

Research Note

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Numerical analysis of vibration and transient behaviour of laminated composite curved shallow shell structure: An experimental validation

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Experimental vibration; Carbon/epoxy composite; HSDT; FEM; ANSYS; Transient behaviour. Abstract. The natural frequency and transient responses of carbon/epoxy layered composite plate structure were analysed through the instrumentality of two higher-order mid-plane kinematic models in this article. The mathematical formulation of the layered composite structure was further utilised to develop a computer programme in MATLAB-15.0 to evaluate the mentioned responses. The practical relevance of the present higher-order models was established via comparing the present numerical results computed using suitable MATLAB computer code with the in-house experimental test data. Additionally, the fundamental frequency and transient responses of the carbon fibre-reinforced epoxy composite plate structure were simulated via finite-element package (ANSYS) by means of the ANSYS Parametric Design Language (APDL) code. The simulated frequencies were compared with those of the present experimental and MATLAB results. Finally, the significance of the proposed higher-order kinematics was established via solving a different kind of illustrations to investigate the influence of various geometrical and material parameters on the dynamic responses of layered composite structure, discussed in detail.

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1. Introduction

The quest of designers to achieve lightweight structural materials with reliable strength and stiffness properties has led to the development of advanced materials, such as laminated composite. Owing to its superior characteristics, laminated structures possess many applications in high-performance engineering fields, such as aerospace, marine, automotive, and civil infrastructure. The layered composites can provide tailor-made properties due to their stacking sequence and layer thickness that, in turn, enhance the final structural performances during their service condition. Today, the laminated composite panel that creates the necessity for the accurate analysis and design of the final finished product rapidly replaces most of the structural components. In general, most of the structures are exposed to the low/high amplitude of vibration under the dynamic loading, and accurate prediction of the desired responses (fundamental frequency and transient response) is of great importance. The Finite-Element Method (FEM) is a potential method established as a versatile numerical tool from the last few decades to analyse the laminated structural problems due to their inherent materials and geometrical complexities. Many studies on the development of the numerical model are reported where the structural responses are obtained

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using different theories, such as classical as well as firstand higher-order shear deformation theories, including the modified kinematic models, for accurate analysis. Further, to improve the accuracy of the numerical results and reduce the experimental cost, studies have continued every now and then. Some of the very relevant and important studies have been reviewed and presented in the following line, and few key deficiencies in the former studies are pointed out.

Mallikarjuna and Kant [1] proposed a Finite-Element (EF) model in higher-order shear deformation theory to analyse the dynamic behavior of the layered composite plate structure. Similarly, the FEM solutions for frequency values of the doublycurved shell panel were computed by Chakravorty et al. [2] using the First-order Shear Deformation Theory (FSDT). Further, the fundamental frequency responses of the fibre-reinforced layered polymer composite plate were examined by Chakraborty and Mukhopadhyay [3] experimentally (impact excitation) and numerically using commercial FE package (NISA). Ahmadian and Zangeneh [4] analysed the dynamic characteristics of the rectangular layered composite plate by means of a superelement. The dynamic responses of thick multilayered composite flat/curved panel and sandwich plate were reported using the new HSDT [5,6] kinematic model. Further, Cugnoni et al. [7] computed the experimental modal test of Glass/Polypropylene composite plate and performed the validation test by comparing them with numerical results of the thin- and thicklayered composite shell panels computed via FSDT as well as the HSDT kinematic models. Tornabene et al. [8] obtained the free vibration responses of layered composite shell panel using a 2D higher-order general formulation. Jeyaraj et al. 9 analysed the vibration and acoustic responses of an isotropic rectangular plate under the harmonic load, including the thermal environment, using different types of commercial FE packages (ANSYS and SYSNOISE). Mehar et al. [10] computed the natural frequency responses of the functionally graded carbon nanotube (FG-CNT) composite plate using the HSDT mid-plane kinematics. The free vibration and time-dependent displacement behaviour of the layered structure were examined based on the FSDT kinematics using an eight-node quadrilateral serendipity element [11-13]. Later, Shokrollahi and Shafaghat [14] examined the dynamic behaviours of the hybrid metal-composite thick trapezoidal plates using global Ritz method in FSDT kinematics. The displacement kinematic using FSDT was also considered by Kerur and Ghosh [15] to compute the frequency responses of the layered composite panel and the active vibration control with integrated Active Fiber Composite (AFC) layer. Kumar and Raju [16] analysed the dynamic responses of the cross- and angle-ply layered composite structures via a mathematical model based on HSDT. The dynamic behaviour of the square laminated plate with edges containing randomly and unidirectionally aligned short fibers was investigated by Eruslu and Aydogdu [17] using the FSDT kinematics. Similarly, the static and dynamic responses of the layered composite shallow shell panel using higher-order FEM model were examined by Sahoo et al. [18] and validated with the corresponding experimental values. Hirwani et al. [19] reported theoretical and experimental vibration analyses of debonded shell structure using different kinematic models in association with FEM. The free vibration behaviour of the layered composite beam was analysed by Li et al. [20] using refined HSDT kinematics. The static and dynamic [21-34] characteristics of layered composite as well as sandwich structure using mathematical model developed based on various theories such as new trigonometric plate [35-37], new sinusoidal higher-order plate [38,39], and hyperbolic shear deformation theory [40,41] were analysed by Tounsi and his co-authors. The Large-amplitude dynamic behaviour of curved panel was reported by Shooshtari and Razavi [42] using a Donnell shell theory. Milan et al. [43] computed the dynamic characteristic of the carbon/epoxy layered composite flat plate structure using ANSYS.

