Developing Simulation Based Optimization Mechanism for Novel Stochastic Reliability Centered Maintenance Problem

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Abstract

This research investigates joint scheduling of maintenance and production planning. This novel integrated problem takes benefit of reliability-centered maintenance (RCM) for monitoring and managing maintenance function of a stochastic complex production-planning problem namely flexible job shop scheduling problem (FJSP). The developed RCM works based on stochastic shocking of machines during their process time. In fact, it implements condition based maintenance approach regulated according to stochastic reliability concept. Comparison of the system reliability with critical levels determines the failure statues of the machines. It activates two main types of reaction called preventive and corrective maintenance. Considering breakdown of the system between inspection intervals makes the proposed model more realistic. Moreover, maintenance activity times and their duration are considered stochastically. Because of the high complexity level for this joint system, simulation-based optimization (SBO) approach is proposed for solving the problem. This SBO searches the feasible area through genetic algorithm (GA) and biogeography based optimization (BBO) algorithm. Different test problems, statistical methods, novel visualizations are used to discuss the problem and the algorithm explicitly.

Keywords

Reliability-centered maintenance (RCM), stochastic production model, condition-based maintenance (CBM), shocking mechanism, biogeography based optimization (BBO)

1- Introduction

Production plans and the maintenance activities are part and parcel. However, most of the production and scheduling problems assume all times machine availability [1]. In contrast to this assumption, real world problems face with lots of situations in which machines break down or need maintenance. [2]. Moreover, inefficient maintenance can cause one third of maintenance costs wasted due to unnecessary or improper maintenance activities [3]. Nonetheless, maintenance issues has no considerable portion of the literature in production and manufacturing problems.

On the other hand, the maintenance and reliability have rich literature on modeling and optimization [4-9]. Thus, taking benefit of this opportunity to realize and reinforce production-planning problems is of interest. One of these opportunities is a method called reliability centered maintenance (RCM). Actually, the main goal of this paper is consistent introducing of RCM to production problem because of its importance in real environment. RCM has various industrial applications in maintenance and reliability
literatures, including power distribution systems, subsea pulpiness, steel plants, chemical industry, transportation, water distribution, and concrete bridge decks inspection [10-15].

RCM controls functionally of the systems to reach a desired level by monitoring its reliability [16]. Moreover, it prioritizes maintenance activities by ranking the failures according to their effects on system reliability. In fact, RCM monitors the reliability of the system continuously and determines type of the required maintenance activities according to the levels of the reliability [16]. Condition-based maintenance (CBM), in many cases, conducts the task of monitoring. CBM development owes to the recent emerging technologies such as radio frequency identification (RFID), micro-electro-mechanical system (MEMS), wireless tele-communication, and product embedded information devices (PEID) [17]. Next, the literature about joint scheduling of maintenance and production planning is reviewed.

1-1 Integration of maintenance and general production problems

proposed a multi-component system under changing job shop with preventive maintenance consideration. Ozkok [48] investigated hull structure production process in a fixed position shipyard company with machine breakdown consideration. Chouikhi et al. [49] integrated a single-unit system with CBM and they optimized the cost of maintenance and inspection time by determining the optimal inspection. They assumed that both corrective and preventive maintenance actions are perfect which means after such actions, the system becomes as good as new one. Besides, they assumed that durations of inspection, corrective maintenance, and preventive maintenance could be negligible. Kim and Ozturkoglu [50] developed a joint scheduling of single machine problem with multiple preventive maintenances. They proposed ant colony optimization and particle swarm optimization in order to solve this problem. Ying et al. [51] introduced different SMPS considering maintenance activity between two sequential jobs. Lin et al. [52] evaluated reliability of a multistate flexible FSSP with stochastic capacity. Huang and Yu [53] developed a two-stage multiprocessor FSSP with maintenance and clean production aims. Cui and Lu [54] investigated flexible maintenance in SMPS, solved their problem through earliest release date-longest processing time (ERD-LPT) and branch, and bound (B&B) algorithm.

1-2 Integration of maintenance and flexible job shop scheduling problem (FJSP)
Flexible job shop scheduling problem is a popular and complex flexible manufacturing problem [55,56]. In classical FJSP, most researches assume that all machines are available during their working process. Both areas of the optimization problems, i.e. model development [57-61] and solving method extension [62-76] can be found in the classical literature of FJSP. Demir and Isleyen [77] performed a comprehensive evaluation on the various mathematical models presented for the FJSP. Zribi and Borne [78] assumed unavailability of machines due to preventive maintenance. Gao et al. [79] proposed a preventive maintenance for FJSP where the period of maintenance tasks are non-fixed and should be determined during the scheduling procedure. Wang and Yu [80] developed FJSP considering maintenance activities either flexible in a time window or fixed beforehand. Moradi et al. [81] integrated FJSP and preventive maintenance by optimizing unavailability and makespan. Mokhtari and Dadgar [82] introduced a joint FJSP and PM model that assumes the failure rates are time varying. In their model the duration of PM activities are fixed. Ahmadi et al. [83] studied random machine breakdown in FJSP with simulation considerations. The related important studies are summarized in Table 1.

