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A capital flow-constrained lot-sizing problem with trade credit

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Dynamic programming.

Abstract. This paper incorporates capital flow constraints and trade credit into lot-sizing problems. A capital flow constraint is different from a traditional capacity constraint; when a manufacturer begins to produce a certain number of products, its present capital should not be less than the total production costs of that period; otherwise, the manufacturer must decrease production quantity or suspend production, or he/she could delay payment using trade credit. Moreover, the capital of each period should also be greater than zero to avoid bankruptcy. A mathematical model is formulated for the single-item lot-sizing problem. Based on dynamic programming, this mixed integer problem is approximated to a traveling salesman problem to find the longest route and we divide the model into sub-linear problems without integer variables, and propose a dynamic programming algorithm with heuristic adjustment to solve it. An interior point algorithm can easily solve sub-linear problems. The proposed algorithm could obtain optimal solutions under certain situations. Numerical analysis shows that the proposed algorithm has small optimality deviation percentage under other situations and enjoys computation efficiency advantage, as compared with CPLEX 12.6.2. It also indicates that the capital flow constraints and the application of trade credit in lot-sizing problems could affect optimal production decisions.

1. Introduction

A lot-sizing problem is a production planning activity that considers the best use of production resources to achieve production goals over a certain length of planning horizon and decide the optimal timing and level of production [1,2]. Making correct decisions on lot-sizing directly affects total production costs and production efficiency, which are important for a manufacturer to survive and compete on the market.

Wagner and Whitin [3] proposed an $O(T^2)$ algorithm based on dynamic programming to solve the single-item uncapacitated lot-sizing problem, where T is the length of planning horizon. Wagelmans et al. [4] developed an $O(TlgT)$ algorithm for the Wagner-Whitin cases. There is now a sizable literature on this area extending the basic model. For Capacitated Lot-Sizing Problems (CLSP), it is generally difficult to exactly obtain an optimal solution because of the computational complexity. Solution approaches to CLSP include mathematical programming heuristics, Lagrangian heuristics, decomposition and aggregation heuristics, metaheuristics, and problem-specific greedy heuristics [5]. For multi-level lot-sizing problems, metaheuristics are often used for the solution, such as tabu search algorithm, genetic algorithm, simulated annealing algorithm, and so forth. Comprehensive reviews of lot-sizing problems can be found in [5-9].

There are also some papers that deal with lot-sizing problems with a profit maximization approach. Aksen et al. [10] proposed a forward recursive dynamic

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programming algorithm to solve a single-item lot-sizing problem with immediate lost sales in a profit maximization model. Aksen [11] considered loss of goodwill in the uncapacitated lot-sizing problem, assuming that unsatisfied demands in a given period can cause a shrinkage in the demand of the next period. Absi et al. [12] investigated the multi-item capacitated lot-sizing problem with setup times and lost sales and, then, used a Lagrangian relaxation of the capacity constraints to divide the problem into single-item uncapacitated lot-sizing sub-problems. Sereshti and Bijali [13] discussed the general lot-sizing and scheduling problem with demand choice flexibility and evaluated the efficiency of two profit maximization models.

However, seldom have previous studies on lot-sizing problems considered the influence of capital constraints on production planning. In practice, companies may encounter capital shortage problems, especially Small- and Medium-sized Enterprises (SMEs). Once capital is in shortage, the manufacturer has to suspend or reduce production and cannot provide a sufficient number of products for their customers in time. A report showed the shortage of capital accounting for 17% of companies' bankruptcy in Australia in 2008 [14]. The survey data in Elston and Audretsch [15] suggested that 84% of high-tech entrepreneurs in US experienced capital shortage at some time. Based on another report done in UK in 2013, SMEs identified capital flow shortage as one of the two main factors affecting their business [16].

Trade credit is widely used in market transactions that constitutes a major source of short-term financing to prevent capital shortage. Studies found that, in countries outside of USA, trade credit accounted for approximately 20% of all investment financed externally; in USA, trade credit was used by approximately 60% of small businesses [17]. Another survey revealed that trade credit owed by Australian businesses was estimated to be over 80 billion dollars in 2013, accounting for around 8% of their total liabilities; for some industries such as construction, retail and wholesale trades, trade credit generally exceeded 25% of their total assets [18].

Researching on the trade credit in supply chain management problems has lasted for many years. Goyal [19] established an EOQ model based on trade credit. Subsequently, trade credit was taken into account in many inventory problems considering factors such as deteriorating items, allowable shortage, linked to order quantity, inflation, and so forth [20]. Some representative studies are listed here. Teng et al. [21] extended the constant demand to a linear non-decreasing demand function of time in a trade credit problem. Liao et al. [22] established an EOQ model for deteriorating items with two warehouses, and the delayed payment was permitted only when the ordering quantity met a given threshold. Jaggi et al. [23] formulated an inventory model with fully backlogged allowable shortages considering pay-off time for the retailer. Ouyang et al. [24] proposed an integrated inventory model with capacity constraint and a permissible delay payment period, which is order-size dependent. Yadav et al. [25] investigated a retailer's optimal policy under inflation in a fuzzy environment with trade credit. Zhou et al. [26] studied an uncooperative order model for items with trade credit, inventory-dependent demand, and limited displayed-shelf space. Reviews of the trade credit literature can be found in [20,27].

To the best of our knowledge, for the multi-period lot-sizing problem, no studies have considered capital flow constraints and trade credit in the models; for many works regarding trade credit, they are usually based on EOQ models and not designed for multi-period lot-sizing problems. Since capital shortage is a commonly encountered problem for many small- and medium-sized enterprises and trade credit is a widely used option to deal with capital shortage, this paper formulates a profit maximization model for the single-item lot-sizing problem. In the problem, it is assumed that the initial capital of the manufacturer in each period should not be less than the total production costs of that period, guaranteeing production continuity; further to that, the manufacturer can use trade credit to delay payment to their suppliers to alleviate capital pressure. To make the problem solvable, it is assumed that trade credit is in a simple form: The supplier offers a fixed length and fixed interest rate of trade credit, and the manufacturer is not allowed to pay back the trade credit in advance.

The main contributions of this paper are three-fold: (1) Capital flow constraints are introduced to a traditional lot-sizing problem; (2) Trade credit is also incorporated into the problem to alleviate capital shortage; (3) A polynomial algorithm and some heuristic adjustments are devised to solve the problem. Our model can help a manufacturer make lot-sizing decisions, when encountering capital shortage problems.

The remainder of this paper is organized as follows. Section 2 lists the notations and assumptions of our paper. Section 3 formulates the mathematical model. Section 4 presents a dynamic programming model of the problem. Section 5 analyses some properties, and Section 6 provides a solving algorithm. Section 7 implements the computational study. Section 8 concludes the paper and outlines future extensions.