It can be clearly observed from the above reviews that many numerical attempts have been already made to investigate the dynamic responses of the layered structure via different numerical as well as analytical techniques. It should be noted that the investigation of the fundamental frequency, including the timedependent displacement responses of the lavered composite structure, using the HSDT model and the subsequent validation with experimental and simulation (ANSYS) results, is very limited in numbers. Hence, to address the issue and overcome the shortcomings of the former researchers, the current study aims to work as a bridge between the gaps. In this regard, the present research focuses on the development of numerical model to compute the fundamental frequencies and timedependent deflections of the layered composite structure using two different HSDT models. Further, the responses are evaluated using the homemade computer code in MATLAB software and experiments on carbon/epoxy layered composite. The responses obtained using the theoretical and experimental are utilised for a comparison purpose to establish the requirement of the currently developed higher-order models. Again, the structural responses are computed via simulation model through structural simulation software (ANSYS) and compared with both the numerical and experimental values. Finally, sensitivity analysis is carried out on different parameters, such as shell panel geometries (cylindrical, spherical, flat, hyperboloid, and elliptical), including the other geometrical and material parame-



Figure 1. Geometry of laminated curved panel.

ters, to show their effects on the frequency and timedependent displacement responses.

2. Theoretical formulation

2.1. Geometry of the panel

For the present investigation, a layered composite doubly-curved shell panel with N number of orthotropic layers of equal thickness has been considered, as presented in Figure 1. The following geometrical parameter of the panel is considered: the length as a, the breadth as b, and the thickness as h. The displacement continuity within the layered composite is considered based on HSDT kinematics [44], where the in-plane displacement functions are defined as a cubic function of thickness co-ordinate, and the displacement field function in the thickness direction is assumed to be constant or linearly varying throughout the thickness. In addition, the necessary assumptions on the current modelling purpose (uniform layer thickness, bonding between the layers, elastic behaviour of individual composite constituent, etc.) are made, similar to the reference [44].

2.2. Displacement field and strain displacement relation

The displacement continuum for the first HSDT model (Model-1) is presented in Eq. (1), where displacement function in the thickness direction is considered constant through the thickness:

$$u(x, y, z, t) = u_0(x, y, t) + z\theta_x(x, y, t) + z^2\phi_x(x, y, t) + z^3\lambda_x(x, y, t),$$

$$\nu(x, y, z, t) = \nu_0(x, y, t) + z\theta_y(x, y, t) + z^2\phi_y(x, y, t) + z^3\lambda_y(x, y, t),$$

$$w(x, y, z, t) = w_0(x, y, t).$$
(1)

Further to the above, another HSDT kinematic model say, Model-2, is also employed for the current mathematical modelling of the layered composite panel structure, where the displacement function through the thickness is assumed to be varying linearly [5]:

$$\begin{split} u(x, y, z, t) = &u_0(x, y, t) + z\theta_x(x, y, t) + z^2\phi_x(x, y, t) \\ &+ z^3\lambda_x(x, y, t), \\ \nu(x, y, z, t) = &\nu_0(x, y, t) + z\theta_y(x, y, t) + z^2\phi_y(x, y, t) \\ &+ z^3\lambda_y(x, y, t), \end{split}$$

$$w(x, y, z, t) = w_0(x, y, t) + z\theta_z(x, y, t),$$
(2)

where u, ν , and w represent the translation of any point (within the laminate) along x, y, and z directions, respectively; t denotes the time, u_0 , ν_0 , and w_0 are the mid-plane displacements, θ_x and θ_y denote the rotations of the normal to the mid-plane about yand x directions, respectively. The rest of the terms $\phi_x, \phi_y, \lambda_x, \lambda_y$, and θ_z are the higher-order terms of Taylor series expansion, considered to maintain the actual profile of shear stress (parabolic variation) through the thickness of the layered composite structure.

