The NDEA–MOP Model in the Presence of Negative Data Using Fuzzy Method

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Abstract

In this study, the multi-objective programming (MOP) method was used to solve network DEA (NDEA) models with assumption that, negative data is considered for the proposed NDEA model which consists of semi-negative and semi-positive input and output. At first, two stage and then k stage production models were formulated with consideration of negative data. In the multi-objective programming, two separate objective functions including the divisional efficiencies and the overall efficiency of the organization are modeled. In comparison to conventional DEA with negative data, the advantage of the proposed NDEA models is consideration of intermediate processes and products, in order to calculate the organization's overall efficiency. However, in conventional DEA, sub-stages of the organizations are neglected. To measure the efficiencies of an organization regarding interactive internal process, two case studies were investigated by application of the NDEA-MOP method with negative data. Case study 1 is focused on units with two stages having semi-negative and semi-positive indexes. In case study 2, units with three stages are evaluated. These units also have semi-negative and semi-positive indexes. The overall efficiency of each unit is calculated using the proposed models. Fuzzy approach as a solution procedure is applied.

Keywords: Data envelopment analysis; network DEA; semi-positive data; semi-negative data; overall efficiency; Fuzzy method.

1. Introduction

Nowadays, performance assessment of industrial and economical units plays important role in achieving their managerial success and continuous progress. Data envelopment analysis (DEA) is a nonparametric method used to analyze and evaluate the performance of Decision MakingUnits (DMUs) which converts multiple inputs into multiple outputs and takes the qualitative and quantitative measures into account. In recent years, extensive application of DEA is observed in several contexts such as health care, education, manufacturing, retailing, banking, etc. In the conventional DEA model, two types of models namely 1) the aggregation and 2) separation approaches are applied to measure efficiencies. In the aggregation model, divisions are aggregated into a single company, the DMU is evaluated as a black box and the internal linking activities are neglected. Therefore, it is not possible to evaluate the performance of individual division. In the separation model, each division in a DMU is considered as a separate unit and the linking activities between divisions are completely ignored. Thus, efficiencies of the organization's linking processes via both mentioned methods cannot be evaluated [1]. The network DEA (NDEA) model was proposed by Lewis and Sexton [2] to overcome the weakness of the traditional DEA model. This model has a multi-stage structure which accounts for both divisional efficiencies and the overall efficiency in a unified framework. Also, it considers internal interaction within DMUs where the intermediate measures among the stages play crucial roles in evaluation of the efficiency. In recent years, the attention of a large number of researchers have been drawn to efficiency assessment in multistage production processes, where each DMU transforms some external inputs to final outputs by the intermediate products. Details of some researches in this field can be found in Despotis and Koronakos [3], Carayannis et al. [4], Jarosz et al. [5], and Gang et al. [6]. The first DEA model, CCR, was proposed by Charnes et al. [7] with assumption of constant-returns-to-scale. The evolutionary form of this model, named BCC [8], is proposed by extending to variable-returns-to-scales. *n* DMUs is considered (*j*=1,…….,n) under assessment. Each DMU consumes m inputs (*i*=1,…, *m*) and produces s outputs (*r*=1,…,s), denoted by ($x_{ij}, x_{2j}, \ldots, x_{mj}$) and ($y_{1j}, y_{2j}, \ldots, y_{sj}$) respectively. The efficiency of DMU_k can be calculated by the CCR and BCC models as Equations (1) and (2):

$$Max \quad E_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \qquad \text{(CCR)} \qquad (1)$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \;, \quad j = 1,2,\ldots,n$$

$$u_r, v_i \geq \varepsilon \quad ,r = 1,2,\ldots,s \;; \quad i = 1,2,\ldots,m$$

$$Max \quad E_k = \frac{\sum_{r=1}^{s} u_r y_{rk} - u_o}{\sum_{i=1}^{m} v_i x_{ik}} \qquad \text{(BCC)} \qquad (2)$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj} - u_0}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \;, \quad j = 1,2,\ldots,n$$

$u_r, v_i \geq \varepsilon \quad , r = 1,2,\dots,s \;\; ; \quad i = 1,2,\dots,m$

u_0 Unrestricted in sign

In Equations (1) and (2), E_k is the objective function which is maximized for every DMU_k individually; u_r and v_i are weights of the outputs and inputs respectively; X_{ik} and Y_{rk} are the *i-th* input and *r-th* output of DMU_k; and ε is a small positive value which indicates positive weights; u_0 is the intercept of the production function in the BCC model. Previous researches have documented different methods for solving network DEA. Cheng et al. [9] studied about deriving a common set of weights by multi-objective programming (MOP) model based on a compensatory data envelopment analysis (DEA) model, in order to rank all DMUs. In order to solve it, the MOP model is transformed into a single objective programming (SOP) using a fuzzy programming method. Thereafter, the SOP model is solved by the proposed approximation algorithm. Kao et al. [10] proposed the multi-objective programming (MOP) method in order to solve network DEA (NDEA). Two types of NDEA–MOP models, BCC–MOP and CCR–MOP, are assessed. Divisional and the overall efficiency of the organization are measured without neglecting the efficiencies of its subunits. Matin and Azizi [11] measured performance of production systems by a new unified generalized network DEA model when interrelationships between individual sub-processes are considered. General network DEA model is evaluated by some illustrative numerical examples. Wang et al [12] constructed two-stage DEA model and then use a fuzzy multi-objective for evaluating the performance of US bank holding companies (BHCs). This paper analyzed the relationship between BHCs performance and their intellectual capital (IC).Despotis and Koronakos [13] assess efficiency of a two-stage network using a novel DEA approach. In the proposed method, unique and unbiased efficiency scores for the individual stages are calculated. Afterwards, the aggregation method a posteriori is applied in order to compose stages to obtain the efficiency of the overall system. Halkos et al. [14] reviewed the classification of two-stage DEA models, as well as their mathematical formulations, and main applications. The simple case of these models such as the two-stage models and general models such as the network DEA models were analyzed. These models are categorized into four models named independent, connected, relational and game theoretic two-stage DEA models. Lee and Li [15] studied Fuzzy multiple objective programming and compromise programming with Pareto optimum. In recent times, evolutionary algorithms have become a widely used methodology in MOP. The main aim of this study was to solve network DEA by applying the multi objective programming (MOP) method. At first, a two stage production system is assumed in order to convert some input products in the first stage and use these outputs as inputs of the second stage for producing final outputs. In this paper at first, negative data is considered for the proposed NDEA model which consists of semi-negative and semi-positive input and output. Thereafter, the k stage production process with consideration of negative data is formulated by MOP. MOP is concerned with finding the solutions in which a set of objective functions are simultaneously optimized, meaning that it is not possible to improve any objective without degrading some others. Many practical applications such as pattern classification can be posed as MOP problems.

