{\delta} = \frac{\pi_r^{D*}}{\pi^{C*}}, \overline{\delta} = (1 - \frac{\pi_m^{D*}}{\pi^{C*}})]$, a profit sharing contract results in increasing the profit of both parties in comparison with the decentralized model.

Proof. The retailer prefers the profit sharing contract if $\pi_r^{PS*} - \pi_r^{D*} \geq 0$. Since $\pi_r^{PS*} = \delta\pi^{C*}$, it is satisfied when $\delta \geq \frac{\pi_r^{D*}}{\pi^{C*}} = \underline{\delta}$. The manufacturer prefers this contract if $(1-\delta)\pi^{C*} - \pi_m^{D*} \geq 0$; it is equal to $\delta \leq \left(1 - \frac{\pi_m^{D*}}{\pi^{C*}}\right) = \overline{\delta}$.■

4.3. Optimal utility function

It is assumed that $\Delta\pi_r$ and $\Delta\pi_m$ are extra revenue of the retailer and manufacturer, respectively. The relations between the profits of both players are:

$$\Delta\pi_r = \pi_r^{PS*} - \pi_r^{D*},$$

$$\Delta\pi_m = \pi_m^{PS*} - \pi_m^{D*},$$

$$\Delta\pi = \Delta\pi_r + \Delta\pi_m. \tag{25}$$

It is assumed that the retailer's utility function of $\Delta\pi_r$ is u_r, while manufacturer's utility function of $\Delta\pi_m$ is u_m. In this study, analogous to [10], it is assumed that the two players have equal bargaining capabilities. Thus, Nash bargaining equilibrium model is utilized for determining extra revenue of both players. Consequently, the optimum bargaining result is achieved from optimizing Eq. (26):

$$\text{Max } u_m(\Delta\pi_m) * u_r(\Delta\pi_r), \tag{26}$$

s.t.

$$u_m(\Delta\pi_m) = (\Delta\pi_m)^{b_m},$$

$$u_r(\Delta\pi_r) = (\Delta\pi_r)^{b_r},$$

$$\Delta\pi = \Delta\pi_r + \Delta\pi_m. \tag{27}$$

We have assumed that two players are averse to risk, and $0 < b_m < 1$ and $0 < b_r < 1$. Therefore, optimization model in Eq. (27) results in Eqs. (28) and (29):

$$\Delta\pi_r = \frac{b_m}{b_r + b_m}\Delta\pi, \tag{28}$$

$$\Delta\pi_m = \frac{b_r}{b_r + b_m}\Delta\pi. \tag{29}$$

After determining $\Delta\pi_m$ and $\Delta\pi_r$, we can calculate δ as Eq. (30):

$$\delta^* = \frac{\Delta\pi_r^* + \pi_r^{D*}}{\pi^{C*}}. \tag{30}$$

Two players distribute profit at the end of each time interval according to the contract. Executing this model might face some administrative obstacles, compared to the revenue sharing model. However, in this situation, each player prefers this scenario to the decentralized scenario.

5. Numerical example

In this section, we present a numerical example to illustrate the theoretical results and explore the differences between *Centralized* (C), *Decentralized* (D), and *fully Decentralized* (DN) scenarios; see, Figures 2-4. Moreover, the sensitivity analysis of *Revenue Sharing (RS)* and *Profit Sharing (PS)* is carried out; see Figure 5. The data for this numerical example is presented in Table 1.

In Figure 2(a), the total profits of different scenarios of C, D, and DN with respect to customer loyalty to retailer channel (θ) are illustrated. It is concluded that the total profits of three scenarios are convex with respect to the customer loyalty (θ). Moreover, it is shown that the centralized model is the dominant option for all levels of customer loyalty (θ). Additionally, for $\theta < \theta_1$, the decentralized scenario (D) is preferred to fully decentralized scenario (DN). However, if $\theta > \theta_1$, this preference becomes reverse. Figure 2(b) shows that retailer price increases by increasing the customer loyalty to retailer channel, whereas network marketing price has reverse behavior with respect to customer loyalty, Propositions 4 and 8.