Further, another model is developed in ANSYS package via the batch input technique of ANSYS Parametric Design Language (APDL), code named as Model-3. The simulation model is discretised using a well-defined SHELL-281 element. The SHELL-281 element is an eight-node element with six degrees of freedom at each node and is suitable for the analysis of thin- to moderately thick-layered structures. The displacement kinematics of the simulation model based on FSDT [45] is shown in Eq. (3):

$$u(x, y, z, t) = u_0(x, y, t) + z\theta_x(x, y, t),$$

$$\nu(x, y, z, t) = \nu_0(x, y, t) + z\theta_y(x, y, t),$$

$$w(x, y, z, t) = w_0(x, y, t) + z\theta_z(x, y, t).$$
(3)

Further, the constitutive equation is expressed in the following line for any kth lamina within the laminate

which is oriented at an arbitrary angle " θ " about any principal material axes:

$$\{\sigma_{ij}\} = [\overline{Q}_{ij}] \{\varepsilon_{ij}\}, \qquad (4)$$

 $\{\sigma_{ij}\}, \{\varepsilon_{ij}\}, \text{and } [\overline{Q}_{ij}] \text{ represent the stress tensor, strain tensor, and reduced stiffness matrix, respectively. The expansion of the strain tensor can be presented further as follows:$

$$\left\{\varepsilon_{ij}\right\} = \left\{\begin{array}{c}\varepsilon_{xx}\\\varepsilon_{yy}\\\varepsilon_{zz}\\\gamma_{xy}\\\gamma_{xz}\\\gamma_{yz}\\\gamma_{yz}\end{array}\right\} = \left\{\begin{array}{c}\left(\frac{\partial u}{\partial x} + \frac{w}{R_{x}}\right)\\\left(\frac{\partial v}{\partial y} + \frac{w}{R_{y}}\right)\\\left(\frac{\partial w}{\partial z}\right)\\\left(\frac{\partial w}{\partial z}\right)\\\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\right)\\\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u}{R_{x}}\right)\\\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{v}{R_{y}}\right)\end{array}\right\},\quad(5)$$

where R_x and R_y are the principal radii of curvature in x and y axes, respectively.

2.3. Finite-element formulation

The FEM is a potential tool to analyse the structural responses of the layered composite structure with complex geometry. In the present formulation, the necessary discretisations of the proposed model of the layered structure have been performed through the use of suitable FEM steps using a nine-node isoparametric element. Now, "d" as a displacement vector at an arbitrary point on the mid-surface of the panel structure for all three models can be expressed in a generalised form as follows:

$$d = \sum_{i=1}^{n} N_i(x, y) d_i,$$
 (6)

where $\{d_i\} = \{u_{0i}v_{0i}w_{0i}\theta_{xi}\theta_{yi}\phi_{xi}\phi_{yi}\lambda_{xi}\lambda_{xi}\}^T$, $\{d_i\} = \{u_{0i} v_{0i}w_{0i}\theta_{xi}\theta_{yi}\theta_{zi}\phi_{xi}\phi_{yi}\lambda_{xi}\lambda_{yi}\}^T$, and $\{d_i\} = \{u_{0i} v_{0i}w_{0i}\theta_{xi}\theta_{yi}\theta_{zi}\}^T$ are displacement field functions for the corresponding models (Model-1, Model-2, and Model-3) utilised in the current analysis. Similarly, N_i represents the shape functions of the *i*th node.

Now, the strain tensor can be represented in the matrix form as follows:

$$\{\varepsilon\} = [T] \{\bar{\varepsilon}\}, \tag{7}$$

where [T] and $\{\bar{\varepsilon}\}$ denote the thickness coordinate matrix and mid-plane strain. Now, the mid-plane strain vector can be further explored and written as follows:

$$\{\bar{\varepsilon}\} = [B_L] \{d_i\},\tag{8}$$

where $[B_L]$ denotes the matrix of a general strain displacement relation according to the type of displacement field model.

