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A novel correlation coefficient of intuitionistic fuzzy sets based on the connection number of set pair analysis and its application

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Abstract. Set Pair Analysis (SPA) is an updated theory of dealing with uncertainty, which overlaps with other theories of uncertainty such as probability, vague, fuzzy, and Intuitionistic Fuzzy Set (IFS). Considering the fact that the correlation coefficient plays an important role during a decision-making process, after revealing the weakness of the existing correlation coefficients between the IFSs, this study proposes a novel correlation coefficient and weighted correlation coefficients formulation to measure the relative strength of different IFSs. To do so, firstly, corresponding to each intuitionistic fuzzy number, the connection number of the SPA theory is formulated in the form of the identity, discrepancy, and contrary degrees; moreover, measurements of the novel correlation coefficient are defined. Pairs of identity, discrepancy, and contrary of the connection number are taken as a vector representation during the formulation. Lastly, a decision-making approach based on the proposed measures is presented, illustrated by two numerical examples in pattern recognition and medical diagnosis.

1. Introduction

Success of human beings in life depends on the ability of decision-making (DM) and recognition of precise situations, demonstrating that DM plays an important role in human life. However, due to the increasing complexity of the system, it is difficult for decision-makers to make a perfect decision, since most of preferences and values during the DM process are imbued with uncertainty. To handle it, Zadeh [1] introduced the concept of Fuzzy Set (FS) theory to deal with uncertain or ambiguous data by assigning a membership value corresponding to each element whose range is between 0 and 1. Following their successful study, researchers have become preoccupied, in their extensions and out of them, with Intuitionistic Fuzzy Set (IFS) and Interval-Valued Intuitionistic Fuzzy Set (IVIFS) theories, as proposed by Atanassov [2] and Atanassov and Gargov [3], respectively. They involved the membership and non-membership degrees simultaneously, and many scholars have widely used them. Research on this subject can be classified roughly into four main topics: aggregation operators, measures, decision-making methods, and preference relation. For instance, Xu and Yager [4] presented a geometric aggregation operator, while Xu [5] presented a weighted averaging operator for aggregating different Intuitionistic Fuzzy Numbers (IFNs). Garg [6] presented a generalized intuitionistic fuzzy interactive geometric aggregation operator using Einstein t-norm and t-conorm operations. Xu and Chen [7], Xu [8]

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developed some arithmetic and geometric aggregation operators, namely interval-valued intuitionistic fuzzy weighted averaging and geometric operators, respectively, for aggregating the interval-valued intuitionistic fuzzy information. Garg et al. [9] presented an entropy-based approach to solving the decision-making problem under fuzzy environment. Garg [10], further, developed a new generalized, improved score function to rank the IVIFSs. Later on, Garg [11,12] extended the theory of the IFS to the Pythagorean fuzzy set, which is less than one, to relax the condition of the sum of their membership function to square sum of its membership functions. He presented generalized geometric as well as averaging aggregation operators. Apart from that, various researchers pay more attention to the decision-making process to aggregate different alternatives using different aggregation operators [13-23] and their corresponding references.

On the other hand, a correlation plays an important role in statistics and engineering sciences. Through correlation analysis, the joint relationship of two variables can be examined with the aid of a measure of interdependency of the two variables. Chiang and Lin [24] discussed the correlation of fuzzy data based on conventional statistics and derived a formula for the correlation coefficient of fuzzy sets. Liu and Kao [25] studied the correlation coefficient of fuzzy numbers using a mathematical programming approach based on the classical definition of correlation coefficients. Later, Hong [26] studied the correlation coefficient of fuzzy numbers by applying T_w-based (the weakest t-norm) algebraic operations. Wang and Li [27] introduced the correlation of interval-valued fuzzy numbers. Furthermore, Gerstenkorn and Manko [28] introduced the correlation coefficient of IFSs, whereas Bustince and Burillo [29] discussed the concepts of correlation and correlation coefficient of Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs). Hong [30] generalized the concepts of correlation and correlation coefficient of IVIFSs to a general probability space and extended the results of Bustince and Burillo [29]. Xu et al. [31] defined a correlation coefficient of IFSs from the set-theoretic viewpoint. Xu [32] provided a survey on correlation analysis of IFSs and proposed a new method for deriving the correlation coefficients of IFSs, enjoying some advantages over the existing methods. Garg [33] defined the concepts of correlation and correlation coefficients of Pythagorean fuzzy sets. Garg [34], further, presented a novel accuracy function under the interval-valued Pythagorean fuzzy set to solve the decision-making problems.

Based on these studies, it has been observed that their proposed correlation coefficients have several drawbacks. For instance, measures of the existing correlation coefficients are independent of the degree of the membership or non-membership when one of the membership and non-membership degrees is zero in IFSs. In addition, the effect of a change of the degree of the membership function on their index values remains unaffected. Therefore, it gives an inconsistent model which is unable to rank different IFSs on the respective scales. Although the above studies have been widely used by researchers, their credibility is not guaranteed. In order to handle the uncertainties in a more precise way, Zhao [35] introduced the Set Pair Analysis (SPA) for handling the uncertainties specific to the data in which certainty and uncertainty are studied as one system. SPA is a modified uncertainty theory in which both certainties and uncertainties are treated as an integrated certain-uncertain system. In SPA, the main component is the Connection Number (CN) which represents three aspects of identity, discrepancy, and contrary degrees to describe the certainty and uncertainty of an element. The main principal of SPA is to analyze the feature of set pair and construct a connection degree for them. The concept of the arithmetic operations between the connection numbers was defined in Liu et al. [36]. Wang and Gong [37] proposed a decision-making method based on the set-pair analysis to solve the decision-making problems with ascertained criteria weight and the criteria value being an interval random variable. Hu and Yang [38] proposed a dynamic stochastic multi-attribute decision-making approach based on the cumulative prospect theory and SPA. Xie et al. [39] presented a CN under an interval-valued fuzzy set by taking the positive and negative ideal schemes. Fu and Zhou [40] proposed a triangular fuzzy MCDM method based on SPA. Kumar and Garg [41] presented a Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method under the IVIFS environment based on the connection number of the SPA. Apart from these, some researchers [42-50] have solved the fuzzy decision-making problem under SPA.

Thus, based on the above observations, it has been observed that SPA is one of the most successful theories of handling the uncertainties and certainties in the system. Motivated by this, the present paper develops the correlation coefficient measures based on the connection number of the SPA to solve the multi-criteria group decision-making problems where preferences related to different alternatives are taken in the form of the IFNs. In order to achieve it, firstly, the shortcoming of the existing correlation coefficient measures under the IFSs environment has been highlighted; then, connection numbers corresponding to IFNs have been proposed to overcome these shortcomings. Based on these connection numbers, a new informational energy and the covariance between the two CNs have been defined; hence, the correlation coefficient and a weighted correlation coefficient have been proposed. Then, accordingly, the problem of MCDM from the fields of pattern recognition and medical diagnosis has been solved to validate the effectiveness and applicability of the proposed decision method.

The rest of the paper is organized as follows. Section 2 describes the basic concepts of the IFSs and their corresponding existing correlation coefficients. Section 3 presents correlation coefficients and weighted correlation coefficients based on the connection number of the SPA. Section 4 highlights the shortcomings of the existing correlation coefficients. Section 5 describes the decision-making method based on the proposed correlation coefficients under the SPA theory. Section 6 demonstrates the proposed correlation coefficients with numerical examples related to pattern recognition and medical diagnoses. Finally, Section 7 presents a concrete conclusion.

2. Preliminaries

In this section, some basic concepts about the IFSs are defined over non-empty universal set X.

2.1. Intuitionistic Fuzzy Set (IFS)

An IFS A is defined as a set of ordered pairs over a universal set X given by [2]:

$$A = \{\langle x, u_A(x), v_A(x)\rangle \mid x \in X\},$$

where $u_A : X \to [0,1]$ and $v_A : X \to [0,1]$ represent the degrees of membership and non-membership of x to A such that for any $x \in X$, $u_A(x)+v_A(x) \leq 1$. Moreover, the intuitionistic index of x to A is defined as $\pi_A(x) = 1 - u_A(x) - v_A(x)$, Usually, the pair $\langle u_A(x), v_A(x)\rangle$ is called an Intuitionistic Fuzzy Number (shorted by IFN) and is often simplified as $\langle u, v\rangle$ where $u \in [0,1], v \in [0,1], u+v \leq 1$.

