

Sharif University of Technology

Scientia Iranica

Transactions D: Computer Science & Engineering and Electrical Engineering www.scientiairanica.com



A generalized entropy-based two-phase threshold algorithm for noisy medical image edge detection

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Received 22 January 2016; received in revised form 5 July 2016; accepted 22 August 2016

KEYWORDS

Image edge detection; Hill entropy; Thresholding; Canny edge detection; Medical imaging; Image analysis. **Abstract.** Edge detection in medical imaging is a significant task for object recognition of human organs and is considered a pre-processing step in medical image segmentation and reconstruction. This article proposes an efficient approach based on generalized Hill entropy to find a good solution for detecting edges under noisy conditions in medical images. The proposed algorithm uses a two-phase thresholding: firstly, a global threshold calculated by means of generalized Hill entropy is used to separate the image into object and background. Afterwards, a local threshold value is determined for each part of the image. The final edge map image is a combination of these two separate images based on the three calculated thresholds. The performance of the proposed algorithm is compared to Canny and Tsallis entropy using sets of medical images corrupted by various types of noise. We used Pratt's Figure Of Merit (PFOM) as a quantitative measure for an objective comparison. Experimental results indicated that the proposed algorithm displayed superior noise resilience and better edge detection than Canny and Tsallis entropy methods for the four different types of noise analyzed, and thus it can be considered as a very interesting edge detection algorithm on noisy medical images.

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1. Introduction

In image analysis and computer vision systems, the edges of objects in an image contain important information that can be used as low-level features [1]. When noise exists in the image, an accurate detection of these contours becomes a very hard and timeconsuming task [2]. Producing the continuous contours of the object boundaries is a fundamental step in image processing and computer vision, especially in the field of feature detection and feature extraction [3], and it is the main goal of edge detection algorithms. The detection of these edges is a critical pre-processing step for a variety of tasks, including object recognition [4,5] and segmentation [6-9].

An edge can be defined as a boundary that divides an area of an image into two regions [1] or a single pixel with a local discontinuity in intensity [2]. So, different algorithms of edge detection can select the edges in various ways of representation, and the goodness or appropriateness of them will depend on the definition of those edges.

Edge detection is widely used in medical diagnosis [10] and all different medical imaging modalities (X-rays [11], ultrasonography [12], computed tomography [13], magnetic resonance imaging [14], nuclear medicine [15] or microscopy [16]).

The type of noise that can be encountered in all these different medical imaging modalities can be very different (salt & pepper noise [17], additive white Gaussian noise [18], Poisson noise [19] or speckle

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noise [20]), and since none of the existing methods produces the optimum results for all images with different types of noise, finding an optimal method for edge detection is still an active field of research. Therefore, a robust algorithm is required to overcome the noise and consider the global structure of the edges to reduce broken ones.

Entropy is a concept in information theory which is used to measure the amount of information in a message [21-25]. Entropy is defined in terms of the probabilistic behavior of a source of information. Compared with other methods, the most important advantages of entropy-based approaches for edge detection are the ease of its implementation and the few number of operations [26]. Entropy-based approaches are very stable and efficient in noisy environments [26]. Their comparison with other edge-detection algorithms has shown that they have high capability and have been successfully applied to problems of edge detection in noisy image [26].

This article presents a new edge detection algorithm based on generalized Hill entropy for noisy images with the goal of extracting continuous edges and reducing the number of broken ones by means of a two-phase thresholding (see Figure 1 for an schematic overview of the presented algorithm).

The structure of this paper is as follows: Section 2 introduces the related work to edge detection algorithms; Section 3 presents the proposed edge detection approach based on generalized Hill entropy; Section 4 contains experiments and Section 5 contains results and discussion.

2. Related work

Many edge detection algorithms have been proposed. The results are usually examined either by visual inspection as a qualitative measure [1,26] or quantitatively by different indexes [27-35]. Some of these algorithms utilize a linking technique collecting pixels that belong to a set of edges [36-38]. In this context, the linking technique cannot be perfect except for simple shapes like lines or circles [36]; the use of image filters presents drawback which may affected image features, and thus the localization ability of an edge detector becomes poor [39].