2. Notations and assumptions

The following notations and assumptions are used to develop the mathematical model of the paper. Some notations will be presented later, if required.

2.1. Notations

T	Length of planning horizon;
t	Index of a period, $t = 1, 2, .., T$;
d_t	Demand in period t;
p_t	Selling price in period t;
c_t	Unit production cost (variable cost) in period t;
s_t	Setup cost in period t;
h_t	Unit inventory cost in period t;
π_t	Unit penalty cost for lost sales in period t;
B_0	Initial capital;
I_0	Initial inventory;
k	Trade credit length, constant value;
r	Trade credit interest rate, constant value;
B_t	End-of-period capital in period t, decision variable;
I_t	End-of-period inventory in period t, decision variable;
y_t	Production quantity in period t, decision variable;
w_t	Quantity of lost sales in period t, decision variable;
z_t	Binary variable whether or not using trade credit in period t, decision variable.

2.2. Assumptions

The following assumptions are adopted in our paper:

(i) When a production process begins, the manufacturer purchases raw materials from a supplier, resulting in variable production costs. If trade credit is used, the payment of raw materials to the supplier can be postponed after trade credit length;

(ii) Since suppliers tend to take higher risks for longer trade credit length or give new credit to customers while old trade credit is not paid back, then it is assumed that trade credit length, k, is smaller than the length of planning horizon; when the manufacturer decides to apply a new trade credit in period t, he/she should not have unpaid old trade credit; the manufacturer should not have unpaid trade credit in the last period; the manufacturer could not pay back the trade credit in advance;

(iii) Capital in the beginning of period t should not be less than the total production costs in period t, i.e., $B_{t-1} \geq s_t + (1 - z_t)c_t y_t$;

(iv) End-of period capital for any period t should not be below zero, i.e., $B_t \geq 0$;

(v) Initial inventory for period 1 is 0, i.e., $I_0 \geq 0$;

(vi) No backorder is allowed;

(vii) The manufacturer can decide the realized quantities for customer's demands, i.e., he/she can decide the lost sales for any period t, but has to pay the penalty cost of lost sales.

Assumptions (i) and (ii) define the trade credit usage in the proposed lot-sizing problem. Assumptions (iii) and (iv) define the capital flow constraints. Assumptions (v) and (vi) are also the standard assumptions of the Wagner-Whitin model [3]. Assumption (vii) indicates that the manufacturer can decide how many products to provide for the customers.

3. Mixed integer model

Capital flow for any period t meets Eq. (1):

$$B_t = \begin{cases} B_{t-1} + p_t(d_t - w_t) - z_{t-k+1}c_{t-k+1}y_{t-k+1} \\ \quad (1+r)^k - (1 - z_t)c_t y_t \\ \quad -(h_t I_t + \pi_t w_t + s_t x_t) \qquad t \geq k \\ \\ B_{t-1} + p_t(d_t - w_t) - (1 - z_t)c_t y_t \\ \quad -(h_t I_t + \pi_t w_t + s_t x_t) \qquad t < k \end{cases} \tag{1}$$

Eq. (1) is divided into two terms, because when $t \geq k$, the manufacturer may have to pay the principal and the interest of trade credit occurring in period $t - k + 1$, which is $z_{t-k+1}c_{t-k+1}y_{t-k+1}(1 + r)^k$; when $t < k$, $t - k + 1 \leq 0$ and the manufacturer cannot pay back the trade credit. The realized sales in period t are given by $d_t - w_t$, i.e., demand minus lost sales in t. The revenue in period t is $p_t(d_t - w_t)$. The variable production cost in period t is $(1-z_t)c_t y_t$, meaning that if the manufacturer uses trade credit in period t, he/she does not pay the variable production cost. Other costs, including inventory costs, lost sales penalty costs, and setup costs, are given by $h_t I_t + \pi_t w_t + s_t x_t$.

Based on Eq. (1), the final capital B_T is derived:

$$\begin{aligned} B_T = &B_0 + \sum_{t=1}^{T} \ [p_t(d_t - w_t) - (h_t I_t + \pi_t w_t + s_t x_t) \\ &- (1 - z_t)c_t y_t] - \sum_{t=k+1}^{T} z_{t-k+1}c_{t-k+1}y_{t-k+1} \\ &(1 + r)^k. \end{aligned} \tag{2}$$

The mixed integer mathematical model of capital flow constrained lot-sizing problem with trade credit (CFLSP-TC) is formulated as follows:

Model CFLSP-TC

Max :

$$B_T - B_0, \tag{3}$$

s.t. :

$t = 1, 2, ..., T$

(1), (2)

$$y_t \le M x_t, \tag{4}$$

$$(1 - z_t) c_t y_t + s_t x_t \le B_{t-1}, \tag{5}$$

$$w_t \le d_t, \tag{6}$$

$$I_t = I_{t-1} + y_t - d_t + w_t, \tag{7}$$

$$z_t \le x_t, \tag{8}$$

$$\sum\nolimits_{i=t}^{t+k-1} z_i \le 1, \tag{9}$$

$$\sum\nolimits_{i=T-k+2}^{T} z_i = 0, \tag{10}$$

$$I_0 = 0, I_t \ge 0, \tag{11}$$

$$w_t \ge 0, y_t \ge 0, \tag{12}$$

$$x_t \in \{0, 1\}, z_t \in \{0, 1\}. \tag{13}$$

The objective is to maximize the final capital increment determined by Eq. (3). Constraint (1) is the capital flow balance, and Constraint (2) is the final capital expression. Constraint (4) enforces setups with positive production amounts in each period. Constraint (5) is the capital flow constraint, i.e., the initial capital in period should not be less than its total production costs in period t. If $z_t = 1$, the manufacturer has to pay the variable production cost of period t. It also ensures the non-negativity of B_{t-1}, avoiding bankruptcy. Constraint (6) ensures that any lost sales w_t in period t cannot exceed demand quantity d_t of that period. Constraint (7) provides inventory balance of two consecutive periods. Constraints (8)-(10) show Assumptions (i) and (ii): The manufacturer uses trade credit only in the production launching periods; the manufacturer can have at most one unpaid trade credit in a period and should not have unpaid trade credit in the last period; further to that, he/she cannot pay back trade credit in advance. Constraint (11) represents Assumptions (v) and (vi). Constraints (12) and (13) guarantee the non-negativity and binarity of variables, respectively.