According to the CCR and BCC models, objective functions including the overall and divisional efficiencies within a DMU are optimized. Two case studies were evaluated to indicate the benefits of NDEA-MOP. In order to calculate efficiency of units in the presence of semi-negative and semi-positive indexes, various models are presented. Also, different views are suggested in order to calculate the performance two and multi -stage units. The difference between this article and other articles is that the present study deals with two and multi-stage units having semi-negative and semi-positive indexes. In other word, this paper is focused on units which are combination of these two states. The remainder of this paper is structured as follows: In section 2 method of Izadikhah and Farzipoor Saen [16] is discussed. In section 3 two-stage DEA model is discussed. In section 4, a model for calculation of overall efficiency of units with two stages in the presence of semi-positive and semi-negative data is proposed. In section 5, units are extended to k-stages and a method is stated for calculating the overall efficiency of units. Fuzzy approach [17] as a solution procedure is proposed in two sections. In section 6, two case studies are implemented to examine the network DEA models with multi-objective programming. In section 7 Results and discussion is given and the conclusion section is given at the end of the paper.

2. Non-radial efficiency in two stage network DEA with negative data

Consider n units under assessment of DMU_j $(j = 1, \dots, n)$ with two stage network structure as shown in Fig. 1. Stage 1 consume X_j as input and produce Z_j as output and stage 2 consume Z_j as input and produce y_j as output. Izadikhah and Farzipoor Saen [16] using the idea of Chen and Zhu [18] presented non-radial model to calculate efficiency of each stages and the overall efficiency of a unit in the presence of negative data. This model for the evaluation DMU_p unit is as follows.

$$Max\ w_1\left(\frac{1}{m}\sum_{i=1}^{m}\frac{\theta_i}{\bar{\theta}_p}+\frac{1}{D}\sum_{d=1}^{D}\frac{\varphi_d}{\bar{\varphi}_p}\right)+w_2\left(\frac{1}{D}\sum_{d=1}^{D}\frac{\alpha_d}{\bar{\alpha}_p}+\frac{1}{s}\sum_{r=1}^{s}\frac{B_r}{\bar{B}_p}\right) \tag{3}$$

s.t. (Stage 1)

$$\sum_{j=1}^{n}\lambda_j x_{ij} \le x_{ip} - \theta_i |x_{ip}| \quad ; \quad i = 1, \dots, m$$

$$\sum_{j=1}^{n}\lambda_j z_{dj} \ge \tilde{z}_{dp} + \varphi_d |\tilde{z}_{dp}| \quad ; \quad d = 1, \dots, D$$

$$\sum_{j=1}^{n}\lambda_j = 1$$

$$\lambda_j \ge 0 \quad ; \quad j = 1, \dots, n$$

(Stage 2)

$$\sum_{j=1}^{n} \mu_j d_{dj} \leq \tilde{z}_{dp} - \alpha_d |\tilde{z}_{dp}| \quad ; \quad d = 1, \dots, D$$

$$\sum_{j=1}^{n} \mu_j y_{rj} \geq y_{rp} + B_r |y_{rp}| \quad ; \quad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \mu_j = 1$$

$$\mu_j \geq 0 \quad ; \quad j = 1, \dots, n$$

Where

$$\bar{\theta}_p = Max_i \left\{ \frac{x_{ip} - x_{iL}}{|x_{ip}|} \quad ; \quad x_{ip} \neq 0 \ , i = 1, \dots, m \right\}$$

$$\bar{\alpha}_p = Max_d \left\{ \frac{z_{dp} - z_{dL'}}{|z_{dp}|} \quad ; \quad z_{dp} \neq 0 \ , d = 1, \dots, D \right\}$$

$$\bar{\varphi}_p = Max_d \left\{ \frac{z_{dL} - z_{dp}}{|z_{dp}|} \quad ; \quad z_{dp} \neq 0 \ , d = 1, \dots, D \right\}$$

$$\bar{B}_p = Max_r \left\{ \frac{y_{rL'} - y_{rp}}{|y_{rp}|} \quad ; \quad y_{rp} \neq 0 \ , r = 1, \dots, s \right\}$$

And

$$x_{iL} = Min_j\{x_{ij}\} \quad ; \quad i = 1, \dots, m \qquad , \qquad z_{dL'} = Min_j\{z_{dj}\} \quad ; \quad d = 1, \dots, D$$

$$z_{dL} = Max_j\{z_{dj}\} \quad ; \quad d = 1, \dots, D \qquad , \qquad y_{rL'} = Max_j\{y_{rj}\} \quad ; \quad r = 1, \dots, s$$

In the presented model, $w_1 + w_2 = 1$ and overall efficiency of DMU_p is calculated as follows.

$$\theta^* = 1 - \left(\frac{w_1}{2} \left[\frac{1}{m} \sum_{i=1}^{m} \frac{\theta_i^*}{\bar{\theta}_p} + \frac{1}{D} \sum_{d=1}^{D} \frac{\varphi_d^*}{\bar{\varphi}_p} \right] + \frac{w_2}{2} \left[\frac{1}{D} \sum_{d=1}^{D} \frac{\alpha_d^*}{\bar{\alpha}_p} + \frac{1}{s} \sum_{r=1}^{s} \frac{B_r^*}{\bar{B}_P} \right] \right)$$

$$\theta^* = 1 - [w_1(1 - \theta_1^*) + w_2(1 - \theta_2^*)]$$

Where θ_1^* and θ_2^* are efficiency of stage 1 and stage 2 respectively.

Note that, in model 3, $\tilde{z}$ is unknown decision variable which is calculated from solving the model. In this paper, a method is presented which consider general state of a two-stage model (i.e, stage 2 has external input) and k-stage model for non-negative indexes.

3. Two-stage DEA model

Consider n DMUs under assessment. Fig. 1 shows the network structure of each DMU where, X_j expresses input of first stage and Z_j represents output of first stage which is considered as input of second stage and Y_j takes into account as output of second stage.

Where$X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$, $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$, $and Z_j = (z_{1j}, z_{2j}, \dots, z_{tj})$

(Figure 1 near here)

4. Two-stage model NDEA-CCR in the presence of semi-positive and semi-negative data

It was assumed that I is input series which is positive in all DMUs and $\bar{I}$ is input series, that is positive in some DMUs and negative in other DMUs, so that $m = |I| + |\bar{I}|$ and $I \cap \bar{I} = \emptyset$ Furthermore, T is z_{tj} series, that is positive in all decision making units and $\bar{T}$ is z_{tj} series which is positive in some units and negative in others as $t = |T| + |\bar{T}|$ and $T \cap \bar{T} = \emptyset$. In addition, R is output series which is positive in all DMUs, $\bar{R}$ is output series, that is positive in some DMUs and negative in others such that $s = |R| + |\bar{R}|$ and $R \cap \bar{R} = \emptyset$. The efficiency of each DMU can be calculated as follows:

$$E_j^{(1)} = \frac{\sum_{t\epsilon T} \mu_t z_{tj} + \sum_{t\epsilon \bar{T}} \bar{\mu}_t z_{tj}^{(1)} - \sum_{t\epsilon \bar{T}} \hat{\mu}_t z_{tj}^{(2)}}{\sum_{i\epsilon I} v_i x_{ij} + \sum_{i\epsilon \bar{I}} \bar{v}_i x_{ij}^{(1)} - \sum_{i\epsilon \bar{I}} \hat{v}_i x_{ij}^{(2)}} \tag{4}$$

$$E_j^{(2)} = \frac{\sum_{r\in R} u_r y_{rj} + \sum_{r\epsilon \bar{R}} \bar{u}_r y_{rj}^{(1)} - \sum_{r\epsilon \bar{R}} \hat{u}_r y_{rj}^{(2)}}{\sum_{t\epsilon T} \mu_t z_{tj} + \sum_{t\epsilon \bar{T}} \bar{\mu}_t z_{tj}^{(1)} - \sum_{t\epsilon \bar{T}} \hat{\mu}_t z_{tj}^{(2)}} \tag{5}$$