Recently, many papers have been published in the area of image edge detection [27-35] that tests its importance as follows: Lopez-Molina et al. [27] presented a study that focuses on the improvements of edge detection by using Anisotropic Diffusion (AD) instead of Gaussian Linear Filtering (GLF); in [28], the modified scheme is presented to improve the performance of traditional Canny edge detection through an adaptive filter based on bi-dimensional general auto-regression model; in [29], Lopez-Molina et al. presented a novel edge detection framework based on the measurement of grey level changes using a new class of functions called relief functions; in [30], a new edge detection method that combines smoothing spline algorithm and gray-



Figure 1. Schematic overview of the proposed algorithm where different steps can be observed. Original image (a) is divided into two parts (object (b) and background (c)) following the threshold value, t_1 , calculated by means of Hill entropy (1st step of the algorithm). Afterwards, the threshold values t_2 and t_3 are calculated from object (b) and background (c), respectively, by Hill entropy (the 2nd step of the algorithm). The final edge map (d) is obtained by applying the threshold values t_1 , t_2 , and t_3 to the original image.

moments operator is introduced; in [31], Ray presented a scheme for unsupervised edge detection that uses the highly efficient Absolute Difference Mask (ADM) algorithm to generate the initial edge image and uses a subsequent modified non-maximal suppression scheme to optimize the edge output resulting in the final edge map; in [32], Lopez-Molina et al. presented a multiscale method for edge detection based on increasing Gaussian smoothing, the Sobel operators, and coarseto-fine edge tracking; in [33], a study focusing on the edge detection process by the generation of a fuzzy representation of the edges is shown; in [34], a new approach is presented for edge detection using a combination of Bacterial Foraging Algorithm (BFA) and a probabilistic derivative technique derived from Ant Colony Systems; in [35], a new PSO-based approach is presented to detect edges in noisy images and reduce the broken and jagged ones by means of a developed penalized fitness function based on the possibility score of a curve fitted on an edge and its curvature cost.

Every edge detection algorithm has its advantages and disadvantages and does not appear to be an optimal edge detector that could be able to detect the edges of any image type and show high resistance to noise. Therefore, efficient algorithms are required to explore edge detection under challenging conditions with high resistance to noise, reducing the shortcomings of traditional edge detectors at the same time.

Some of the most popular edge detectors are Sobel [40] and Prewitt [41] based on the first-order derivative of the pixel intensities or the Laplacian-of-Gaussian (LoG) [42,43] edge detector that uses instead the second-order differential operators to detect the location of edges. However, these algorithms tend to be sensitive to noise, which is actually a high frequency phenomenon. In 1986, Canny [44] proposed an edge detector, which combines a smoothing function with zero-crossing-based edge detection, resulting into an algorithm more resilient to noise than the previously mentioned ones.

Entropy is an uncertainty measure introduced by Shannon into information theory to describe how much information is contained in a source governed by a probability law [23], and it has played an important role in recent work on edge detection algorithms: in [45], Xiao et al. proposed a Gray-Level & Gradient Magnitude (GLGM) histogram for thresholding. GLGM histogram employs Fibonacci quantized gradient magnitude to effectively characterize spatial information by applying entropic image thresholding; Singh and Singh [26] proposed an algorithm based on Shannon entropy for edge detection in gray level images obtaining acceptable results; in [46], El-Khamy et al. used the relationship of the probability partition and the fuzzy 2-partition of the image gradient to select the optimal gradient-threshold, then it selects the algorithm that assures that the entropy reaches a minimum value; in [47], El-Sayed, presented a new algorithm for edge detection using both Shannon entropy and Tsallis entropy and in [48], Elaraby et al. proposed a new algorithm for edge detection of images based on hybrid types of entropy.

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3. The proposed approach based on generalized hill entropy

In this section, we discuss the generalized Hill entropy by which our proposed edge-detection algorithm is based. Let $p_1, p_2, ..., p_k$ be the probability distribution of a discrete source. Therefore, $0 \le p_i \le 1$, i =1, 2, ..., k, and $\sum_{i=1}^{k} p_i = 1$, where k is the total number of states. The entropy of a discrete source is often obtained from the probability distribution.