It should be noted that if trade credit length $k = 0$, model CFLSP-TC is a capital flow-constrained lot-sizing problem without trade credit (CFLSP); therefore, CFLSP is a special case of CFLSP-TC. The single-item capacitated lot-sizing problem was shown by Bitran and Yanasse [28] to be NP-hard. As for the proposed problem, the capital flow constraint equation (Eq. (5)) is a capacity constraint by removing $s_t x_t$ and z_t from the model and replacing B_{t-1} with C_t, where C_t is the capacity constraint in period t. Therefore, the capital flow constraint is a special type of capacity constraints, and models CFLSP-TC and CFLSP are also NP-hard problems.

4. Dynamic programming model

A dynamic programming model is designed for the proposed problem in order to analyse some properties.

4.1. States

In any period t, its states are: initial inventory level I_{t-1}, initial capital B_{t-1}, initial trade credit state Ω_{t-1}, and initial trade credit payable state L_{t-1}. To describe Ω_t, the definition of *trade credit duration* is provided.

Definition 1. In any period t, the number of periods from the last period that uses trade credit to the last production launching period is called *trade credit duration*.

Trade credit state Ω_t is a binary set representing the production plan since the last period uses trade credit when its trade credit duration is smaller than trade credit length, k. In the binary set, 1 means 'launching production' and 0 means 'not launching production' in this period. Apparently, $\Omega_0 = \phi$. $|\Omega_t|$ is used to represent the number of elements in set Ω_t.

Figure 1 shows clearly Ω_4 and $|\Omega_4|$ in 4 periods of

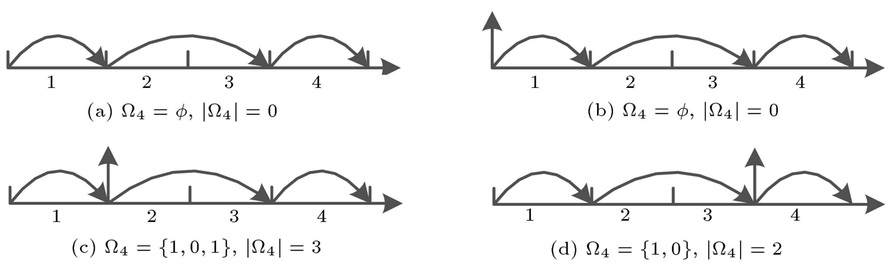

Figure 1. Meanings of Ω_4 and $|\Omega_4|$ for a production plan with different trade credit usage (the up arrow represents the beginning of a period that uses trade credit.)

a production plan with trade credit length $k = 3$ and different trade credit usage. If there are no preceding periods in period t that use trade credit, or trade credit duration is longer than k, Ω_t is an empty set, which is shown by Figure 1(a) and (b). In Figure 1(a), its trade credit duration is 0; in Figure 1(b), its trade credit duration is 4; therefore, the trade credit states are both empty sets. In Figure 1(c), its trade credit duration is 3; in Figure 1(d), its trade credit duration is 1; the trade credit states are given by the binary sets.

Trade credit payable state L_t is the payable accounts of the manufacturer for the trade credit at the end of period t. Apparently, $L_0 = 0$.

4.2. Actions

The actions in period t include production quantity y_t, demand realized quantity v_t, and possible use of trade credit z_t. The lower bounds for y_t, v_t, and z_t are all zeros; the upper bounds are presented below:

$$\bar{y}_t = \max\{0, (B_{t-1} - s_t)/c_t\}, \tag{14}$$

$$\bar{v}_t = \bar{d}_t, \tag{15}$$

$$\bar{z}_t = \begin{cases} 1 & |\Omega_{t-1}| = 0 \quad \text{or} \quad |\Omega_{t-1}| \geq k \\ 0 & 1 \leq |\Omega_{t-1}| \leq k-1 \end{cases} \tag{16}$$

The upper bounds for z_t in Eq. (16) imply that the manufacturer could not use trade credit if $1 \leq |\Omega_{t-1}| \leq k-1$, because it has unpaid trade credit.

4.3. States transition function

A unit step function is defined $K(x)$: $K(x) = 1$ if $x > 0$, $K(x) = 0$ if $x \leq 0$. The state transition equations are:

$$\Omega_t = \begin{cases} \Omega_{t-1} \cup \{0\} & z_t = 0,\ |\Omega_{t-1}| \neq 0 \\ \varnothing & z_t = 0,\ |\Omega_{t-1}| = 0 \\ \{1\} & z_t = 1 \end{cases} \tag{17}$$

$$L_t = \begin{cases} L_{t-1} & z_t = 0, 0 < |\Omega_t| < k \\ c_t y_t & z_t = 1, 0 < |\Omega_t| < k \\ 0 & |\Omega_t| = 0 \quad \text{or} \quad |\Omega_t| \geq k \end{cases} \tag{18}$$

$$I_t = I_{t-1} + y_t - v_t, \tag{19}$$

$$B_t = \begin{cases} B_{t-1} + p_t v_t - (1 - z_t) c_t y_t - h_t I_t - s_t K(y_t) \\ \quad -\pi_t (d_t - v_t) - L_t (1+r)^k \qquad |\Omega_t| = k \\ \\ B_{t-1} + p_t v_t - (1 - z_t) c_t y_t - h_t I_t - s_t K(y_t) \\ \quad -\pi_t (d_t - v_t) \qquad |\Omega_t| \neq k \end{cases} \tag{20}$$

4.4. Immediate profit

The immediate profit in period t is the capital increment during this period. The given initial states B_{t-1}, I_{t-1}, Ω_{t-1}, L_{t-1}, actions y_t, v_t, z_t, and immediate profit ΔB_t can be expressed as follows:

$$\Delta B_t(I_{t-1}, \Omega_{t-1}, L_{t-1}, B_{t-1}) = B_t - B_{t-1}. \tag{21}$$

4.5. Functional equation

$f_t(I_{t-1}, \Omega_{t-1}, L_{t-1}, B_{t-1})$ is defined as the maximum capital quantity increment during period $t+1, t+2, ..., T$, given initial states B_{t-1}, I_{t-1}, Ω_{t-1}, L_{t-1}. When $t = 1, 2, ..., T$, the functional equation is expressed as follows:

$$f_t(I_{t-1}, \Omega_{t-1}, L_{t-1}, B_{t-1}) = \max_{0 \leq y_t \leq \bar{y}_t, 0 \leq v_t \leq \bar{v}_t, 0 \leq z_t \leq \bar{z}_t} \{\Delta B_t + f_{t+1}(I_t, \Omega_t, L_t, B_t)\}. \tag{22}$$

The boundary condition of the functional equation is $f_{T+1}(I_T, \Omega_T, L_T, B_T) = 0$. The dynamic programming model is equivalent to Model CFLSP-TC, in which the objective is $f_1(I_0, \Omega_0, L_0, B_0)$.