Figure 2(c) shows that by increasing customer loyalty to retailer channel, confidence level in centralized

Table 1. Data for numerical example.

$\alpha_0 = 10 * 10^6$	$\theta = [0.3 - 0.8]$	$\alpha_1 = 6 * 10^4$	$\alpha_2 = 1.5 * 10^4$
$\beta_N = 7 * 10^4$	$\beta_r = 8 * 10^4$	$C = 30$	$h = 0.25 * C$
$\eta_1 = 10^6$	$\lambda_N = 0.3$	$N_N = 10^5$	$\eta_2 = 200$
$k = 50$	$\gamma = 0.3$	$b_R = 0.5$	$b_N = 0.5$

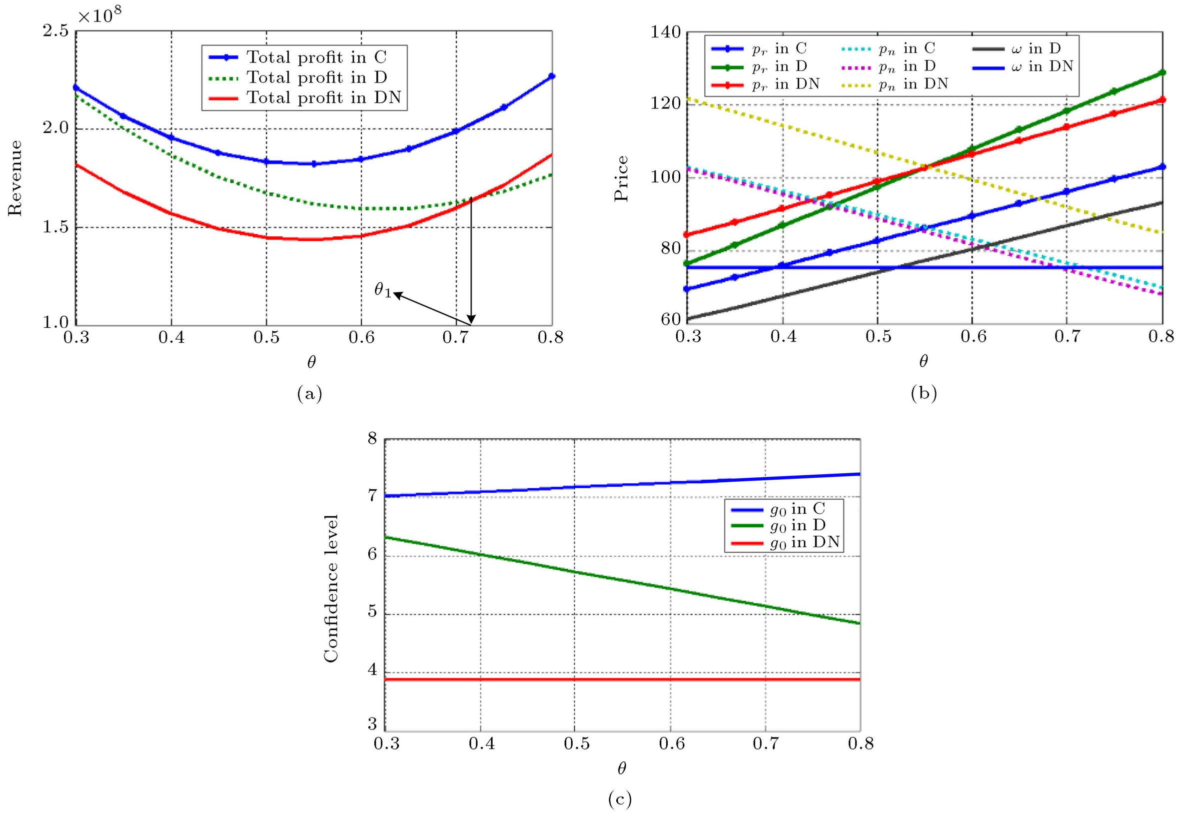

Figure 2. The impact of customer loyalty to retailer channel (θ) on total profit, channel prices, and confidence level (Centralized (C), Decentralized (D), and Fully centralized (DN) Models).