The Shannon entropy [23] is defined as follows:

$$H(p) = -\sum_{i=1}^{k} p_i \ln(p_i).$$
 (1)

This formalism has been shown to be restricted to the domain of validity of the Boltzmann-Gibbs-Shannon (BGS) statistics. These statistics seem to describe nature when effective microscopic interactions and microscopic memory are short-ranged. Generally, systems that comply with BGS statistics are called extensive systems. If the physical system can be decomposed into two statistically independent subsystems A and B, the probability of the joined system is $p^{A+B} = p^A \cdot p^B$. It has been verified that the Shannon entropy has the extensive property (additive) [23]:

$$H(A + B) = H(A) + H(B).$$
 (2)

The generalized entropy of Hill [49-51] is defined as:

$$N_{\alpha} = \left(\sum_{i=1}^{W} p_i^{\alpha}\right)^{\frac{1}{1-\alpha}}, \text{ for } \alpha \ge 0 \text{ and } \alpha \ne 1.$$
(3)

Hill entropy is non-extensive in such a way that for a statistical independent system, the entropy of the system is defined by the following pseudo additive entropic rule:

$$N_{\alpha}(A+B) = N_{\alpha}(A) + N_{\alpha}(B) + (1-\alpha) \cdot N_{\alpha}(A) \cdot B_{\alpha}(B).$$
(4)

The concept of entropy becomes increasingly important in image processing, when an image can be interpreted as an information source with the probability law given by its image histogram [52-56].

For an image with k gray-levels, let $p_1, p_2, ..., p_t, p_{t+1}, ..., p_k$ be its probability distribution, where p_t is the normalized histogram (i.e., $p_t = h_t/((M \times N)))$

and h_t is the gray level histogram. Using this distribution, we can derive two probability distributions, one for the object (class A) and the other for the background (class B) as follows:

$$p_A : \frac{p_1}{P_A}, \frac{p_2}{P_A}, ..., \frac{p_t}{P_A},$$
$$p_B : \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, ..., \frac{p_k}{P_B},$$
(5)

$$P_A = \sum_{i=1}^{t} p_i, \qquad P_B = \sum_{i=t+1}^{k} p_i,$$
 (6)

where t is the threshold value.

In terms of the definition of Hill entropy, the entropy of object pixels (N^A_{α}) and background pixels (N^B_{α}) can be defined as follows:

$$N_{\alpha}^{A} = \sum_{i=1}^{W} \left(\frac{p_{i}^{\alpha}}{P_{A}}\right)^{\frac{1}{1-\alpha}},$$
$$N_{\alpha}^{B} = \sum_{i=1}^{W} \left(\frac{p_{i}^{\alpha}}{P_{B}}\right)^{\frac{1}{1-\alpha}}.$$
(7)

Hill entropy, $N_{\alpha}(t)$, is parametrically dependent upon threshold value t for the object and the background. It is formulated as the sum of each entropy, allowing the pseudo-additive property for statistically independent systems, as in Eq. (4). We try to maximize information measure between the two classes (object and background). When $N_{\alpha}(t)$ is maximized, luminance level (t) that maximizes the function is considered to be the optimum threshold value. This can be done with low-computational effort:

$$t^{\text{opt}} = Arg \max \left[N^A_{\alpha}(t) + N^B_{\alpha}(t) + (1 - \alpha) \right]$$
$$.N^A_{\alpha}(t).N^B_{\alpha}(t) \right], \tag{8}$$

when $\alpha \to 1$, the threshold value in Eq. (8) equals the same value found by Shannon entropy. Thus, this proposed method includes Shannon's method as a special case. The following expression can be used as a criterion function to obtain the optimal threshold when $\alpha \to 1$:

$$t_{Sh}^{\text{opt}} = Arg \max\left[N_{\alpha}^{A}(t) + N_{\alpha}^{B}(t)\right],\tag{9}$$

where t_{Sh}^{opt} is the Shannon entropy.

By using a spatial filtering mask of size 3×3 (the smallest possible meaningful size), the probability of each central pixel of the window can be determined

by entropy as $H(Cpix) = -p_c \ln(p_c)$, where p_c is the probability of central pixel Cpix of the binary image under the window.