5. Mathematical properties

Based on the dynamic programming model, some mathematical properties are presented. The algorithm proposed in the current paper exploits these properties to solve the problem. First, the definitions of *production cycle* and *production round* are given in this paper.

Definition 2. In a production plan, if the manufacturer launches production at the beginning of period m and does not launch new production until the end of period t $(m \leq t \leq T)$, period t to period t will be called a production cycle.

Definition 3. In a production plan, for one or more consecutive production cycles that last from the beginning of m to the end of period $t (m \leq t \leq T)$, if the initial inventory for period m and the end-of-period inventory for period t are zeros, period m to period t will be called a production round.

Lemma 1. For any production in starting period t, when I_{t-1} is fixed and if $|\Omega_{t-1}| = 0$ or $|\Omega_{t-1}| \geq k$, $f_t(I_{t-1}, \Omega_{t-1}, L_{t-1}, B_{t-1})$ is a non-decreasing function of B_{t-1}.

Proof. When I_{t-1} is fixed, I_{t-1} has no influence on the final capital increment. If $|\Omega_{t-1}| = 0$ or $|\Omega_{t-1}| \geq k$, according to Eq. (18), $L_{t-1} = 0$. Therefore:

$$f_t(I_{t-1}, \Omega_{t-1}, L_{t-1}, B_{t-1}) = f_t(I_{t-1}, \Omega_{t-1}, 0, B_{t-1})$$
$$= \max_{0 \leq y_t \leq \bar{y}_t, 0 \leq v_t \leq \bar{v}_t, 0 \leq z_t \leq \bar{z}_t} \{\Delta B_t + f_{t+1}(I_t, \Omega_t, L_t, B_t)\}. \tag{23}$$

When B_{t-1} increases, from Eqs. (14), (15), and (16), the upper bounds for v_t and z_t remain unchanged, while the upper bound for y_t remains constant or expands. There always exist actions y'_t, v'_t, and z'_t that make $f'_t(I_{t-1}, \Omega_{t-1}, 0, B'_{t-1})$

not lower than $f_t(I_{t-1}, \Omega_{t-1}, 0, B_{t-1})$. Therefore, $f_t(I_{t-1}, \Omega_{t-1}, L_{t-1}, B_{t-1})$ is non-decreasing with B_{t-1} for fixed I_{t-1}, $|\Omega_{t-1}| = 0$ or $|\Omega_{t-1}| \geq k$.

Lemma 2. When trade credit length $k = 0$, for any production starting period $t+1$ ($t+1 = 1, 2, ..., T$), if all the variable production costs are equal, i.e., $c_{t+1} = c$, the optimal solution satisfies $I_t x_{t+1} = 0$.

Proof. If there is a solution that does not satisfy Lemma 2, i.e., $I_t > 0$ and $x_{t+1} = 1$, assume period $t+1'$ of the former production cycle begins in period m ($1 \leq m \leq t$). The production plan is shown in Figure 2.

According to the capital flow expression as in Eqs. (1) and (2):

$$B_t = B_{m-1} + \sum_{i=m}^{t} p_i(d_i - w_i) - s_m - c_m y_m - \sum_{i=m}^{t} h_i I_i,$$

and:

$$B_T = B_t + \sum_{i=t+1}^{T} [p_i(d_i - w_i) - h_i I_i - s_i x_i - c_i y_i].$$

If the production quantity in period m decreases I_t and production quantity in period $t+1$ increases I_t, the solution is still feasible. B_t and B_T change to the following:

$$B'_t = B_t + \sum_{i=m}^{t} h_i I_i + c_m I_t,$$

$$B'_T = B_T + \sum_{i=m}^{t} h_i I_i + c_m I_t - c_{t+1} I_t.$$

If all the variable production costs are equal, $c_m = c_{t+1}$. Therefore, $B'_T > B_T$. The final capital increases, $I_t > 0$, and $c_{t+1} = 1$ is not optimal.

Lemma 3. For the trade credit length $k \neq 0$, for any production starting in period $t+1$ ($t+1 = 1, 2, ..., T$), if all the variable production costs are equal and $\min\{h_i | i = 1, 2, ..., T\} \geq (1+r)^k - c$, the optimal solution also satisfies $I_t x_{t+1} = 0$.

Proof. If there is a solution that does not satisfy Lemma 3, i.e., $I_t > 0$ and $x_{t+1} = 1$, assume that period $t+1$'s former production cycle also begins in period m ($1 \leq m \leq t$), as shown in Figure 2. If the production quantity in period m decreases I_t and production quantity in period $t+1$ increases I_t, the solution is still feasible. According to Eqs. (1) and (2), B_T changes to the following:

If $z_m = 0$ and $z_{t+1} = 0$,

$$B'_T = B_T + \sum_{i=m}^{t} h_i I_i + c_m I_t - c_{t+1} I_t$$

If $z_m = 0$ and $z_{t+1} = 1$,

$$B'_T = B_T + \sum_{i=m}^{t} h_i I_i + c_m I_t - c_{t+1} I_t (1+r)^k,$$

If $z_m = 1$ and $z_{t+1} = 0$,

$$B'_T = B_T + \sum_{i=m}^{t} h_i I_i + c_m I_t (1+r)^k - c_{t+1} I_t,$$

If $z_m = 1$ and $z_{t+1} = 1$,

$$B'_T = B_T + \sum_{i=m}^{t} h_i I_i + c_m I_t (1+r)^k - c_{t+1} I_t (1+r)^k.$$

When all the variable production costs are equal, $B'_T > B_T$ for the situations $z_m = 0$ and $z_{t+1} = 0$; $z_m = 1$ and $z_{t+1} = 0$; $z_m = 1$, and $z_{t+1} = 1$.

If $\min\{h_i | i = 1, 2, ..., T\} \geq (1+r)^k - c$ is also satisfied, $B'_T > B_T$ for the situation $z_m = 0$ and $z_{t+1} = 1$. Therefore, Lemma 3 is proven.

Lemmas 2 and 3 are also known as the *zero-inventory ordering policy,* in which the initial inventory of a production launching period is always zero. Based on Lemmas 1 to 3, Model CFLSP-TC is converted into a traveling salesman problem, which finds the longest route as shown in Figure 3 (in this example, $T = 4$). $BB_{m,n}$ is the maximum capital increment in a production round from period m to period n with initial capital B^*_{m-1}, initial trade credit state meeting $|\Omega_{m-1}| = 0$ or $|\Omega_{m-1}| \geq k$, and end-of-period trade credit state meeting $|\Omega_{n-1}| = 0$ or $|\Omega_{n-1}| \geq k$.

