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A Lagrangian relaxation approach to fuzzy robust multi-objective facility location network design problem

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Abstract. This study considers a multi-objective combined budget constrained Facility Location/Network Design Problem (FL/NDP), in which system uncertainty is considered. The most obvious practical examples of the problem are territorial designing and locating of academies, airline networks, and medical service centers. In order to assure network reliability versus uncertainty, an efficient robust optimization approach is applied to model the proposed problem. The formulation is minimizing the total expected costs, including transshipment costs, Facility Location (FL) costs, and fixed cost of road/link utilization as well as minimizing the total penalties of uncovered demand nodes. Then, in order to consider several system uncertainties, the proposed model is changed to a fuzzy robust model by suitable approaches. An efficient sub-gradient based Lagrangian relaxation algorithm is applied. In addition, a practical example is studied. In the following, a series of experiments, including several test problems, is designed and solved to evaluate of the performance of the algorithm. The obtained results emphasize that considering practical factors (e.g. several uncertainties, system disruptions, and customer satisfaction) in modelling of the problem can lead to significant improvement of the system yield and, subsequently, more efficient utilization of the established network.

1. Introduction

Nowadays, most of the production industries, logistic systems, and supply chains constantly look for systems to simultaneously decrease several total costs and improve the efficiency of their structures. FL, Network Design (ND), and allocation of demands to them are the problems, which can obviously help to achieve cost reduction and performance improvement goals, efficiently.

In this paper, we combine 3 topics, namely, FL, ND, and facility covering, and study a multi-objective FL/NDP with regards to system uncertainty. The proposed problem is considered as a two-objective-function mathematical model such that the first objective function optimizes the fixed costs and operational costs and the second objective function optimizes the total penalty costs for any nodes that cannot be covered. The model simultaneously optimizes the network costs and uncovering costs. Therefore, minimizing the total costs and optimizing the customer satisfaction are studied simultaneously. Accordingly, the proposed model amplifies the joint profits of customers and the network.

Several applications of the proposed problem can be found in different industries, services, and logistics systems. There are some Supply Chains (SCs) and logistics systems with a variety of uncertainties in various practical problems. Simultaneously considering

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SC, logistics systems, ND, and a variety of system uncertainties is critical in order to ameliorate the efficiency, productivity, and system safety. Some of the well-known examples are airline and railroad networks, pipelines for gas and water, and delivering services systems such as fire stations, education centers, universities, and health care centers. As a concluding point, the best examples are SCs of different spare parts, food products, petrochemicals, etc.

The rest of the paper is organized as follows: In Section 2, a brief review of the relevant literature on the proposed problem is provided, and the research gap and our contribution are explained in the following. In Section 3, the mathematical model description and its formulation are presented. In details, first, the assumptions and notations are described. Then, the multi-objective facility location network design problem is described and, finally, the fuzzy robust model formulation is proposed. In Section 4, general overview of the Lagrangian relaxation approach is presented. In Section 5, the proposed approach for the fuzzy robust multi-objective facility location network design problem is explained. A practical example and experimental analysis are discussed in Section 7. Finally, conclusions and future works are explained.

2. Literature review and research gap

2.1. Background

To clearly describe our contribution, 3 main trends in the literature can be reviewed that may be interesting for comparison: (1) the FL/ND problem, (2) the multi-objective FL Problems (FLPs), and (3) the FL with parameter uncertainty. Clearly, these 3 research trends are greatly related to the topic of this paper, i.e. locating of facility concerning the ND multi-objective problem and parameter uncertainty.

The first trend of research is on the FL problem with respect to ND. Several studies have considered FL, ND, and demand allocation problems, separately. By simultaneously considering the FL and ND, the proposed problem can be clearly explained in a more practical modeling and formulation. The literature emphasizes that the classical FLPs consist of the uncapacitated FLPs, the maximum covering [1], the p-center and p-median [2], and the set covering problems [3]. However, some studies have been done on the FLPs with regards to the ND topic. Drezner and Wesolowsky [4] presented some two-objective formulations for designing of an initial network with potential bi-directional links, each of which could be either opened at a predetermined cost or not. Rahmaniani and Ghaderi [5] formulated a Capacitated FL/ND Problem (CFL/NDP) with multi-type transshipment links and multi-type construction costs. Their mathematical formulation minimized the operational and total transshipment costs. Also, Ghaderi and Jabalameli [6] formulated a dynamic version of uncapacitated budget constrained FL/NDP with regards to the planning horizon. This study was extended by Ghaderi [7] with respect to equity-based objective. At each time period, his model optimized the summation of the total dynamic costs of the maximum travel time. In another study, Rahmaniani and Ghaderi [8] effectively solved the CFL/NDP by several simple suitable meta-heuristics according to the Variable Neighborhood Search (VNS) approach. Hassanzadeh Amin and Baki [9] proposed a multi-objective mixed-integer linear programming model for a closed-loop supply chain network by considering global factors, including exchange rates and customs duties under uncertain demand. A solution approach based on fuzzy programming was developed for solving the optimization problem. Taleizadeh et al. [10,11] developed a multi-product multi-constraint supply chain problem under fuzzy environment. Taleizadeh et al. [12] developed a single-product single-period problem under fuzzy demand in which a hybrid fuzzy simulation and meta-heuristics algorithm was applied to optimize the problem.

The second trend of research is on multi-objective FLPs. Nozick [13] modelled a fixed-charge FLP considering some coverage constraints to minimize cost and maintain a convenient service level, simultaneously, in recognizing FLs. Also, they presented and tested two Lagrangian relaxations according to two heuristics. Villegas et al. [14] formulated the Colombian coffee supply network as a bi-objective (coverage and cost) uncapacitated FLP. They designed two multi-objective evolutionary algorithms according to the Pareto Archive Evolution Policy (PAEP) and the Non-dominated Sorting Genetic Algorithm (NSGA II). Farhan and Murray [15] presented some mathematical formulations that simultaneously considered some issues, including partial regional service, coverage range, and the distance of the potential demand in facility siting. Taleizadeh et al. [16] developed a manufacturing system with scrap and rework process. They considered a mutli-product single-manufacturing facility to manufacture the items. Murray et al. [17] presented two models for the Location Set Covering Problem (LSCP) as LSCP-Explicit and LSCP-Implicit. In the LSCP-Implicit approach, it was presumed that each request region could be partially covered by one facility, i.e. the whole demands of one region could be covered by more than one facility. Jabalameli and Mortezaei [18] considered a limited capacity for each link for transferring the demands and formulated an extension of the CFL/NDP. Maliszewski et al. [19] combined the p-dispersion model with other FL objectives as 4 multi-objective models. The objectives were relevant to siting critical assets, such as the p-center, p-

median, p-maxian, and max-cover models. Taleizadeh et al. [20] and Taleizadeh and Pentico [21] studied two incentive policies in inventory and supply chain system. Xifeng et al. [22] proposed a methodology that provided a multi-objective function optimization model for the classical Uncapacitated FLP (UFLP) in order to characterize the trade-off among economic, service, and environmental considerations. Eskandarpour et al. [23] developed a multi-objective post-sales ND formulation with regards to tactical and strategic decisions, which was conducive to optimize the environmental pollution, all tardiness, and all fixed and variable costs, simultaneously. Ozgen and Bahadir [24] studied an uncertain four-stage SC network and simultaneously minimized the maximum qualitative factors of benefit and total costs by combining a fuzzy analytical hierarchical process approach and a two-phase possibility linear mathematical programming approach. Sadjadi et al. [25] studied a capacitated multi-product multi-level reverse logistics ND with fuzzy returned products. Their model determined both locations of the facilities and treatment activities. Afshari et al. [26] studied the forward and reverse streams in a multi-objective, multi-period, multi-commodity green distribution-service network. They simultaneously optimized sustainability of the system, client euphoria, and profitability by optimizing FL agreements. They simultaneously minimized the location and operational costs, and maximized the customer sustainable satisfaction. Pasandideh et al. [27] formulated a multi-period multi-product three-level SC network in which the expected and the variance of the total location and operational costs were simultaneously minimized. Tavakkoli-Moghaddam et al. [28] formulated a multi-objective facility location problem with congestion and pricing policies. Their formulation dealt with situations in which immobile service facilities were congested by a stochastic demand following M/M/m/k queues. Zhang et al. [29] modelled a multi-objective public health-care facility location problem in which health-care facilities should be located to improve the equity of accessibility, raise the total accessibility for the entire population, reduce the population that fell outside the coverage range, and decrease the cost of building new facilities. Hajipour et al. [30] formulated a multi-objective multi-layer facility location-allocation model with congested facilities using classical queuing systems. The goal was to determine the optimal number of facilities and service allocation at each layer. They dealt with three objective functions aiming at:

1. Minimizing the sum of aggregate travel and waiting times;
2. Minimizing the cost of establishing the facilities;
3. Minimizing the maximum idle probability of the facilities.