Where $\mu_t, \bar{\mu}_t, \hat{\mu}_t, v_i, \bar{v}_i, \hat{v}_i, u_r, \bar{u}_r, \hat{u}_r$ are weights of inputs and outputs and are all non-negative. In addition, for every $t\epsilon\bar{T}$ we have $z_{tj} = z_{tj}^{(1)} - z_{tj}^{(2)}$ where $z_{tj}^{(1)} \geq 0\,, z_{tj}^{(2)} \geq 0.$

Furthermore:

$$z_{tj}^{(1)} = \begin{cases} z_{tj} & z_{tj} \geq 0 \\ 0 & z_{tj} < 0 \end{cases} \quad ; \quad z_{tj}^{(2)} = \begin{cases} 0 & z_{tj} \geq 0 \\ -z_{tj} & z_{tj} < 0 \end{cases} \qquad (6)$$

For every $r\epsilon\bar{R}$ we have $y_{rj} = y_{rj}^{(1)} - y_{rj}^{(2)}$ where $y_{rj}^{(1)} \geq 0, y_{rj}^{(2)} \geq 0$ and

$$y_{rj}^{(1)} = \begin{cases} y_{rj} & y_{rj} \geq 0 \\ 0 & y_{rj} < 0 \end{cases} \quad ; \quad y_{rj}^{(2)} = \begin{cases} 0 & y_{rj} \geq 0 \\ -y_{rj} & y_{rj} < 0 \end{cases} \qquad (7)$$

In addition, for $i\epsilon\bar{I}$ we have $x_{ij} = x_{ij}^{(1)} - x_{ij}^{(2)}$ where $x_{ij}^{(1)} \geq 0, x_{ij}^{(2)} \geq 0$

$$x_{ij}^{(1)} = \begin{cases} y_{rj} & x_{ij} \geq 0 \\ 0 & x_{ij} < 0 \end{cases} \quad ; \quad x_{ij}^{(2)} = \begin{cases} 0 & x_{ij} \geq 0 \\ -x_{ij} & x_{ij} < 0 \end{cases} \qquad (8)$$

The following model is used to calculate efficiency of DMU_o

$$Max\ E_o^1 \qquad (9)$$

$$Max\ E_o^2$$

$$s.t.\ E_j^{(k)} \leq 1 \quad ; \quad k = 1,2 \quad ; \quad j = 1, \dots, n$$

All variables are non-negative. There are different methods to solve the two objective model 9. In this study Fuzzy method [10] is applied. General algorithm of this method is presented in section 5. By putting k=2 in the presented algorithm, model 9 can be solved.

5. K-stage model NDEA-CCR in the presence of semi-positive and semi-negative data

Consider *n* decision making unit (DMUs) under assessment in which each has a network structure as shown in Fig. 2. Each DMUs consists of k stage. Let m_k, r_k be the number of inputs and outputs of the *k-th* stage. Assuming that $r_k = \hat{r}_k + \overline{r_k}$, $m_k = \widehat{m}_k \cup \overline{m}_k$ where $\hat{r}_k$ represents the number of outputs, all DMUs are positive in the *k-th* stage and $\bar{r}_k$ indicates the number of outputs as some DMUs are positive while others are negative in the *k-th* stage. If the *k*,*h* stages are linked to each other, they are denoted by (*k*, *h*). The intermediate product, $z_j^{(k,h)}$, which is an output of the *k-th* stage plays input role for the *h-th* stage and its number is equal to L_k. Similarly, it can be written as $L_k = \hat{L}_k + \bar{L}_k$ where, $\hat{L}_k$ shows the number of intermediate product which is positive for the *k-th* stage in all DMU_j and $\bar{L}_k$ is the number of intermediate product which is positive for *k-th* stage in some DMUs and negative in others.

The multi-objective NDEA-CCR model in the presence of negative data is defined as follows:

(Figure 2 near here)

$Max\ E_o^1$

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$$Max\ E_o^K = \frac{E_o^{K,Y}}{E_o^{K,X}} \tag{10}$$

$k=1,\dots, K$

$E_j^k \leq 1 \quad ; \quad k=1,\dots,K \ ; \ j=1,\dots,n$

Where:

$$E_j^{k,Y}=\sum_{r=1}^{\hat{r}_k}u_r^k y_{rj}^k+\sum_{r=1}^{\bar{r}_k}\bar{u}_r^k\,y_{rj}^{(1)k}-\sum_{r=1}^{\bar{r}_k}\hat{u}_r^k y_{rj}^{(2)k}+\sum_{\forall(k,h)}\sum_{p=1}^{t(k,h)}\mu_h^k\,z_{pj}^{(k,h)}$$

$$+\sum_{\forall(k,h)}\sum_{p=1}^{\bar{t}(k,h)}\bar{\mu}_h^k\,z_{pj}^{(1)(k,h)}-\sum_{\forall(k,h)}\sum_{p=1}^{\bar{t}(k,h)}\hat{\mu}_h^k\,z_{pj}^{(2)(k,h)} \quad (11)$$

$$E_j^{k,X}=\sum_{i=1}^{\widehat{m}_k}v_i^k\,x_{ij}^k+\sum_{i=1}^{\overline{m}_k}\bar{v}_i^k x_{ij}^{(1)k}-\sum_{i=1}^{\overline{m}_k}\hat{v}_i^k x_{ij}^{(2)k}+\sum_{\forall(g,k)}\sum_{q=1}^{t(g,k)}w_g^t\,z_{qj}^{(g,k)}$$

$$+\sum_{\forall(g,k)}\sum_{q=1}^{\bar{t}(g,k)}\overline{w}_g^k\,z_{qj}^{(1)(g,k)}-\sum_{\forall(g,k)}\sum_{q=1}^{\bar{t}(g,k)}\widehat{w}_g^k z_{qj}^{(2)(g,k)} \quad (12)$$