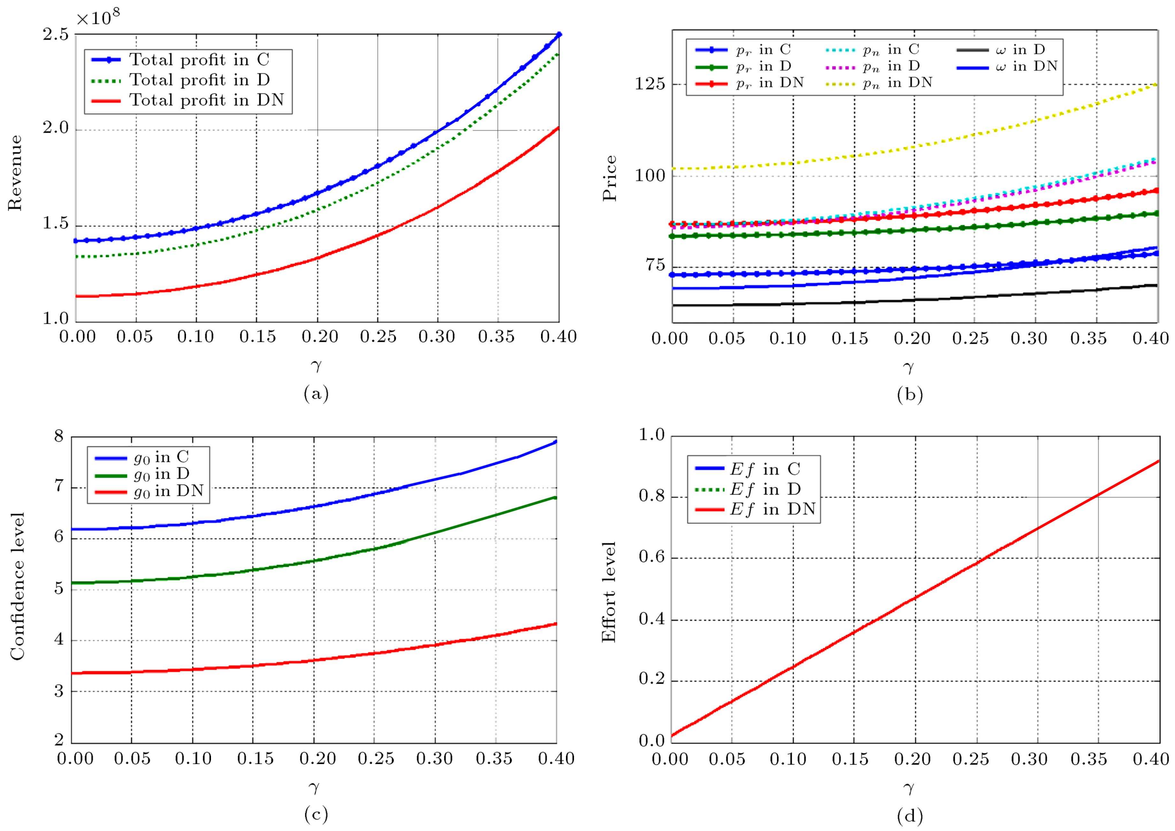

Figure 3. The impact of probability of sale (γ) on total profit, channel prices, and confidence level.