When the probability of central pixel $p_c = 1$, then the entropy of that pixel equals zero. Thus, if the gray level of all pixels under the window is homogeneous, $p_c = 1$ and H = 0. In this situation, the central pixel is not an edge pixel. At $p_c = 7/9$ or $p_c = 8/9$, the variety for gray level of pixels under the window is low. For these cases, the central pixel is not considered an edge pixel. In the remaining cases, where $p_c \leq 6/9$, the variety of gray level of pixels under the window is high, and thus we can assume that we are on an edge pixel.

The proposed algorithm based on generalized hill entropy can be summarized as follows:

- 1. Input: A digital gray-scale image I of size $M \times N$.
- Let f(a, b) be the original gray value of the pixel at point (a, b):

$$a = 1, 2, ..., M, b = 1, 2, ..., N.$$

- 3. Calculate the probability distribution p_i , $0 \le i \le 255$ for every image pixel.
- 4. For all $t \in \{0, 1, ..., 255\}$: 4.1. Calculate P_A , P_B , p_A , and p_B as:

$$p_A : \frac{p_1}{P_A}, \frac{p_2}{P_A}, \dots, \frac{p_t}{P_A},$$
$$p_B : \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, \dots, \frac{p_z}{P_B},$$
$$P_A = \sum_{i=1}^t p_i, \text{ and } P_B = \sum_{i=t+1}^z p_i$$

4.2. Calculate the optimum threshold value t^* as:

$$t^{*} = Arg \max \left[N_{\alpha}^{A}(t) + N_{\alpha}^{B}(t) + (1 - \alpha) \cdot N_{\alpha}^{A}(t) \cdot N_{\alpha}^{B}(t) \right].$$

5. Create a binary image: For all a, b:

If $I(a,b) \leq t^*$ then f(a,b) = 0 else f(a,b) = 1.

- 6. Create a mask, S, with 3×3 , $\beta = (m-1)/2$ and $\gamma = (n-1)/2$.
- 7. Create an $M \times N$ output image, g for all a and b, set g(a, b) = f(a, b).
- 8. Checking for edge pixels:

For all $b \in \{\gamma + 1, ..., N - \gamma\}$, and

$$a \in \{\beta + 1, ..., M - \beta\}, sum = 0.$$

- For all $k \in \{-\gamma, ..., \gamma\}$, and $j \in \{-\beta, ..., \beta\}$, If f(a, b) = f(a + j, b + k), then sum = sum + 1. If (sum > 6), then g(a, b) = 0; else, g(a, b) = 1.
- 9. Output: The edge detection image g of I.

3.1. Time complexity

Time complexity of the proposed approach is $O(M \times$ $N \times m \times n$, where $M \times N$ is the number of pixels in **I**, and $m \times n$ is the number of pixels in w's window. As can be observed, time complexity for computing each threshold values t_1, t_2 , and t_3 is linear with the number of pixels. The reason is that computing the gray-level histogram of I takes $O(M \times N)$, and finding t^{opt} takes $O(k^2)$, where k is the number of gray-level in I. Since $k^2 \ll M \times N$, $O(M \times N + k^2)$ leads to $O(M \times N)$. The algorithm shows that building the binary image is also linear in the number of pixels, i.e. $O(M \times N)$. Finally, it is well-known that the time complexity of the application of a spatial discrete filter of order $m \times n$ in an $M \times N$ image can be done in $O(M \times N \times m \times n)$. As a result, this last step dominates the time complexity of the process, i.e. $O(4 \times M \times N + M \times N \times m \times n)$ leads to $O(M \times N \times m \times n)$.

The overall time complexity of Canny edge detector is $O(M \times N \log[M \times N])$. Thus, the computational complexity of the proposed solution is asymptotically equivalent to the simplest techniques.

4. Experimental design

All the analyses were performed using MATLAB R2012b (The Mathworks, Inc., Natick, MA, USA)

running on an Intel CoreTM 2 Duo 2.20 GHz personal computer with 3 GB of RAM.