Where $y_{rj}^k=y_{rj}^{(1)k}-y_{rj}^{(2)k};\ \forall\, r\in\{1,\dots,\bar{r}_k\}$; $y_{rj}^{(1)k}\geq 0$; $y_{rj}^{(2)k}\geq 0$ where

$$y_{rj}^{(1)k}=\begin{cases}y_{rj}^k & y_{rj}^k\geq 0\\ 0 & y_{rj}^k<0\end{cases}\quad;\quad y_{rj}^{(2)k}=\begin{cases}0 & y_{rj}^k\geq 0\\ -y_{rj}^k & y_{rj}^k<0\end{cases} \quad (13)$$

And $x_{ij}^k=x_{ij}^{(1)k}-x_{ij}^{(2)k};\forall\, i\in\{1,\dots,\overline{m}_k\}$; $x_{ij}^{(1)k}\geq 0$; $x_{ij}^{(2)k}\geq 0$ where

$$x_{ij}^{(1)k}=\begin{cases}x_{ij}^k & x_{ij}^k\geq 0\\ 0 & x_{ij}^k<0\end{cases}\quad;\quad x_{ij}^{(2)k}=\begin{cases}0 & x_{ij}^k\geq 0\\ -x_{ij}^k & x_{ij}^k<0\end{cases} \quad (14)$$

And $z_{qj}^{(g,k)}=z_{qj}^{(1)(g,k)}-z_{qj}^{(2)(g,k)};\forall\, q\in\{1,\dots,\bar{t}\}$; $z_{qj}^{(1)(g,k)}\geq 0$; $z_{qj}^{(2)(g,k)}\geq 0$ where

$$z_{qj}^{(1)(g,k)}=\begin{cases}z_{qj}^{(g,k)} & z_{qj}^{(g,k)}\geq 0\\ 0 & z_{qj}^{(g,k)}<0\end{cases}\quad;\quad z_{qj}^{(2)(g,k)}=\begin{cases}0 & z_{qj}^{(g,k)}\geq 0\\ -z_{qj}^{(g,k)} & z_{qj}^{(g,k)}<0\end{cases} \quad (15)$$

To solve the above model (Model 10) through fuzzy method [10], the following operation was performed:

Step 1: The ideal answer for each of the objective function is obtained.

$$E_o^{k*}=maxE_o^k \quad (16)$$

$$s.t.\ \ E_j^k\leq 1\,, j=1,\dots,n$$

All weights are non-negative.

The optimal answer of model (16) is calculated for $k = 1, \dots, K$ and its optimal value is named E_o^{k*}.

Step 2. The anti-ideal answer for each of the objective function is obtained.

$$E_o^{k-} = min\ E_o^k \tag{17}$$

$$s.t.\ \ E_j^k \geq 1\ , j = 1, \dots, n$$

All weights are non-negative.

The optimal answer of model (17) is calculated for $k = 1, \dots, K$ and its optimal value is named E_o^{k-}.

Step 3. The membership function for each of the objective function for ideal and anti-ideal answers is defined as follows:

$$\mu(E_o^k) = \frac{E_o^k - E_o^{k-}}{E_o^{k*} - E_o^{k-}} \tag{18}$$

Note that since, $E_o^{(k)*}$ is ideal efficiency of DMU_o. Therefore, $0 \leq E_o^{(k)*} \leq 1$.

$E_o^{(k)-}$ (The distance to the border of inefficiency) is anti-ideal efficiency of DMU_o. So that, $E_o^{(k)-} \geq 1$. Since $E_o^{(k)*} \leq E_o^{(k)} \leq E_o^{(k)-}$, it can be easily proved for $E_o^{(k)*} - E_o^{(k)-} \neq 0$ and always$0 \leq \mu E_o^{(k)} \leq 1$. For the case of $E_o^{(k)*} - E_o^{(k)-} = 0$, a unit is added to the denominator of $\mu E_o^{(k)}$ in order to overcome problem of becoming zero denominator.

Step 4. The following model is solved

$$max\ \lambda = min\ \{\mu(E_o^k)\} \tag{19}$$

$$s.t.\ \ \lambda \leq \mu(E_o^k) \qquad\qquad k = 1, \dots, K$$

$$\mu(E_o^k) = \frac{E_o^k - E_o^{k-}}{E_o^{k*} - E_o^{k-}} \qquad\qquad k = 1, \dots, K$$

All original constraints.

Step 5. $E_o = \sum_{k=1}^{k} w_k E_o^k$ is considered overall efficiency of DMU_o unit, where w_k is weight of the *k-th* stage.

$\sum_{k=1}^{k} w_k = 1$. Note that Since, for each k, $0 \leq E_o^{(k)} \leq 1$. Therefore, efficiency of is always in (0, 1]. If$E_o = 1$, then we say that DMU_o is efficient and for each $i \neq j$, if $E_j \geq E_i$, then i-th unit is more efficient than j-th unit.

6. Case studies

In this section, two numerical examples are investigated. In the first case, an example with two stages is evaluated. This data includes semi-positive and semi-negative indexes. In the second case an example with k stages is studied that makes a CCR-Mop model. The fuzzy method stated in section 3 is used for finding the optimum answer.

6.1. Case 1: Example of two stages NDEA-CCR with semi-positive and semi-negative data

In this section, units consisting of two stages are considered. According to Fig. 3, X_j is the input of stage 1 and its outputs are Z_j and $\bar{Z}_j$ which are positive outputs and semi-positive and semi-negative outputs, respectively. These outputs serve as inputs of stage 2. Also, $\bar{x}_j$ is external input of stage 2. The outputs of stage 2 are Y_j and $\bar{Y}_j$ which are positive outputs and semi-positive and semi-negative outputs respectively. These data is presented in Table 1.

(Figure 3 near here)

(Table 1 near here)

Regarding Table 1 stage 1 includes three positive inputs and stage 2 has two positive outputs y_{1j}, y_{2j} and two semi-positive and semi-negative outputs $\bar{y}_{1j}, \bar{y}_{2j}$. Output of stage 1 (input of stage 2) includes two positive outputs z_{1j}, z_{2j} and two semi-positive and semi-negative outputs $\bar{z}_{1j}, \bar{z}_{2j}$.

Model 8 is used to calculate efficiency of DMU$_o$ ($o\epsilon\{1, \dots, 10\}$). In this model E_j^1, E_j^2 are calculated as follows.

$$E_j^1 = \frac{\mu z_j + \bar{\mu}_1 \bar{z}_{1j}^{(1)} - \hat{\mu}_1 \bar{z}_{1j}^{(2)} + \bar{\mu}_2 \bar{z}_{2j}^{(1)} - \hat{\mu}_2 \bar{z}_{2j}^{(2)}}{\upsilon_1 x_{1j} + \upsilon_2 x_{2j}} \tag{20}$$

$$E_j^2 = \frac{u_1 y_{1j} + u_2 y_{2j} + \bar{u}_1 \bar{y}_{1j}^{(1)} - \hat{u}_1 \bar{y}_{1j}^{(2)} + \bar{u}_2 \bar{y}_{2j}^{(1)} - \hat{u}_2 \bar{y}_{2j}^{(2)}}{\mu z_j + \bar{\mu}_1 \bar{z}_{1j}^{(1)} - \hat{\mu}_1 \bar{z}_{1j}^{(2)} + \bar{\mu}_2 \bar{z}_{2j}^{(1)} - \hat{\mu}_2 \bar{z}_{2j}^{(2)} + \bar{\upsilon} \bar{x}_j}$$

Where model 8 for evaluation of DMU$_1$ is as follows:

$$Max\ E_o^1 \tag{21}$$

$$Max\ E_o^2$$

$$s.t.\ E_j^{(k)} \leq 1 \quad ; \quad k = 1,2 \quad ; \quad j = 1, \dots, 10$$

In order to solve the above two objective model, the algorithm presented in Section 5 is used.