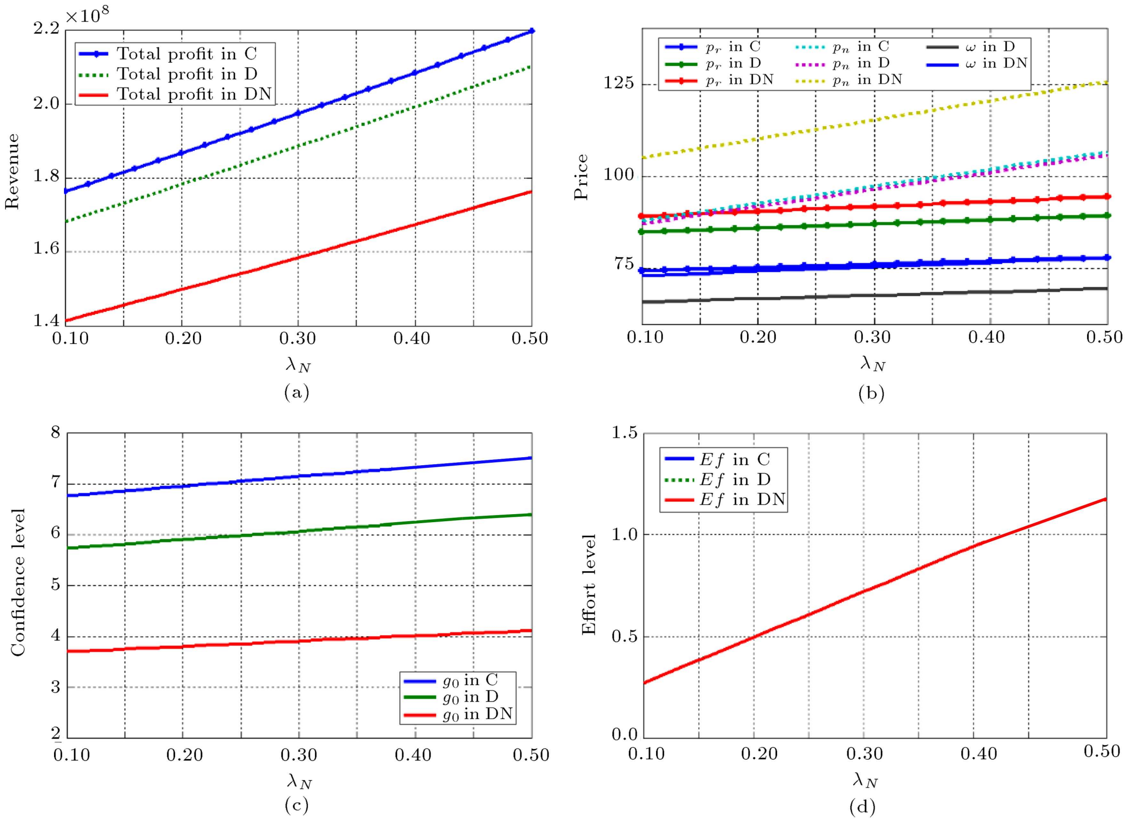

Figure 4. The impact of commission coefficient (λ_N) on total profit, channel prices, and confidence level.

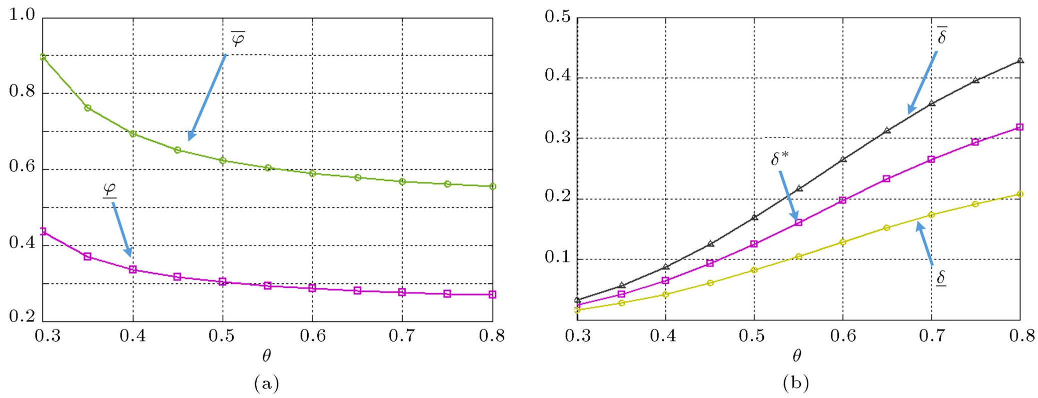

Figure 5. Acceptable ranges of φ, and δ in revenue sharing and profit sharing scenarios.

scenario increases, whereas in the decentralized model (D), it decreases. In addition, it is seen that confidence level in fully decentralized model (DN) is independent of customer loyalty.