The total strain energy of the laminated panel is expressed in the following line using the strain and stress tensors and is expressed as follows:

$$U = \frac{1}{2} \iint \left[\int_{-h/2}^{+h/2} \{\varepsilon\}^T \{\sigma\} dz \right] dx dy.$$
(9)

Eq. (9) can be modified further by substituting stress and strain relation, presented as follows:

$$U = \frac{1}{2} \iint \left(\{\bar{\varepsilon}\}^T [D] \{\bar{\varepsilon}\} \right) dx dy, \tag{10}$$

where $[D] = \int_{-h/2}^{+h/2} [T]^T [Q_{ij}][T] dz.$

The kinetic energy of the layered panel is presented in terms of mass density and velocity as follows:

$$T = \frac{1}{2} \int_{V} \rho\left\{\dot{d}\right\}^{T} \left\{\dot{d}\right\} dV, \tag{11}$$

where $\{\dot{d}\}$ and ρ represent the global velocity vector and mass density, respectively.

The free vibrated composite panel equation is formed using the necessary energy functional and solved via Hamilton's principle. Finally, the equation of motion is expressed in the following line:

$$d\int_{t_1}^{t_2} (T-U)dt = 0,$$
(12)

where T represents the kinetic energy, and U represents the strain energy.

Now, Eq. (12) can be rewritten by substituting value of d, T, and U from Eqs. (6), (10), and (11):

$$[M]\{\ddot{d}_i\} + [K]\{d_i\} = 0, \tag{13}$$

where [M], [K], \tilde{d}_i , and d_i denote the mass matrices, stiffness matrix, acceleration, and the displacement, respectively. The stiffness and mass matrix can be written further as follows:

$$[K] = \int_{A} [B_L]^T [D] [B_L] dA,$$

$$[M] = \int_{A} [N]^T [N] \rho dA.$$
 (14)

Further, the final governing equation used to evaluate the fundamental frequency response of the system in an eigenvalue form can be expressed as follows:

$$([K] - \omega^2[M])\{d\} = 0, \tag{15}$$



Figure 2. Representation of end conditions: (a) Simply-supported, (b) clamped, and (c) free end condition.

where ω is the fundamental frequency.

Eq. (15) can be solved by applying different sets of constraint conditions (Figure 2(a)-(c)) at the edges in order to decrease the number of unknowns and avoid any rigid body motion.

2.4. Newmark's integration scheme for transient analysis

In order to obtain the time-dependent deflection responses, Newmark's integration scheme has been employed. The governing equation of the transient responses accounts for the effects of the inertia, damping, and static deflection. The final form of the governing equation can be presented as follows:

$$[M]\ddot{d} + [C]\dot{d} + [K]d = [F], \tag{16}$$

where [M] represents the mass matrix, [C] denotes damping matrix, [K] represents the stiffness matrices, and [F] is the externally applied load vector. Further, the above-mentioned time-dependent motion equation is solved to compute the maximum deflection parameter at the centre of the panel for total time 'T'. The total time is divided into small time zones, e.g. time steps Δt and the values are calculated for each time step. Different integration parameters, such as α , δ , and a_0 to a_7 of used Newmark's integration, are assumed to be the same as those defined in [46]. The expression for the effective stiffness matrix at each time step is expressed as follows:

$$[\hat{K}] = [K] + a_0[M].$$
(17)

Similarly, the expression for the final load matrix and successive time step $(t + \Delta t)$ implementation of the present analysis is presented in the following lines:

$${}^{t+\Delta t}[\hat{F}] = {}^{t+\Delta t}[F] + [M](a_0{}^t d + a_2{}^t \dot{d} + a_3{}^t \ddot{d}).$$
(18)

Further, the expression of the displacement, acceleration, and velocity can be presented as follows:

$$[\hat{K}]^{t+\Delta t} d = {}^{t+\Delta t} [\hat{F}],$$

$${}^{t+\Delta t} \ddot{d} = a_0 ({}^{t+\Delta t} d - {}^t d) - a_2{}^t \dot{d} - a_3{}^t \ddot{d},$$

$${}^{t+\Delta t} \dot{d} = {}^t \dot{d} + a_6{}^t \ddot{d} + a_7{}^{t+\Delta t} \ddot{d}.$$
(19)

3. Results and discussion

The results and their corresponding discussions are reported in three major subsections. In the first section, the consistency and accuracy of the proposed HSDT models are examined via computing the dynamic and free vibration responses of different mesh refinements, which are utilised for the comparison with those of the earlier published responses. Subsequently, the second subsection describes the experimental evaluation of elastic properties and experimental dynamic responses of carbon/epoxy layered composite structure. After the validation of the present numerical model, some new numerical illustrations are computed theoretically to show the importance of the present model and the influence of the other design parameters on the dynamic responses.