2.2. Correlation coefficient based on IFSs

For any two IFSs $A = \{\langle x_t, u_A(x_t), v_A(x_t)\rangle \mid x_t \in X\}$ and $B = \{\langle x_t, u_B(x_t), v_B(x_t)\rangle \mid x_t \in X\}$ in the universe of discourse X, the informational energies of these sets are denoted by $E_{IFS}(A)$ and $E_{IFS}(B)$, respectively, which are defined as follows [28]:

$$E_{IFS}(A) = \sum_{t=1}^{n}[u_A^2(x_t) + v_A^2(x_t)],$$

$$E_{IFS}(B) = \sum_{t=1}^{n}[u_B^2(x_t) + v_B^2(x_t)].$$

The correlation of IFSs A and B is defined as follows:

$$C_{IFS}(A,B) = \sum_{t=1}^{n}\Big(u_A(x_t)u_B(x_t) + v_A(x_t)v_B(x_t)\Big).$$

Thus, accordingly, the correlation coefficients of A and B are defined as follows:

$$K_{IFS_1}(A,B) = \frac{C_{IFS}(A,B)}{\sqrt{E_{IFS}(A)\cdot E_{IFS}(B)}} = \frac{\sum_{t=1}^{n}\Big(u_A(x_t)u_B(x_t) + v_A(x_t)v_B(x_t)\Big)}{\sqrt{\sum_{t=1}^{n}\big(u_A^2(x_t)+v_A^2(x_t)\big)\cdot\sum_{t=1}^{n}\big(u_B^2(x_t)+v_B^2(x_t)\big)}}. \quad (1)$$

On the other hand, Xu et al. [31] suggested an alternative form of the correlation coefficients of IFSs A and B by Eqs. (2) and (3) as shown in Box I. The function K_{IFS} satisfies the following properties:

(P1) $0 \leq K_{IFS}(A,B) \leq 1$;

(P2) $K_{IFS}(A,B) = K_{IFS}(B,A)$;

(P3) $K_{IFS}(A,B) = 1$ if $A = B$.

Example 2.1. Let A_1, A_2, and A_3 be three IFSs defined in $X = \{x_1, x_2\}$ by: $A_1 = \{(x_1, 0.10, 0.15), (x_2, 0.10, 0.15)\}$, $A_2 = \{(x_1, 0.20, 0.30), (x_2, 0.20, 0.30)\}$, and $A_3 = \{(x_1, 0.30, 0.45), (x_2, 0.30, 0.45)\}$. Then, it is easy to check if $K_{IFS_1}(A_1, A_2) = K_{IFS_1}(A_2, A_3) = K_{IFS_1}(A_3, A_1)$.

$$K_{IFS_2}(A,B) = \frac{\sum_{t=1}^{n}\Big(u_A(x_t)u_B(x_t) + v_A(x_t)v_B(x_t)\Big)}{\max\Big\{\{\sum_{t=1}^{n}\big(u_A^2(x_t)+v_A^2(x_t)\big)\}^{1/2}, \{\sum_{t=1}^{n}\big(u_B^2(x_t)+v_B^2(x_t)\big)\}^{1/2}\Big\}}, \quad (2)$$

and:

$$K_{IFS_3}(A,B) = \frac{\sum_{t=1}^{n}\Big(u_A(x_t)u_B(x_t) + v_A(x_t)v_B(x_t) + \pi_A(x_t)\pi_B(x_t)\Big)}{\max\Big\{\sum_{t=1}^{n}\big(u_A^2(x_t)+v_A^2(x_t)+\pi_A^2(x_t)\big), \sum_{t=1}^{n}\big(u_B^2(x_t)+v_B^2(x_t)+\pi_B^2(x_t)\big)\Big\}}. \quad (3)$$

Box I

Example 2.2. Consider two IFNs $A = \langle 0.4, 0\rangle$ and $B = \langle 0, 0.5\rangle$. Then, the correlation coefficient between them, by using Eq. (1), is $K_{IFS_1}(A,B) = \frac{(0.4)(0)+(0.5)(0)}{\sqrt{0.4^2+0^2}\sqrt{0.5^2+0^2}} = 0$. On the other hand, if we replace IFN A with $A_1 = \langle 0.6, 0\rangle$, then the correlation coefficient between A_1 and B is $K_{IFS_1}(A_1,B) = \frac{(0.6)(0)+(0)(0.5)}{\sqrt{0.6^2+0^2}\sqrt{0^2+0.5^2}} = 0$. Hence, the existing index is independent of the change of the membership degree in IFS.

Example 2.3. Let two IFNs $A = \langle 0.4, 0.3\rangle$ and $B = \langle 0, 0.5\rangle$, then the correlation coefficient becomes:

$$K_{IFS_1}(A,B) = \frac{(0.4)(0) + (0.3)(0.5)}{\sqrt{0.4^2 + 0.3^2}\sqrt{0^2 + 0.5^2}} = 0.6.$$

On the other hand, if IFNs A and B are replaced with $A_1 = \langle 0.1, 0.4898\rangle$ and $B_1 = \langle 0.45189, 0.21398\rangle$, respectively, then the correlation coefficient becomes:

$$K_{IFS_1}(A_1,B_1) = \frac{(0.1)(0.45189) + (0.4898)(0.21398)}{\sqrt{0.1^2 + 0.4898^2}\sqrt{0.45189^2 + 0.21398^2}} = 0.6.$$

Hence, it has been concluded that, by changing the membership and non-membership degrees of IFSs, the correlation coefficient remains the same; therefore, it is inconsistent and, hence, unable to rank the alternatives. Therefore, there is a need to enhance these measures.

2.3. Set pair analysis

Zhao [35] presented a novel concept of the Set Pair Analysis (SPA) to deal with both of the certainty and uncertainty degrees as one system based on the quantitative analysis. The main principal of SPA is to analyze the features of set pair and construct a connection degree for them. The set pair of two interrelated sets A and B denoted by $H(A,B)$ for a given problem W and Connection Number (CN), represented by $\mu(H,W)$, is characterized by the the three components, namely "identity", "discrepancy", and "contrary", and are defined as follows:

$$\mu(H,W) = (S/N) + (F/N)i + (P/N)j, \tag{4}$$

where N is the total number of features, in which S represents the "identity" features, P is the "contrary" features and $F = N - S - P$ is the "discrepancy" features of sets A and B. Herein, we define the ratio $S/N = a$ (identity degree), $F/N = b$ (discrepancy degree), and $P/N = c$ (contrary degree). Then, Eq. (4) becomes:

$$\mu(H,W) = a + bi + cj. \tag{5}$$

Herein, i is coefficient of "discrepancy degree" such that $i \in [-1, 1]$, j is the coefficient of "contrary degree", and $j = -1$. It is clearly seen that $0 \le a, b, c \le 1$ and $a + b + c = 1$.

3. The proposed correlation coefficient based on connection numbers

Let $A = \langle u, v\rangle$ be an IFS where u and v represent the degrees of membership and nonmembership values, respectively, of A such that $u, v \in [0, 1]$ and $u + v \le 1$. Then, connection number μ corresponding to set A is defined as follows:

$$\mu(H,A) = a + bi + cj, \tag{6}$$

where $a = u(1 - v)$ is the "identity", $b = 1 - u(1 - v) - v(1 - u)$ is the "discrepancy", and $c = v(1 - u)$ is the "contrary" degrees.

Let $A = \{\langle x_t, u_A(x_t), v_A(x_t)\rangle \mid x_t \in X\}$ be an IFS in the universe of discourse X, and its corresponding connection number is defined as $\mu_A = \{a(x_t) + b(x_t)i + c(x_t)j \mid x_t \in X\}$ where $a(x_t), b(x_t), c(x_t) \in [0, 1]$ and $a(x_t) + b(x_t) + c(x_t) = 1$; for every $x_t \in X$, we define:

$$E(A) = \sum_{t=1}^{n} \left(a^2(x_t) + b^2(x_t) + c^2(x_t)\right), \tag{7}$$

which is called the informational energy of set A.