B^*_n is defined as the maximum end-of-period capital in period n given $|\Omega_n| = 0$ or $|\Omega_n| \geq k$, and

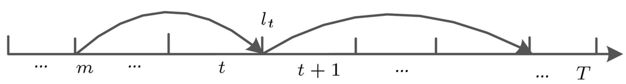

Figure 2. Production plan sketch when $I_t > 0$ and $x_{t+1} = 1$.

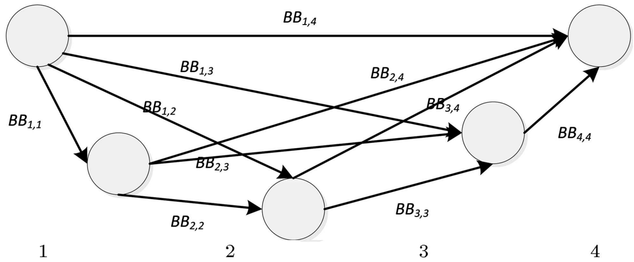

Figure 3. Traveling salesman problem for model CFLSP-TC.

$B_0^* = B_0$. The following recursive equation is proposed for the traveling salesman problem.

$$B_n^* = \max_{1 \le m \le n} \left[B_{m-1}^* + BB_{m,n} \right]. \tag{24}$$

Lemma 4. For any period $t+1$ ($t+1 = 1, 2, ..., T$), given initial states $I_t = 0$, $|\Omega_t| = 0$ or $|\Omega_t| \ge k$, $BB_{t+1,T} = \max_{B_T} f_{t+1}(I_t, \Omega_t, L_t, B_t)$.

Proof. If $I_t = 0$, $|\Omega_t| = 0$ or $|\Omega_t| \ge k$, $f_{t+1}(I_t, \Omega_t, L_t, B_t) = f_{t+1}(0, \Omega_t, 0, B_t)$. From Lemma 1, obviously, $f_{t+1}(0, \Omega_t, 0, B_t)$ is a non-decreasing function of B_t. Therefore, $f_{t+1}(0, \Omega_t, 0, B_t)$ gets its maximum value when $B_t^* = B_t$. Maximum value for it is represented by $\max_{B_T} f_{t+1}(I_t, \Omega_t, L_t, B_t)$. Since the manufacturer should not have unpaid trade credit in the last period, $|\Omega_T| = 0$ or $|\Omega_T| \ge k$. By definition, $BB_{t+1,T}$ has the same meaning with $\max_{B_T} f_{t+1}(I_t, \Omega_t, L_t, B_t)$ given $I_t = 0$, $|\Omega_t| = 0$ or $|\Omega_t| \ge k$. Therefore, Lemma 4 is proved.

Lemma 5. In any period t, given $I_t = 0$, $|\Omega_t| = 0$ or $|\Omega_t| \ge k$, the optimal production plan from periods 1 to t is part of the optimal production plan from periods 1 to T.

Proof. Based on Eq. (22) and the proof of Lemma 4, given $I_t = 0$, $|\Omega_t| = 0$ or $|\Omega_t| \ge k$, $f_{t+1}(I_t, \Omega_t, L_t, B_t)$ gets the maximum value when $B_t^* = B_t$, and B_t equals B_t^* when the production plan from periods 1 to t is optimal. Therefore, what is optimal for period 1 to period t is also optimal for $f_{t+1}(I_t, \Omega_t, L_t, B_t)$ given $I_t = 0$, $|\Omega_t| = 0$ or $|\Omega_t| \ge k$, and the optimal production plan from periods 1 to t is part of the optimal production plan from periods 1 to T.

Theorem 1. If all the variable production costs are equal, i.e., $c_t = c$, and $\min\{h_i | i = 1, 2, ..., T\} \ge (1+r)^k - c$, recursive equation (Eq. (24)) can obtain the optimal solution for model CFLSP-TC.

Proof. When the variable production costs are equal, Lemma 3 shows that the problem satisfies the *zero-inventory ordering policy*. Hence, the optimal production plan is a combination of several production rounds.

Recursive equation (Eq. (24)) in fact enumerates all the possible production rounds given all the possible initial trade credit states that meet $|\Omega_t| = 0$ or $|\Omega_t| \ge k$. In Eq. (24), maximum initial capital is always selected for the computation of capital increment of each production round. Lemma 4 shows that this way of computation guarantees the maximums capital increment for the latter production; Lemma 5 shows that the optimal production plan from periods 1 to t is part of the total optimal production plan given $|\Omega_t| = 0$ or $|\Omega_t| \ge k$. Lemma 4 and Lemma 5 together indicate that the optimal production plan is a combination of several production rounds with initial trade credit states and end-of-period trade credit states that meet $|\Omega_{t_1}| = 0$ or $|\Omega_{t_1}| \ge k$, and $|\Omega_{t_2}| = 0$ or $|\Omega_{t_2}| \ge k$ (t_1 is the beginning period of the production round, and t_2 is the end period of the production round).

These are the same properties as the Wagner-Whitin case [3]. Recursive equation (Eq. (24)) enumerates all the possible combinations of production rounds; the one that gives the maximum final capital is the optimal solution.

Theorem 2. If all the variable production costs are equal, i.e., $c_t = c$, recursive equation (Eq. (24)) can obtain the optimal solution for Model CFLSP.

Proof. Model CFLSP is a special case of Model CFLSP-TC without considering trade credit. The *zero-inventory ordering policy* is also put into effect when all variable production costs are equal. For Model CFLSP, trade credit states satisfy $|\Omega_t| = 0$. Therefore, Lemmas $1-5$ also hold for Model CFLSP, and Eq. (24) can obtain the optimal solution.

If the parameter values do not satisfy the conditions of Theorem 1 or Theorem 2, Eq. (24) only obtains an approximate solution. The *zero-inventory ordering policy* is not optimal, and, in some situations, it is better to hold some inventory when launching a new production cycle. Therefore, a heuristic adjustment is devised, as shown by Corollary 1, to bring the approximate solution closer to optimality.

Corollary 1. In a feasible solution, assume that solutions are x_t, y_t, w_t, and z_t. For any two adjacent production cycles, assume that the former production cycle begins in period t_1, and the latter production cycle begins in period t_2. If $B_{t_1-1} - s_{t_1} - (1-z_t)c_{t_1}y_{t_1} > 0$, $ct_1(1+z_{t_1}r)^k + \sum_{i=t_1}^{t_2-1} h_i < c_{t_2}(1+z_{t_2}r)^k$, then it is better to move some production quantity Δy_{t_2} from y_{t_2} to y_{t_1} to obtain the final capital if the production plan is still feasible after the adjustment.