Please Insert Table 1 here

1-3 Gap analysis

According to the literature, a rare portion of the production studies is devoted to FJSP, CBM and RCM. Therefore, this research reinforces FJSP problem through RCM concept. Real world assumptions, rarely considered in the literature, are assumed in the developed RCM. For instance, it assumes breakdown possibility between inspection intervals. It also considers maintenance occurrence and duration time stochastically. In addition, it assumes stochastically recovery level of the system after preventive maintenance. Moreover, we use both types of maintenance strategies, called corrective maintenance (CM) and preventive maintenance (PM). CBM is used to detect the level of the reliability [84]. The structure of the paper is as follows. Section 2 presents the related literature review of the problem. Section 3 discusses the elements of the proposed joint problem. The simulation-based approach related to
the proposed RCM is developed in Section 4. Section 5 presents the proposed problem and its solving methodology through numerical examples. Finally, Section 7 concludes the paper.

2- Preliminaries of the developed joint problem

The considered production problem is stochastic version of the simple FJSP. FJPS has two tasks allocating operations to machines and determining the sequence of allocated operations to each machine [72, 79]. Simple FJSP is consisted of \( n \) jobs \( J(J_i, i \in \{1,2,\ldots,n\}) \), each job \( J_i \) includes \( n_i \) operations \( O_{ij}, j \in \{1,2,\ldots,n_i\} \) that are processed on \( m \) machines \( M(M_k, k \in \{1,2,\ldots,m\}) \). The FJSP objective function of this paper is makespan \( (C_{\text{max}}) \) given below:

\[
C_{\text{max}} = \max \{C_k|k = 1, \ldots, n\},
\]

(1)

where \( C_k \) denotes complementation time of machine \( k \) [74].

Figure 1 illustrates FJSP example by three jobs, four machines and nine operations. Figure includes a table and a related Gant chart. In the table, the numbers present the processing times of operations on machines in addition to their set of capable machines. The ‘inf’ symbols means the machine cannot operate the corresponding operation. The Gant chart depicts combination of the sequence and the assignment for a sample solution.

This research realizes the basic FJSP production planning problem through considering the real stochastic nature of the maintenance function. The heart concept of the proposed approaches is RCM. RCM determines and classifies the failure modes and tries to keep the reliability of the system in a level where the occurrence of these modes are prevented [16]. Actually, it monitors the system status predictively to recognize the mode and do the required qualified actions as consequence [85-87].

Please Insert Figure 1 here

The monitoring mechanism of the proposed RCM works based on the CBM approach. CBM determines the maintenance activities according to the actual condition of the systems [85]. In addition, developed RCM mimics the shocking process [86] that degrades the considered reliability function of the machines stochastically. In other words, CBM monitors the reliability degradation caused by stochastic shocking process. Simultaneously, it predicts and determines the appropriate maintenance actions according to the reliability statues of the machines [16, 85]. The failures considered in the research are of both types CM and PM. Now, in case the reliability status falls beneath the first critical threshold \( L \), CBM suggests to have PM and if it gets inferior to failure rate \( LL \), a failure or breakdown occurs [87].

Please Insert Figure 2 here

Figure 2 illustrates reliability deteriorating and failure modes schematically. This figure plots the manner of the reliability from two aspects. In the upper part, it introduces the stochastic variables of the problem, while in the lower part, on a generally similar figure, it focuses on the maintenance activities according to
the state of reliability. The $S$ values in the figure denote the shock times that reduce machine reliability within simulation process. This example encompasses seven shocks, i.e. $S1$ to $S7$, presented on the horizontal axis. The $M$ values, i.e. $M1$ and $M2$, denotes the time of the $j^{th}$ maintenance activity on the machine.

After shocks $S1$ to $S3$, reliability of the machine is still higher than $L$. Therefore, machine requires no maintenance activity. Then, the fourth stochastic shock ($S4$) decreases the reliability of machine to the preventive maintenance bound $L$. Therefore, on the inspection time $2T$ the PM maintenance activity is recognized. The PM maintenance activity recovers and improves the degradation level in $M1$. The machine works in this level of the reliability until the $S5$ occurs. Since the reliability level of machine after shock $S5$ is higher than the $L$, no maintenance activity is required. However, $S6$ degrades the machine even less than $LL$ thus a corrective maintenance should be done. This corrective maintenance has two main distinctive differences with PM, namely 1) happening between the inspection intervals that causes breakdown of the machines and 2) improving the reliability to a new machine reliability level or reliability zero in $M2$.

In Figure 2, the number represents the stochastic event types that occur during the working process of the machine as follows.

Number 1 is a stochastic variable that denotes machine reliability level ($Rel_m$) or $rel$ and follows exponential distribution with parameter ($RL ~ Exp(\eta))$. In fact, this number is a function of degradation of machine in each time ($D_m(t)$) according to function of Eq.2. In this equation, $\beta_0$ and $\beta_1$ are reliability deterioration rates and weighted average of critical levels i.e. $DM=(L+4*LL)/5$. Machine degradation ($DL_m$) or $D_m(t)$ follows exponential distribution with parameter ($DL_m ~ Exp(\eta)$). It should be noticed that in the equations of this paper, $DL_m$ and $D_m(t)$ denote the machine degradation and $RL_m$ and $Rel_m(t)$ denote machine reliability.

$$Rel_m(t) = \frac{e^{-\beta_0 D_m(t)}}{1 + e^{-\beta_1 (D_m(t)-DM)}}$$

Number 2 denotes the PM duration ($PMD$) and it follows lognormal distribution ($PMD ~ log normal(\mu_{PM}, \sigma_{PM})$).