Assume that two sets:

$$A = \{(x_t, a_1(x_t), b_1(x_t), c_1(x_t)) \mid x_t \in X\},$$

and:

$$B = \{(x_t, a_2(x_t), b_2(x_t), c_2(x_t)) \mid x_t \in X\},$$

are defined over the universe of discourse X where $a_1(x_t), a_2(x_t), b_1(x_t), b_2(x_t), c_1(x_t), c_2(x_t) \in [0, 1]$ for every $x_t \in X$. Then, the following so-called correlation of CNs A and B is defined as follows:

$$C(A,B) = \sum_{t=1}^{n} \left[a_1(x_t) \cdot a_2(x_t) + b_1(x_t) \cdot b_2(x_t) + c_1(x_t) \cdot c_2(x_t)\right]. \tag{8}$$

From the above, it is obvious that the correlation of CNs satisfies the following properties:

(P1) $C(A, A) = E(A)$;

(P2) $C(A, B) = C(B, A)$.

Definition 3.1. Let A and B be the sets of CNs corresponding to the IFSs on a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ denoted by $A = \{(x_t, a_1(x_t), b_1(x_t), c_1(x_t)) \mid x_t \in X\}$ and $B = \{(x_t, a_2(x_t), b_2(x_t), c_2(x_t)) \mid x_t \in X\}$, respectively. Then, the correlation coefficient between A and B is given by Eq. (9) as shown in Box II.

Theorem 3.1. For any two sets of CNs A and B defined over the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, correlation coefficient K_1 satisfies the following properties.

$$K_1(A,B)=\frac{C(A,B)}{\sqrt{E(A)\cdot E(B)}}=\frac{\sum_{t=1}^{n}\left[a_1(x_t)\cdot a_2(x_t)+b_1(x_t)\cdot b_2(x_t)+c_1(x_t)\cdot c_2(x_t)\right]}{\sqrt{\sum_{t=1}^{n}\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right)}\sqrt{\sum_{t=1}^{n}\left(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\right)}}. \tag{9}$$

Box II

(P1) $K_1(A,B)=K_1(B,A)$;

(P2) $0\leq K_1(A,B)\leq 1$;

(P3) $A=B\Leftrightarrow K_1(A,B)=1$;

(P4) If $A\subseteq B\subseteq C$, for a connection number C then $K_1(A,C)\leq K_1(A,B)$ and $K_1(A,C)\leq K_1(B,C)$.

Proof.

(P1) It is straightforward.

(P2) The inequality in $K_1(A,B)\geq 0$ is evident; thus, as shown in Box III we prove $K_1(A,B)\leq 1$. By Cauchy-Schwarz inequality, we have:

$$(x_1y_1+x_2y_2+\ldots+x_ny_n)^2$$
$$\leq (x_1^2+y_1^2+\ldots+x_n^2)$$
$$\cdot(y_1^2+y_2^2+\ldots+y_n^2)$$

where $(x_1+x_2+\ldots+x_n)\in R^n$ and $(y_1+y_2+\ldots+y_n)\in R^n$. Then we obtain the relation shown in Box IV.

Let the following notations be:

$$\sum_{t=1}^{n}a_1^2(x_t)=\xi_1,\qquad \sum_{t=1}^{n}b_1^2(x_t)=\rho_1,$$

$$\sum_{t=1}^{n}c_1^2(x_t)=\eta_1,\qquad \sum_{t=1}^{n}a_2^2(x_t)=\xi_2,$$

$$\sum_{t=1}^{n}b_2^2(x_t)=\rho_2,\qquad \sum_{t=1}^{n}c_2^2(x_t)=\eta_2.$$

So the above inequality becomes:

$$K_1(A,B)\leq\frac{\sqrt{\xi_1\xi_2}+\sqrt{\rho_1\rho_2}+\sqrt{\eta_1\eta_2}}{\sqrt{\xi_1+\rho_1+\eta_1}\cdot\sqrt{\xi_2+\rho_2+\eta_2}}.$$

By squaring on both sides, we get:

$$K_1^2(A,B)\leq\frac{\left(\sqrt{\xi_1\xi_2}+\sqrt{\rho_1\rho_2}+\sqrt{\eta_1\eta_2}\right)^2}{(\xi_1+\rho_1+\eta_1)\cdot(\xi_2+\rho_2+\eta_2)}.$$

By subtracting 1 from both sides, we obtain the relation shown in Box V. Thus, we have $0\leq K_1(A,B)\leq 1$.

(P3) Sufficient condition is obviously true. Now, necessary condition is proven.

So, let equality sign hold in the proof of (P2) if we have $a_1(x_t)=ha_2(x_t)$, $b_1(x_t)=hb_2(x_t)$ and $c_1(x_t)=hc_2(x_t)$ for some positive real number h. However, according to the condition of CN, we have $a_1(x_t)+b_1(x_t)+c_1(x_t)=$

$$K_1(A,B)=\frac{\sum_{t=1}^{n}\Big(a_1(x_t)\cdot a_2(x_t)+b_1(x_t)\cdot b_2(x_t)+c_1(x_t)\cdot c_2(x_t)\Big)}{\sqrt{\sum_{t=1}^{n}\Big(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\Big)\cdot\sum_{t=1}^{n}\Big(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\Big)}}.$$

Box III

$$K_1(A,B)\leq\frac{\left(\sum_{t=1}^{n}a_1^2(x_t)\sum_{t=1}^{n}a_2^2(x_t)\right)^{1/2}+\left(\sum_{t=1}^{n}b_1^2(x_t)\sum_{t=1}^{n}b_2^2(x_t)\right)^{1/2}+\left(\sum_{t=1}^{n}c_1^2(x_t)\sum_{t=1}^{n}c_2^2(x_t)\right)^{1/2}}{\sqrt{\left(\sum_{t=1}^{n}a_1^2(x_t)+\sum_{t=1}^{n}b_1^2(x_t)+\sum_{t=1}^{n}c_1^2(x_t)\right)}\cdot\sqrt{\left(\sum_{t=1}^{n}a_2^2(x_t)+\sum_{t=1}^{n}b_2^2(x_t)+\sum_{t=1}^{n}c_2^2(x_t)\right)}}.$$

Box IV

$$K_1^2(A,B)-1\leq\frac{-\left(\left(\sqrt{\xi_1\rho_2}-\sqrt{\rho_1\xi_2}\right)^2+\left(\sqrt{\rho_1\eta_2}-\sqrt{\eta_1\rho_2}\right)^2+\left(\sqrt{\xi_1\eta_2}-\sqrt{\eta_1\xi_2}\right)^2\right)}{(\xi_1+\rho_1+\eta_1)\cdot(\xi_2+\rho_2+\eta_2)}\leq 0.$$

Box V

$1=a_2(x_t)+b_2(x_t)+c_2(x_t)$ which implies that $h=1$. Hence, $A=B$.

(P4) $A\subseteq B\subseteq C$ which implies that $a_1(x_t)\leq a_2(x_t)\leq a_3(x_t)$, $b_1(x_t)\geq b_2(x_t)\geq b_3(x_t)$, and $c_1(x_t)\geq c_2(x_t)\geq c_3(x_t)$. Geometrically, if $A\subseteq B\subseteq C$, then the angle between A and C should be larger than the angles between A and B and between B and C for any element x_t and is a decreasing function within the interval $[0,\pi/2]$. Thus, relations $K_1(A,C)\leq K_1(A,B)$ and $K_1(A,C)\leq K_1(B,C)$ can be obtained by Eq. (9)

Example 3.1. Let A and B be any two sets of CNs defined in $X=\{x_1,x_2,x_3\}$ such that $A=\{(x_1,0.25,0.35,0.40),(x_2,0.30,0.45,0.25),(x_3,0.60,0.20,0.20)\}$, $B=\{(x_1,0.75,0.15,0.10),(x_2,0.55,0.35,0.10)$, and $(x_3,0.40,0.20,0.40)\}$. Then, using Eq. (7), the informational energy of A is written as follows:

$$E(A)=\sum_{t=1}^{n}\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right)$$
$$=(0.25^2+0.35^2+0.40^2)$$
$$+(0.30^2+0.45^2+0.25^2)$$
$$+(0.60^2+0.20^2+0.20^2)=1.1400.$$