Step 1: solve these two models.

$$E_1^{(1)*} = Max E_1^{(1)}$$

$$s.t.\ \ E_j^{(k)} \leq 1 \ \ ; \quad k = 1,2 \quad ; \ \ j = 1, \ldots, 10 \tag{22}$$

$$E_1^{(2)*} = Max E_1^{(2)}$$

$$s.t.\ \ E_j^{(k)} \leq 1 \ \ ; \quad k = 1,2 \quad ; \ \ j = 1, \ldots, 10 \tag{23}$$

All variables are assumed non-negative in both models. Assume that $E_1^{(1)*}$, $E_1^{(2)*}$ are ideal efficiency of stage 1 and stage 2 related to DMU_o . The results of models 22 and 23 are shown in Table 2. (Note the first row of Table 2).

Step 2: Solve the following models.

$$E_1^{(1)-} = Min\ E_1^{(1)} \tag{24}$$

$$s.t.\ \ E_j^{(1)} \geq 1 \ \ ; \quad k = 1,2 \quad ; \ \ j = 1, \ldots, 10$$

$$E_1^{(2)-} = Min\ E_1^{(2)} \tag{25}$$

$$s.t.\ \ E_j^{(1)} \geq 1 \ \ ; \quad k = 1,2 \quad ; \ \ j = 1, \ldots, 10$$

All variables in both models are assumed non-negative. Assume that $E_1^{(1)-}$, $E_1^{(2)-}$ are anti-ideal efficiency of stage 1 and stage 2 related to DMU_o that its results are given in first row of Table 2.

Step 3: The membership function for each of the steps 1 and 2 using an ideal and anti-ideal efficiency value are defined as follows.

$$\mu\left(E_1^{(1)}\right) = \frac{E_1^{(1)} - E_1^{(1)-}}{E_1^{(1)*} - E_1^{(1)-}} \tag{26}$$

$$\mu\left(E_1^{(2)}\right) = \frac{E_1^{(2)} - E_1^{(2)-}}{E_1^{(2)*} - E_1^{(2)-}}$$

Where $0 \leq \mu\left(E_1^{(1)}\right) \leq 1$, $0 \leq \mu\left(E_1^{(2)}\right) \leq 1$

Step 4: Solve the following model

$$Max\ \lambda \tag{27}$$

$$s.t.\ \ \lambda \leq \mu\left(E_1^{(1)}\right)$$

$$\lambda \leq \mu\left(E_1^{(2)}\right)$$

$$\mu\left(E_1^{(k)}\right) = \frac{E_1^{(k)} - E_1^{(k)-}}{E_1^{(k)*} - E_1^{(k)-}} \quad ; \quad k = 1,2$$

$$E_1^{(1)} = \frac{\mu z_1 + \bar{\mu}_1 \bar{z}_{11}^{(1)} - \hat{\mu}_1 \bar{z}_{11}^{(2)} + \bar{\mu}_2 \bar{z}_{21}^{(1)} - \hat{\mu}_2 \bar{z}_{21}^{(2)}}{v_1 x_{11} + v_2 x_{21}}$$

$$E_1^{(2)} = \frac{u_1 y_{11} + u_2 y_{21} + \bar{u}_1 \bar{y}_{11}^{(1)} - \hat{u}_1 \bar{y}_{11}^{(2)} + \bar{u}_2 \bar{y}_{21}^{(1)} - \hat{u}_2 \bar{y}_{21}^{(2)}}{\mu z_1 + \bar{\mu}_1 \bar{z}_{11}^{(1)} - \hat{\mu}_1 \bar{z}_{11}^{(2)} + \bar{\mu}_2 \bar{z}_{21}^{(1)} - \hat{\mu}_2 \bar{z}_{21}^{(2)} + \bar{v} \bar{x}_j}$$

$$E_j^{(1)} \leq 1 \quad ; \quad k = 1,2 \quad ; \quad j = 1, \ldots, 10$$

All variables are non-negative.

Step 5: Assume that $E_1^{(1)}, E_j^{(2)}$ are results of optimum answer of model 27. These results are given in Table 3. In order to calculate efficiency of DMU_1, correlation of

$E_1 = \frac{1}{2} E_1^{(1)} + \frac{1}{2} E_1^{(2)}$ is used. Not that reults are given in Table 3 with consideration of

$w_1 = w_2 = \frac{1}{2}$

(Table 2 near here)

Table 3 near here)

(According to Table 3, overall efficiency of none of units is not unit. So that DMU_3 and DMU_9 have higher efficiency than other units.

6.2. Case 2: Extended electric power companies

This case is related to extended electric power companies with link 3 [19]. In case 1 [19], inputs and outputs are not positive but this case is an extension of case 1 with the exception that two semi-positive and semi-negative outputs are added to stages 2 and 3 (To better understand refer to Fig. 4). Data of this case is given in Table 4, whereX_j, $\bar{X}_j$, $\hat{X}_j$ are inputs of stages 1, 2, and 3 respectively. Y_j^2, $\bar{Y}_j^2$ are positive outputs and semi-positive and semi-negative outputs of stage 2 respectively. Y_j^3, $\bar{Y}_j^3$ are positive outputs and semi-positive and semi-negative outputs of stage 3 respectively. Furthermore, Z_j^1, Z_j^2 are links of stage 1 to stage 2 and stage 2 to stage 3 respectively in which Z_j^1, Z_j^2 have a non-negative value.

(Figure 4 near here)

(Table 4 near here)

$$E_j^{(1)} = \frac{wz_j^1}{vx_j}$$

$$E_j^{(2)} = \frac{Az_j^2 + uy_j^2 + N\bar{y}_j^{2(1)} - D\bar{y}_j^{2(2)}}{wz_j^1 + M\bar{x}_j}$$

$$E_j^{(3)} = \frac{Py_j^3 + Q\bar{y}_j^{(3)1} - S\bar{y}_j^{(3)2}}{Az_j^2 + B\hat{x}_j}$$

At first, models (16) and (17) are used for data of Table 4. Ideal and ant-ideal answers are obtained for each unit and its results are presented in Table 5. Results of model 19 for case 2 are presented in Table 6.