3.1. Stability and accuracy investigation

The stability of the present numerical models, i.e. Model-1 and Model-2, and the simulation model, i.e. Model-3, has been examined by analysing the vibration and time-dependent deflection responses of layered composite flat plate structure for various mesh sizes, as shown in Figures 3(a) and 3(b), respectively. The non-dimensional natural frequencies of simply-supported two-layer and four-layer symmetric cross-ply $(0^{\circ}/90^{\circ})$ layered flat panels for five different thickness ratios are obtained using the geometrical parameter, the same



Figure 3(a). Convergence of non-dimensional frequency of a simply-supported two-layered cross-ply laminated flat panel: (i) Model-1, (ii) Model-2, and (iii) Model-3.



Figure 3(b). Convergence of non-dimensional frequency of a simply-supported four-layered symmetric cross-ply symmetric laminated flat panel: (i) Model-1 (ii) Model-2, and (iii) Model-3.

		Non-dimensional frequency						
Lamination Scheme	a/h	Model-1	Model-2	Model-3	Analytical [5]	3D Elasticity [47]	Analytical [48]	Analytical [49]
	4	7.9398	7.9043	7.4616	7.8908	8.3546	8.3546	8.0889
	10	10.4134	10.4955	10.2501	10.4156	10.5680	10.5680	10.4610
$(0^{\circ}/90^{\circ})$	20	11.0730	11.1693	11.0126	11.0509	11.1052	11.1052	11.0639
	50	11.3221	11.4209	11.2631	11.2537	11.2751	11.2751	11.2558
	100	11.3854	11.487	11.3005	11.2837	11.3002	11.3002	11.2843
	4	9.0866	9.2694	9.0672	9.271	9.3235	10.2032	9.3949
	10	15.1775	15.134	15.0329	15.0949	15.1073	15.9405	15.1426
(0°/90°) _s	20	17.6754	17.7127	17.6301	17.6434	17.6457	17.9938	17.6596
	50	18.7214	18.7833	18.6754	18.6713	18.6718	18.7381	18.6742
	100	18.9384	19.0044	18.8426	18.8355	18.8356	18.8526	18.8362

 Table 1. Non-dimensional frequency of a simply-supported two-layered and four-layered symmetric cross-ply laminated flat panels.

as in [5] and M1 properties. The non-dimensional natural frequency throughout the analysis is obtained via the formulae: $\bar{\omega} = \frac{\omega a^2}{h} \sqrt{\frac{\rho}{E_t}}$, where ω represents the fundamental frequency responses, if not stated otherwise. It has been seen from the convergence test that the responses are converging well with different mesh sizes. Accordingly, a mesh size of (6×6) is utilised for further investigation. In addition, the responses of five side-to-thickness ratios (a/h=4, 10, 20, 50 and 100) are obtained, and then they are compared with those of the analytical and numerical responses of references [5,47-49], as depicted in Table 1. Based on the comparison, it is inferred that the present results show good agreement with the published results of different solution approaches.

3.2. Experimental validation

In the present section, the free vibration responses are obtained experimentally for carbon/epoxy angleply flat panels of two- and four-layer types under two support conditions (SFSF and CFFF), and they are compared with the present FE responses obtained using all the three models. The comparison study is depicted in Table 2. In this example, the elastic properties of two- and four-layer composites are examined experimentally, namely M2 and M3, as presented in Table 3. For the evaluation, three specimens (along longitudinal, transverse, and inclined directions (45° to the longitudinal direction)) are prepared following the instruction given in the ASTM D3039/D3039M [50]. The specimens are tested using Universal Testing Machine (UTM) INSTRON-1195 at NIT, Rourkela. All the tests have been performed by fixing the loading rate as 1 mm/min. The UTM and the broken (tested) specimens of the carbon/epoxy layered composite are



Figure 4(a). Tensile testing machine (INSTRON 1195).

provided in Figures 4(a) and 4(b), respectively. It is necessary to mention that Poison's ratio for the current analysis is the same as that in [51]. Additionally, the shear modulus for each set of the laminate has been obtained via the general formula available in [52]:

$$\left(G_{lt} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_l} - \frac{1}{E_t} - \frac{2v_{12}}{E_l}}\right).$$

Now, the vibration test is conducted using the homemade experimental set-up at parent Institute (NIT Rourkela), and the corresponding data are recorded via cDAQ-9178 (National Instruments). The instrument is an eight-channel compact data acquisition