The informational energy of B is given as $E(B)=1.3900$. Now, by using Eq. (8), the correlation between sets A and B is written as follows:

$$C(A,B)=\sum_{t=1}^{n}\left[a_1(x_t)\cdot a_2(x_t)+b_1(x_t)\cdot b_2(x_t)\right.$$
$$\left.+c_1(x_t)\cdot c_2(x_t)\right]=(0.25\times 0.75$$
$$+0.35\times 0.15+0.40\times 0.10)$$
$$+(0.30\times 0.55+0.45\times 0.35$$
$$+0.25\times 0.10)+(0.60\times 0.40$$
$$+0.20\times 0.20+0.20\times 0.40)=0.9875.$$

Hence, the correlation coefficient between the sets of CNs A and B is given by:

$$K_1(A,B)=\frac{C(A,B)}{\sqrt{E(A)\cdot E(B)}}=\frac{0.9875}{1.1400\times 1.3900}$$
$$=0.7845.$$

Furthermore, in order to stress the importance of the proposed correlation coefficient with respect to the existing ones, the correlation measure has been computed for the above taken examples, where the existing measures have failed to rank the alternatives, as below.

Example 3.2. If the proposed measure is applied to the above considered Example 2.1, then we get $K_1(A_1,A_2)=0.9759$, $K_1(A_2,A_3)=0.9839$ and $K_1(A_3,A_1)=0.9215$. Thus, the proposed measure classifies the IFSs and, hence, is able to identify the best one.

Example 3.3. If the proposed correlation coefficient measure is applied to Example 2.2, i.e., by taking two IFNs $A=\langle 0.4,0\rangle$ and $B=\langle 0,0.5\rangle$ then $K_1(A,B)=0.5883$. On the other hand, if IFN A is replaced with $A_1=\langle 0.6,0\rangle$, then the correlation coefficient between A_1 and B is $K_1(A_1,B)=0.3922$. Hence, the proposed measure simultaneously considers the effect of the change of the membership degree on IFS during the analysis.

Example 3.4. If the proposed measures are applied to the data as considered in Example 2.3, then we get $K_1(A,B)=0.8026$, and if the replaced IFNs are used, $A_1=\langle 0.1,0.4898\rangle$ and $B_1=\langle 0.45189,0.21398\rangle$, then we get $K_1(A_1,B_1)=0.7865$. Therefore, the proposed measure has sufficiently considered the impact of the change in the degrees of membership and non-membership on the correlation coefficient measure and, hence, has overcome the shortcomings of the existing measures.

Definition 3.2. Let A and B be the set of two CNs. Then, the correlation coefficient is defined by Eq. (10) as shown in Box VI.

Theorem 3.2. Correlation coefficient, $K_2(A,B)$, between the two sets of CNs A and B, satisfies the following properties:

(P1) $K_2(A,B)=K_2(B,A)$;

$$K_2(A,B)=\frac{C(A,B)}{\max\{E(A),E(B)\}}=\frac{\sum\limits_{t=1}^{n}\left[a_1(x_t)\cdot a_2(x_t)+b_1(x_t)\cdot b_2(x_t)+c_1(x_t)\cdot c_2(x_t)\right]}{\max\left\{\sum\limits_{t=1}^{n}\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right),\sum\limits_{t=1}^{n}\left(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\right)\right\}}. \qquad (10)$$

Box VI

$$K_3(A,B)=\frac{C_\omega(A,B)}{\sqrt{E_\omega(A)\cdot E_\omega(B)}}=\frac{\sum\limits_{t=1}^{n}\omega_t\Big(a_1(x_t)\cdot a_2(x_t)+b_1(x_t)\cdot b_2(x_t)+c_1(x_t)\cdot c_2(x_t)\Big)}{\sqrt{\sum\limits_{t=1}^{n}\omega_t\Big(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\Big)\cdot\sum\limits_{t=1}^{n}\omega_t\Big(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\Big)}}, \qquad (11)$$

and:

$$K_4(A,B)=\frac{C_\omega(A,B)}{\max\{E_\omega(A),E_\omega(B)\}}=\frac{\sum\limits_{t=1}^{n}\omega_t\Big(a_1(x_t)\cdot a_2(x_t)+b_1(x_t)\cdot b_2(x_t)+c_1(x_t)\cdot c_2(x_t)\Big)}{\max\left\{\sum\limits_{t=1}^{n}\omega_t\Big(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\Big),\sum\limits_{t=1}^{n}\omega_t\Big(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\Big)\right\}}. \qquad (12)$$

Box VII

(P2) $0\le K_2(A,B)\le 1$;

(P3) $A=B\Leftrightarrow K_2(A,B)=1$;

(P4) If $A\subseteq B\subseteq C$ for a connection number C then $K_2(A,C)\le K_2(A,B)$ and $K_2(A,C)\le K_2(B,C)$.

Proof. Properties (P1), (P3), and (P4) are straight forward; thus, we omit them here. Also, $K_2(A,B)\ge 0$ is evident. Now, from Theorem 3.1, we have $(C(A,B))^2\le E(A)\cdot E(B)$. Therefore, $C(A,B)\le\max\{E(A),E(B)\}$; thus, $K_2(A,B)\le 1.\square$

However, in many practical situations, different sets may have taken different weights; thus, weight ω_t of element $x_t\in X$ $(t=1,2,\dots,n)$ should be taken into account such that $\omega_t>0$, $\sum\limits_{t=1}^{n}\omega_t=1$. In the following, a weighted correlation coefficient is developed between CNs in which by Eqs. (11) and (12) as shown in Box VII, the above formulated $K_1(A,B)$ and $K_2(A,B)$ are extended to the weighted correlation coefficient.

It can be easily deduced that if $\omega=(1/n, 1/n,\dots,1/n)^T$, then Eqs. (11) and (12) reduce to the correlation coefficient defined in Eqs. (9) and (10), respectively. It is easy to see that the weighted correlation coefficients $K_3(A,B)$ and $K_4(A,B)$ between the sets of CNs satisfy the properties of $0\le K_3(A,B)\le 1$ and $0\le K_4(A,B)\le 1$.

Theorem 3.3. Let $\omega=(\omega_1,\omega_2,\dots,\omega_n)^T$ be the weight vector of x_t $(t=1,2,\dots,n)$, such that $\omega_t>0$ and $\sum_t\omega_t=1$. Then, the weighted correlation coefficient, $K_k(A,B)(k=3,4)$, between the sets of CNs A and B defined by Eqs. (11) and (12), satisfies the following properties:

(P1) $K_k(A,B)=K_k(B,A)$;

(P2) $0\le K_k(A,B)\le 1$;

(P3) $A=B\Leftrightarrow K_k(A,B)=1$;

(P4) If $A\subseteq B\subseteq C$ for a connection number C, then $K_k(A,C)\le K_k(A,B)$ and $K_k(A,C)\le K_k(B,C)$.

Proof. Proof follows the above theorem; hence, it is omitted here.

4. Drawbacks of the existing correlation coefficients

In this section, some limitations inherent in the existing methods of correlation coefficients for Atanassov IFSs are shown. Advantages of the proposed correlation coefficients have been discussed. To show the limitations of the existing correlation coefficients, a few examples are presented in this section. For convenience, assume that the weights of elements, $x_t\in X$, are all equal.

4.1. Correlation coefficient of Zeng and Li [51]

Assume that there are three patterns defined in the form of IFS with the degrees of membership and non-

$$ZL(A,B)=\frac{\sum_{t=1}^{n}\Big(u_A(x_t)u_B(x_t)+v_A(x_t)v_B(x_t)+\pi_A(x_t)\pi_B(x_t)\Big)}{\sqrt{\sum_{t=1}^{n}\left(u_A^2(x_t)+v_A^2(x_t)+\pi_A^2(x_t)\right)\cdot\sum_{t=1}^{n}\left(u_B^2(x_t)+v_B^2(x_t)+\pi_B^2(x_t)\right)}}. \quad (13)$$

Box VIII

membership $\langle u(x_t), v(x_t)\rangle$ in $X=\{x_1,x_2,x_3\}$ defined as follows:

$$A_1=\{\langle x_1,0.4,0.5\rangle,\langle x_2,0.7,0.1\rangle,\langle x_3,0.3,0.3\rangle\},$$

$$A_2=\{\langle x_1,0.5,0.4\rangle,\langle x_2,0.7,0.2\rangle,\langle x_3,0.4,0.3\rangle\},$$

$$A_3=\{\langle x_1,0.4,0.5\rangle,\langle x_2,0.7,0.1\rangle,\langle x_3,0.4,0.3\rangle\}.$$

Assume that a sample of $B=\{\langle x_1,0.1,0.1\rangle,\langle x_2,1.0,0.0\rangle,\langle x_3,0.0,1.0\rangle\}$ is given.