Number 3 represents the improving or recovery level through PM ($RLPM$) activity, calculated as Eq.3, and it follows lognormal distribution ($RLPM ~ log normal(\mu_{PM}', \sigma_{PM}')$).

$$Rel_{new} = Rel_{old} + RLPM; \quad LL < Rel_{old} \leq L$$

Number 4 denotes the CM duration ($CMD$) and it follows lognormal distribution ($CMD ~ log normal(\mu_{CM}, \sigma_{CM})$).

Number 5 represents the improving or recovery level through CM ($RLCM$) activity, calculated as Eq.4, that entirely removes the reliability of machine or makes it one.

$$Rel_{new} = Rel_{old} + RLCM; \quad Rel_{old} \leq LL$$

Number 6 denotes the stochastic time between two shocks ($TBS$) and it follows an exponential distribution ($TBS ~ Exp(\lambda)$).

Please Insert Figure 3 here
Figure 3 illustrates a brief explanation of the explained reliability modification process.

3- Simulation-Based Optimization (SBO) Algorithm

The proposed SBO has two main elements, including optimization algorithm and simulation process. Two different meta-heuristic algorithms namely GA and BBO conduct the optimization algorithm. Accordingly, this section is classified into three parts. The first two parts introduce the mentioned elements respectively and the third one integrates the whole elements and operators together.

3-1 Optimization algorithm of the SBO

Before developing the optimization algorithms separately, let us explain them comparatively. GA and BBO, as population-based algorithms, have many similarities. Both algorithms include a set of individuals, called chromosomes and Habitats, respectively. The fitness values of the individuals are called fitness and high suitability index (HSI), respectively. Other detailed comparisons of the algorithms are refereed to [84].

3-1-1 The BBO algorithm

BBO mimics the migration term of biogeography science [88, 89]. The solution or habitat structure in this paper is a vector as long as the number of the operations or total number of the operations (TNOP). Each cell of this vector is an ordered pair in which the upper object is the operation name while the lower object is the assigned machine to that operation. Moreover, the first row of the solution structure shows the sequence of the operations for operating on machines. Figure 4 illustrates a sample of solution structure related to the Gant chart of Figure 1.

Please Insert Figure 4 here

BBO implements different strategies in its mutation operator. In sequencing sub-vector (SSV), it applies a hybrid strategy, including swap, reversion and insertion, through a random process, as shown in Figure 5.

Please Insert Figure 5 here

For the assignment sub-vector (MASV), BBO implements through machine changing from the capable table of each operation as Figure 6.

Please Insert Figure 6 here

For executing the migration, in sequencing part, permutation operator conducts the migration as Figure 7, and in assignment part, mask operator plays the role as Figure 8.

3-1-2 The Genetic Algorithm (GA) operators

GA implements reproduction, mutation and crossover as the conductive operators for searching the search space. Reproduction operator copies a set of the elite chromosome to the next generation [90].
3-2 The simulation agent of the algorithm

As mentioned in the developed scheduling model, the proposed FJSP contains different stochastic components, such as $RL$, $PMD$, $RLPM$, $CMD$, $RLCM$ or $TBS$, to encompass a realistic version of the RCM. These variables change the states of the solutions dynamically.

SBO, as a powerful tool of optimization, has a finger in almost every pie or term of the stochastic programming [84]. Two general class of stochastic optimization problems exist in the literature, namely the parametric (static) and the control (dynamic) ones. The static optimization includes a set of static parameters for all states. However, in the control optimization, solutions change according to dynamic states [84]. Here, because of the stochastic nature of problem, dynamic strategy controls the simulation process. Figure 9 plots the general structure of the proposed SBO.

Please Insert Figure 9 here

The input of Figure 8 is a solution from the optimization process and its output is the simulated version of the objective function. This SBO conducts a loop of simulation runs ($Numsim$) to obtain average and the standard deviation of solutions for reporting a more robust solution. In this flowchart, $dt$ regulates sample time of the simulation. Moreover, $VT$ and $LVT$ denote predetermined length between visit times and the obtained last visit time, respectively. Besides, the terms $IJS(j)(i)$, $IJF(j)(i)$, and $IMB(m)$ in Figure 10 to Figure 12 are binary logical variables that represent ‘is operation $j$ of job $i$ started’, ‘is operation $j$ of job $i$ finished’, and ‘is machine $m$ busy’, respectively.

Please Insert Figure 10 here
Please Insert Figure 11 here

The reliability updating function of Figure 10 determines the level of the reliability for machines and the maintenance decision. Figure 11 includes the logic of the maintenance decision determination.

Please Insert Figure 12 here
Please Insert Figure 13 here

According to schedules, machine and job statues determination functions are activated as given in Figure 12 and Figure 13, respectively. These functions determine the start and finish statues of jobs plus the business statues of machines at each moment of simulation.

Please Insert Figure 14 here

The job statue function includes the shocking time determination functions. Figure 14 illustrates the proposed shocking logic. SBO at these shock times updates the reliability level of machines during the operating times for the related assigned operations. Of course, they have impact on the types of the maintenance decisions according to the reliability level obtained after the shock times.

4-Computational Results

This section provides us with the numerical examples of the problem to have a detailed view about the developed stochastic problem and the simulation based algorithms. The general information of these test problems are described in Section 2 and their detailed descriptions are in a file, called $RCM$, placed in
Researchgate Site of the first two authors. In this section, the proposed SBO is compared with genetic algorithm (GA).