Karasakal and Ahmet [31] proposed a bi-objective facility location model that investigated both service to uncovered demands and partial coverage. Due to the limited number of facilities to be opened, some of the demand nodes might not be within full or partial coverage distance of a facility. Their objectives were maximization of full and partial coverage, and minimization of the maximum distance between uncovered demand nodes and their nearest facilities.

The third trend of research is on the FL with parameter uncertainty. In general, by formulating a scenario-based stochastic program, this approach has traditionally been incorporated into the FLP. One of the first studies was done by Sheppard [32] who proposed a scenario-based approach to FLPs. Tsiakis et al. [33] studied a multi-level multi-product SC with regards to the scenario-based demand uncertainty. Shishebori and Jabalameli [34] and Shishebori et al. [35] studied the reliable FL/NDP with respect to system disruptions. They developed a Mixed Integer Nonlinear Programming (MINLP) formulation for the problem. In another study, Shishebori et al. [36] considered the cost of system failures as the maximum accpetable disruption cost and formulated the reliable FL/NDP as MINLP. Shishebori and Yousefi-Babadi [37] studied a reliable and robust health care station location ND problem, which simultaneously took an investment budget constraint, system disruptions, and uncertain parameters into account. Lu et al. [38] formulated a reliable facility location design, which allowed disruptions at different locations to be correlated with an uncertain joint distribution. They distributionally applied robust optimization to minimize the expected cost under the worst-case distribution with predetermined marginal disruption probabilities. Their worst-case distribution had a practical interpretation with disruption propagation and its sparse structure allowed solving the problem efficiently. Huang and Di [39] modeled an uncapacitated facility location problem with customers' positions subject to experts' estimations. They used some uncertain variables to explain the estimations of customers' positions and proposed an expected distance minimization model. Jalali et al. [40] formulated a bi-objective reliable FLP with multiple capacity levels in a three-echelon SCM, while there was a constraint on the coverage levels. Also, they considered a provider-side uncertainty for Distribution-Centers (DCs). Their goal was to find a near-optimal solution including suitable locations of DCs and plants, the fraction of satisfied customer demands, and the fraction of items sent to DCs to minimize the total cost and maximize fill rate, simultaneously. Keyvanshokooh et al. [41] proposed a novel hybrid robust-stochastic programming approach for a flexible closed-loop supply chain networks design in order to cover demands and returns based on market conditions. They applied

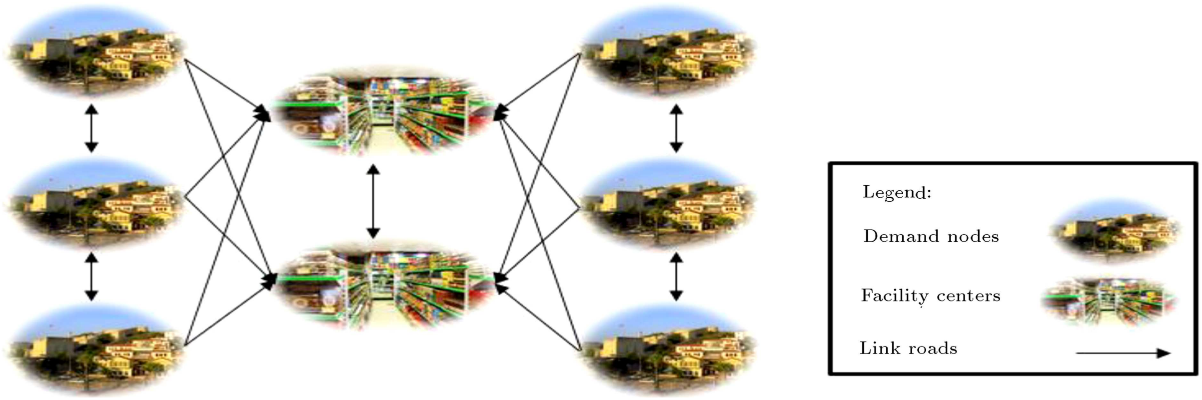

Figure 1. A general scheme of a FL/NDP.

Latin Hypercube Sampling with backward reduction for scenario generation.

In the background, another approach to the parameter uncertainty is fuzzy theory. In several decision making states, great grade of uncertainties, as fuzziness aspect, is enmeshed in the data collection. Some studies introduced the fuzzy theory and its application [42,43]. Also, some studies explained the application of fuzzy optimization approaches to Linear Programming (LP), Goal Programming (GP), and other mathematical programming problems [43-54].

2.2. Research gap

Although much effort has been done for understanding and mitigating several practical factors in different FLPs, early models assumed these practical factors separately. Accordingly, separately assuming these factors can lead to unreal models, which cannot be suitable and useful in the practical environment. Moreover, sometimes these models do not adequately capture the simultaneous impacts of several practical factors in different FLPs.

A significant point in the formulating of FLPs is to develop a suitable programming model, which can obtain a reasonable practical solution. Four remarkable subjects, which can help to achieve this goal, are FL, ND, demand uncertainty, and multi-objectivity. We briefly explain the role of these subjects in formulating FLPs, separately.

The contribution of this paper, versus the past studies, is investigating an integrated scenario-based approach to FLPs with regards to the mentioned topics, which is conducive to improve the efficiency of FLPs. The problem is named fuzzy robust multi-objective FL/NDP (FR/MO/FL/NDP). Simultaneously, considering 3 practical topics, namely, ND, multi-objectivity, and parameter uncertainty, to propose the mathematical modeling of FLPs is the main motivation of this research. To the best of the authors' knowledge, this integrated subject has not been studied yet.

3. Mathematical formulation and description

3.1. Definition

In a geographic area, there are a set of towns as demand nodes and a set of roadways as transshipment links. In this region, there are some facility centers (e.g. chain stores). Some new facility centers should be located; some transshipment links should be selected for utilization such that the total FL costs, fixed costs of link utilization, and the variable transshipment costs are minimized. On the other hand, it is desired to maximize the covering of demand nodes. Thus, the first objective optimizes the fixed cost (including FL costs and link utilization costs) and operational costs (including the variable transshipment costs), and the second objective optimizes the total penalty costs for any nodes that cannot be covered. One point that should be considered is that the number of demands of each node has stochastic behavior, i.e. this parameter is not certain. Figure 1 illustrates a scheme of an FL/NDP.

The major assumptions of the problem can be described as follows: (I) All facility centers and transshipment roads/links are capacitated; (II) Each customer is either completely covered or not covered at all; (III) The fixed cost of road/link utilization includes the fixed costs of vehicle rent, the fixed contract costs with vehicle transportation companies, the toll costs, and other marginal costs on that road/link. This definition of cost only depends on the quality and distance of the considered road/link; (IV) All transshipment costs and the demand of the node j are uncertain and have stochastic behavior; (V) A specific robust weight of solution variance is defined for each objective function; (VI) The number of facilities is predetermined.

3.2. Notifications

Sets:

Φ Set of demand nodes

Δ Set of transshipment roads/links

Parameters:

P_f Number of new facilities to locate;

γ_i Fixed location cost of a facility at node i ($i \in \Phi$);

p_{ij} Fixed cost of link utilization (i,j) ($(i,j) \in \Phi$);

t_{ij} Transshipment cost of a unit flow on link (i,j) ($(i,j) \in \Delta$);

LIB Maximum level of investment budget constraint;

d_i Demand value at node i, ($i \in \Phi$);

α_i Capacity of facility i ($i \in \Phi$);

M A big number;

β_{ij} Capacity of road/link (i,j) ($(i,j) \in \Delta$);

Π_i Penalty of the uncoverage of demands of node i ($i \in \Phi$);

k_{ij} 1 if the facility at node j covers the demand of node i ($i,j \in \Phi$), 0 otherwise.