To investigate the new algorithm effectiveness, we have compared it to Canny algorithm, as it is considered the optimal edge detector [57], and to Tsallis entropy, as it has, theoretically, similar properties. The three algorithms have been applied to different medical images with various types of noise. This section also describes image sets and performance measurements that have been used in the experiments.

4.1. Image sets

Image set includes one sagittal and one axial T1 weighted Magnetic Resonance (MR) images of a human brain, one axial T1 weighted MR image of an intervertebral disc, and one sagittal MR image of a whole human spine (see Figure 2). To explore the performance of the new algorithm in noisy environments, these images are corrupted by four types of noise: salt & pepper (noise density = 0.05), Gaussian (zero mean noise with variance = 0.01), Poisson, and speckle (uniformly distributed random noise with mean = 0 and variance = 0.04) noise.

4.2. Quantitative performance measurement

The performance of the edge detection on noisy images is evaluated by comparing the result provided by



Figure 2. Medical images used in this work to test the performance of the proposed algorithm compared with that of Canny and Tsallis entropy edge detection algorithms. These original images (the first left column) were corrupted by various types of noise (salt & pepper, Gaussian, Poisson and speckle): (a) Sagittal T1 weighted MR image of a human brain, (b) axial T1 weighted MR image of a human brain, (c) axial T1 weighted MR image of an intervertebral disc, and (d) sagittal MR image of a whole human spine.

Canny and Tsallis entropy with that of our proposed method on the same images without noise, considered as reference for the optimal edge detection.

For an objective comparison of the localization accuracy of the edge detection algorithms, Pratt's Figure Of Merit (PFOM) is used as a quantitative measure [58]. PFOM is defined as:

$$R = \frac{1}{\max(I_I, I_A)} \sum_{i=1}^{I_A} \frac{1}{1 + \beta d(i)^2},$$
(10)

where I_I and I_A indicate the number of ideal and actual edge points in ideal edge map and the generated edge map images, d(i) is the distance between pixel *i* in the generated edge map and the nearest ideal edge point in the ideal edge map, and β is a constant scale factor which is typically set to 1/9. This measure is an index to compute the localization accuracy of edge detection algorithms. A larger value of R indicates stronger performance.

5. Results and discussion

This section presents the results and discussion of the comparison (qualitatively and quantitatively) of the proposed algorithm with Canny and Tsallis entropy algorithms. The results obtained from these three methods are included in Subsection 5.1 for a visual analysis and in Subsection 5.2 for a quantitative analysis. The results of each edge map depend on the thresholding values selected by means of the generalized Hill entropy for that image under different noise types. Table 1 displays the values of three thresholds for all four images (for the original image and for the four different added types of noise).

5.1. Subjective comparison

For a qualitatively comparison of our proposed algorithm and Canny and Tsallis entropy, we applied the three algorithms to the images previously described as corrupted by different types of noise. The resulting edge maps are shown in Figure 3.

The resulting images show that the proposed algorithm performed better than Canny and Tsallis entropy on the studied set of images. The boundary of the objects is more clearly defined after applying our algorithm than using Canny or Tsallis entropy algorithms for edge detection, and this shows that the proposed algorithm is resistant to noise. Canny algorithm, even with post-processing, did not work well for these noisy images and many noise spots in the resulting images were observed.

Table 1. Threshold values selected by generalized Hill entropy for the different analyzed images (for the original image and for the four different added types of noise). Image 1: sagittal T1 weighted MR image of a human brain; Image 2: axial T1 weighted MR image of a human brain; Image 3: axial T1 weighted MR image of an intervertebral disc; and Image 4: sagittal MR image of whole human spine.