4-1 Parameter tuning
Parameters of the algorithms are tuned through Taguchi method [91].

Please Insert Table 2 here
Please Insert Table 3 here

Table 2 and Table 3 show the determined levels of parameters of BBO and GA.

4-2 Algorithms’ Outputs
Tables 4 and 5 present the outputs of the algorithms for the developed stochastic problem for GA and BBO, respectively. Moreover, these tables include the results of the algorithms for simple version of the problem as a lower bound validation. The lower bound model is the simple version of the FJSP with any stochastic parameter or maintenance consideration. Obviously, in such situation both of the Cmax and execution time of the algorithm present lower bound values for the developed stochastic problem. The simple problem does not encounter with PM, CM, or breakdown. Moreover, it does not need inspection. Therefore, Cmax values are just due to the main operations and are at worst case equal to the stochastic version. In terms of execution time, less time is required for processing only some operations in comparison with the case in which different maintenance components are also inserted beside the operations.

In each table and for the main developed problem, because of the stochastic nature of the problems, each test problem is run several times and the average (CmaxMean), standard deviation of Cmax (CmaxSTD) values, and average execution times (Time) are reported. In the simple model part of the tables, Diff1 is difference value of the Cmax in stochastic model and simple lower bound model (i.e. Diff1=CmaxMean-Cmax). Similarly, Diff2 shows differences of time values of the models (i.e. Diff2=Time1-Time2).

Please Insert Table 4 here
Please Insert Table 5 here

In both Tables 4 and 5, the last columns represent the average values of the columns. Since Cmax, standard deviation and Time objective functions are all minimization, the less value denotes the best one.

Please Insert Figure 15 here
Please Insert Figure 16 here

Figures 15 compares the algorithms on the three metrics average Cmax (CmaxMean), average time, and average standard deviation of obtained simulated solutions. As it is clear, only in time metric GA is better than BBO. Figure 16 does the comparison for obtained outputs of the algorithms on the deterministic version or lower bound problem.

Please Insert Figure 17 here
As can be seen in Fig.17, algorithms do not have difference on $C_{\text{max}}$. Besides, although they have same trend in time part, the vertical dimension of the algorithm’s outputs are different.

*Please Insert Table 6 here*
*Please Insert Table 7 here*

Tables 6 and 7 conduct the statistical tests for the simple and stochastic versions. In fact, they prove that the algorithms in terms of $C_{\text{max}}$ are non-dominated and in terms of time, GA is superior. Figures 18 compares the convergence plot of GA and BBO for the stochastic and simple problem for the mentioned metrics. Moreover, the real time novel reliability-monitoring illustration is also presented in Figure 19 for problem FJSP9. GA is used for drawing these figures. This developed and innovative figure illustrates the developed reliability-centered maintenance approach in detailed. In this figures, whenever a task is assigned to a machine, its reliability degrades during the task operation. Then, according to the mentioned logic behind the PM and CM, suitable maintenance reaction is taken.

*Please Insert Figure 18 here*
*Please Insert Figure 19 here*

### 4.3 Discussion
As mentioned in Figure 2, our RCM problem assumes two determining levels, i.e. $L$ and $LL$. These levels are tuned as 0.81 and 0.11, respectively. According to this figure, six stochastic components are considered in the proposed RCM to make it realistic. These components and variables are also shown in Figure 20 on the main selected problem FJSP9. In fact, this figure is Figure 19 but in reliability part, it just reports the outputs of the Machine2 for presentation simplicity.

*Please Insert Figure 20 here*

Number (1) or $RL$ and Number (6) or $TBM$ in Figure 20 depicts a set of reliability degradations and set of shocks, respectively, due to activation of operation 1.1 on Machine2. However, since the values of these variables are too little, the associated values are presented all together for a specific operation. $RL$ is regulated according to the function of Figure 3 and Figure 10. Shock times of $TBM$ are generated according to Figure 14. Besides, the (3) values show the effect of the PM ($RLPM$) on the reliability level of machine and they cause PM with duration denoted by Number (2). The PM occurs when the degradation level goes less than the $L$ level at the time or before the inspection times. Inspection times are presented in Gant chart part of figure. CM recovery level ($RLCM$) and their durations are pointed by number (5) and (4), respectively. CM happens when the reliability level violates $LL$ level. The activation of PM or CM and their durations are done by the maintenance decision function given in Figure 11. In the Gant chart part of the figure, machines and jobs are scheduled through Figure 12 and Figure 13, respectively. Figure 9 manages the whole simulation task. Numbers (7) and (8) in this figure show the wasted time according to the maintenance requirement recognized with the autonomous detection engine of the simulation algorithm. It means that during the periods shown by numbers 7 and 8, operations $O_{3,3}$ and $O_{4,4}$ had been started respectively, since, they had been degraded in the reliability figure. However, since their reliability levels had become less than $LL$ and $L$, respectively, they require CM and PM. Therefore, their main operations are interrupted and the maintenance operations are started. Of course, since the jobs are not resumable in our problem, they are started from beginning after their maintenance activities. To sum up, these figures prove that designed algorithm can control the process autonomously.
5- Conclusion

This research focused on the maintenance consideration in production problems. A stochastic FJSP was developed by considering a modern maintenance system called RCM. This autonomous RCM monitors reliability level permanently and decides which maintenance activity should be done. Since the developed problem needs real time checking of stochastic events, it is so complicated. Therefore, two SBO mechanisms, namely GA and BBO, are developed to conduct the optimization problem. The required main and sub functions of the proposed algorithms were described in detail with sufficient examples. According to these results, proposed RCM took benefit from its considered CBM concept properly. Moreover, it handles the considered assumptions and constraints during the optimization process completely. Moreover, different innovative and novel visualization techniques illustrated the proposed logics of the stochastic problem explicitly. Future work of this research may control the cost term of the maintenance within a multi-objective problem or develop other stochastic techniques, based on decomposition, to handle the same problem.