Proof. If $B_{t_1-1} - s_{t_1} - (1-z_t)c_{t_1}y_{t_1} > 0$, period t_1 has residual production capacity, which can produce more. After the moving adjustment, the final capital changes to the following:

$$B'_T = B_T + \left[c_{t_2}(1+z_{t_2}r)^k - c_{t_1}(1+z_{t_1}r)^k - \sum_{i=t_1}^{t_2-1} h_i \right] \Delta y_{t_2}.$$

If $c_{t_1}(1+z_{t_1}r)^k + \sum_{i=t_1}^{t_2-1} h_i < c_{t_2}(1+z_{t_2}r)^k$, $B'_T >$

$$\Delta y_{t_2} = \begin{cases} \min\left\{\frac{B_{t_1-1}-s_{t_1}}{c_{t_1}(1-z_{t_1})}, \frac{B_{t_2-1}-(1-z_{t_2})c_{t_2}y_{t_2}-s_{t_2}}{c_{t_2}(1+z_{t_2}r)^k-c_{t_1}(1+z_{t_1}r)^k-\sum_{i=t_1}^{t_2-1}h_i}\right\} & z_{t_1}=0 \\ \frac{B_{t_2-1}-(1-z_{t_2})c_{t_2}y_{t_2}-s_{t_2}}{c_{t_2}(1+z_{t_2}r)^k-c_{t_1}(1+z_{t_1}r)^k-\sum_{i=t_1}^{t_2-1}h_i} & z_{t_2}=1 \end{cases} \quad (25)$$

Box I

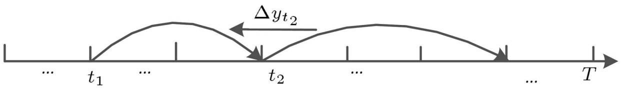

Figure 4. Heuristic adjustment of Corollary 1.

B_T. If the production plan is still feasible after the adjustment, the final capital increases.

This heuristic adjustment step is shown in Figure 4. To judge whether the production plan is still feasible after the adjustment, it is required to judge whether the end-of-period capital in each period is above zero or not.

The moving production quantity Δy_{t_2} can be obtained by Eq. (25) as shown in Box I.

When $z_{t_1} = 0$, the first term in the min bracket is the maximum production increment that period t_1 can provide; the second term in the min bracket is the maximum production decrement that period t_2 can accept; the moving production quantity Δy_{t_2} is the minimum value of the two terms. When $z_{t_1} = 1$, the manufacturer uses trade credit in period t_1, and period t_1 can provide production increment finitely; therefore, only the second term is required in Eq. (25).

6. Sub-linear problems and algorithm

6.1. Computation of $BB_{m,n}$

In recursive equation (Eq. (25)), in order to compute $BB_{m,n}$, several sub-linear problems without integer variables are formulated. Since the production round from periods m to n can include several production cycles, $bb_{m,n}$ can be used to represent the maximum capital increment for a specific production round from periods m to n. If $|\Omega_n| = 0$, the manufacturer does not use trade credit from periods m to n. There is only one production cycle required in the production round to compute $BB_{m,n}$. The maximum capital increment is computed by Model Sub-1.

Model Sub-1 ($|\Omega_n| = 0$):

$$\text{Max} \quad bb_{m,n} = \sum_{t=m}^{n} [p_t v_t - (h_t I_t + c_m v_t) - \pi_t(d_t - v_t)] - s_m, \quad (26)$$

s.t. :

$t = m, m+1, ..., n$

$$c_m \sum_{i=m}^{n} v_i + s_m \leq B_{m-1}, \quad (27)$$

$$B_t \geq 0, \quad (28)$$

$$I_{m-1} = 0, I_n = 0, \quad (29)$$

$$0 \leq v_t \leq d_t. \quad (30)$$

Objective function in Eq. (26) maximizes the capital increment from periods m to n, where $d_t - v_t$ is the lost sales in period t. Constraints (27) and (28) represent capital flow constraints assumptions in this paper. In Constraint (28), B_t is the expression of v_t, which can be deducted by Eqs. (1), (35), and (36). It is very lengthy, thus, we omit its full expressions here. Constraint (29) means that initial inventory and final inventory of the production cycle are zeros, which is a heuristic step if parameter values do not satisfy Theorem 1 or 2. Constraint (30) provides the lower and upper bounds of variables.

If the computation of Model Sub-1 does not obtain a feasible solution, we set $v_t = 0$ ($t = m, m+1, ..., n$) and $BB_{m,n} = 0$, which means that it is better not to launch production for the production cycle from periods m to n.

When $|\Omega_n| \geq k$, the manufacturer uses trade credit in period m. For a specific production plan in the production round from periods m to n, assume that there are l production cycles, and the production launching periods are $t_1, t_2, ..., t_l$ (for easy of expression, we set $m = t_1$, $n = t_l - 1$). When $|\Omega_n| \geq k$, the production plan from periods m to n is shown in Figure 5. The maximum capital increment is computed by Model Sub-2.

Model Sub-2 ($|\Omega_n| \geq k$):

$$\text{Max } bb_{m,n} = \sum_{t=m}^{n} [p_t v_t - h_t I_t - \pi_t(d_t - v_t)]$$

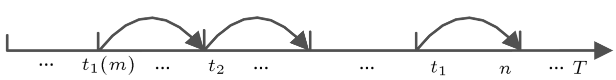

Figure 5. Production plan from period m to period n when $|\Omega_n| \geq k$.

Initialization: $t=1$. $1 \times T$ zero matrices x, y, w, z, B^*. $T \times T$ zero matrices BB, bb^i $(i=1,2,...,2^k)$.

Step 1: For $n=t,t+1,...,T$, get the value of m based on trade credit state Ω_n. Compute $bb_{m,n}$ from sub-linear problems Model Sub-1, and Model Sub-2. For the ith production plan in Ω_n, record its value in $bb^i(m,n)$.

Step 2: According to Eq. (34), calculate $BB_{m,n}$ and record its value in $BB(m,n)$; according to Eq. (24), calculate B_t^* and record its value in $B^*(t)$. Obtain the production plan from periods 1 to t: $x(1:t)$, $y(1:t)$, $w(1:t)$, $z(1:t)$.

Step 3: $t=t+1$, repeat Steps 1 and 2 until $t=T$.

Step 4: Return to check if production plan meets Corollary 1. If the condition is met, make plan adjustments. Obtain the final production plan: $x(1:T)$, $y(1:T)$, $w(1:T)$, $z(1:T)$ and final capital B_T^*.