Decision variables:

V_i 1 if a facility center is located at node i ($i \in \Phi$), 0 otherwise;

U_{ij} 1 if a road/link (i,j) is selected for utilization ($(i,j) \in \Delta$), 0 otherwise;

N_{ij} Demand value on road/link (i,j) ($(i,j) \in \Delta$);

W_i 1 if demand of node i is not covered ($i \in \Phi$), 0 otherwise.

3.3. Model formulation

3.3.1. The first objective

In the first objective, the total cost includes fixed cost of facilities locating, fixed cost of links utilizing, and variable transshipping cost. The first objective function is organized as follows:

$$\text{Minimize} \quad \sum_{i\in\Phi} \gamma_i V_i + \sum_{(i,j)\in\Delta} p_{ij} U_{ij} + \sum_{(i,j)\in\Delta} t_{ij} N_{ij},$$

where $\sum_{i\in\Phi} \gamma_i V_i$ and $\sum_{(i,j)\in\Delta} p_{ij} U_{ij}$ are the fix costs and $\sum_{(i,j)\in\Delta} t_{ij} N_{ij}$ is the transshipment costs.

3.3.2. The second objective

The second objective function optimizes the total penalty costs of demand nodes that are not covered by other opened facilities. The second objective can be arranged as:

$$\text{Minmize} \quad \sum_{i\in\Phi} \Pi_i W_i.$$

When $W_i = 1$, node i is not covered by opened facilities; thus, the penalty cost is equal to $\Pi_i W_i$.

3.3.3. The multi-objective counterpart formulation

According to the assumptions and notations, the MIP model of the Multi-Objective FL/NDP (MO/FL/NDP) can be expressed as:

((MO/FL/NDP)):

$$Obj(I) = \text{Minimize} \sum_{i\in\Phi} \gamma_i V_i + \sum_{(i,j)\in\Delta} p_{ij} U_{ij} + \sum_{(i,j)\in\Delta} t_{ij} N_{ij}, \tag{1}$$

$$Obj(II) = \text{Minimize} \sum_{i\in\Phi} \Pi_i W_i, \tag{2}$$

s.t.

$$\sum_{(i,j)\in\Delta} N_{ji} - \sum_{(i,j)\in\Delta} N_{ij} + d_i \leq \alpha_i + M(1 - V_i) \quad \forall i \in \Phi, \tag{3}$$

$$\sum_{(i,j)\in\Delta} N_{ij} - \sum_{(i,j)\in\Delta} N_{ji} \leq d_i + M(V_i) \qquad \forall i \in \Phi, \tag{4}$$

$$\sum_{(i,j)\in\Delta} N_{ij} - \sum_{(i,j)\in\Delta} N_{ji} \geq d_i - M(V_i) \qquad \forall i \in \Phi, \tag{5}$$

$$\sum_{j\in\Phi} k_{ij} V_j + W_i \geq 1 \qquad \forall i \in \Phi, \tag{6}$$

$$\sum_{i\in\Phi} f_i V_i + \sum_{(i,j)\in\Delta} c_{ij} U_{ij} \leq LIB, \tag{7}$$

$$U_{ij} + U_{ji} \leq 1 \qquad \forall (i,j) \in \Delta, \tag{8}$$

$$\sum_{i\in\Phi} V_i = P_f, \tag{9}$$

$$N_{ij} \leq \beta_{ij} U_{ij} \qquad \forall (i,j) \in \Delta, \tag{10}$$

$$V_j, W_i \in \{0,1\} \qquad \forall i,j \in \Phi, \tag{11}$$

$$U_{ij} \in \{0,1\} \qquad \forall (i,j) \in \Delta, \tag{12}$$

$$N_{ij} \geq 0 \qquad \forall (i,j) \in \Delta. \tag{13}$$

Eqs. (1) and (2) are objective functions. The first one presents minimization of the total costs, including FL, road/link utilization, and transshipment costs; the second one illustrates minimization of the total penalty costs of demands uncovering. Constraints (3) - (5) present the flow conservation constraints. Constraint (6) ensures that $W_i = 1$ when the demand point, i is covered by no facilities. The investment budget constraint is illustrated in Constraint (7). Constraints (8) presents that the transshipment link cannot be selected for utilization in both directions of (i,j) and (j,i). Constraint (9) presents the predetermined number of facilities that have to be opened. Constraint (10) is the capacity of the links. Constraints (11)-(13) are non-negativity and standard integrality constraints of decision variables.

3.3.4. Robust optimization approach

Basically, most of the SCs and logistic systems can be affected by two broad categories of risk. The first category is parameter (e.g. demand, fixed costs, operational costs, and lead times) uncertainty and the second category is system disruptions (e.g. economic disruptions, strikes, natural disasters, and terrorist attacks). One of the most efficient approaches to deal with the first category is Robust Optimization (RO). This approach has been applied broadly in the area of uncertain control and optimization since the late 1990s. The most common RO approaches are Mulvey et al. [55] , Ben-Tal and Nemirovski [56-58], and Bertsimas and Sim [59,60]. As the Mulvey approach is easily understandable for many designers/architects and it has a scenario-based problem, this robust approach is considered, which is conducive to cope with related parameter uncertainties. For more details of this robust optimization approach, the readers can refer to Mulvey et al. [55], Yu and Li [61], Leung et al. [62], Bozorgi-Amiri et al. [63], and Shishebori and Yousefi-Babadi [37].

3.3.5. The robust multi-objective counterpart formulation

The robust MO/FL/NDP (R/MO/FL/NDP) model is an uncertain model, which can be reduced by using Eqs. (26)-(28). Accordingly, the R/MO/FL/NDP can be modelled as follows (before the representation of the robust model, some new nomenclature should be introduced):

Sets:

Ω Set of uncertain scenarios; $\Omega = \{1,2,...,s\}$

Parameters:

λ_1 Determined weight of the solution variance of the first objective;

λ_2 Determined weight of the solution variance of the second objective;

d_i^s Demand at node i under uncertain scenario s $(i \in \Phi\ , s \in \Omega)$;

t_{ij}^s Transshipment cost of a unit flow on road/link (i,j) under uncertain scenario, s $((i,j) \in \Delta, s \in \Omega)$;

π^s Probability of uncertain scenario s $(S \in \Omega)$.

Decision variables:

θ_1^s Robust method variable for the first objective function;

θ_2^s Robust method variable for the second objective function;

$N_{i,j}^s$ Demand value of transshipment road/link (i,j) under uncertain scenario s $((i,j) \in \Delta, s \in \Omega)$.

R/MO/FL/NDP:

$$\min Z_1 = \sum_{s\in\Omega} \pi^s.\xi^s + \lambda_1 \sum_{s\in\Omega} \pi^s.\left[\left(\xi^s - \sum_{s'\in\Omega} \pi^{s'}.\xi^{s'}\right) + 2\theta_1^s\right] + \sum_{i\in\Phi} \gamma_i V_i + \sum_{(i,j)\in\Delta} p_{ij} N_{ij}, \tag{14}$$

$$\min Z_2 = \sum_{s\in\Omega} \pi^s.\varsigma^s + \lambda_2 \sum_{s\in\Omega} \pi^s\left[\left(\varsigma^s - \sum_{s'\in\Omega} \pi^{s'}.\varsigma^{s'}\right) + 2\theta_2^s\right], \tag{15}$$

s.t.