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List of Tables

Table 1. Literature review of the integration of scheduling and maintenance

Table 2. The factor levels of BBO.

Table 3. The factors levels of GA.

Table 4. Outputs of the algorithms on test problems.

Table 5. Outputs of the algorithms on test problems.

Table 6. T-test for comparing GA and BBO on the metrics of the Table 4.

Table 7. T-test for comparing GA and BBO on the metrics of the Table 5.
List of figures

Figure 1. The machine capability table and Gant chart of a related feasible solution.

Figure 2. The maintenance activities due to the degradation level.

Figure 3. The proposed reliability modification model.

Figure 4. The proposed solution habitat (solution) vector of the example Fig.1.

Figure 5. The proposed hybrid SSV operator of the habitat.

Figure 6. MASV operator of BBO.

Figure 7. Proposed migration of sequencing.

Figure 8. Proposed migration of assignment.

Figure 9. The overall flowchart of the proposed simulation part of SBO.

Figure 10. Proposed reliability updating function.

Figure 11. Proposed maintenance decision function.

Figure 12. Proposed machine statuses determination function.

Figure 13. Proposed job statuses determination function.

Figure 14. Proposed shock creation function.

Figure 15. Comparison of algorithms on the stochastic problem with maintenance considerations.

Figure 16. Comparison of algorithms on the simple problem without maintenance considerations.

Figure 17. Comparison of algorithms with their obtained lower bound.

Figure 18. Comparing convergence plot of GA and BBO for three problems.

Figure 19. Real time reliability level according to the Gant chart evolution for FJSP9.

Figure 20. Description of the main result of the outputs on FJSP9.
<table>
<thead>
<tr>
<th>Ref. #</th>
<th>Year</th>
<th>Scheduling Types</th>
<th>Objectives</th>
<th>Types of Maintenance</th>
<th>Solving Methodologies</th>
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<td>2005</td>
<td>Single Machine</td>
<td>TWCT</td>
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<td>GA</td>
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<td>2005</td>
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<td>Pan et al. [39]</td>
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<td>Single Machine</td>
<td>MWT</td>
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<td>2011</td>
<td>Parallel Machine</td>
<td>TWCT</td>
<td>*</td>
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<td>Lee [16]</td>
<td>1999</td>
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<td>Allaoui and Artiba [23]</td>
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<th>Reference</th>
<th>Year</th>
<th>Type</th>
<th>Objective</th>
<th>Method(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naderi et al. [35]</td>
<td>2011</td>
<td>Flexible flow shop</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Huang &amp; Yu [50]</td>
<td>2016</td>
<td>Flow shop</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Zribi et al. [34]</td>
<td>2008</td>
<td>Job Shop</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Mati [38]</td>
<td>2010</td>
<td>Job Shop</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Ben Ali [42]</td>
<td>2011</td>
<td>Job Shop</td>
<td>$C_{\text{max}}$, $\text{Cost}$</td>
<td>*</td>
</tr>
<tr>
<td>Zhou et al. [44]</td>
<td>2011</td>
<td>Job Shop</td>
<td>$C_{\text{max}}$, $\text{Cost}$</td>
<td>*</td>
</tr>
<tr>
<td>Zribi and Borne [71]</td>
<td>2005</td>
<td>FJSP</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Gao et al. [72]</td>
<td>2006</td>
<td>FJSP</td>
<td>$C_{\text{max}}$, TWL, CWL</td>
<td>*</td>
</tr>
<tr>
<td>Wang and Yu [73]</td>
<td>2010</td>
<td>FJSP</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Moradi et al. [74]</td>
<td>2011</td>
<td>FJSP</td>
<td>$C_{\text{max}}$, Unavailability</td>
<td>*</td>
</tr>
<tr>
<td>Mokhtari, and Dadgar [75]</td>
<td>2015</td>
<td>FJSP</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
<tr>
<td>Ahmadi et al. [80]</td>
<td>2016</td>
<td>FJSP</td>
<td>$C_{\text{max}}$, Stability</td>
<td>*</td>
</tr>
<tr>
<td>This Study</td>
<td></td>
<td>FJSP</td>
<td>$C_{\text{max}}$</td>
<td>*</td>
</tr>
</tbody>
</table>

* BBO & GA
**Table 2.** The factor levels of BBO.

<table>
<thead>
<tr>
<th></th>
<th>A: Iteration Size</th>
<th>B: Population Size</th>
<th>C: Mutation Rate</th>
<th>D: E Rate</th>
<th>A: I Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>0.1</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>30</td>
<td>0.2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>50</td>
<td>0.3</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**Table 3.** The factors levels of GA.