Zeng and Li [51] defined the correlation measure between IFSs A and B by Eq. (13) as shown in Box VIII. Hence, accordingly, the suitable classifier for sample B from A_1, A_2 and A_3 patterns can be identified using Eq. (13). The results corresponding to each pattern are obtained as follows:

$$ZL(A_1,B)=ZL(A_2,B)=ZL(A_3,B)=0.6262.$$

On the other hand, to identify which pattern belongs to sample B by using Eq. (9), we convert the intuitionistic fuzzy values of the given patterns and sample into the CNs according to Eq. (6) as follows:

$$A_1=\{\langle x_1,0.20,0.50,0.30\rangle,\langle x_2,0.63,0.34,0.03\rangle,\langle x_3,0.21,0.58,0.21\rangle\},$$

$$A_2=\{\langle x_1,0.30,0.50,0.20\rangle,\langle x_2,0.56,0.38,0.06\rangle,\langle x_3,0.28,0.54,0.18\rangle\},$$

$$A_3=\{\langle x_1,0.20,0.50,0.30\rangle,\langle x_2,0.63,0.34,0.03\rangle,\langle x_3,0.28,0.54,0.18\rangle\},$$

$$B=\{\langle x_1,0.09,0.82,0.09\rangle,\langle x_2,1.00,0.00,0.00\rangle,\langle x_3,0.00,0.00,1.00\rangle\}.$$

Then, based on these CNs, Eq. (9) is utilized, and the measure values are obtained as follows:

$$K_1(A_1,B)=0.6879,$$

$$K_1(A_2,B)=0.6534,$$

$$K_1(A_3,B)=0.6777.$$

Thus, based on these results, correlation coefficient of Zeng and Li [51] cannot classify this sample. On the other hand, the SPA theory is able to accommodate the required information about the sample and, hence, find if sample B belongs to pattern A_1 according to the recognition principle.

4.2. Correlation coefficient of Szmidt and Kacprzyk [52]

Szmidt and Kacprzyk [52] proposed a correlation coefficient for two Atanassov's IFSs, A and B, to express not only a relative strength but also a positive or negative relationship between A and B. Suppose that there exists a random sample $x_1,x_2,\ldots,x_n\in X$. Then, correlation coefficient $r(A,B)$ is given by:

$$r(A,B)=\frac{r_1(A,B)+r_2(A,B)+r_3(A,B)}{3}, \quad (14)$$

where:

$$r_1(A,B)=\frac{\sum_{t=1}^{n}(u_A(x_t)-\overline{u_A})(u_B(x_t)-\overline{u_B})}{\sqrt{\sum_{t=1}^{n}(u_A(x_t)-\overline{u_A})^2.\sum_{t=1}^{n}(u_B(x_t)-\overline{u_B})^2}},$$

$$r_2(A,B)=\frac{\sum_{t=1}^{n}(v_A(x_t)-\overline{v_A})(v_B(x_t)-\overline{v_B})}{\sqrt{\sum_{t=1}^{n}(v_A(x_t)-\overline{v_A})^2.\sum_{t=1}^{n}(v_B(x_t)-\overline{v_B})^2}},$$

$$r_3(A,B)=\frac{\sum_{t=1}^{n}(\pi_A(x_t)-\overline{\pi_A})(\pi_B(x_t)-\overline{\pi_B})}{\sqrt{\sum_{t=1}^{n}(\pi_A(x_t)-\overline{\pi_A})^2.\sum_{t=1}^{n}(\pi_B(x_t)-\overline{\pi_B})^2}},$$

where:

$$\overline{u_A}=\frac{1}{n}\sum_{t=1}^{n}u_A(x_t),\qquad \overline{u_B}=\frac{1}{n}\sum_{t=1}^{n}u_B(x_t),$$

$$\overline{v_A}=\frac{1}{n}\sum_{t=1}^{n}v_A(x_t),\qquad \overline{v_B}=\frac{1}{n}\sum_{t=1}^{n}v_B(x_t),$$

$$\overline{\pi_A}=\frac{1}{n}\sum_{t=1}^{n}\pi_A(x_t),\qquad \overline{\pi_B}=\frac{1}{n}\sum_{t=1}^{n}\pi_B(x_t).$$

Based on the above formulated correlation coefficient, as shown, it is not able to find a good correlation between the intuitionistic fuzzy sets. For instance, consider A and B as two Atanassov's IFSs in $X = \{x_1, x_2, x_3\}$ defined as:

$$A = \{\langle x_1, 0.1, 0.2\rangle, \langle x_2, 0.2, 0.1\rangle, \langle x_3, 0.29, 0.0\rangle\},$$

$$B = \{\langle x_1, 0.1, 0.3\rangle, \langle x_2, 0.2, 0.2\rangle, \langle x_3, 0.29, 0.1\rangle\}.$$

Then, by applying Eq. (14), we get $r(A, B) = 1$, which means that these two IFSs are perfectly correlated, which is not true as seen from the structure of sets A and B. On the other hand, if our proposed method is applied to find the correlation coefficient between A and B using Eqs. (9) and (10), then we get $K_1(A, B) = 0.9922$ and $K_2(A, B) = 0.9298$, respectively. These results show that A and B are not perfectly correlated.

4.3. Correlation coefficient of Xu et al. [31]

Xu et al. [31] defined the correlation coefficient in the environment of intuitionistic fuzzy sets by considering the degrees of membership and non-membership as well as the degree of hesitation between the two intuitionistic sets A and B given in Box IX.

Consider the following three patterns A_1, A_2 and A_3 represented in the form of intuitionistic fuzzy sets:

$$A_1 = \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.1\rangle, \langle x_3, 0.3, 0.3\rangle\},$$

$$A_2 = \{\langle x_1, 0.5, 0.4\rangle, \langle x_2, 0.7, 0.2\rangle, \langle x_3, 0.4, 0.3\rangle\},$$

$$A_3 = \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.1\rangle, \langle x_3, 0.4, 0.3\rangle\}.$$

Assume that a sample $B = \{\langle x_1, 0.1, 0.1\rangle, \langle x_2, 1, 0\rangle, \langle x_3, 0, 1\rangle\}$ is given. Then, by using the correlation coefficient of Xu et al. [31] defined in Eq. (15), the following results are obtained:

$$Xu_1(A_1, B) = Xu_1(A_2, B) = Xu_1(A_3, B) = 0.4398.$$

Thus, based on their results, it has been concluded that their correlation coefficient cannot be used to classify sample B with three patterns A_1, A_2, and A_3.

To remove this problem, the intuitionistic fuzzy values of the given patterns and sample are converted into the CNs according to Eq. (6) as follows:

$$A_1 = \{\langle x_1, 0.20, 0.50, 0.30\rangle, \langle x_2, 0.63, 0.34, 0.03\rangle, \langle x_3, 0.21, 0.58, 0.21\rangle\},$$

$$A_2 = \{\langle x_1, 0.30, 0.50, 0.20\rangle, \langle x_2, 0.56, 0.38, 0.06\rangle, \langle x_3, 0.28, 0.54, 0.18\rangle\},$$

$$A_3 = \{\langle x_1, 0.20, 0.50, 0.30\rangle, \langle x_2, 0.63, 0.34, 0.03\rangle, \langle x_3, 0.28, 0.54, 0.18\rangle\},$$

$$B = \{\langle x_1, 0.09, 0.82, 0.09\rangle, \langle x_3, 1.00, 0.00, 0.00\rangle, \langle x_3, 0.00, 0.00, 1.00\rangle\}.$$

Eq. (10) is utilized to obtain the correlation coefficient as follows:

$$K_2(A_1, B) = 0.4817,$$

$$K_2(A_2, B) = 0.4445,$$

$$K_2(A_3, B) = 0.4705.$$

Thus, it has been concluded that sample B can be classified with pattern A_1.