(Table 5 near here)

(Table 6 near here)

(Table 7 near here)

The fuzzy method of Kao and his co-authors is used and the membership function value is calculated for each unit of assessment. Also, the overall efficiency of each DMU_j is calculated via $E_j = \sum_{k=1}^{3} w_k E_j^k$ with assumption of $w_1 = w_2 = w_3 = \frac{1}{3}$ and its results are presented in Table 7**.** In this case, the weight of the entire stages is assumed to be identical. As shown in Table 7, DMU_3 is efficient and the other DMUs are inefficient.

8. Conclusion

Measuring efficiency of units under assessment is one of the valuable goals of data envelopment analysis. Since all input and output indexes cannot be positive, extended models are proposed by which the efficiency of units can be calculated. Also, some methods are presented for measuring two and multi stage network DEA structure which are applied for positive data. The difference between this article and other articles which are focused on calculating units with two and multi-stages is that a model is proposed in this paper which is able to calculate the efficiency of network DEA in the presence of semi-positive and semi-negative indexes. Two case studies for the presented work are stated. In the first example, units with two stages having semi-positive and semi-negative indexes are considered, in the second example, units with 3 stages are selected. This ensures that some outputs of stage 2 and stage 3 are semi-negative and semi-positive. Thereafter, the proposed method is applied to calculate the overall efficiency of units under assessment. To calculate the overall efficiency of units with more than two stages, solving MOP model is necessary. Fuzzy programming method as a solution procedure is proposed. The question that arises is whether it can be offered as a method to calculate the overall efficiency of multi stage NDEA, so that solving MOP problem is not needed.

Izadikhah and Farzipoor Saen [16] presented a model in order to calculate efficiency of two stage network in the presence of negative data. Their two stage network structure consists of input, output and intermediate indexes. The strong point of the proposed method is to calculate efficiency of k-stage network in the presence of negative data. So that networks have external input for k-stage in addition to consumed input of stage 1.

In future research, multi-objective programming (MOP) can be applied in the presence of interval data which are not crisp. Such problems can be solved using fuzzy techniques. Furthermore, this research can be expanded in solving MOP with consideration of interval negative data.

Nomenclature

e_j	Efficiency of jth stage
$e_j^{(1)}, e_j^{(2)}$	Efficiency of first stage, Efficiency of second stage
$E_j^{k,X}$	Efficiency of *k-th* stage in input related to DMU_j
E_k	Efficiency of *k-th* stage
E_o^K	Efficiency of *k-th* stage related to DMU_o
E_o^{k*}	The ideal efficiency of *k-th* stage related to DMU_o
E_o^{k-}	The anti-ideal efficiency of kth stage related to DMU_o
$E_o^{K,X}$	Efficiency of *k-th* stage in input related to DMU_o
$E_o^{K,Y}$	Efficiency of *k-th* stage in output related to DMU_o
I	input series which is positive in all DMUs
$\bar{I}$	input series, that is positive in some DMUs and negative in other DMUs
L_k	the number of intermediate product
$\bar{L}_k$	is the number of intermediate product which is positive for *k-th* stage in some DMUs and negative in others

$\hat{L}_k$	shows the number of intermediate product which is positive for the *k-th* stage in all DMU_j
$\bar{r}_k$	the number of outputs as some DMUs are positive while others are negative in the *k-th* stage
$\hat{r}_k$	the number of outputs that all DMUs are positive in the *k-th*
R	output series which is positive in all DMUs
$\bar{R}$	output series, that is positive in some DMUs and negative in others
T	z_{tj} series, that is positive in all DMUs
$\bar{T}$	z_{tj} series which is positive in some units and negative in others
u_o	Constant return to scale
$u_r, \bar{u}_r, \hat{u}_r$	weights of outputs which are non-negative
u_r^k	Output weight of the *k-th* stage
$\bar{u}_r^k$	Output weight of the *k-th* stage for semi-positive and semi-negative
$v_i, \bar{v}_i, \hat{v}_i$	weights of inputs which are non-negative
$\bar{v}_i^k, \hat{v}_i^k, v_i^k$	input weights of *k-th* stage
$\overline{w}_g^k, \widehat{w}_g^k$	Intermediate weight of *k-th* stage for semi-positive and semi-negative input
w_g^t	Intermediate weight of *t-th* stage which is positive
w_k	weight of the *k-th* stage
x_{ij}	Input of *j-th* stage
$x_{ij}^{(1)}, x_{ij}^{(2)}$	Input of *j-th* stage when $\bar{I}$ belong to negative and positive set
x_{ij}^k	Input of *k-th* stage related to DMU_j
$x_{ij}^{(1)k}, x_{ij}^{(2)k}$	Input of *j-th* stage for semi-positive and semi-negative
x_{ik}	Input of *k-th* stage
x_{io}	Input of stage related to DMU_o
$x_{io}^{(1)}, x_{io}^{(2)}$	Input of stage related to DMU_o when $\bar{I}$ belong to negative and positive set
X_j	input of stage 1 which is positive output
$\bar{X}_j$	input of stage 1 which is semi-positive and semi-negative input
y_{rj}	Output of jth stage
$y_{rj}^{(1)}, y_{rj}^{(2)}$	Output of *j-th* stage when $\bar{R}$ belong to negative and positive set
$y_{rj}^{(1)k}, y_{rj}^{(1)k}$	Output of the *k-th* stage related to DMU_j for semi-positive and semi-negative
y_{rj}^k	Output of the *k-th* stage related to DMU_j
y_{rk}	Output of kth stage
y_{ro}	Output of stage related to DMU_o
$y_{ro}^{(1)}, y_{ro}^{(2)}$	Output of stage related to DMU_o when $\bar{R}$ belong to negative and positive set
Y_j	outputs of stage 2 which is positive output
$\bar{Y}_j$	outputs of stage 2 which is semi-positive and semi-negative output
Z_j	Ouputs of of stage 1 which is positive output
$\bar{Z}_j$	Ouputs of of stage 1 which is semi-positive and semi-negative output
$z_{pj}^{(k,h)}$	intermediate product related to kth and hth stage for DMU_j
$z_{pj}^{(1)(k,h)}, z_{pj}^{(2)(k,h)}$	intermediate variable of *k-th* and *h-th* stage related to DMU_j
$z_{qj}^{(g,k)}$	Intermediate stage related to DMU_j from k and g loop which is positive
$z_{qj}^{(1)(g,k)}, z_{qj}^{(2)(g,k)}$	Intermediate stage related to DMU_j from k and g loop which is from semi-positive and semi-negative data set
z_{tj}	intermediate variable of jth stage
$z_{tj}^{(1)}, z_{tj}^{(2)}$	intermediate variable of jth stage when $\bar{T}$ belongs to negative and positive set
$\mu(E_o^k)$	The membership function for each of the objective function for ideal and anti-ideal answers related to DMU_o
$\bar{\mu}_h^k$	intermediate weights *k-th* stage
$\hat{\mu}_h^k$	intermediate weight of *k-th* stage
$\mu_t, \bar{\mu}_t, \hat{\mu}_t$	weights of intermediate stage which are non-negative
ε	Positive and small variable