<table>
<thead>
<tr>
<th></th>
<th>A: Iteration Size</th>
<th>B: Population Size</th>
<th>C: Crossover Rate</th>
<th>D: Mutation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>30</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>50</td>
<td>0.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 4.** Outputs of the algorithms on test problems.

<table>
<thead>
<tr>
<th>Problem #</th>
<th>GA</th>
<th></th>
<th></th>
<th>BBO</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{\text{maxMean}}$</td>
<td>Time</td>
<td>$C_{\text{maxSTD}}$</td>
<td>$C_{\text{maxMean}}$</td>
<td>Time</td>
<td>$C_{\text{maxSTD}}$</td>
</tr>
<tr>
<td>FJSP1</td>
<td>108</td>
<td>440.02</td>
<td>0</td>
<td>108</td>
<td>873.81</td>
<td>0.0011</td>
</tr>
<tr>
<td>FJSP2</td>
<td>193</td>
<td>924.09</td>
<td>15.202</td>
<td>173.5</td>
<td>1907.36</td>
<td>98.005</td>
</tr>
<tr>
<td>FJSP3</td>
<td>93</td>
<td>683.67</td>
<td>0</td>
<td>93</td>
<td>1429.61</td>
<td>0</td>
</tr>
<tr>
<td>FJSP4</td>
<td>146.75</td>
<td>728</td>
<td>13.22</td>
<td>141.05</td>
<td>1365.07</td>
<td>16.97</td>
</tr>
<tr>
<td>FJSP5</td>
<td>167.7</td>
<td>1255.8</td>
<td>25.243</td>
<td>212.35</td>
<td>2871.43</td>
<td>30.759</td>
</tr>
<tr>
<td>FJSP6</td>
<td>76</td>
<td>690.4</td>
<td>0</td>
<td>76</td>
<td>1222.08</td>
<td>0</td>
</tr>
<tr>
<td>FJSP7</td>
<td>261.9</td>
<td>1277.74</td>
<td>4.596</td>
<td>265</td>
<td>2347.27</td>
<td>21.637</td>
</tr>
<tr>
<td>FJSP8</td>
<td>169.2</td>
<td>1197.9</td>
<td>12.727</td>
<td>183.1</td>
<td>2200</td>
<td>3.889</td>
</tr>
<tr>
<td>FJSP9</td>
<td>266.95</td>
<td>2702</td>
<td>30.193</td>
<td>252.8</td>
<td>5244.42</td>
<td>47.16</td>
</tr>
<tr>
<td>FJSP10</td>
<td>204.05</td>
<td>2496.35</td>
<td>8.414</td>
<td>203.09</td>
<td>4550.73</td>
<td>21.99</td>
</tr>
</tbody>
</table>

**Average** | 168.655 | 1239.597 | 10.959 | 170.789 | 2401.178 | 26.14111 |
Table 5. Outputs of the algorithms on test problems.

<table>
<thead>
<tr>
<th>Problem #</th>
<th>GA</th>
<th>BBO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{max}$</td>
<td>Time2</td>
</tr>
<tr>
<td>FJSP1</td>
<td>108</td>
<td>2.17</td>
</tr>
<tr>
<td>FJSP2</td>
<td>149</td>
<td>2.65</td>
</tr>
<tr>
<td>FJSP3</td>
<td>93</td>
<td>2.67</td>
</tr>
<tr>
<td>FJSP4</td>
<td>133</td>
<td>2.73</td>
</tr>
<tr>
<td>FJSP5</td>
<td>154</td>
<td>3.66</td>
</tr>
<tr>
<td>FJSP6</td>
<td>76</td>
<td>6.02</td>
</tr>
<tr>
<td>FJSP7</td>
<td>203</td>
<td>6.82</td>
</tr>
<tr>
<td>FJSP8</td>
<td>139</td>
<td>7.45</td>
</tr>
<tr>
<td>FJSP9</td>
<td>167</td>
<td>11.37</td>
</tr>
<tr>
<td>FJSP10</td>
<td>154</td>
<td>11.7</td>
</tr>
<tr>
<td>Average</td>
<td>137.6</td>
<td><strong>5.724</strong></td>
</tr>
</tbody>
</table>

Table 6. $T$-test for comparing GA and BBO on the metrics of the Table 4.

<table>
<thead>
<tr>
<th>Metric Name</th>
<th>P-Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{max}$ ($C_{max_{mean}}$)</td>
<td>0.943</td>
<td>They are not considerably different</td>
</tr>
<tr>
<td>$Time1$</td>
<td>0.044</td>
<td>GA outperforms BBO</td>
</tr>
<tr>
<td>$Standard Deviation$ ($C_{max_{STD}}$)</td>
<td>0.150</td>
<td>They are not considerably different</td>
</tr>
</tbody>
</table>

Table 7. $T$-test for comparing GA and BBO on the metrics of the Table 5.

<table>
<thead>
<tr>
<th>Metric Name</th>
<th>P-Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{max}$</td>
<td>0.926</td>
<td>They are not considerably different</td>
</tr>
<tr>
<td>$Time2$</td>
<td>0.000</td>
<td>GA outperforms BBO</td>
</tr>
</tbody>
</table>
Figure 1. The machine capability table and Gant chart of a related feasible solution.
Figure 2. The maintenance activities due to the degradation level.
Calculate the Reliability level of machines $\text{Rel}(m)$ in each new sample time

\begin{align*}
\text{If} \quad \text{Rel}(m) & \geq L \\
\text{Rel}(m)_{\text{new}} & = \text{Rel}(m)_{\text{old}} \\
\text{Else if} \quad LL & \leq \text{Rel}(m) \leq L \\
\text{Rel}(m)_{\text{new}} & = \text{Rel}(m)_{\text{old}} + \text{RLPM} \\
\text{Else} \\
\text{Rel}(m)_{\text{new}} & = \text{Rel}(m)_{\text{old}} + RLCM = 1
\end{align*}

End

End

Figure 3. The proposed reliability modification model.