$$\xi^s - \sum_{s\in\Omega} \pi^s.\xi^s + \theta_1^s \geq 0 \qquad \forall s \in \Omega, \tag{16}$$

$$\varsigma^s - \sum_{s\in\Omega} \pi^s.\varsigma^s + \theta_2^s \geq 0 \qquad \forall s \in \Omega, \tag{17}$$

$$\textstyle\sum_{(j,i)\in\Delta} N_{ji}^s - \sum_{(i,j)\in\Delta} N_{ij}^s + d_i^s \leq \alpha_i + M(1-V_i) \qquad \forall i \in \Phi, \forall s \in \Omega, \tag{18}$$

$$\textstyle\sum_{(i,j)\in\Delta} N_{ij}^s - \sum_{(i,j)\in\Delta} N_{ji}^s \leq d_i^s + M(V_i) \qquad \forall i \in \Phi, \forall s \in \Omega, \tag{19}$$

$$\textstyle\sum_{(i,j)\in\Delta} N_{ij}^s - \sum_{(i,j)\in\Delta} N_{ji}^s \geq d_i^s - M(V_i) \qquad \forall i \in \Phi, \forall s \in \Omega, \tag{20}$$

$$N_{ij}^s \leq \beta_{ij} U_{ij} \qquad \forall (i,j) \in \Delta, \forall s \in \Omega, \tag{21}$$

$$N_{ij}^s \geq 0 \qquad \forall (i,j) \in \Delta, \forall s \in \Omega, \tag{22}$$

$$\theta_1^s, \theta_2^s \geq 0 \qquad \forall s \in \Omega, \quad \text{and integer}, \tag{23}$$

and Constraints (6)-(9), (11), (12), where $\xi^s = \sum_{(i,j)\in\Delta} t_{ij}^s\, N_{ij}^s$, and $\varsigma^s = \sum_{i\in\Phi} \Pi_i^s\, W_i$.

Constraints (14)-(15) present the robust objective function by Muley et al. [55] approach. Constraints (16)-(17) represent the linearization of the objective function in the RO method. Constraints (6)-(9), (11), (12), (21), and (22) have been described previously. In addition, Constraint (23) illustrates the general integrality constraints of decision variables.

3.3.6. Fuzzy goal programming

The weighted ε-constraint, Goal Programming (GP), Pareto optimal, and the LP-norm are the most popular methods in the literature for treating multi-objective optimization problems [63-67]. In multi-objective problems, since the objective functions have several conflicts with each other, obtaining an optimal solution, which can optimize all of the objective functions altogether, is very difficult in most of the times (or impossible sometimes). Accordingly, instead of obtaining an optimal solution, finding an efficient solution, which can satisfy all of the objective functions near optimality, can significantly help several decision makers to approach some practical reasonable solutions for different industries/services environments. Due to its numerous applications in solving multi-objective problems and being easily understandable to several decision makers, the GP is employed. Moreover, due to the capability of fuzzy set theory for finding the suitable crisp solutions in uncertain environments, Fuzzy Goal Programming (FGP) is employed for solving the R/MO/FL/NDP.

The first studies applied the concept of membership functions and, accordingly, proposed an FGP formulation [45,68,69]. If the ith goal of the multi-objective mathematical model is obtained and the Decision Maker (DM) is generally satisfied, then its membership function is equal to 1; otherwise, the membership function has a value between 0 and 1. Accordingly, the membership functions have values in the interval [0, 1]. The readers can study the detailed formulation of FGP problem in [43,44,70].

3.3.7. The fuzzy robust goal programming counterpart formulation

The necessary sets and variables can be defined as follows:

Sets:

T Set of objective functions, $T = 1, 2$

Variables:

ψ Min {memberships}

FR/MO/FL/NDP:

$$\max \psi, \tag{24}$$

s.t.

$$\psi \leq 1 - \frac{(Z_t - L_t)}{(U_t - L_t)} \qquad \forall t \in T, \tag{25}$$

and Constraints (6)-(9), (11), (12), (16)-(23), where ψ is named the value of decision maker satisfaction [44]. The more this index approaches 1, the better it becomes.

3.3.8. Complexity

Property 1 establishes that the FR/MO/FLNDP problem is NP-hard, since it has the P-median problem and capacitated location-allocation problem, which are themselves NP-hard, as a special case.

Property 1. The FR/MO/FLNDP problem is NP-hard.

Proof. The FR/MO/FLNDP is an extension of the MO/FLNDP. If we remove the second objective of the MO/FLNDP, and if we remove Eqs. (3)-(5), (8), and (10) and set $p_{ij} = u_{ij} = 0$ and LIB $= \infty$, then the FR/MO/FLNDP reduces to the classical P-median problem. Since the P-median problem is NP-hard [71], because the FR/MO/FLNDP is an extension of the classical P-median problem, it is also an NP-hard problem.

On the other hand, it can be said that the FR/MO/FLNDP is an extension of the MO/FLNDP. If we remove the second objective of the MO/FLNDP, and if we remove Eqs. (3)-(5), (8), and (10) and set $u_{ij} = 0$ and LIB $= \infty$, then the FR/MO/FLNDP reduces to the classical capacitated location-allocation problem. Since the capacitated location-allocation problem is NP-hard [72], the FR/MO/FLNDP is an extension of the capacitated location-allocation problem; therefore, it is an NP-hard problem.

4. Lagrangian relaxation

Solving the FR/MO/FL/NDP by GAMS software is very time consuming, especially for large-scale problems. Accordingly, it can be concluded that proposing an efficient solution approach is necessary. Regarding the efficiency of the Lagrangian relaxation approach for solving several FLPs, this approach is applied and customized. In the following, the proposed customized algorithm for the FR/MO/FL/NDP is described in details.

4.1. General points

The literature on the Lagrangian Relaxation (LR) dates back to 1970. In 1970 and 1971, Held and Karp [72,73] applied an LR approach based on the minimum spanning trees in order to design a well-organized algorithm for the Traveling Salesman Problem (TSP). This successful application motivated some researchers. Fisher [74-76] was one of the researchers who developed the application of this algorithm to the general integer programming models and the scheduling problems. Since then, this algorithm has been applied to efficiently solve several practical mathematical programming models. In the following, some studies related to the subject of this study are mentioned. Fisher [74-76] provided great discussions on the numerical perspectives and applications of LRs.

The LR was successfully applied in solving different types of FLPs (FLP). CFLP is one of the several types of FLPs that have been solved by the LR approach, see [73,77-84].

Also, LR has been successfully applied for other types of FLPs, see [85-89].

4.2. The main structure

The combinatorial optimization problem P is modeled as an Integer Programming (IP) mathematical formulation:

(P):

$$Z = \min \delta\Theta, \tag{26}$$

s.t.

$$T\Theta \leq \chi, \tag{27}$$

$$\Lambda\Theta = \vartheta, \tag{28}$$

$$\Theta \geq 0, \qquad \text{and integer}, \tag{29}$$

where Θ, χ, and ϑ are matrices with dimensions $n \times 1$, $k \times 1$, and $m \times 1$, respectively. Also, the remaining matrices have related similar dimensions. Assume that LR_P represents the problem P with relaxing of Constraint (28) and it is added as a penalty to the objective. The optimal value of LR_P is illustrated by Z_{LP}.

Suppose that the constraints of the (P) are categorized as two constraint sets: I) $\Lambda\Theta = \vartheta$, II) $T\,\Theta \leq \chi$; therefore, the Lagrangian problem can be solved easily.

$$LR_P: \ Z_D(\Gamma) = \min, \quad \delta\Theta + \Gamma(\Lambda\Theta - \vartheta), \tag{30}$$

s.t.

$$T\,\Theta \leq \chi, \tag{31}$$

$$\Theta \geq 0, \qquad \text{and integer}, \tag{32}$$

where $\Gamma = (\Gamma_1, ..., \Gamma_m)$ is the vector of Lagrangian coefficients. Z^* presents the obtained solution to the problem (P) from the CPLEX solver of GAMS software. Since Constraint (33), as a complex constraint, is relaxed, the problem (LR_P) can be solved easier than the problem (P). Suppose that the set $XX = \{\Theta | T\Theta \leq \chi, \Theta \geq 0 \quad \text{and integer}\}$ presents the possible bounded solution to the LR_Γ and (P) is a feasible problem. Accordingly, $Z_D(\Gamma)$ will be bounded for all values of $\tilde{\lambda}_l$ (i.e., $\forall \Gamma_v \in \Gamma, \quad l = 1, 2, ..., m$).