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Figure captions

Fig 1. Two-stage network model

Fig. 2. K-stage network model

Fig. 3. Two-stage network model with external input

Fig. 4. Three-stage network model

Table captions

Table 1. The sample data of case 1

Table. 2. Ideal and anti-ideal efficiency of subunit of case 1

Table3. Overall efficiency units of case 1

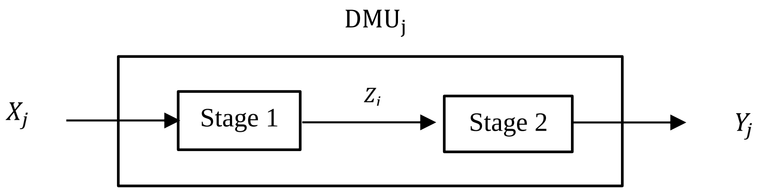
Table3. Overall efficiency units of case 1 (continue)

Table 4. The sample data of case 2

Table 5. Ideal and anti-ideal efficiency of subunits of case 2

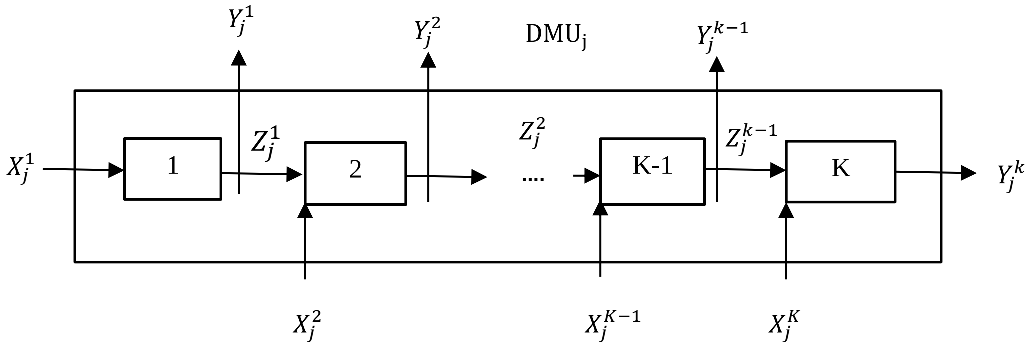
Table 6. Results of model 19 for case 2

Table 7. Overall efficiency of case 2-results of model 19



DMU$_j$
X_j
Stage 1
z_i
Stage 2
Y_j

Fig. 1.



Y_j^1
Y_j^2
DMU$_j$
Y_j^{k-1}
X_j^1
1
Z_j^1
2
Z_j^2
....
K-1
Z_j^{k-1}
K
Y_j^k
X_j^2
X_j^{K-1}
X_j^K

Fig. 2.

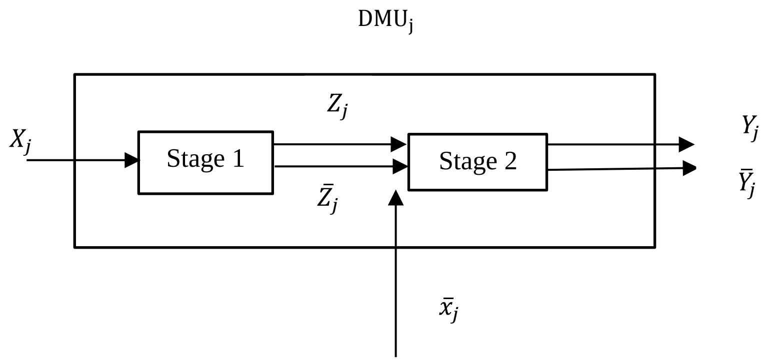

DMU$_j$
Z_j
X_j
Stage 1
Stage 2
Y_j
$\bar{Z}_j$
$\bar{Y}_j$
$\bar{x}_j$

Fig. 3.

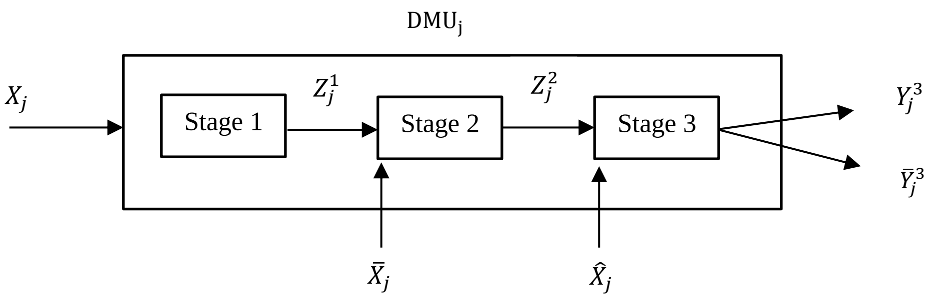

DMU$_j$
X_j
Stage 1
Z_j^1
Stage 2
Z_j^2
Stage 3
Y_j^3
$\bar{Y}_j^3$
$\bar{X}_j$
$\hat{X}_j$

Fig. 4.

Table 1

DMUj	Stage 1			Link			Stage 2			
	x_{1j}	x_{2j}	$\bar{x}_j$	z_j	$\bar{z}_{1j}$	$\bar{z}_{2j}$	y_{1j}	y_{2j}	$\bar{y}_{1j}$	$\bar{y}_{2j}$
1	0.838	0.277	0.962	0.894	0.362	-0.410	0.879	0.337	0.177	-0.423
2	1.233	0.132	0.443	0.678	0.188	-0.932	0.538	0.180	0.915	-0.240
3	0.321	0.045	0.482	0.836	-0.207	0.595	0.911	0.198	-0.488	0.413
4	1.483	0.111	0.467	0.869	-0.516	0.518	0.570	0.491	0.437	0.547
5	1.592	0.208	1.073	0.693	-0.407	0.689	1.086	0.372	-0.549	-0.994
6	0.790	0.139	0.545	0.966	0.269	-0.918	0.722	0.253	0.401	0.398
7	0.451	0.075	0.366	0.647	0.257	0.888	0.509	0.241	-0.533	0.371
8	0.408	0.074	0.229	0.756	-0.103	-0.474	0.619	0.097	0.522	-0.825
9	1.864	0.061	0.691	1.191	0.402	0.443	1.023	0.380	-0.456	0.467
10	1.222	0.149	0.327	0.792	-0.187	-0.674	0.769	0.178	-0.309	0.702

Table 2

DMUj	$E_j^{(1)*}$	$E_j^{(2)*}$	$E_j^{(1)-}$	$E_j^{(2)-}$	$E_j^{(1)*}-E_j^{(1)-}$	$E_j^{(2)*}-E_j^{(2)-}$
1	0.75807	1	1.31914	1	-0.56107	ε
2	0.43852	1	2.28041	1	-1.84189	ε
3	1	0.99998	1	1	ε^*	-2E-05
4	0.41578	0.9999	2.40515	1	-1.98937	-1E-04
5	0.25053	1	3.99161	1	-3.74108	ε
6	0.73791	1	1.35519	1	-0.61728	ε
7	1	0.51801	1	1.08145	ε	-0.56344
8	0.71148	0.99972	1.40553	1	-0.69405	-0.00028
9	1	0.99994	1	1	ε	-6E-05
10	0.28545	1	3.50318	1	-3.21773	ε