Probability of sales (γ) is an important parameter in network marketing. Thus, we investigate its effect on the results. In the following, the probability of sales changes in the range of [0.0-0.4] and customer loyalty is fixed at $\theta = 0.4$; other parameters are as in Table 2.

According to Figure 3(a), when probability of sales (γ) increases, the total profit also increases for all scenarios. Moreover, Figure 3(b) shows that network marketing and retailer prices as well as wholesale price are increasing functions of probability of sales (γ), while network marketing price is more sensitive. In addition, confidence level and effort level increase by increasing probability of sales (γ), which means that lager probability of sales (γ) motivates distributors in network marketing to work more. It is another expression of Proposition 1.

In the following, we perform sensitivity analysis of the commission coefficient. It changes in the range of [0.1-0.5] and customer loyalty is fixed at $\theta = 0.4$; other parameters are as in Table 1.

Figure 4(a) shows that the total profit in all scenarios is increasing function of commission coefficient

Table 2. Sensitivity analysis of the effect of customer loyalty to retailer channel (θ) on total profit in different scenarios.

θ	Total profit in C scenario	Total profit in D scenario	Total profit in DN scenario
0.3	$2.21 * 10^8$	$2.17 * 10^8$	$1.82 * 10^8$
0.4	$1.96 * 10^8$	$1.86 * 10^8$	$1.57 * 10^8$
0.5	$1.83 * 10^8$	$1.67 * 10^8$	$1.45 * 10^8$
0.6	$1.84 * 10^8$	$1.59 * 10^8$	$1.45 * 10^8$
0.7	$1.99 * 10^8$	$1.63 * 10^8$	$1.59 * 10^8$
0.8	$1.26 * 10^8$	$1.76 * 10^8$	$1.86 * 10^8$

Table 3. Sensitivity analysis of the effects of commission coefficient (λ_N) and probability of sale (γ) on total profit.

γ	Total profit in C scenario	Total profit in D scenario	Total profit in DN scenario	λ_N	Total profit in C scenario	Total profit in D scenario	Total profit in DN scenario
0	$1.42 * 10^8$	$1.33 * 10^8$	$1.13 * 10^8$	0.1	$1.73 * 10^8$	$1.68 * 10^8$	$1.41 * 10^8$
0.1	$1.47 * 10^8$	$1.39 * 10^8$	$1.17 * 10^8$	0.2	$1.84 * 10^8$	$1.76 * 10^8$	$1.48 * 10^8$
0.2	$1.64 * 10^8$	$1.58 * 10^8$	$1.31 * 10^8$	0.3	$1.95 * 10^8$	$1.86 * 10^8$	$1.56 * 10^8$
0.3	$1.95 * 10^8$	$1.84 * 10^8$	$1.56 * 10^8$	0.4	$2.06 * 10^8$	$1.97 * 10^8$	$1.65 * 10^8$
0.4	$2.43 * 10^8$	$2.34 * 10^8$	$1.96 * 10^8$	0.5	$2.17 * 10^8$	$2.07 * 10^8$	$1.74 * 10^8$

(λ_N). Moreover, according to Figure 4, by increasing commission coefficient (λ_N), channel prices as well as wholesale and effort levels also increase. It is a confirmation of Propositions 4 and 8.

Figure 5(a) shows that $\underline{\varphi}, \overline{\varphi}$ are decreasing functions of customer loyalty (θ) within the interval. Moreover, Figure 5(b) shows that $\overline{\delta}$ is more sensitive than $\underline{\delta}$ to θ.

For the numerical example, sensitivity analysis of total profit in different scenarios versus the important parameters such as *customer loyalty to retailer channel* (θ), *commission coefficient* (λ_N), and *probability of sale* (γ) is presented in Tables 2 and 3. Amounts of these parameters influence distribution channel architecture.