Accordingly, we have $Z_D(\Gamma) \leq Z^*$. Also, if Θ^* presents the optimal solution to problem P, it will be concluded that:

$$Z_D(\Gamma) \leq \delta\Theta^* + \Gamma(\Lambda\Theta^* - \vartheta) = Z^*. \tag{33}$$

Therefore, if $\Lambda\Theta = \Xi$ is substituted by $\Lambda\Theta \leq \Xi$ in (P), then $\Gamma \geq 0$ is needed and the affirmation leads to:

$$Z_D(\Gamma) \leq \delta\Theta^* + \Gamma(\Lambda\Theta^* - \vartheta) \leq Z^*. \tag{34}$$

In a similar way, $\Gamma \leq 0$ is necessary for establishing the equation $Z_D(\Gamma) \leq Z^*$. Mainly, there is not any guarantee for finding the suitable Γ such that $Z_D(\Gamma) = Z^*$; however, it can be easily observed that the suitable Γ is found most of the times.

Since $Z_D(\Gamma) = Z^*$, (LR_P) can be substituted with (LR_Γ) in order to provide some efficient lower bounds in exact algorithms (e.g. the B & B algorithm) for (P). This is known as the most usual application of the LR_Γ and other applications can be defined, e.g. it can be an indicator for selecting the suitable variables and the next branch of search operation. Perturbing nearly feasible solutions to (LR_Γ) can lead to suitable possible solutions to (P) [87,90].

As it was mentioned, $Z_D(\Gamma)$ presents a lower bound of the problem Z. To catch an upper bound of the problem, the value of Θ, obtained from solving $Z_D(\Gamma)$, is substituted in the main (P) and an upper bound is obtained. In the following, the proposed LR approach is presented.

5. The proposed approach

Since the highest complexities of the model (FR/MO/FL/NDP) are related to Constraints (18)-(20), they are relaxed. Accordingly, the Sub-Gradient based Lagrangian Relaxation (SGbLR) of the model (FR/MO/FL/NDP) can be rewritten as follows:

SGbLR/FR/MO/FL/NDP:

$$\max \psi$$

$$+ \sum_{i\in\Phi, s\in\Omega} \Gamma^1_{i,s} \Bigg[\Bigg(\sum_{(i,j)\in\Delta} N^s_{ji} - \sum_{(i,j)\in\Delta} N^s_{ij} + d^s_i \Bigg)$$

$$- (\alpha_i + M(1 - V_i)) \Bigg]$$

$$+ \sum_{i\in\Phi, s\in\Omega} \Gamma^2_{i,s} \Bigg[\Bigg(\sum_{(i,j)\in\Delta} N^s_{ij} - \sum_{(i,j)\in\Delta} N^s_{ji} \Bigg)$$

$$- (d^s_i + M(V_i)) \Bigg]$$

$$+\sum_{i\in\Phi,s\in\Omega}\Gamma^3_{i,s}\Bigg[-\Bigg(\sum_{(i,j)\in\Delta}N^s_{ij}-\sum_{(i,j)\in\Delta}N^s_{ji}\Bigg)-(d^s_i-M(V_i))\Bigg],\tag{35}$$

s.t. Constraints (6)-(9), (11), (12), (16), (17), (21)-(23), and (25), where, in the objective function (Constraint (35)), the values of $\Gamma^1_{i,s}$, $\Gamma^2_{i,s}$, $\Gamma^3_{i,s}$ are $\Gamma^1_{i,s}\geq 0,\Gamma^2_{i,s}\geq 0,\Gamma^3_{i,s}\leq 0$, because $\Gamma^3_{i,s}$, which is related to Constraint (30), should be $\Gamma^3_{i,s}\leq 0$ in order to satisfy the inequation $Z_D(\Gamma)\leq Z$.

Several methods (e.g. sub-gradient, surrogate sub-gradient, coefficients correction, column generation, bundle, cutting plane) have been proposed for updating Lagrangian coefficients. However, the sub-gradient method is known as a popular method in the literature [74]. This method has had a suitable performance for several mathematical programming problems. The sub-gradient method is a repetitive approach, which starts with some initial values for the Lagrangian coefficients. Then, the values of these coefficients are changed via a systematic procedure. The goal is maximizing the lower bound, obtained by the Lagranian problem (SGbLR/FR/MO/FL/NDP), via finding the best values of the Lagrangian coefficients. Therefore, the Sub-Gradient based Lagrangian Relaxation (SGbLR) approach is used to solve the presented FR/MO/FL/NDP mode.

Suppose that the constraint $\sum_{j\in\Phi}\Lambda_{ij}\Theta_j\geq\Xi_i,\forall i\in\Phi$, is selected to be added to the objective function. Accordingly, the sub-gradient method of Stage 2 in Figure 2 is briefly outlined as follows:

Step 1: η is a parameter that is determined by

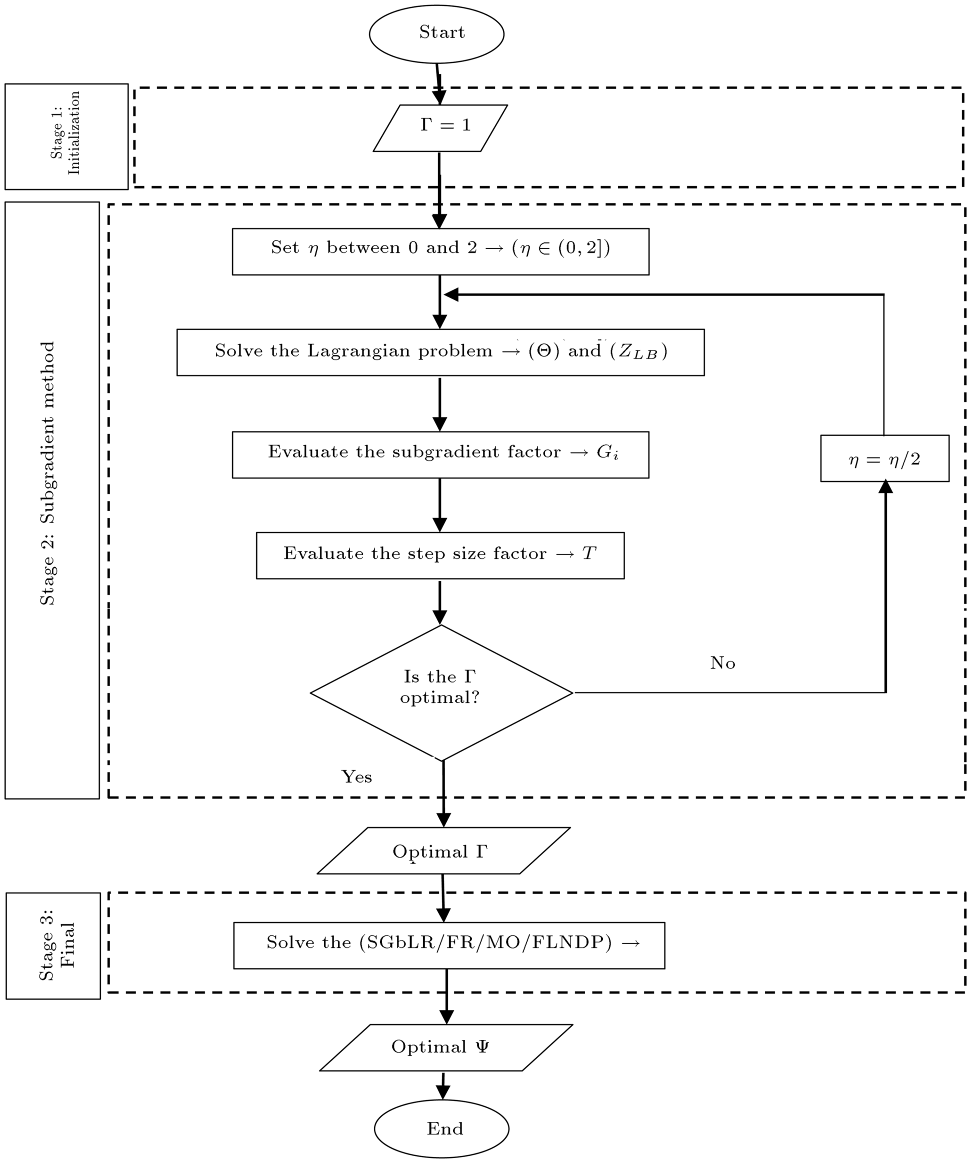

Figure 2. The flowchart of the proposed SGbLR approach.