	Original	Salt & pepper	Gaussian	Poisson	Speckle
Image 1					
t_1	138	131	137	141	142
t_2	83	78	68	87	88
t_3	191	185	189	95	188
Image 2					
t_1	125	119	122	126	142
t_2	68	64	63	60	75
t_3	209	162	191	93	197
Image 3					
t_1	89	88	116	95	117
t_2	46	46	61	50	60
t_3	160	151	189	175	197
Image 4					
t_1	88	75	106	62	98
t_2	45	40	57	46	49
t_3	162	143	181	172	173



Noisy images

Figure 3. Resulting edge map images obtained by applying the proposed algorithm and Canny and Tsallis entropy edge detection algorithms: (a,b,c,d) Results of the edge detection algorithms applied to the four images studied in this work. For each image, the top row (.1) shows the results after applying the proposed algorithm for edge detection, the central row (.2) shows the results after applying Tsallis entropy algorithm, while the bottom row (.3) shows the results after applying Canny algorithm edge detection. Results are shown for the original images and for the same images after having been corrupted by salt & pepper, Gaussian, Poisson, and speckle noise.

5.2. Objective comparison

To quantitatively compare the new algorithm with the Canny and Tsallis entropy algorithms, the localization accuracy was calculated using Pratt's Figure Of Merit **Table 2.** Objective results (PFOM: Pratt's Figure Of Merit) for the proposed algorithm versus Tsallis entropy and Canny for all four images for the different added types of noise. Image 1: sagittal T1 weighted MR image of a human brain; Image 2: axial T1 weighted MR image of a human brain; Image 3: axial T1 weighted MR image of an intervertebral disc; Image 4: sagittal MR image of a whole human spine.

	PFOM			
	Proposed Tsallis		Canny	
	algorithm	entropy		
Image 1				
Salt & pepper	68.3631	64.2356	37.0105	
Gaussian	75.0607	70.5479	56.0638	
Poisson	92.1007	90.1235	88.2085	
$\operatorname{Speckle}$	77.3533	71.5478	73.0216	
Image 2				
Salt & pepper	57.7091	52.8796	40.0925	
Gaussian	72.7233	70.5487	34.6264	
Poisson	96.6391	92.4593	89.0899	
$\operatorname{Speckle}$	84.4332	81.4568	79.6043	
Image 3				
Salt & pepper	63.7627	60.1236	51.5896	
Gaussian	69.7806	63.8745	52.7855	
Poisson	94.7062	92.9658	85.7877	
S peckle	90.6569	86.7845	77.4794	
Image 4				
Salt & pepper	51.1280	48.5892	40.2650	
Gaussian	64.8006	60.8754	48.5146	
Poisson	96.6102	91.4583	89.8145	
S peckle	94.9092	92.4569	88.9649	

(PFOM) to the resulting images after applying different types of noise.

For the test images, we see the value of PFOM in Table 2. A high value of PFOM means that most of the edges are detected (better performance). The best performance is achieved when the images suffer from Poisson noise, followed by speckle noise and Gaussian noise. The worst performance of the proposed algorithm is achieved when the test images suffer from salt & pepper noise. The same occurs for the Canny and Tsallis entropy algorithms, and these results repeat for all the images analyzed in this work.

From the visual and quantitative results, we can conclude that the proposed algorithm is competitive compared to the Canny and Tsallis entropy methods in terms of average performance. Furthermore, the proposed algorithm produces a better solution than Canny and Tsallis entropy for edge detection for every type of image under all types of noise, which indicates its potential and reliability. We infer that the proposed algorithm can be a very interesting option for problemspecific edge detection of medical images under noisy conditions.

6. Conclusion

In this article, a novel algorithm for detecting edges in medical images was presented. A 2-phase thresholding technique based on generalized Hill entropy was used to estimate threshold values required for the proposed algorithm. The first phase permitted us to determine a global threshold value that divides the image into two parts, called object and background. In the second step, a local threshold value was determined for each part of the image, merging the results in the final stage to get the edge map for the image. The proposed algorithm was examined and compared with Canny and Tsallis entropy algorithms on different medical images with four different types of noise. Subjective and objective measures to determine accuracy were used for a comparison of this algorithm. Experimental results showed that the proposed edge detector is more robust under noisy conditions than Canny and Tsallis entropy for all presented images under every type of noise.

Acknowledgements

The first author thanks Erasmus Mundus Action 2 WELCOME Program Third Call under Grant #WELC1104034.

This work was supported in part by the Spanish Ministerio de Economía y Competitividad (MINECO) and by FEDER funds under Grant BFU2015-64380-C2-2-R.

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