6. Conclusion

In this paper, we provided a framework to study the integration of network marketing into a distribution system. Network marketing enforces the sales effort by increasing the motivation of distributors. Consequently, motivated distributors increase their effort level to attract more customers. As endorsement, in all scenarios, the total profit of system was an increasing function of commission coefficient (λ_N).

Moreover, the results showed that training the distributors could increase the performance of distribution system through raising the probability of sales (γ), which increased the profit function of system.

The results showed that in dual channel with no coordination, centralized scenario was the most profitable among all scenarios; however, this might not be applicable to all cases. Moreover, decentralized scenario made more profit than the fully decentralized scenario in normal situation. Furthermore, there was a threshold of customer loyalty degree to retailer channel (θ) that made fully decentralized scenario preferred to decentralized scenario.

Coordination-based scenarios as well as revenue and profit sharing contracts improved the performance of decentralized scenario. The total profit of distribution system in these coordination-based contracts could attain the maximum level, similar to centralized scenario. We also proved that profit or revenue sharing contract was accepted within some specified ranges of φ and δ, as presented in Propositions 11 and 12.

Our analysis might have some limitations. We assumed that all information was known and demand was deterministic.

Therefore, this research can be extended in several directions in future work. First, for uncertain data, some uncertainty approaches can be applied to make it more realistic; second, we can consider random demand instead of deterministic demand; third, internet channel can be included as the third channel of distribution system; fourth, other types of network marketing can be chosen as base model for analyzing; and the last, but not the least, the dynamic pricing of retailer and network channel is also a subject worthy of further investigation.

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Appendix A

Proof of Proposition 2

The Hessian matrix of p_r and p_N is as follows:

$$H_N = \begin{bmatrix} -2\alpha_1 & 2\alpha_2 & \beta_N \\ 2\alpha_2 & -2\alpha_1 & \beta_r \\ \beta_N & \beta_r & -\eta \end{bmatrix},$$

$|H^c_{N1}| = -2\alpha_1 < 0$, and $|H^c_{N2}| = 4\alpha_1^2 - 4\alpha_2^2 > 0$. Therefore, the Hessian matrix is negative definite.

Appendix B

Proof of Proposition 3

From Proposition 2:

$$\frac{\partial P_N}{\partial g_0} = \frac{\alpha_1\beta_N + \alpha_2\beta_r}{2\alpha_1^2 - 2\alpha_2^2} > 0 \quad \text{and}$$

$$\frac{\partial P_r}{\partial g_0} = \frac{(\alpha_1\beta_r + \alpha_2\beta_N)}{2\alpha_1^2 - 2\alpha_2^2} > 0.$$

Since $\alpha_1\beta_N + \alpha_2\beta_r < \alpha_1\beta_r + \alpha_2\beta_N$, then $\frac{\partial P_N}{\partial g_0} > \frac{\partial P_r}{\partial g_0}$.

Appendix C

Proof of Proposition 4

From Eq. (9):

$$\frac{\partial g_0}{\partial \theta} = \frac{\left(\frac{\partial M_{H-C}}{\partial \theta} M_{L-C} - \frac{\partial M_{L-C}}{\partial \theta} M_{H-C}\right)}{M_{L-C}^2} = \frac{\frac{\partial M_{H-C}}{\partial \theta}}{M_{L-C}},$$

since $\frac{\partial M_{L-C}}{\partial \theta} M_{H-C} = 0$.

On the other hand, from Proposition 2:

$$-\frac{\partial B_{2r}}{\partial \theta} = \frac{1}{2}\left(\frac{\alpha_0}{\alpha_1 + \alpha_2}\right) > 0,$$

$$\frac{\partial B_{2N}}{\partial \theta} = -\frac{1}{2}\left(\frac{\alpha_0}{\alpha_1 + \alpha_2}\right) < 0, \quad \text{and}$$

$$\frac{\partial A_{2r}}{\partial \theta} = \frac{\partial A_{2N}}{\partial \theta} = 0.$$