Support	Lamination	Mode	Frequency (Hz)				
$\operatorname{condition}$	\mathbf{scheme}	no.	Experiment	Model-1	Model-2	Model-3	
		1	88	91.36	97.31531	90.403	
		2	180	169.8	174.1709	169.34	
	$[\pm 45^{\circ}]$	3	377	380.1	409.3941	364.75	
		4	437	438.4	468.2937	435.7	
\mathbf{SFSF}		5	484	475.5	501.2964	463.28	
		1	161	165.577	175.1591	164.41	
		2	307	319.80	326.2955	320.40	
	$[\pm 45^{\circ}]_{s}$	3	667	683.90	730.8636	664.59	
		4	827	817.30	869.75	815.80	
		5	868	873.70	911.9091	862.04	
		1	33.5	32.79	35.45	32.58	
		2	81.5	79.88	81.73	78.933	
	$[\pm 45^{\circ}]$	3	210	207.38	217.93	200.67	
		4	249	257.44	262.14	253.05	
\mathbf{CFFF}		5	302	296.73	308.84	287.93	
		1	62.5	60.07	63.96	59.77	
		2	152	151.40	153.23	149.52	
	$[\pm 45^{\circ}]_{s}$	3	378	371.47	383.40	362.75	
		4	485	477.87	494.8	470.59	
		5	561	549.98	562.36	536.25	

Table 2. Natural frequency (Hz) of two-layered and four-layered symmetric angle-ply carbon/epoxy laminated composites flat panel under SFSF and CFFF support.

Table 3. Material properties.							
	Material-1	Material-2	Material-3				

Properties	Material-1	Material-2	Material-3	Material-4
	(M1)	(M2)	(M3)	(M4)
Young's modulus in x direction (E_l)	40 E_t	$6.695~\mathrm{GPa}$	6.469 GPa	$53.5~\mathrm{GPa}$
Young's modulus in y direction (E_t)	$1~{ m GPa}$	$6.314~\mathrm{GPa}$	$5.626~\mathrm{GPa}$	$2.1~\mathrm{GPa}$
Young's modulus in z direction (E_z)	E_t	$6.314~\mathrm{GPa}$	$5.626~\mathrm{GPa}$	$53.5~\mathrm{GPa}$
Shear modulus (G_{lt})	$0.6 E_t$	$2.7~\mathrm{GPa}$	$2.05~\mathrm{GPa}$	$1.05~\mathrm{GPa}$
Shear modulus (G_{tz})	$0.5 E_t$	$1.35~\mathrm{GPa}$	$1.025~\mathrm{GPa}$	$1.05~\mathrm{GPa}$
Shear modulus (G_{lz})	$0.6 E_t$	$2.7~\mathrm{GPa}$	$2.05~\mathrm{GPa}$	$1.05~\mathrm{GPa}$
Poisson's ratio (ν_{lt})	0.25	0.17	0.17	0.25
Poisson's ratio (ν_{lz})	0.25	0.17	0.17	0.25
Poisson's ratio (ν_{tz})	0.25	0.17	0.17	0.25
Density (ρ)	$1900 {\rm ~kgm^{-3}}$	$1900 {\rm ~kgm^{-3}}$	$1900 {\rm ~kgm^{-3}}$	$800 \ {\rm kgm^{-3}}$

system that can be used for a variety of mechanical measurements (temperature, strain, load, pressure, torque, acceleration, and acoustics) and a pictorial form of the same, as presented in Figure 5(a). The frequency responses are recorded for the carbon/epoxy composite flat panel structure under SFSF support condition. Initially, the panel is excited with an

electronic impact hammer at any arbitrary point on the structure (Figure 5(b)), and the output signal is sensed via an accelerometer mounted on the structural panel. The accelerometer is a type of sensor that captures the acceleration, converts it into an analogue voltage signal, and processes to a cDAQ where the analogue signal is further converted into digital signal via the



Figure 4(b). Carbon/epoxy composite flat panel specimen after tensile test.



Figure 5(a). CDAQ 9178 national instruments and display.



Figure 5(b). Experimental set-up for free vibration analysis.

inbuilt analogue-digital converter. Now, the signal is processed further via a well-known signal processing software, named LABVIEW. The LABVIEW operates through three main panels: front panel, block diagram, and the connector panel. The front panel is also called the user interface panel where the recorded data can be seen in the form of a graph or numeric as per user's interest. Further, the block diagram, which is a programming window, and the necessary programming can be changed, called Virtual Instrument (VI). This can be performed to process the input signal and get the desired form of output. Herein, the block diagram is mainly used to capture the frequency responses of the laminate structure, whose details are provided in Figure 5(c). The input acceleration signal coming from the cDAQ now passes through a power spectrum module, as shown in the block diagram, to convert it into the time-domain and frequency-domain forms. The necessary frequency responses obtained from the acceleration signal are kept for the future use, i.e. validation purposes. Finally, the captured frequency responses of carbon/epoxy layered composite plate have been compared with the numerical and simulated responses computed from the proposed and ANSYS models, as shown in Table 2. The comparison study clearly indicates the accuracy and necessity of the current HSDT models (Model-1 and Model-2) instead of FSDT model, i.e. Model-3.