5. Decision-making approach based on the proposed correlation coefficients

This section parents a method for the decision-making problems by means of the proposed correlation coefficient or weighted correlation coefficient between two or more IFSs. In this respect, assume that there are m alternatives, denoted by $A_1, A_2, \ldots, A_m$, which are evaluated under the set of n criteria, denoted by $C_1, C_2, \ldots, C_n$, and their preferences are given in the form of IFNs $\alpha_{kt} = \langle u_{kt}, v_{kt}\rangle; k = 1, 2, \ldots, m; t = 1, 2, \ldots, n$ where u_{kt} and v_{kt}, respectively, represent the degree of how much A_k can satisfy and dissatisfy criteria C_t such that $u_{kt} + v_{kt} \leq 1$. Thus, the rating values corresponding to each alternative are represented in the form of IFNs over the universal set X as follows:

$$A_k = \{\langle x_t, u_{k1}(x_t), v_{k1}(x_t)\rangle \mid x_t \in U\};$$

$$k = 1, 2, \ldots, m; \qquad t = 1, 2, \ldots, n.$$

$$Xu_1(A, B) = \frac{\sum_{t=1}^{n} \Big(u_A(x_t)u_B(x_t) + v_A(x_t)v_B(x_t) + \pi_A(x_t)\pi_B(x_t) \Big)}{\max\left\{ \sum_{t=1}^{n} \left(u_A^2(x_t) + v_A^2(x_t) + \pi_A^2(x_t) \right), \sum_{t=1}^{n} \left(u_B^2(x_t) + v_B^2(x_t) + \pi_B^2(x_t) \right) \right\}}. \tag{15}$$

Box IX

Let ω_t $(t = 1, 2, \ldots, n)$ be the weight of criterion C_t such that $\omega_t > 0$ and $\sum_{t=1}^{n} \omega_t = 1$. In the decision-making process, the concept of an ideal point has been used to access the best alternative. Although the ideal alternative does not exist in the real world, it does provide a useful theoretical construct to evaluate the alternatives. Therefore, each ideal alternative A^* is proposed as IFN $\alpha_t^* = \langle 1, 0 \rangle$ for $t = 1, 2, \ldots, n$. Then, various steps involved in the proposed approach to finding the best alternative(s) are summarized as follows:

Step 1: Collect all the information corresponding to each alternative in terms of IFNs. Hence, an overall intuitionistic fuzzy decision matrix D is expressed as follows:

$$D = \begin{pmatrix} \langle u_{11}, v_{11} \rangle & \langle u_{12}, v_{12} \rangle & \ldots & \langle u_{1n}, v_{1n} \rangle \\ \langle u_{21}, v_{21} \rangle & \langle u_{22}, u_{22} \rangle & \ldots & \langle u_{2n}, u_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle u_{m1}, v_{m1} \rangle & \langle u_{n2}, v_{n2} \rangle & \ldots & \langle u_{mn}, v_{mn} \rangle \end{pmatrix} \quad (16)$$

Step 2: Convert this intuitionistic fuzzy normalized decision matrix into its equivalent connection number as $\mu_{kt} = a_{kt} + b_{kt}i + c_{kt}j$ where $a_{kt} = u_{kt}(1 - v_{kt})$, $b_{kt} = 1 - u_{kt}(1 - v_{kt}) - v_{kt}(1 - u_{kt})$ and $c_{kt} = v_{kt}(1 - u_{kt})$.

Step 3: Compute $K_1(A_k, A^*)$ or $K_2(A_k, A^*)$ or $K_3(A_k, A^*)$ or $K_4(A_k, A^*)$ measure between alternatives A_k $(k = 1, 2, \ldots, m)$ and the ideal alternative A^* using Eqs. (9), (10), (11) and (12), respectively, where A^* is the ideal alternative whose connection number is given by $\mu_{A^*} = 1 + 0i + 0j$.

Step 4: Rank all of the alternatives with respect to the value of $K_1(A_k, A^*)$ or $K_2(A_k, A^*)$ or $K_3(A_k, A^*)$ or $K_4(A_k, A^*)$ $(k = 1, 2, \ldots, m)$.

Step 5: Choose the best alternative with respect to the maximum value of $K_1(A_k, A^*)$ or $K_2(A_k, A^*)$ or $K_3(A_k, A^*)$ or $K_4(A_k, A^*)$ $(k = 1, 2, \ldots, m)$.

6. Illustrative Example

In this section, an example related to the decision-making problem, from the field of pattern recognition and medical diagnosis, is taken to demonstrate the effectiveness of the proposed method.

Example 6.1: Pattern Recognition. Consider three known patterns C_1, C_2, and C_3 represented in the form of IFSs on $X = \{x_1, x_2, x_3\}$ as follows:

$$C_1 = \{\langle x_1, 1.0, 0.0 \rangle, \langle x_2, 0.8, 0.0 \rangle, \langle x_3, 0.7, 0.1 \rangle\},$$

$$C_2 = \{\langle x_1, 0.8, 0.1 \rangle, \langle x_2, 1.0, 0.0 \rangle, \langle x_3, 0.9, 0.0 \rangle\},$$

$$C_3 = \{\langle x_1, 0.6, 0.2 \rangle, \langle x_2, 0.8, 0.0 \rangle, \langle x_3, 1.0, 0.0 \rangle\}.$$

Consider an unknown pattern B which will be recognized, where:

$$B = \{\langle x_1, 0.5, 0.3 \rangle, \langle x_2, 0.6, 0.2 \rangle, \langle x_3, 0.8, 0.1 \rangle\}.$$

The target of this problem is to classify pattern B in one of classes C_1, C_2 and C_3. To do so, firstly, the IFSs are converted to CNs of given patterns; then, sample and their corresponding numbers are summarized as below:

$$C_1 = \{\langle x_1, 1.0, 0.0, 0.0 \rangle, \langle x_2, 0.8, 0.20, 0.0 \rangle, \langle x_3, 0.63, 0.34, 0.03 \rangle\},$$

$$C_2 = \{\langle x_1, 0.72, 0.26, 0.02 \rangle, \langle x_2, 1.0, 0.0, 0.0 \rangle, \langle x_3, 0.9, 0.1, 0.0 \rangle\},$$

$$C_3 = \{\langle x_1, 0.48, 0.44, 0.08 \rangle, \langle x_2, 0.8, 0.20, 0.0 \rangle, \langle x_3, 1.0, 0.0, 0.0 \rangle\},$$

$$B = \{\langle x_1, 0.35, 0.50, 0.15 \rangle, \langle x_2, 0.48, 0.44, 0.08 \rangle, \langle x_3, 0.72, 0.26, 0.02 \rangle\}.$$

Now, the proposed correlation coefficient indexes, K_1 and K_2, have been computed from B to C_k $(k = 1, 2, 3)$, and their results are given as follows:

$$K_1(C_1, B) = 0.7755, \qquad K_1(C_2, B) = 0.8350,$$

$$K_1(C_3, B) = 0.9223, \qquad K_2(C_1, B) = 0.6221,$$

$$K_2(C_2, B) = 0.6395, \qquad K_2(C_3, B) = 0.7544.$$

Thus, from these two proposed correlation coefficient indexes, it can be concluded that pattern B belongs to pattern C_3.

On the other hand, if it is assumed that weights of x_1, x_2 and x_3 are 0.5, 0.3, and 0.2, respectively, then correlation coefficients K_3 and K_4 can be utilized for obtaining the most suitable pattern as follows:

$$K_3(C_1, B) = 0.7104, \quad K_3(C_2, B) = 0.8129,$$

$$K_3(C_3, B) = 0.9262, \quad K_4(C_1, B) = 0.5270,$$

$$K_4(C_2, B) = 0.6224, \quad K_4(C_3, B) = 0.7842.$$

Thus, ranking order of the three patterns is C_3, C_2, and C_1; hence, C_3 is the most desirable pattern classified with B.