After some simplification, we have:

$$\frac{\partial M_{H-C}}{\partial \theta} = \frac{\partial B_{2r}}{\partial \theta}(\beta_r - \beta_N) > 0,$$

and since $M_{l-CWO} > 0$, $\frac{\partial g_0}{\partial \theta} > 0$. From Proposition 2

$$\frac{\partial P_r}{\partial \theta} = A_{2r}\frac{\partial g_0}{\partial \theta} + g_0\frac{\partial A_{2r}}{\partial \theta} + \frac{\partial B_{2r}}{\partial \theta}.$$

Since $\frac{\partial g_0}{\partial \theta} > 0$, $\frac{\partial A_{2r}}{\partial \theta} = 0$ and $\frac{\partial B_{2r}}{\partial \theta} > 0$, then $\frac{\partial P_r}{\partial \theta} > 0$.

Appendix D

Proof of Proposition 7

From Proposition 6:

$$\frac{\partial P_N}{\partial g_0} = \frac{\alpha_1\beta_N + \alpha_2\beta_r}{2\alpha_1^2 - 2\alpha_2^2} > 0,$$

$$\frac{\partial P_r}{\partial g_0} = \frac{\alpha_1\beta_r + \alpha_2\beta N}{2\alpha_1^2 - 2\alpha_2^2} > 0.$$

Since $\alpha_1\beta_N + \alpha_2\beta_r < \alpha_1\beta_r + \alpha_2\beta_N$, then $\frac{\partial \omega}{\partial g_0} > \frac{\partial p_N}{\partial g_0}$.

Appendix E

Proof of Proposition 8

From Eq. (15):

$$\frac{\partial g_0}{\partial \theta} = \frac{\left(\frac{\partial M_{H-D}}{\partial \theta} M_{L-DN} - \frac{\partial M_{L-D}}{\partial \theta} M_{H-D}\right)}{m_{l-d}^2}$$

$$= \frac{\frac{\partial M_{H-D}}{\partial \theta}}{M_{L-D}},$$

since:

$$\frac{\partial M_{L-D}}{\partial \theta} = 0.$$

On the other hand, from Proposition 6:

$$-\frac{\partial B_{3\omega}}{\partial \theta} = \frac{1}{2}\left(\frac{\alpha_0}{\alpha_1 + \alpha_2}\right) > 0,$$

$$\frac{\partial B_{3N}}{\partial \theta} = -\frac{1}{2}\left(\frac{\alpha_0}{\alpha_1 + \alpha_2}\right) < 0,$$

$$\frac{\partial B_{3N}}{\partial \theta} = -\frac{\partial B_{3\omega}}{\partial \theta},$$

$$\frac{\partial A_{3\omega}}{\partial \theta} = \frac{\partial A_{3N}}{\partial \theta} = 0.$$

After simplification:

$$\frac{\partial M_{H-D}}{\partial \theta} = \frac{1}{4}\left(\frac{\beta_r \alpha_1 - 2\alpha_1 \beta_N - \alpha_2 \beta_r}{\alpha_1(\alpha_1 + \alpha_2)}\right)\alpha_0$$

If $\beta_N > \frac{\beta_r(\alpha_1 - \alpha_2)}{2\alpha_1}$, then $\frac{\partial M_{H-D}}{\partial \theta} < 0$ and $M_{L-D} > 0$. Therefore, $\frac{\partial g_0}{\partial \theta} < 0$. Similarly, if $\beta_N > \frac{\beta_r(\alpha_1 - \alpha_2)}{2\alpha_1}$, then $\frac{\partial P_N}{\partial \theta} < 0$, since:

$$\frac{\partial p_N}{\partial \theta} = A_{3N}\frac{\partial g_0}{\partial \theta} + g_0 \frac{\partial A_{3N}}{\partial \theta} + \frac{\partial B_{3N}}{\partial \theta},$$

$$\frac{\partial g_0}{\partial \theta} < 0, \quad \frac{\partial A_{3N}}{\partial \theta} = 0, \quad \frac{\partial B_{3N}}{\partial \theta} < 0.$$

Biographies

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