Box II

$$-c_{t_1}\sum_{i=t_1}^{t_2} v_i(1+r)^k$$

$$-\sum_{i=2}^{l} c_{ti}\sum_{j=t_i}^{t_{i+1}-1} v_j - \sum_{i=1}^{l} s_{t_i}, \qquad (31)$$

s.t. :

$t=m,m+1,...,n$

(28), (29), (30)

$$s_m \le B_{m-1}, \qquad (32)$$

$$s_{t_i}+c_{t_i}\sum_{j=t_i}^{t_{i+1}-1} v_j \le B_{t_i}-1 \qquad i=1,2,...,l. \quad (33)$$

In Model Sub-2, Constraints (32) and (33) are the capital flow constraints for launching production under the assumptions of our paper. If the computation of Model Sub-2 does not obtain a feasible solution, set $v_t=0$ $(t=m,m+1,...,n)$ and $bb_{m,n}=0$. Because $BB_{m,n}=0$ is the maximum capital increment in the production round:

$$BB_{m,n}=\max\{bb_{m,n}|\,|\Omega_n|=0 \quad \text{or} \quad |\Omega_n|\ge k\}. \quad (34)$$

For $t=m,m+1,...,n$, the relation among v_t, y_t, and w_t is shown as in Eqs. (35) and (36):

$$y_t=\sum_{j=t_i}^{t_{i+1}-1} v_j, \qquad i=2,3,...,l, \qquad (35)$$

$$w_t=d_t-v_t. \qquad (36)$$

We also have the following lemma regarding the number of computations done for $BB_{m,n}$.

Lemma 6. There are at most 2^k sub-linear problems $bb_{m,n}$ to compute $BB_{m,n}$.

Proof. When $|\Omega_n|\ge k$, by the definition of trade credit state and trade credit duration, trade credit duration for period n could be: $0,1,...,k-1$; there are at most $2^0+2^1+...+2^{k-1}=2^k-1$ combinations of production rounds in Ω_n. When $|\Omega_n|=0$, there are only one production round in Ω_n. Therefore, there are at most $2^{k-1}+1=2^k$ sub-linear problems to compute $BB_{m,n}$.

6.2. Algorithm DPH

Based on Theorem 1, Corollary 1, and sub-linear problems computations, the following dynamic programming algorithm is proposed with heuristic adjustment (DPH) in Box II to solve our problem.

6.3. Computation complexity of the algorithm

During the recursion of Eq. (24), there are $T(T+1)/2$ computations of $BB_{m,n}$. By Lemma 6, there are at most 2^k computations of sub-linear problems for each $BB_{m,n}$. Therefore, there are $2^kT(T+1)/2$ computations of sub-linear problems. The computational complexity of our algorithm is $O(2^kT^2\psi)$, where ψ is the computational complexity of the algorithm for the sub-linear problems.

Without integer variables, a polynomial interior point algorithm can solve the sub-linear problems. ψ is $O(T^{3.5}L_0)$ for the commonly used interior point algorithm, where L_0 denotes the total length of the binary coding of the input data. Details of the complexity of interior point algorithm could be found in [29]. This is one of the reasons why we removed integer variables from original model and divided it into sub-linear problems.

Because trade credit length is generally small compared with planning horizon length T, 2^k can be viewed as a constant number, and our algorithm is polynomial in this sense. Since Model CFLSP is a special case of Model CFLSP-TC, our algorithm can also solve the capital flow constrained problem without trade credit, and its computational complexity is $O(T^2\psi)$.

7. Numerical analysis

In our numerical experiments, the linear programming algorithm is the interior point algorithm in MATLAB based on the paper of Zhang [30]. Our algorithm is coded in MATLAB 2014a and ran on a desktop computer with an Intel (R) Core (TM) i7-3770 CPU, at 3.40 GHz, 4GB of RAM at a 32-bit Windows 7 operating system.

7.1. An example about the effects of capital and trade credit

Assume that trade credit length, $k = 3$, interest rate, $r = 7\%$, initial capital, $B_0 = 200$, and planning horizon length, $T = 8$. Values of other parameters are listed in Table 1.

The solution of our algorithm is shown in Figure 6. Because the problem satisfies the optimal conditions in Theorem 1, the solution is optimal. In the optimal solution, the manufacturer uses trade credit in period 2 and pays back the trade credit at the end of period 4.

The optimal production arrangement plan without trade credit is shown in Figure 7. Because all the variable production costs are equal, the solution is also optimal according to Theorem 2.

Figures 6 and 7 demonstrate whether or not using trade credit affects the optimal production plan. Figure 8 shows the optimal capital increment as the changing of trade credit settings and quantity of the initial capital.

In Figure 8(a), as initial capital increases, the final optimal capital increment also increases, yet remains stable when the initial capital is large enough. The reason is that when the initial capital is sufficient, capital flow constraints are not production capacity constraints, and the results are the same as the cases without capital constraints. In Figure 8(b), the final capital increment varies as the trade credit length changes. In Figure 8(c), the final capital increment decreases as trade credit interest rate increases, which is reasonable. All those figures illustrate that trade credit settings and capital do exert influence on the optimal production plan and final capital increment.

Table 1. Parameter values of the example.

p_t	30	22	12	14	30	12	12	40
c_t	10	10	10	10	10	10	10	10
h_t	5	5	5	5	5	5	5	5
s_t	100	100	100	100	100	100	100	100
π_t	2	2	2	2	2	2	2	2
d_t	9	12	9	25	9	20	20	25

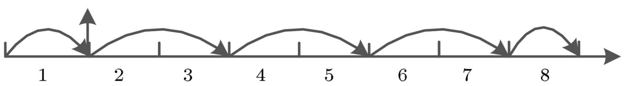

Figure 6. Optimal production plan using trade credit.

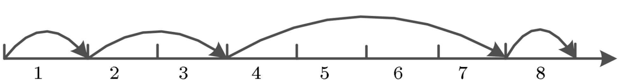

Figure 7. Optimal production plan without using trade credit.

7.2. Computational comparisons of the proposed algorithm with CPLEX

A numerical experiment is performed to test the deviation of the proposed algorithm from the optimal solutions on a large set of randomly generated problems. The randomized scheme of test problem generation is presented in Table 2, which is similar to the test beds in Aksen's work [11], except that we also list some levels for the initial capital, trade credit length, interest rate, and unit penalty cost in the comparison.

In Table 2, regarding the initial capital levels, $B_0 = s_1 + c_1 d_1$ guarantees that the manufacture has enough capital for the production of the first period; $B_0 = s_1 + c_1(d_1 + d_2)$ guarantees capital for the production of the first two periods; $B_0 = s_1 + c_1(d_1 + d_2 + d_3)$ guarantees capital for the production of the first three periods. There are totally 864 test problems for a certain planning horizon length T.