3.3. Transient response

Now, the present models are extended to compute the transient behaviour of three different flat panel cases. In the present comparison study, the transient behaviors of a single-layered orthotropic plate and a four-layered square angle-ply laminated composite plate with simply-supported edges are examined under uniformly distributed step load $(q_0 = 0.1 \text{ N/mm}^2)$ The transient responses are computed using the present numerical models (Model-1 and Model-2) as well as the simulation model ANSYS (Model-3) by setting the time step to 10 μ s. For this analysis, the panel dimension of 250 mm length and 5 mm height is taken with M4 material properties, as given in Table 3. The transient responses of single-layer and four-layer angleply $(\pm 45^{\circ})_{\rm s}$ layered composite panels are compared with the available published responses [53,54], as plotted in Figures 6(a) and 6(b), respectively. The figures clearly show that the responses are in close agreement with the previously reported responses.

3.4. Numerical examples

The convergence and validation results clearly indicate that the present developed HSDT models are capable enough to analyse the time-dependent deflection and vibration characteristics of the layered composite structure with adequate accuracy. Now, the models are



Figure 5(c). Block diagram of the LABVIEW software.



Figure 6(a). Central deflection versus time response of single-layered orthotropic laminated flat panel to a step load (0.1 N/mm²).

utilised further to solve a new example to enhance the quantitative understanding on influence of the design parameters (the thickness ratios, the curvature ratios, the support conditions) on dynamic responses of the layered composite plate/shell structure.

3.4.1. Influence of support conditions on fundamental frequency response

It is well known that different support conditions of the composite structure affect the overall stiffness and, further, the structural responses significantly. In the current example, the influence of the support



Figure 6(b). Central deflection versus time response of simply-supported four-layered angle-ply laminated flat panel to a step load (0.1 N/mm^2) .

condition on frequency responses of the square fourlayer cross-ply laminated plates with M1 material properties has been investigated. The responses are evaluated for five different end conditions: CCCC (alledge clamped), SCSC (two-edge simply-supported and two-edge clamped), SSSS (all-edge simply-supported), CFCF (two-edge clamped and two-edge free), and CFFF (one-edge clamped and others free, i.e. cantilever) and five side-to-thickness ratios (a/h = 2, 4,10, 20, 50, and 100) using Model-1 and presented in Figure 7. It is observed from the Figure that the nonlinear vibration responses are in the ascending order with CFFF, SSSS, SCSC, CFCF, and CCCC end



Figure 7. Effect of support conditions on the non-dimensional frequency of four-layered symmetric cross-ply laminated composite flat panel.

conditions irrespective of the side-to-thickness ratio. It is due to the increase in the overall stiffness with the increasing number of constraints.

3.4.2. Influence of curvature ratio on fundamental frequency response

The shell panel can be easily categorized into a deep or shallow shell panel, based on its curvature ratio. As the shell panel changes its geometry from shallow to deep, the stretching and bending energies change, i.e. the stretching energy becomes high compared to bending energy, which significantly affects the structural response. In this illustration, fundamental frequency responses of different shell panels (cylindrical, spherical, hyperboloid, and ellipsoid) with simply-supported boundaries have been analysed for fine different curvature ratio (R/a = 20 to 40, 60, 80 and 100) using Model-1, as presented in Figure 8. The frequencies



Figure 8. Effect of curvature ratio on the non-dimensional frequency of four-layered symmetric cross-ply laminated composite panel.

of four-layer symmetric cross-ply layered composite structure are obtained with M1 material properties by taking a/h = 20. It has been noted from the present example that the non-dimensional fundamental frequency responses are decreasing for each of shell geometries, except the hyperboloid panel due to the unequal curvature.

