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Topological and non-topological soliton solutions to the 1 + 3-dimensional Gross-Pitaevskii equation with quadratic potential term

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Abstract. This paper carries out the integration of the 1 + 3-dimensional Gross-Pitaevskii Equation (GPE) in presence of quadratic potential term to obtain the 1-soliton solutions. The solitary wave ansatz method is employed to integrate the considered equation. Parametric conditions for the existence of the soliton solutions are determined. Both non-topological (bright) and topological (dark) soliton solutions are reported and we observed that the existence condition for bright and dark soliton solutions were opposite to each other. Finally, the two integrals of motion of the governing model equation have been extracted.

1. Introduction

Solitary waves have been the focus of robust research interest because of their potential applications in several fields of physics and mathematics. Such applications range from plasmas physics, fluid dynamics, fiber-optic communications, and photonics, in general, to Bose-Einstein Condensates (BECs) and nuclear physics and in a variety of natural phenomena from water and plasma waves to crystal lattice vibrations and energy transport in proteins [1,2]. The study of solitons in various physical systems reveals many exciting problems from both fundamental and applied points of view. The basic tactic one may adopt to predict, control, and quantify the underlying features of a system under study is to model the system in terms of mathematical equations, which are generally nonlinear and find exact analytic solutions to such model equations using some suitable methods. The importance of obtaining the exact solutions, if available, to those nonlinear equations is that they facilitate the verification of numerical solutions and aid in the stability analysis of solutions. From a theoretical point of view, dilute BECs are an attractive topic due to the fact that the many-body system can be characterized to first-order by a mean-field macroscopic order parameter or wave-function. Furthermore, this macroscopic wave-function is well-described by a nonlinear Schrödinger-type equation, called the Gross-Pitaevskii Equation (GPE), which is essential for the description of BECs [3]. It was introduced by Gross and Pitaevskii for unrelated problems, but has later been found useful in different quantum systems [4,5]. Solutions to GPE are of great interest, because they can be applied to a variety of physical systems. Various solutions to GPE have been discovered, including localized (solitary) waves. However, the proven stable soliton solutions to GPE exist only in 1 + 1 dimensions and there are no known exact stable solitons in higher dimensions. It is well known that in 1 + 1 dimensions, this equation is completely integrable by Inverse Scattering Transform technique

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(IST). In $1+2$ and $1+3$ dimensions, the equation is not integrable. Very recently, T. Mayteevarunyoo et al. constructed families of vortical, quadrupole, and fundamental solitons in a 2-dimensional nonlinear-Schrödinger/Gross-Pitaevskii equation, which modeled BECs or photonic crystals [6]. The equation included the attractive or repulsive cubic nonlinearity and an anisotropic periodic potential. There are many new results that are being constantly reported in various journals in this area [7-9].

The integrability aspects of various kinds of nonlinear evolution equations were studied for several years by the classical method of IST that was a monopoly for decades. But, it is no longer the case. In fact, nowadays, there are various modern methods of integrability that are used to integrate these different kinds of nonlinear evolution equations. Some of these common methods of integrability are F-expansion method, Projective Ricatti equation method, Lie symmetry analysis method, G'/G method of integrability, He's semi-inverse variational principle, tanh-coth method, and many more [10-17]. However, one needs to be careful in applying these methods of integrability as it could lead to incorrect results. This fact was pointed out by Kudryashov et al. in 2009 [18]. Although a closed-form soliton solution can be obtained by these techniques, a couple of shortcomings of these methods, as opposed to the classical method of IST, are that these methods cannot compute the conserved quantities of these equations nor can they lay down an expression of the soliton radiation. However, the fact that soliton solutions can be obtained is itself a big blessing.

In this paper, the solitary-wave ansatz method will be applied to recover the topological and non-topological soliton solutions to the generalized $1+3$-dimensional GPE with quadratic potential term. It should be noted that this method of soliton ansatz is very similar to the exponential function method.