Because Model CFLSP-TC is a non-linear mixed-integer programming problem, CPLEX cannot obtain its optimal solution directly. All the possible values of z_t $(t = 1, 2, ..., T)$ are enumerated; then, they are solved by CPLEX and the maximum final capital increment is selected as the optimal solution. Therefore, the computational time of the optimal solutions by CPLEX is very time-consuming which also increases greatly as T increases. For $T = 8$, the total computation time of the 864 test problems by CPLEX is 1.6 hours; for $T = 10$, the total computation time is 6.7 hours; for $T = 12$, the total computation time is 29.5 hours. Moreover, an attempt is made to solve our problem by some meta-heuristics such as genetic algorithm and simulated annealing. However, it is difficult for those meta-heuristics to obtain a feasible solution, because the capital flow constraints are not fixed and related with many parameter's values, such as initial capital, selling price, and values of decision variables, such as production quantity, lost sales quantity, etc. Therefore, the proposed algorithm is only compared with CPLEX for $T = 8$, $T = 10$, and $T = 12$ for trade credit lengths lasting from 1 to 4. The experimental results are shown in Table 3.

Table 3 shows that the average deviation of our algorithm from the optimal solution for Model CFLSP-TC (capital flow-constrained lot-sizing problem with trade credit) is rather low. Of all the test problems, maximum deviation optimality percentage of our algorithm is 4.71%, and mean deviation optimality percentage of our algorithm is lower than 0.1%. Of all

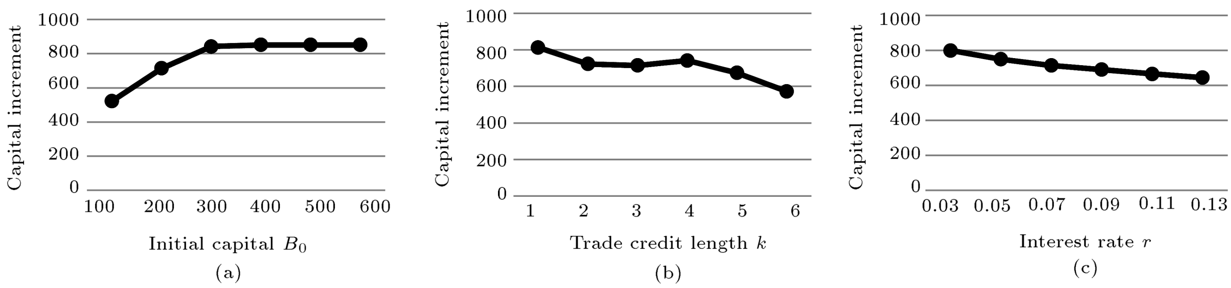

Figure 8. Optimal capital increment for different initial capital and trade credit settings.

Table 2. Randomized generation scheme for the test problem.

Parameter	Levels	Values
Trade credit length	4	1. $k = 1$ 2. $k = 2$ 3. $k = 3$ 4. $k = 4$
Interest rate	3	1. $r = 0.11$ 2. $r = 0.07$ 3. $r = 0.03$
Demand distributions	3	1. Exponential with $\mu = 150$ 2. Normal with $\mu = 150$, $\sigma = 40$ 3. Normal with $\mu = 150$, $\sigma = 10$
Unit production cost Unit holding cost	2	1. Both constant: $c_t = 13$ and $h_t = 1$ 2. Both seasonally varying
Selling price	2	1. Uniformly distributed in [15, 25] 2. Both seasonally varying
Initial capital	3	1. $B_0 = s_1 + c_1 d_1$ 2. $B_0 = s_2 + c_1(d_1 + d_2)$ 3. $B_0 = s_1 + c_1(d_1 + d_2 + d_3)$
Unit penalty cost	2	1. Constant: $\pi_t = 2$ 2. Constant: $\pi_t = 4$
Setup cost	1	Constant: $s_t = 1000$

the 2592 test cases, there are only 85 cases in which the proposed algorithm could not reach optimality. In general, the proposed algorithm can get 96.72% optimal cases. For Model CFLSP, it can be solved by CPLEX directly. The proposed algorithm is compared with CPLEX for different planning horizon lengths. The results are shown in Table 4.

Table 4 shows that, for Model CFLSP (capital flow-constrained lot-sizing problem without trade credit), the deviation optimality percentage of our algorithm is also very small. When the problem size is small, the branch and bound method in CPLEX can compute them very fast. However, when the problem size is large, CPLEX needs large memory to store matrices, substantially increasing the computation time. The proposed algorithm runs faster than CPLEX for large-sized problems of Model CFLSP.

8. Conclusions and outlook

Capital shortage is a common problem faced by many small- and medium-size companies. Therefore, it is necessary to consider capital flow constraints when making operational decisions. Trade credit is a widely used option for many companies to deal with capital shortage. This paper formulates a single-item lot-sizing model considering capital flow constraints and trade credit. Based on dynamic programming, an algorithm

Table 3. Performance of the proposed algorithm for Model CFLSP-TC.

T	k	Num. of cases	Num. of non-optimal cases	Average deviation	Max deviation
8	$k=1$	216	0	0%	0%
	$k=2$	216	6	0.06%	3.36%
	$k=3$	216	4	0.03%	4.20%
	$k=4$	216	3	0.02%	3.19%
10	$k=1$	216	8	0.04%	3.63%
	$k=2$	216	11	0.06%	4.71%
	$k=3$	216	13	0.04%	3.28%
	$k=4$	216	11	0.04%	2.81%
12	$k=1$	216	2	0.01%	0.28%
	$k=2$	216	9	0.05%	4.03%
	$k=3$	216	10	0.07%	4.34%
	$k=4$	216	8	0.02%	1.01%
Total		2592	85	0.03%	4.34%

Table 4. Performance of the proposed algorithm for Model CFLSP.

T	Num. of cases	Avg. DPH time (s)	Avg. CPLEX time (s)	Num. of non-optimal cases	Average deviation	Max deviation
12	216	0.62	0.14	4	0.01%	2.45%
24	216	2.68	0.22	10	0.01%	0.81%
36	216	6.34	0.48	9	0.02%	1.42%
60	216	11.92	58.24	8	0.01%	1.47%
72	216	19.99	117.14	5	0.01%	0.20%
Total	1080	8.166	35.24	36	0.01%	2.45%

with heuristic adjustment is proposed to solve the problem. Numerical studies show that capital flow and trade credit usage do affect the optimal production plan and final capital increment in the lot-sizing problem. The proposed algorithm can solve the problem in a shorter time, as compared with commercial solver CPLEX. Under certain conditions, it can obtain the optimal solution. Under other conditions, it has little deviation compared with optimal solutions. Mathematical properties and the proposed algorithm are also suitable for the problem with the given quantity of loan, equaling the increasing capital in some periods in our model.

Future research can be extended in two directions: First is to consider the stochastic demand or the multi-item lot-sizing problems with capital flow constraints; second is to include some other financial behaviours, such as inventory financing, factoring business, etc.

Acknowledgments

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