decision makers between 0 and 2 (i.e. $0 < \eta \leq 2$). Z_{UB} is a possible solution to the main problem, which is acquired by heuristic methods. Also, Γ_ν presents the initial arbitrary Lagrangian coefficients;

Step 2: The Lagrangian problem is figured out with the Lagrangian coefficients of Step 1. Accordingly, the value of the lower bound of the main problem (Z_{LB}) and the value of decision variables (Θ_j) are determined;

Step 3: For each relaxed constraint, a sub-gradient, named G_i, is defined as follows:

$$G_i = \Xi_i - \sum_{j \in \Phi} \Lambda_{ij}\Theta_j, \quad \forall i \in \Phi. \tag{36}$$

Step 4: In order to update of the direction of the SGbLR vector (Γ), a step scale, named T, is defined as follows:

$$T = \frac{\eta(Z_{UB} - Z_{LB})}{\sum_{i \in \Phi} G_i^2}. \tag{37}$$

It is recommended that, at first, η should be set to 2 ($\eta = 2$). If in the predetermined number of iterations, no improvement of $Z_D(\Gamma)$ is obtained, η can be reduced to half its previous value;

Step 5: The value of Γ_i is updated via Eq. (38). Then, we come back to Step 2 and figure out the Lagrangian problem with new Lagrangian coefficients:

$$\Gamma_i = \max(0, \Gamma_i + TG_i), \ \forall i \in \Phi. \tag{38}$$

It is noted that one of the following conditions (or similar cases) can be the termination condition: (I) reducing π until this parameter reaches a predetermined value and (II) doing a predetermined number of iterations.

6. A practical example

Here, a practical example is described with 30 nodes. Five nodes should be selected and constructed among the 30 proposed nodes. Other information (e.g. FL costs and transshipment costs) has been provided by Shishebori and Yousefi-Babadi [37]. The problem is solved by applying the proposed SGbLR approach and, after solving, 0.88 is obtained for ψ.

Figure 3 presents the behavior of the objective value of SGbLR/FR/MO/FL/NDP (ψ) versus P_f.

Based on Figure 3, it is obvious that locating of excessive facilities cannot improve decision maker satisfaction. Figure 3 emphasizes that if 12 facilities of 30 potential facilities are located, the maximum decision maker satisfaction will be attained.

Another important factor, which can affect the objective value, is the value of Budget Constraint (BC).

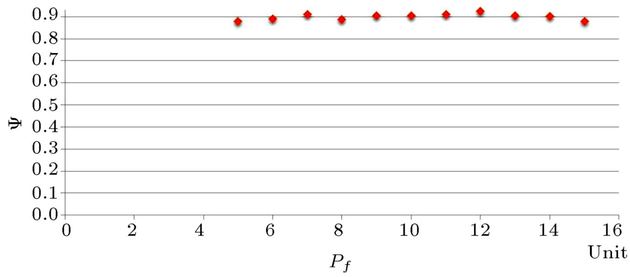

Figure 3. Changes of decision maker satisfaction (Ψ) versus the P_f.

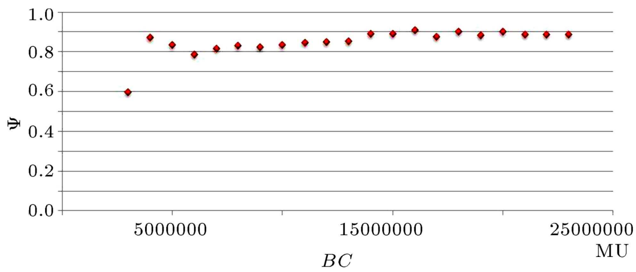

Figure 4. Changes of decision maker satisfaction for different values of investment budget.

Figure 4 shows how the changes of BC can affect the optimal objective value of FR/MO/FL/NDP.

Through Figure 4, it can be clarified that excessive budget investment does not necessarily lead to designer satisfaction. Figure 4 emphasizes that the optimal budget investment value, which can lead to the maximum value of decision maker satisfaction, is 16,000,000 MU (Monetary Unit).

7. Numerical results

In order to evaluate the performance of the SGbLR solution algorithm, relatively comprehensive numerical experiments were done. The algorithm was coded in GAMS 24.1.2 and performed by the CPLEX solver on a computer with a core 2 Due @ 2.0 GHz and 8 GB RAM operating with windows 7.0.

7.1. Experimental design

Fifty-seven test problems with several sizes were solved. These problems were randomly generated by the uniform distribution function with several uncertainty scenarios. According to the historical data, some changing conditions (e.g. climate change, fluctuations in stock market costs, etc.) can lead to uncertainty in some of the input parameters of the proposed model. In this study, the demands of each node as well as the transshipment unit costs are stochastic as some scenarios. Therefore, the value of demand transshipment will be stochastic. These uncertain situations are categorized as follows: excellent, good, medium, and bad with probabilities of 0.25, 0.28, 0.32, and 0.2, respectively. The transshipment cost for each client in kilometers is defined as a proportion of distance for the 4 mentioned scenarios as follows, 0.0040, 0.0035, 0.0025, and 0.0015, respectively. The fixed cost of opening each facility uniformly varies between [27,000,000–48,000,000] MU (Monetary Unit). The fixed cost of link utilization is obtained by multiplying a coefficient (85,000) by the distance between each two nodes. The capacity of the facilities is obtained by multiplying a coefficient (10) by the expected value of node's demand. Also, the robust parameters, including the determined weight of the solution variance of the first objective (λ_1), the determined weight of the solution variance of the second objective (λ_2), and probability of uncertain scenario (π^s), are randomly generated between (0.2,2), (0.2,2), and (0.1,1), respectively. In order to normalize the generated data, the summation of π^s is limited to 1 for all scenarios. It is noted that the procedure for generating these parameters is according to Bozorgi-Amiri et al. [63] and Shishebori and Yousefi-Babadi [37].

7.2. Algorithm performance

Table 1 compares the performance of the CPLEX with that of the proposed SGbLR algorithm. For each algorithm, the table illustrates the objective value ("Value") and the CPU time ("Time"). The CPU time for the proposed algorithm includes the computational time required to solve the SGbLR in Stage 2 in Figure 2, which is then used as an input to the main stage (Stage 3 in Figure 2) of the algorithm. In addition, Table 1 presents the objective function values from the SGbLR ("LRB") and CPLEX ("CB"). "LRB" presents the CPU time of the SGbLR solved in Step 2 of the proposed algorithm. The ratio of the objective value of the best solution obtained from the CPLEX to the corresponding value from the proposed algorithm can be calculated as follows:

$$\text{Ratio}_{\text{Obj.Fun}_{(\psi)}}(\%) = \frac{\text{Objective function}(\psi)_{\text{SGbLR}}}{\text{Objective function}(\psi)_{\text{Cplex}}} \times 100, \qquad (39)$$

$$\text{Ratio}_{\text{Time}}(\%) = \frac{\text{Time}_{\text{SGbLR}}}{\text{Time}_{\text{Cplex}}} \times 100. \qquad (40)$$

In Table 1, values less than 100% in the "Ratio (%)" column (with "Value" and "Time" sub-columns) indicate that the proposed SGbLR algorithm outperforms the CPLEX concerning CPU time and the objective function value, respectively. The applied SGbLR algorithm is faster than the CPLEX for most of the test problems. In addition, no feasible solution in the allowed time (i.e. 6000s) is specified by the symbol "n/a" in the table. It is shown that for a few instances (especially for small-size instances), the CPU time of the SGbLR algorithm is more than that of the CPLEX; because, basically, the SGbLR algorithm is applied to a vast variety of instances with several sizes and the performance of the algorithm is better observable when the amount of the instances grows.

Table 1 shows that the solutions attained by the algorithm are, on average, 0.458% (100% − 99.542% = 0.458%) less than those attained by CPLEX, i.e. it can provide similar or better solutions than CPLEX does for medium and relatively great-scale instances. Moreover, the SGbLR is much faster; it requires only 50.766% of the CPLEX's time, on average, and does not fail to obtain solutions to any of the test problems throughout the time bound, whilst CPLEX fails to do so for 16 out of the 70 test problems.