<table>
<thead>
<tr>
<th>Habitat Sequence</th>
<th>O_{31}</th>
<th>O_{32}</th>
<th>O_{33}</th>
<th>O_{11}</th>
<th>O_{22}</th>
<th>O_{12}</th>
<th>O_{23}</th>
<th>O_{13}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habitat Machine Assignment</td>
<td>M_{3}</td>
<td>M_{2}</td>
<td>M_{1}</td>
<td>M_{2}</td>
<td>M_{1}</td>
<td>M_{3}</td>
<td>M_{2}</td>
<td>M_{1}</td>
</tr>
</tbody>
</table>

Figure 4. The proposed solution habitat (solution) vector of the example Fig.1.

1. Generate a random number between 1 and 3 and call it Rand1/2.
2. Generate a random permutation with TNQP and call it Raniperm1/2.

\text{If} \quad \text{Rand1}=1

\text{Execute Swap Operator}:

a1= Raniperm1(1)

a2= Raniperm1(2)

SSV1=SSV

SSV1(a1)= SSV(a2)

SSV1(a2)= SSV(a1)

\text{Else if} \quad \text{Rand1}=2

\text{Execute Reversion Operator}:

b1= \text{min}(\text{Randperm1}(1), \text{Randperm1}(2))

b2= \text{max}(\text{Randperm1}(1), \text{Randperm1}(2))

SSV1=SSV

\text{Reverse the order of the objects placed between the b1 and b2 in SSV1 or SSV1 (b1:b2)=SSV1(b2::1:b1)}

\text{Else} \quad \text{Rand1}=3

\text{Execute Insertion Operator}:

c1= Raniperm1(1)

c2= Raniperm1(2)

\text{if} \quad C1 < C2

SSV1=[SSV(1:c1) SSV(c2) SSV(c1+1:c2-1) SSV(c2+1:end)]

\text{else}

SSV1=[SSV(1:c2-1) SSV(c2+1:c1) SSV(c2) SSV(c1+1:end)]

End

Figure 5. The proposed hybrid SSV operator of the habitat.

<table>
<thead>
<tr>
<th>Habitat Sequence</th>
<th>O_{31}</th>
<th>O_{32}</th>
<th>O_{33}</th>
<th>O_{11}</th>
<th>O_{22}</th>
<th>O_{12}</th>
<th>O_{23}</th>
<th>O_{13}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habitat Machine Assignment</td>
<td>M_{6}</td>
<td>M_{3}</td>
<td>M_{6}</td>
<td>M_{3}</td>
<td>M_{6}</td>
<td>M_{3}</td>
<td>M_{6}</td>
<td>M_{3}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Habitat Sequence</th>
<th>O_{31}</th>
<th>O_{32}</th>
<th>O_{33}</th>
<th>O_{11}</th>
<th>O_{22}</th>
<th>O_{12}</th>
<th>O_{23}</th>
<th>O_{13}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habitat Machine Assignment</td>
<td>M_{6}</td>
<td>M_{3}</td>
<td>M_{6}</td>
<td>M_{3}</td>
<td>M_{6}</td>
<td>M_{3}</td>
<td>M_{6}</td>
<td>M_{3}</td>
</tr>
</tbody>
</table>

Figure 6. MASV operator of BBO.
**Permutation Crossover**

Generate an integer number \(1-2\) and denote it by \(理念\)

\(EH=\text{emigrated habitat}; \ IH=\text{immigrated habitat}; \ MH=\text{migrated habitat}\)

\(R_i^p=\text{Generate an integer number } \left[1/(TNOP_i^p)\right]\)

\(x_{i1}=EH_i^p\)

\(x_{i2}=EH_i^p+1; \text{end}\)

\(x_{i2}=IH_i^p\)

\(x_{i2}=IH_i^p+1; \text{end}\)

\(MH=[x_{i1} \times x_{i2}]\)

\(MH=[x_{i1} \times x_{i2}]\)

**Figure 7.** Proposed migration of sequencing.

/\!
/
\ Create a random Mask vector with binary values as long as \(TNOP/\)
/\!
/
\(EH=\text{emigrated habitat}; \ IH=\text{immigrated habitat}; \ MH=\text{migrated habitat}\)

For \(i=1:TNOP\)

If \(Mask(i)=0\)

\(MH(i)=EH(i)\)

Else

\(MH(i)=IH(i)\)

End

End

**Figure 8.** Proposed migration of assignment.
Figure 9. The overall flowchart of the proposed simulation part of SBO.
Figure 10. Proposed reliability updating function.

```plaintext
1 for m=1:M
2    if t=0
3        Reliability level (RL) of machine m is zero i.e. RL[m]=1
4    end
5    if t is not shock time of the machine m i.e. t≠shocktime[m]
6        /Note: Shock times are obtained from shock creation function/
7        go to the next machine of the loop i.e. m=m+1
8    else
9        The final DL[m] of machine m is cumulated with a random number of
10           exponential distribution with η mean i.e. RLnew[m]= RLold[m]-exp(η)
11    end
12 According to the updated reliability determine the maintenance decision
13    /Note: through maintenance creation function/
14 end
```