3.4.3. Influence of aspect ratio on fundamental frequency response

The aspect ratio (a/b) of any structural component plays a major role in stiffness and stability behaviour and becomes more important for thin laminated curved panels. In this example, Model-1 is employed to calculate the natural frequency responses of the fourlayer symmetric cross-ply layered composite structure with simply-supported edges for five different aspect ratios (a/b = 1, 1.5, 2, 2.5, and 3). The responses for all different geometries (hyperboloid, spherical, cylindrical, flat and ellipsoid) are computed utilising M1 composite properties and taking a/h = 50 and R/a= 5, as presented in Figure 9. It has been seen from the figure that the non-dimensional fundamental frequency values are increasing as the aspect ratio increases for each of the shell geometries. However, the differences between the results become insignificant after a/b =2 for each of shell geometries, except the flat panel case. It has also been noticed that the highest and lowest frequencies are obtained for the spherical and flat panels, respectively.

3.4.4. Influence of shell geometry on fundamental frequency response

In this illustration, the influence of the shell panel geometries (hyperboloid, flat, spherical, cylindrical, and ellipsoid) on the free vibration responses of laminated composite structure is investigated. For the investiga-



Figure 9. Effect of aspect ratio on the non-dimensional frequency of four-layered symmetric cross-ply laminated composite panel.



Figure 10. Effect of shell geometry on the non-dimensional frequency of four-layered symmetric cross-ply laminated composite panel (R/a = 10).

tion, the frequencies of simply-supported square fourlayer cross-ply symmetric layered composite panel (R/a = 10) with M1 material properties are obtained for six different thickness ratios (a/h = 2, 4, 10, 20, 50, and 100), as depicted in Figure 10. It has been observed from the figure that the variations of the responses are insignificant for thick laminates (a/h = 2, 4, and 10), whereas the differences are pronounced for thin panels (a/h = 20, 50 and 100). In addition, it has been observed that the maximum frequencies are obtained for the spherical geometry, and the least frequency is obtained for the flat panels.

3.4.5. Influence of thickness ratio on time-dependent deflection response

This example has been solved to obtain the timedependent deflection responses of the square, simplysupported four-layer angle-ply $(\pm 45^{\circ})_{\rm s}$ layered composite panel. The mentioned responses are obtained using Model-1 for five different thickness ratios (a/h = 30, 35, 40, 45, and 50) with M4 material properties under the uniform step loading of 0.1 N/mm², as presented in Figure 11. It is observed from the figure that the time-dependent displacement response increases as the thickness ratio increases; thus, the responses' frequencies decrease.

3.4.6. Influence of shell geometries on transient behaviour

The time-dependent transverse central deflection responses of different layered composite shell panels (hyperboloid, cylindrical, flat, ellipsoid, and spherical) are examined in the current example. The responses are calculated for square, simply-supported four-layer angle-ply $(\pm 45^{\circ})_{\rm s}$ layered composite structure (R/a = 10, a/h = 50) under the uniform step loading of 0.1 N/mm² through the use of Model-1 and M4 properties.



Figure 11. Effect of thickness ratio on the central deflection of simply-supported four-layered angle-ply laminated flat panel.



Figure 12. Effect of curvature ratio on the central deflection of simply-supported four-layered angle-ply laminated flat panel.

The calculated responses are presented in Figure 12. The figure shows that the central deflection responses are at maximum for flat panel and at minimum for the spherical shell panel. In addition, it is inferred that the responses of the hyperboloid and flat panel are close to each other in a few instances of time.

4. Conclusions

The fundamental frequency and transient behaviour of the carbon/epoxy layered composite flat/curved shallow shell panels were investigated numerically by developing two FE models in HSDT kinematic. Further, a MATLAB code was prepared based on the proposed models to compute the numerical responses and compare them with the subsequent experimental responses. In addition, a simulation model in ANSYS software was developed, and the response obtained using simulation software was also compared with that of the present experimental and numerical results. The validity of the proposed models was also checked by comparing the present numerical results with the result of the published literature. Additionally, the importance of the present models and the influences of the different design parameter were illustrated by solving a new numerical illustration. Based on the convergence, validation, and parametric study, the following conclusions were drawn and discussed below:

- a) The convergence and validation study of the proposed HSDT models clearly indicate that the presented models are suitable for the fundamental frequency and time dependent-deflection response of the layered composite flat/curved shallow shell structure;
- b) The parametric studies show that different geometries of the shell panel considerably affect both vibration and transient responses;
- c) The fundamental frequencies are at maximum for the spherical and at minimum for the plate structure. In addition, the time-dependent deflection responses are at maximum for the plate and at minimum for the layered spherical composite structure;
- d) The side-to-thickness ratio, constraint conditions, curvature ratio, and aspect ratio affect the fundamental frequency and time-dependent transverse deflection of the flat/curved shallow shell panel significantly.

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