With attention to Table 1 and Figures 5-16, it is clear that the SGbLR algorithm has significantly low CPU time in comparison with CPLEX, while the objective values of both the SGbLR algorithm and CPLEX are almost similar. However, it is observed

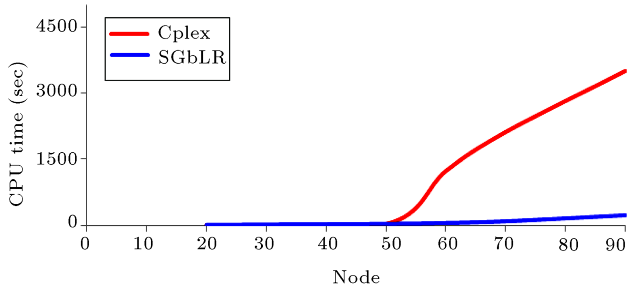

Figure 5. Varying of CPU time for several values of demand nodes with $P_f = 5$.

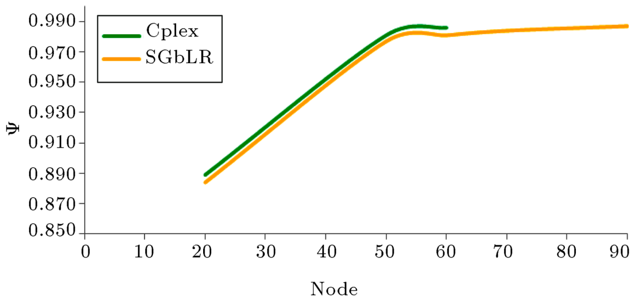

Figure 6. Varying of the Ψ for several values of demand nodes with $P_f = 5$.

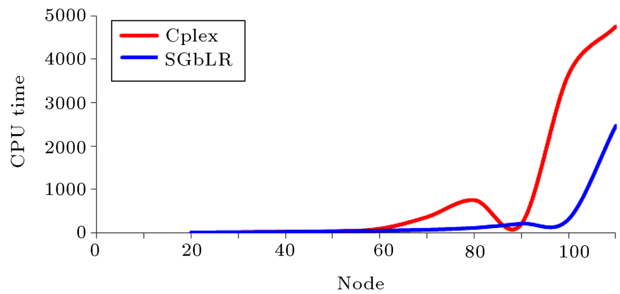

Figure 7. Varying of CPU time for several values of demand nodes with $P_f = 6$.

Table 1. Comparison of the performance of the proposed SGbLR versus CPLEX.

Instance abbreviation	P_f	N	CPLEX		SGbLR		Ratio (%)	
			Value	Time	Value	Time	Value	Time
IS_1	5	20	0.889	7.374	0.884	9.451	99.438	128.167
IS_2		50	0.981	29.044	0.977	33.313	99.592	114.698
IS_3		60	0.986	1231.094	0.981	51.725	99.493	4.202
IS_4		70	n/a	2115.448	0.984	90.313	n/a	4.269
IS_5		90	n/a	3500.400	0.987	226.248	n/a	6.463
IS_6	6	20	0.871	6.003	0.864	9.698	99.196	161.553
IS_7		40	0.968	18.568	0.963	21.361	99.483	115.042
IS_8		50	0.979	30.009	0.978	34.238	99.898	114.092
IS_9		60	0.985	92.750	0.979	48.473	99.391	52.262
IS_{10}		70	0.989	363.976	0.983	70.190	99.393	19.284
IS_{11}		80	0.991	755.562	0.988	112.073	99.697	14.833
IS_{12}		90	0.993	204.162	0.990	212.126	99.698	103.901
IS_{13}		100	n/a	3661.134	0.995	314.245	n/a	8.583
IS_{14}		110	n/a	4758.000	0.995	2469.197	n/a	51.896
IS_{15}	7	20	0.874	7.579	0.853	8.250	97.597	108.853
IS_{16}		40	0.968	31.355	0.953	19.032	98.450	60.698
IS_{17}		50	0.976	20.884	0.943	29.159	96.619	139.624
IS_{18}		60	0.984	94.204	0.979	42.618	99.492	45.240
IS_{19}		70	0.989	92.929	0.982	65.345	99.292	70.317
IS_{20}		80	0.991	253.063	0.986	104.265	99.495	41.201
IS_{21}		90	0.993	387.364	0.990	148.872	99.698	38.432
IS_{22}		100	0.994	309.629	0.989	200.321	99.497	64.697
IS_{23}		110	0.995	784.111	0.992	267.077	99.698	34.061
IS_{24}		120	0.996	633.000	0.992	310.447	99.598	49.044
IS_{25}		150	0.997	1409.328	0.994	714.652	99.699	50.709
IS_{26}		160	0.997	2402.000	0.995	1248.253	99.799	51.967
IS_{27}		170	n/a	5361.509	0.998	181.351	n/a	3.382
IS_{28}		180	n/a	5400.000	0.998	2641.595	n/a	48.918
IS_{29}	8	20	0.897	5.157	0.893	8.804	99.554	170.719
IS_{30}		40	0.969	27.479	0.963	17.489	99.381	63.645
IS_{31}		50	0.976	43.578	0.974	23.587	99.795	54.126
IS_{32}		60	0.984	139.460	0.977	39.115	99.289	28.047
IS_{33}		70	0.985	167.147	0.981	49.408	99.594	29.560
IS_{34}		80	0.989	302.278	0.986	92.482	99.697	30.595
IS_{35}		90	0.991	415.648	0.988	194.196	99.697	46.721
IS_{36}		100	0.993	612.718	0.989	211.041	99.597	34.443
IS_{37}		110	0.995	1037.188	0.991	280.243	99.598	27.019
IS_{38}		120	0.995	788.675	0.991	366.711	99.598	46.497
IS_{39}		130	0.996	1270.595	0.992	479.864	99.598	37.767
IS_{40}		150	0.997	1980.000	0.996	718.667	99.900	36.296
IS_{41}		170	0.998	2402.000	0.995	1243.666	99.699	51.776
IS_{42}		180	n/a	4230.188	0.995	1324.871	n/a	31.319

Table 1. Comparison of the performance of the proposed SGbLR versus CPLEX (continued).

Instance abbreviation	P_f	N	CPLEX		SGbLR		Ratio (%)	
			Value	Time	Value	Time	Value	Time
IS_{43}		190	n/a	4723.669	0.996	2024.569	n/a	42.860
IS_{44}		200	n/a	5435.678	0.998	2244.402	n/a	41.290
IS_{45}	9	50	0.973	49.607	0.969	29.159	99.589	58.780
IS_{46}		60	0.976	101.222	0.972	42.618	99.590	42.103
IS_{47}		70	0.985	143.017	0.981	65.345	99.594	45.690
IS_{48}		80	0.989	245.595	0.986	104.265	99.697	42.454
IS_{49}		90	0.990	205.437	0.987	174.196	99.697	84.793
IS_{50}		100	0.991	361.219	0.990	201.041	99.899	55.656
IS_{51}		110	0.993	539.674	0.991	260.243	99.799	48.222
IS_{52}		120	0.994	859.818	0.993	356.711	99.899	41.487
IS_{53}		130	0.993	1326.328	0.991	459.864	99.799	34.672
IS_{54}		150	0.998	2358.214	0.994	479.453	99.599	20.331
IS_{55}		160	n/a	3865.686	0.996	598.667	n/a	15.487
IS_{56}		170	n/a	4019.557	0.997	1047.030	n/a	26.048
IS_{57}		200	n/a	5115.975	0.998	1674.296	n/a	32.727
IS_{58}	10	80	0.987	161.000	0.987	47.670	100.000	29.609
IS_{59}		90	0.990	214.700	0.989	84.210	99.899	39.222
IS_{60}		100	0.992	221.870	0.992	106.230	100.000	47.879
IS_{61}		120	0.994	449.860	0.991	170.050	99.698	37.801
IS_{62}		140	0.996	712.250	0.994	314.780	99.799	44.195
IS_{63}		170	0.997	1166.610	0.996	590.510	99.900	50.618
IS_{64}		180	0.997	2290.960	0.996	817.990	99.900	35.705
IS_{65}		190	0.997	2297.740	0.997	1068.100	100.000	46.485
IS_{66}		200	0.998	3076.370	0.995	1217.180	99.699	39.565
IS_{67}		230	n/a	6920.730	0.997	3042.070	n/a	43.956
IS_{68}		250	n/a	8120.380	0.997	4779.590	n/a	58.859
IS_{69}		270	n/a	21060.000	0.998	5820.000	n/a	27.635
IS_{70}		300	n/a	25342.657	0.998	6234.340	n/a	24.600
Average			**0.981**	**2125.263**	**0.981**	**696.980**	**99.542**	**50.766**