2. Theoretical model

Here, we consider the generalized GPE in $1+3$ dimensions with constant coefficients in the following form [9]:

$$iU_t+\frac{\beta}{2}(U_{xx}+U_{yy}+U_{zz})+\chi|U|^2U+\alpha r^2U=0, \qquad (1)$$

where U is normalized to the total number of atoms N; $\int |U|^2 d\mathbf{r}=N$. t is the reduced time, i.e. time in the frame of reference moving with the wave packet; $r=\sqrt{x^2+y^2+z^2}$ is the radial position coordinate; and α is the strength of the quadratic potential that can be attractive or expulsive for $\alpha<0$ or $\alpha>0$, respectively. The attractive and expulsive traps are, in general, approximated by a harmonic oscillator potential. The functions β and χ stand for the diffraction and the nonlinearity coefficients, respectively. Generally, nonlinearity coefficient χ has a time-dependence form given as $\chi(t)=4\pi\hbar^2|a_s(t)|/M_0$, which illustrates the interaction function, with $a_s(t)$ being the s-wave scattering length modulated by the Feshbach resonance and M_0 being the atomic mass of the condensate. For the sake of generality, we have kept $\chi(t)$ time-independent by removing time dependence from the scattering length parameter, a_s. Here, in Eq. (1), the first term represents the evolution term and the second, third, and fourth terms, in parentheses, represent the dispersion in x, y, and z directions while the fifth term represents cubic nonlinearity. Solitons are the result of a delicate balance between dispersion and nonlinearity. All coordinates are made dimensionless by the choice of coefficients. When the coefficients are constant, the behavior of solutions toward the GPE strongly depends on the dimensionality of the problem. It is known that this equation supports solitons that are studied in the context of BEC and nonlinear optics [19-21]. Now, Eq. (1) will be integrated to obtain the exact 1-soliton solution [22-35].

3. Mathematical analysis

3.1. Bright 1-soliton

Bright solitons are known as bell-shaped solitons or non-topological solitons. The bright soliton is regarded as a localized intensity peak above a continuous wave background. These kinds of solitary waves are modeled by sech function.

In this paper, Eq. (1) will be integrated by the aid of solitary wave ansatz. Therefore, based on the soliton solution in $1+1$ and $1+2$ dimensions, it is first necessary to write the solution to Eq. (1) in the phase-amplitude form as:

$$U(x,y,z,t)=\frac{A}{\cosh^p[B_1x+B_2y+B_3z-vt]}e^{i(-k_1x-k_2y-k_3z+\omega t+\theta)}. \qquad (2)$$

Here, in Eq. (2), A is the amplitude of the soliton and B_1, B_2, and B_3 are the inverse widths of soliton in the x, y, and z directions, respectively, and v represents the velocity of the soliton. Also, k_1, k_2, and k_3 represent the soliton frequencies in the x, y, and z directions, respectively, while ω represents the solitary wave number and, finally, θ is the soliton phase constant. The exponent p will be derived during the course of derivation of the soliton solution. From Eq. (2), we have:

$$U_t=\left[pvA\frac{\tanh(\eta)}{\cosh^p(\eta)}+\frac{i\omega A}{\cosh^p(\eta)}\right]e^{i\phi}, \qquad (3)$$

$$U_{xx}=\left[\frac{p^2AB_1^2}{\cosh^p(\eta)}-\frac{p(p+1)AB_1^2}{\cosh^{p+2}(\eta)}-\frac{k_1^2A}{\cosh^p(\eta)}+2ik_1pAB_1\frac{\tanh(\eta)}{\cosh^p(\eta)}\right]e^{i\phi}, \tag{4}$$

$$U_{yy}=\left[\frac{p^2AB_2^2}{\cosh^p(\eta)}-\frac{p(p+1)AB_2^2}{\cosh^{p+2}(\eta)}-\frac{k_2^2A}{\cosh^p(\eta)}+2ik_2pAB_1\frac{\tanh(\eta)}{\cosh^p(\eta)}\right]e^{i\phi}, \tag{5}$$

$$U_{zz}=\left[\frac{p^2AB_3^2}{\cosh^p(\eta)}-\frac{p(p+1)AB_3^2}{\cosh^{p+2}(\eta)}-\frac{k_3^2A}{\cosh^p(\eta)}+2ik_3pAB_3\frac{\tanh(\eta)}{\cosh^p(\eta)}\right]e^{i\phi}, \tag{6}$$

where $\eta=B_1x+B_2y+B_3z-vt$ and $\phi=-k_1x-k_2y-k_3z+\omega t+\theta$.