Figure 11. Proposed maintenance decision function.

```plaintext
1 if reliability level of machine m (RL[m]) is less than reliability bound (L) i.e. RL[m]≤L
2    if t is visit time or RL[m] is less than reliability bound (H) i.e. t=VLT & RL[m]≤LL
3        Call the index of the started job and its related operation on machine m as i and j respectively
4        Breakdown of the operation j of job i on machine m
5            to start one type of maintenance activity on that i.e. IJS[i][j]=0
6    if RL[m]>LL
7        Conduct preventive maintenance activity on machine m with stochastic duration number
8            from lognormal distribution i.e. PMD = log normal(μPM,σPM)
9        Increase the reliability level of machine m with a random from lognormal distribution
10           i.e. RLnew = log normal(μPM,σPM) and Relnew = Relold + RLPM; LL < Relold ≤ LL
11        Set the start time of PM job on machine m (STJ[m]) and start time of machine m (STM[m]) to t
12            i.e. STJ[m]=t & STM[m]=t
13        Set the finish time of PM job on machine m (FTJ[m]) and finish time of machine m (STM[m]) to t+PMD
14            i.e. FTJ[m]=t+PMD & STM[m]=t+PMD
15    else if RL[m]≤LL
16        Conduct corrective maintenance activity on machine m with stochastic duration number
17            from lognormal distribution i.e. CMD = log normal(μCM,σCM)
18        Improve the reliability level of machine m entirely and set it to one.
19            i.e. Relnew = Relold + RLCM; Relold ≤ LL
20        Set the start time of CM job on machine m (STJ[m]) and start time of machine m (STM[m]) to t
21            i.e. STJ[m]=t & STM[m]=t
22        Set the finish time of CM job on machine m (FTJ[m]) and finish time of machine m (STM[m]) to t+CMD
23            i.e. FTJ[m]=t+CMD & STM[m]=t+CMD
24    end
25 end
26 Make the machine m busy i.e. IMB[m]=1 /Note: This business lasts to the end of FTM[m]
27 end
```
| for m=1:M |
| if Machine m is busy i.e. IMB[m]=1 |
| if op<FTM[m] |
| Relax machine and change its status to no busy i.e. IMB[m]=0 |
| if Type of the job on machine is not maintenance activity |
| Call the index of the started job and its related operation on machine m as i and j respectively |
| /Note: The index of the maintenance activities does not require calling in this part and they are simply finished/ |
| Change the status of j of job i on machine m to finish i.e. JF[i][j]=1 |
| end |
| else |
| Go to the next machine i.e. m=m+1 |
| end |

Figure 12. Proposed machine statuses determination function.

| for m=1:M |
| if Machine m is busy i.e. IMB[m]=1 |
| Go to the next machine i.e. m=m+1 |
| end |
| for op=1:TNOP |
| Call the job and operation index of the op\textsuperscript{th} ordered operation |
| from the sequence harmony sub vector that is assigned to machine m as i and j respectively |
| /Note: Assignment are determined according to assignment sub part of harmony solution/ |
| if The called operation is started i.e. IJS[i][j]=1 |
| Go to the next operation i.e. op=op+1 |
| else if Called operation is not started and it’s all precedence operations are finished |
| i.e. IJS[i][j]=0 & all[JF[i][j+1]]=1 |
| Called operation can be started i.e. IJS[i][j]=1 |
| Start time of this operation is maximum of the existing simulation time, |
| its final precedence finish time, and the finish time of the final operated job on machine m |
| i.e. STJ[i][j]=Max[t, FTJ[i][j-1], FTM[m]] |
| Finish time of the operation is it’s start time added to it’s processing time on machine m |
| i.e. FTJ[i][j]= STJ[i][j]+PTJ[i][j][m] |
| Start and finish times of the machine m should also be updated |
| i.e. STM(m)=STJ[i][j] & FTM(m)=FTJ[i][j] |
| Shock times should also be determines in this stage through shock creation function |
| end |

Figure 13. Proposed job statuses determination function.

| The starting time of the shocking is equal to start time of the called operation |
| on machine m and is inserted in shock time set of machine m |
| i.e. S_{t}=STJ[i][j] & Shocktimeset(m)=[S_{t}] |
| while S is less than the finished time of the called operation i.e. S\leq FTJ[i][j] |
| l=l+1 |
| Determine the next shock time through exponential distribution with \lambda mean i.e. S_{i}= S_{t}+exp(\lambda) |
| Insert S_{i} in shock time set of machine m i.e. Shocktimeset_{new}(m)=[Shocktimeset_{old} | S_{i}] |
| end |

Figure 14. Proposed shock creation function.
Figure 15. Comparison of algorithms on the stochastic problem with maintenance considerations.

Figure 16. Comparison of algorithms on the simple problem without maintenance considerations.
Figure 17. Comparison of algorithms with their obtained lower bound.
Figure 18. Comparing convergence plot of GA and BBO for three problems.
Figure 19. Real time reliability level according to the Gant chart evolution for FJSP9.
Figure 20. Description of the main result of the outputs on FJSP9.