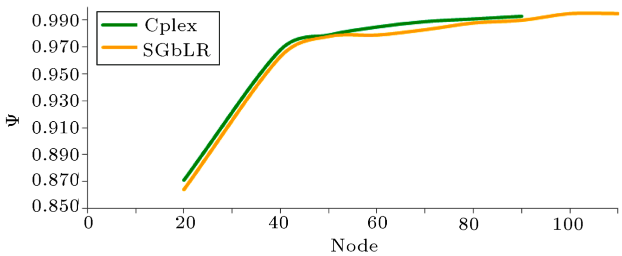

Figure 8. Varying of the Ψ for several values of demand nodes with $P_f = 6$.

that the CPU time of CPLEX has considerably grown with increase in the number of demand nodes and the size of the problem. Moreover, sometimes, the CPLEX cannot find any feasible solution to the problem.

8. Conclusion

The fuzzy robust multi-objective FL/NDP was studied concerning two additional aspects of budget investment constraints and system reliability. The problem was called the fuzzy robust multi-objective FL/NDP (FR/MO/FL/NDP) and formulated as an MIP model. In addition, an efficient algorithm based on SGbLR was applied. The obtained results by the experimental design showed the efficiency of the algorithm versus CPLEX with regards to solution speed, while it still maintained excellent solution quality.

For future research, some directions are recommended. First, we considered the fuzzy aspect of the FR/MO/FL/NDP; however, other aspects of uncertainty (e.g. several probability distributions, intervals,

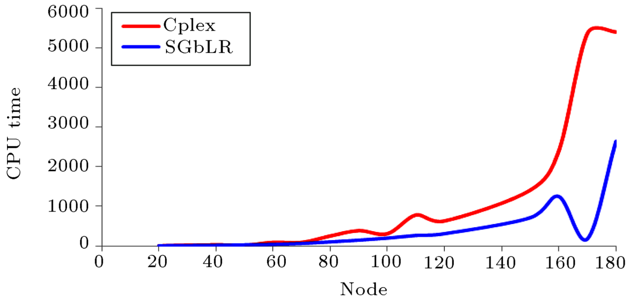

Figure 9. Varying of CPU time for several values of demand nodes with $P_f = 7$.

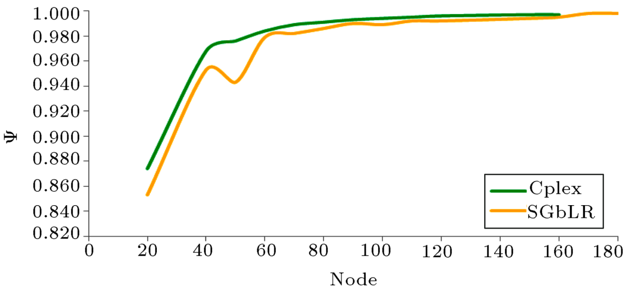

Figure 10. Varying of the Ψ for several values of demand nodes with $P_f = 7$.

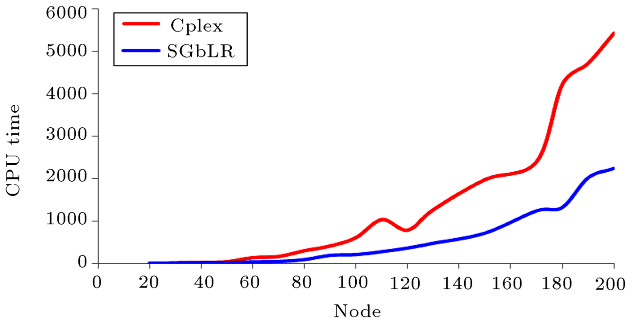

Figure 11. Varying of CPU time for several values of demand nodes with $P_f = 8$.

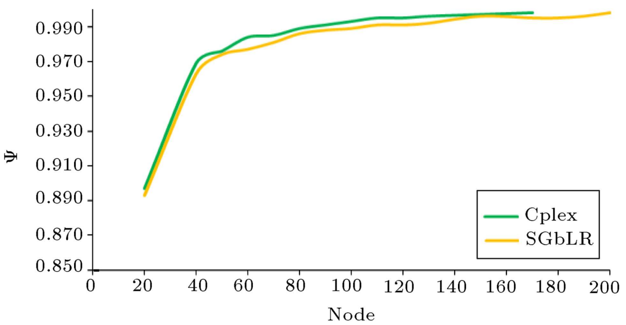

Figure 12. Varying of the Ψ for several values of demand nodes with $P_f = 8$.

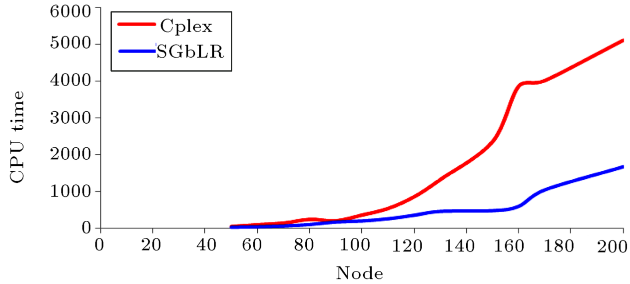

Figure 13. Varying of CPU time for several values of demand nodes with $P_f = 9$.

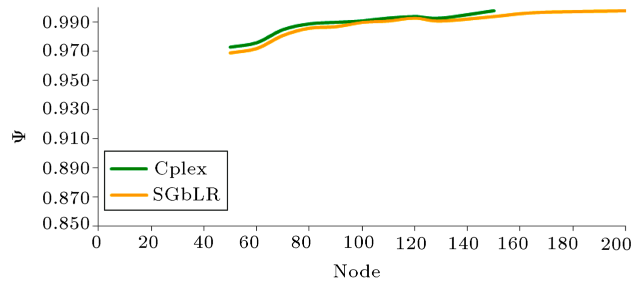

Figure 14. Varying of the Ψ for several values of demand nodes with $P_f = 9$.

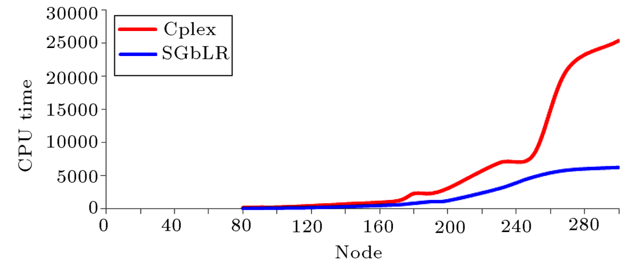

Figure 15. Varying of CPU time for several values of demand nodes with $P_f = 10$.

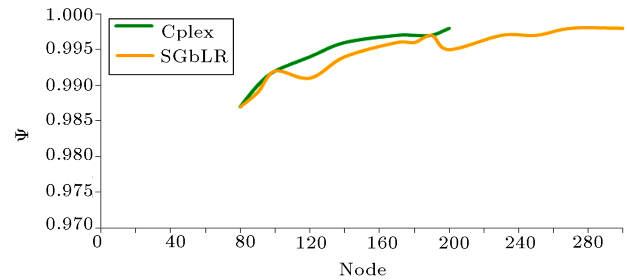

Figure 16. Varying of the Ψ for several values of demand nodes with $P_f = 10$.

etc.) can be considered. Second, different applications of the FR/MO/FL/NDP in several industrial environments and service systems, as a case study, can be considered and studied. Third, and finally, proposing an efficient approach (e.g. Sample Average Approximation (SAA), Monte Carlo simulation, etc.) for the FR/MO/FL/NDP, when the number of scenarios increases, can be another consideration.

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