In order to compare the performance of the proposed approach with those of some existing approaches under the IFS environment, a comparison analysis was conducted based on different approaches as given by the authors in [31,51,53-57]. The results are summarized as below:

(a) If the correlation coefficient measures, as proposed by Li [51] given in Eq. (13), are applied to the considered problem, then their corresponding index values of pattern C_k ($k = 1, 2, 3$) are $ZL(C_1, B) = 0.8882$, $ZL(C_2, B) = 0.9291$, and $ZL(C_3, B) = 0.9710$. Thus, ranking of the alternative is $C_3 \succ C_2 \succ C_1$; hence, C_3 is the most desirable pattern classified with B;

(b) If the cosine similarity measure $CSM(\cdot)$, as proposed by Ye [53], is applied to the considered problem (for more details, see [53]), then its value corresponding to each pattern is $CSM(C_1, B) = 0.9353$, $CSM(C_2, B) = 0.9519$, and $CSM(C_3, B) = 0.9724$. Thus, ranking order is $C_3 \succ C_2 \succ C_1$; hence, the best alternative for classifying pattern B is C_3;

(c) If similarity measure $S_{DC}(\cdot)$, as proposed by Dengfeng and Chuntian [54], is applied to the considered data then its index values for pattern $C_k (k = 1, 2, 3)$ with respect to unknown pattern B is computed as $S_{DC}(C_1, B) = 0.74$, $S_{DC}(C_2, B) = 0.78$, and $S_{DC}(C_3, B) = 0.84$, while by Liu [55] measure denoted by $T(\cdot)$ (for more details, see [55]), $T(C_1, B) = 0.72$, $T(C_2, B) = 0.74$, and $T(C_3, B) = 0.84$ are obtained. Thus, C_3 is the most desirable pattern classified with B;

(d) If the correlation coefficient, as proposed by Xu et al. [31], given in Eq. (15), is applied to the considered problem, then the index values corresponding to these measures are $Xu_1(C_1, B) = 0.7252$, $Xu_1(C_2, B) = 0.7177$, and $Xu_1(C_3, B) = 0.8113$. Therefore, C_3 is the most desirable pattern classified with B;

(e) If distance measure $d(\cdot)$, as proposed by Ejegwa and Modom [56], is applied to the considered problem (for more details, see [56]), then the measure values corresponding to each pattern are obtained as follows: $d(C_1, B) = 0.3200$, $d(C_2, B) = 0.3200$, and $d(C_3, B) = 0.2000$. Therefore, C_3 is the most desirable pattern classified with B;

(f) If similarity measure $S(\cdot)$, as proposed by Hung and Yang [57], is applied to the considered problem, then their respective measure values corresponding to each pattern are obtained, $S(C_1, B) = 0.4700$, $S(C_2, B) = 0.4700$, and $S(C_3, B) = 0.5100$; hence, the ranking of these pattern is $C_3 \succ C_2 = C_1$. Therefore, pattern C_3 is classified with pattern B.

Example 6.2: Medical diagnosis. Consider the dataset in [58] containing four patients: $P = \{Al, Bob, Joe, Ted\}$, five symptoms: $S = \{s_1$(Tempera- ture), s_2 (HeadAche), s_3 (Stomach Pain), s_4 (Cough), s_5 (Chest pain)$\}$, and five diseases: $Q = \{Q_1$(Viral fever), Q_2(Malaria), Q_3(Typhoid), Q_4(Stomach Problem), Q_5 (Chest problem)$\}$. The relations between the patients-the symptoms, denoted by PS, and the symptoms-the diseases, denoted by SQ, are represented in the form of the IFNs as shown in Box X. Then, the target of this problem is to classify patients $P = \{Al, Bob, Joe, Ted\}$ with one of diagnoses $Q = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$, respectively. In order to apply the proposed approach to the considered data, firstly, the information related to the patients with respect to the symptoms and the diagnoses with respect to the symptoms are converted into the connection numbers, and their corresponding results are summarized as shown in Box XI. Now, a developed correlation coefficient K_1 as given in Eq. (9) has been utilized for the above data; hence, the follow-

$$PS = \begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} \langle 0.8, 0.1\rangle & \langle 0.6, 0.1\rangle & \langle 0.2, 0.8\rangle & \langle 0.6, 0.1\rangle & \langle 0.1, 0.6\rangle \\ \langle 0.0, 0.8\rangle & \langle 0.4, 0.4\rangle & \langle 0.6, 0.1\rangle & \langle 0.1, 0.7\rangle & \langle 0.1, 0.8\rangle \\ \langle 0.8, 0.1\rangle & \langle 0.8, 0.1\rangle & \langle 0.0, 0.6\rangle & \langle 0.2, 0.7\rangle & \langle 0.0, 0.5\rangle \\ \langle 0.6, 0.1\rangle & \langle 0.5, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 0.7, 0.2\rangle & \langle 0.3, 0.4\rangle \end{bmatrix} \end{array},$$

and:

$$SQ = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} \langle 0.4, 0.0\rangle & \langle 0.7, 0.0\rangle & \langle 0.3, 0.3\rangle & \langle 0.1, 0.7\rangle & \langle 0.1, 0.8\rangle \\ \langle 0.3, 0.5\rangle & \langle 0.2, 0.6\rangle & \langle 0.6, 0.1\rangle & \langle 0.2, 0.4\rangle & \langle 0.0, 0.8\rangle \\ \langle 0.1, 0.7\rangle & \langle 0.0, 0.9\rangle & \langle 0.2, 0.7\rangle & \langle 0.8, 0.0\rangle & \langle 0.2, 0.8\rangle \\ \langle 0.4, 0.3\rangle & \langle 0.7, 0.0\rangle & \langle 0.2, 0.6\rangle & \langle 0.2, 0.7\rangle & \langle 0.2, 0.8\rangle \\ \langle 0.1, 0.7\rangle & \langle 0.1, 0.8\rangle & \langle 0.1, 0.9\rangle & \langle 0.2, 0.7\rangle & \langle 0.8, 0.1\rangle \end{bmatrix} \end{array}.$$

Box X

$$PS = \begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} \langle 0.72, 0.26, 0.02\rangle & \langle 0.54, 0.42, 0.04\rangle & \langle 0.04, 0.32, 0.64\rangle & \langle 0.54, 0.42, 0.04\rangle & \langle 0.04, 0.42, 0.54\rangle \\ \langle 0.00, 0.20, 0.80\rangle & \langle 0.24, 0.52, 0.24\rangle & \langle 0.54, 0.42, 0.04\rangle & \langle 0.03, 0.34, 0.63\rangle & \langle 0.02, 0.26, 0.72\rangle \\ \langle 0.72, 0.26, 0.02\rangle & \langle 0.72, 0.26, 0.02\rangle & \langle 0.00, 0.40, 0.60\rangle & \langle 0.06, 0.38, 0.56\rangle & \langle 0.00, 0.50, 0.50\rangle \\ \langle 0.54, 0.42, 0.04\rangle & \langle 0.30, 0.50, 0.20\rangle & \langle 0.18, 0.54, 0.28\rangle & \langle 0.56, 0.38, 0.06\rangle & \langle 0.18, 0.54, 0.28\rangle \end{bmatrix} \end{array},$$

and:

$$SQ = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} \langle 0.40, 0.60, 0.00\rangle & \langle 0.70, 0.30, 0.00\rangle & \langle 0.21, 0.58, 0.21\rangle & \langle 0.03, 0.34, 0.63\rangle & \langle 0.02, 0.26, 0.72\rangle \\ \langle 0.15, 0.50, 0.35\rangle & \langle 0.08, 0.44, 0.48\rangle & \langle 0.54, 0.42, 0.04 & \langle 0.12, 0.56, 0.32\rangle & \langle 0.00, 0.20, 0.80\rangle \\ \langle 0.03, 0.34, 0.63\rangle & \langle 0.00, 0.10, 0.90\rangle & \langle 0.06, 0.38, 0.56\rangle & \langle 0.80, 0.20, 0.00\rangle & \langle 0.04, 0.32, 0.64\rangle \\ \langle 0.28, 0.54, 0.18\rangle & \langle 0.70, 0.30, 0.00\rangle & \langle 0.08, 0.44, 0.48\rangle & \langle 0.06, 0.38, 0.56\rangle & \langle 0.04, 0.32, 0.64\rangle \\ \langle 0.03, 0.34, 0.63\rangle & \langle 0.02, 0.26, 0.72\rangle & \langle 0.01, 0.18, 0.81\rangle & \langle 0.06, 0.38, 0.56\rangle & \langle 0.72, 0.26, 0.02\rangle \end{bmatrix} \end{array}.$$