*ε is small amount and positive

Table 3

DMUj	v_1	v_2	v	w	$\bar{w}_1$	$\bar{w}_2$	$\hat{w}_1$	$\hat{w}_2$
1	4.59E3	0	6.14E4	0	8.05 E3	0	0	0
2	9.71E7	0	6.97 E7	0	1.70 E8	0	0	0
3	9.71E7	0	2.30× E11	3.73 E7	7.66 E7	0	0	0
4	9.71E7	0	9.90 E9	3.73 E7	7.66 E7	0	0	0
5	1.04E7	0	1.80 E10	3.99 E7	8.20 E7	0	0	0
6	2.92E4	0	6.03 E6	1.12 E4	2.30 E4	0	0	0
7	2.07E7	0	1.46 E8	7.94 E6	1.63 E7	0	0	0
8	1.27E5	0	1.17 E2	4.89 E4	1.00 E5	0	0	0
9	5.08E2	8.40 E3	1.08 E5	6.47 E2	1.71 E3	0	0	0

Table 3. (continue)

DMUj	u_1	u_2	$\bar{u}_1$	$\bar{u}_2$	$\hat{u}_1$	$\hat{u}_2$	$E_j^{(1)}$	$E_j^{(2)}$	**Overal efficiency (w_1=w_2=1/2)**
1	6.63 E4	3.70 E3	2.80 E4	0	1.53 E4	6.00 E3	0.75807	1	0.879035
2	0	0	7.07 E7	0	1.90 E4	7.28 E6	0.43852	1	0.71926
3	0	0	1.30 E11	0	0	2.60 E10	1	0.99998	0.99999
4	0	0	1.10 E10	0	0	2.10 E10	0.41578	0.99998	0.70788
5	0	0	1.70 E10	0	0	0	0.25053	1	0.625265
6	2.74 E6	1.34 E6	2.46 E6	5.27 E3	0	2.79 E6	0.73791	1	0.868955
7	6.33 E7	1.25 E7	4.16 E7	5.27 E3	0	5.18 E7	1	0.56077	0.780385
8	3.13 E4	0	3.39 E4	3.66 E1	0	0	0.71148	0.99994	0.85571
9	6.94 E4	0	4.19 E4	1.15 E4	0	0	1	0.99999	0.999995

Table 4.

	Stage 1	Stage 2			Stage 3			Link	
DMUj	Input 1 (X_j)	Input 2 ($\bar{X}_j$)	Output 1 (Y_j^2)	Output 2 ($\bar{Y}_j^2$)	Input 3 ($\hat{X}_j$)	Output 3 (Y_j^3)	Output 4 ($\bar{Y}_j^3$)	Link 1 (Z_j^1)	Link 2 (Z_j^2)
1	0.838	0.277	0.879	0.903	0.962	0.337	0.936	0.894	0.362
2	1.233	0.132	0.533	0.097	0.443	0.180	-0.188	0.578	0.188
3	0.321	0.045	0.911	1.355	0.482	0.198	0.906	0.836	0.207
4	1.483	0.111	0.570	1.621	0.467	0.491	0.812	0.869	0.516
5	1.592	0.208	1.086	-1.932	1.073	0.372	-0.092	0.693	0.407
6	0.790	0.139	0.722	-0.737	0.545	0.253	0.064	0.966	0.269
7	0.451	0.075	0.509	0.284	0.366	0.241	0.650	0.647	0.257
8	0.408	0.074	0.619	1.094	0.229	0.097	-1.150	0.756	0.103
9	1.864	0.061	1.023	0.531	0.691	0.380	0.380	1.191	0.402
10	1.222	0.149	0.769	-0.026	0.337	0.178	-0.005	0.792	0.187

Table 5

DMU$_j$	$E_j^{(1)*}$	$E_j^{(2)*}$	$E_j^{(3)*}$	$E_j^{(1)-}$	$E_j^{(2)-}$	$E_j^{(3)-}$
1	0.40963	1	0.97325	2.45077	1	1.01044
2	0.21114	0.7878	1	1.26321	1.2844	1.17199
3	1	1	1	5.98289	12.53931	1.18488
4	0.225	1	1	1.34613	5.17403	3.03264
5	0.16714	1	0.95533	1	1.64535	1
6	0.46951	0.65585	0.98436	2.80905	1.63687	1.339
7	0.55084	0.91166	0.98362	3.29562	2.13869	1.8993
8	0.71148	0.85394	0.98416	4.25668	2.63603	1.22178
9	0.24534	1	0.99045	1.46783	5.2849	1.58622
10	0.24886	0.89274	0.99703	1.48889	1.62641	1.52352

Table 6

DMUj	W	V	A	U	N	D	M	P	Q	S	B	W
1	0.4582	1.19332	0.88437	0.31301	0	0	2.1313	1.01104	0.35708	0	0.70671	0.4582
2	0.31141	0.81103	1.79514	0.01349	0	0	5.97623	3.30896	0	0	1.49552	0.31141
3	1.19617	3.11526	0	0.3017	0.53517	0	0	0	1.10375	0	2.07469	1.19617
4	0.25892	0.67431	1.36079	0	0.18373	0	6.98201	2.03666	0	0	0.63776	0.25892
5	0.24119	0.62814	1.22564	0.03792	0	0	4.00412	1.73229	0	0	0.46707	0.24119
6	0.48604	1.26582	1.34837	0.26357	0	0	3.81644	2.5292	0	0	1.16934	0.48604
7	0.85138	2.21729	2.17825	0.49233	0	0	5.98878	2.43018	0.60643	0	1.20271	0.85138
8	0.94111	2.45098	0	0	0.71012	0	3.89897	4.15336	0	0	4.36681	0.94111
9	0.20599	0.53648	2.47192	0	0.01185	0	1.24E+01	2.60643	0	0.00216	0.0091	0.20599
10	0.31422	0.81833	0.77865	0.36044	0	0	5.04122	3.22966	0	0	2.53529	0.31422

Table 7

DMUj	$E_j^{(1)}$	$E_j^{(2)}$	$E_j^{(3)}$	E_j
1	0.40963	0.59528	0.67495	0.559953
2	0.21114	0.34474	0.59561	0.38383
3	1	1	1	1
4	0.225	1	1	0.741667
5	0.16714	0.54002	0.64441	0.450523
6	0.46951	0.55301	0.63989	0.554137
7	0.55084	0.8104	0.97985	0.780363
8	0.71148	0.77688	0.40288	0.630413
9	0.24534	1	0.99045	0.745263
10	0.24886	0.42278	0.57488	0.415507