Box XI

$$\begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} 0.8360 & \mathbf{0.8914} & 0.8122 & 0.4833 & 0.4499 \\ 0.3474 & 0.2892 & 0.5525 & \mathbf{0.7846} & 0.5449 \\ 0.7583 & 0.6816 & \mathbf{0.8113} & 0.5167 & 0.4661 \\ \mathbf{0.8608} & 0.8493 & 0.7447 & 0.6135 & 0.5647 \end{bmatrix} \end{array}$$

Box XII

ing indices have been computed corresponding to it:

$$\begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} 0.8790 & \mathbf{0.8899} & 0.8102 & 0.4788 & 0.3930 \\ 0.6165 & 0.4308 & 0.7599 & \mathbf{0.9541} & 0.6226 \\ 0.8061 & 0.7054 & \mathbf{0.8696} & 0.5279 & 0.4697 \\ \mathbf{0.8695} & 0.7949 & 0.7362 & 0.6033 & 0.4966 \end{bmatrix} \end{array}.$$

On the other hand, if weights are assigned to 0.15, 0.25, 0.20, 0.15, and 0.25 corresponding to Q_i ($i = 1, 2, \ldots, 5$), respectively, then, by applying correlation coefficient K_3 as given in Eq. (11), the following values are obtained:

$$\begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} 0.8806 & \mathbf{0.8691} & 0.8455 & 0.5275 & 0.3913 \\ 0.6741 & 0.5074 & 0.7809 & \mathbf{0.9530} & 0.5566 \\ 0.8016 & 0.7025 & \mathbf{0.8787} & 0.5411 & 0.4323 \\ \mathbf{0.8654} & 0.7741 & 0.7425 & 0.6501 & 0.5182 \end{bmatrix} \end{array}$$

Based on this analysis, it has been concluded that **Al** suffers from Malaria, **Bob** from Stomach problems, **Joe** from Typhoid, and **Ted** from Viral Fever.

In order to compare the performance of the proposed approach with those of some existing approaches under the IFS environment, a comparison analysis is conducted based on the different approaches as given by the authors in [31,51,56,57]. The results corresponding to these approaches are summarized as below:

(a) If the correlation coefficient measures, as proposed by Zeng and Li [51] given in Eq. (13), are applied to the considered problem, then the indices values, shown in Box XII, are obtained for the patients with respect to the diagnoses. Therefore, based on this analysis, it has been concluded that **Al** suffers from Malaria, **Bob** from Stomach problems, **Joe** from Typhoid, and **Ted** from Viral Fever;

(b) If the correlation coefficient, as proposed by Xu et al. [31], given in Eq. (15), is applied to the considered problem, then the indices values of the patients with respect to the diagnoses are summarized as shown in Box XIII. Thus, based on this analysis, it has been concluded that **Al** suffers from Malaria, **Bob** from Stomach problems, **Joe** from Typhoid and **Ted** from Viral Fever;

(c) If the distance measure, as proposed by Ejegwa

$$\begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} 0.7721 & \mathbf{0.8377} & 0.7941 & 0.4779 & 0.4048 \\ 0.2857 & 0.2741 & 0.4810 & \mathbf{0.6910} & 0.5394 \\ 0.6806 & 0.6591 & \mathbf{0.7708} & 0.4965 & 0.4315 \\ \mathbf{0.8190} & 0.7013 & 0.6692 & 0.5451 & 0.4464 \end{bmatrix} \end{array}.$$

Box XIII

$$\begin{array}{c} \\ Al \\ Box \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} 0.3200 & \mathbf{0.2600} & 0.3200 & 0.4200 & 0.4400 \\ 0.3700 & 0.3700 & 0.3700 & \mathbf{0.3300} & 0.4500 \\ 0.4400 & 0.4400 & \mathbf{0.4200} & 0.4800 & 0.4600 \\ \mathbf{0.3800} & 0.3900 & 0.4400 & 0.4800 & 0.6000 \end{bmatrix} \end{array}.$$

Box XIV

and Modom [56], is applied to the considered problem (for more details, see [56]), then measure values corresponding to each patient with respect to diagnoses Q are obtained, summarized as shown in Box XIV. Therefore, based on this analysis, it has been concluded that **Al** suffers from Malaria, **Bob** from Stomach problems, **Joe** from Typhoid, and **Ted** from Viral Fever;

(d) If the similarity measure, as proposed by Hung and Yang [57], is applied to the considered problem, then their respective measure values are obtained as shown in Box XV. Thus, it has been observed that **Al** suffers from Malaria, **Bob** from Stomach problems, **Joe** from Typhoid, and **Ted** from Viral Fever.

6.1. Advantages of the proposed method

According to the above comparison analysis, the proposed method for addressing the decision-making problems has the following advantages:

(a) The IFS is characterized by the degrees of the membership and non-membership of an element such that their sum is less than 1. However, there may be a situation in which IFS theory be unable to provide the whole information about the situation. On the other hand, SPA theory provides an alternative way to deal with the certainty and uncertainty specific to quantitative analysis of "identity", "discrepancy", and "contrary" degrees of the connection number, such that the sum of their degrees is equal to one. Therefore, SPA theory is more suitable for real scientific and engineering applications;

(b) It has been observed from the existing studies that various researchers [31,51,52] proposed an algorithm using correlation coefficient for IFSs. As mentioned above, these correlation coefficients have some shortcomings, which cannot be represented by IFSs; hence, their corresponding analysis may not give its appropriate results;

(c) The proposed approach represents the intuitionistic fuzzy information using connection degrees, which can simultaneously describe the degrees of membership and non-membership as well as hesi-

$$\begin{array}{c} \\ Al \\ Bob \\ Joe \\ Ted \end{array} \begin{array}{c} \begin{array}{ccccc} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array} \\ \begin{bmatrix} 0.8100 & \mathbf{0.8200} & 0.8000 & 0.5400 & 0.5000 \\ 0.6700 & 0.5400 & 0.7400 & \mathbf{0.9000} & 0.6400 \\ 0.7500 & 0.6800 & \mathbf{0.8200} & 0.6000 & 0.5400 \\ \mathbf{0.8000} & 0.7700 & 0.7100 & 0.6300 & 0.5500 \end{bmatrix} \end{array}.$$

Box XV

tation degree with a simple mathematic depiction. Therefore, the connection degree can be computed without any transformation; hence, their corresponding proposed approach can effectively prevent any loss of information;

(d) The results obtained by the proposed methods might be more accurate since it considers the hesitation degree. The proposed correlation coefficient is more generalized and suitable to solve the real-life problem more accurately than the existing ones.

7. Conclusion

In the present manuscript, correlation coefficients under the set pair analysis theory were presented in terms of its parameters using the connection numbers. The shortcomings of the existing coefficients were highlighted and overcome by the proposed measures. The proposed correlation coefficients suitably utilized the degrees of certainty and uncertainties by the "identity", "discrepancy", and "contrary" degrees in terms of the connection number. Illustrative examples were provided which show that some of the existing correlation coefficients under intuitionistic fuzzy information fail to handle the situation; however, the proposed ones have easily identified it. Furthermore, to deal with the situations where the elements are correlated to each other, weighted correlation coefficients were defined. To demonstrate the efficiency of the proposed coefficients, examples from the field of pattern recognition as well as from the medical diagnosis were selected. Based on these studies, it was concluded that the proposed correlation coefficients could easily handle the real-life decision-making problems with their targets, hence being beneficial for the plant personal.

In the future, the technique can be extended to different fields and some new information measures are proposed. In addition, the methodology could be used to make further improvements to various mathematical programming models such as fuzzy goal programming, cluster analysis, dynamic programming and uncertain programming.

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