Substituting Eqs. (3)-(6) into Eq. (1) and equating the real and imaginary parts yield the following pair of relations:

$$-\frac{\omega A}{\cosh^p(\eta)}+\frac{\beta}{2}\left[\frac{p^2A(B_1^2+B_2^2+B_3^2)}{\cosh^p(\eta)}-\frac{p(p+1)A(B_1^2+B_2^2+B_3^2)}{\cosh^{p+2}(\eta)}-\frac{A(k_1^2+k_2^2+k_3^2)}{\cosh^p(\eta)}\right]+\frac{\chi A^3}{\cosh^{3p}(\eta)}+\frac{\alpha r^2A}{\cosh^p(\eta)}=0, \tag{7}$$

$$pvA\frac{\tanh(\eta)}{\cosh^p(\eta)}+p\beta A(k_1B_1+k_2B_2+k_3B_3)\frac{\tanh(\eta)}{\cosh^p(\eta)}=0. \tag{8}$$

From Eq. (7), noting that $\frac{1}{\cosh^{p+j}(\eta)}$ is a linearly independent function for $j=0,2$, its coefficients must be, respectively, set to zero. Now, from Eq. (7), equating the exponents $p+2$ and $3p$ yields $p=1$. Again, equating the coefficients of $1/\cosh^{3p}(\eta)$ and $1/\cosh^{p+2}(\eta)$ in Eq. (7) gives:

$$A=\sqrt{\beta\sum_{i=1}^{3}(B_i^2)/\chi}. \tag{9}$$

Relation (9) leads to the constraint condition $\beta\sum_{i=1}^{3}(B_i^2)/\chi>0$, which must be valid for the existence of solitary waves in Eq. (1). Finally, equating the coefficients of $1/\cosh^p(\eta)$ in Eq. (7) yield:

$$\omega=\frac{\beta}{2}\sum_{i=1}^{3}(B_i^2-k_i^2)+\alpha(x^2+y^2+z^2). \tag{10}$$

Now, in Eq. (8), setting the coefficients of linearly independent functions $\tanh(\eta)/\cosh^p(\eta)$ equal to zero yields the velocity for bright soliton, given by:

$$v=-\beta\sum_{i=1}^{3}(k_iB_i). \tag{11}$$

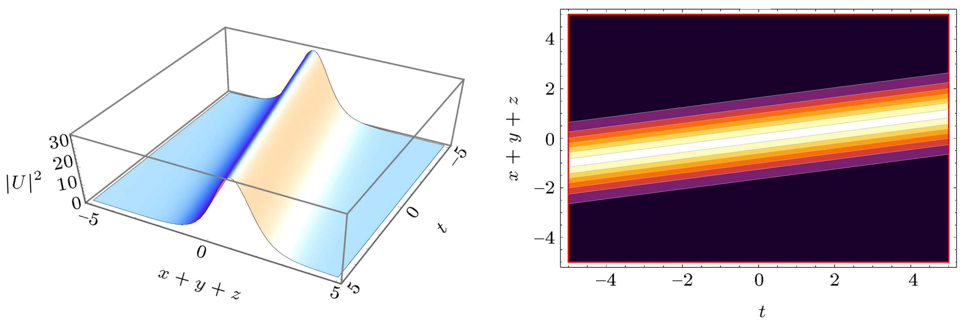
Thus, the 1-soliton solution to the GPE in 1 + 3 dimensions is given by:

$$U(x,y,z,t)=\frac{A}{\cosh^p[B_1x+B_2y+B_3z-vt]}e^{i(-k_1x-k_2y-k_3z+\omega t+\theta)}, \tag{12}$$

where the amplitude is related to the inverse widths in the x, y, and z directions and is given by Eq. (9). The velocity of the soliton is given by Eq. (11), the wave number is given by Eq. (10), and the corresponding intensity of the bright soliton solution takes the form:

$$|U(x,y,z,t)|^2=\frac{\beta\sum_{i=1}^{3}(B_i^2)}{\chi}\mathrm{sech}^2[B_1x+B_2y+B_3z-vt]. \tag{13}$$

Figure 1 shows the numerical simulation of the solution



(a) (b)

Figure 1. Profile of the 1-bright solitary wave solution.

with the parameters chosen by $B_1 = B_2 = B_3 = 1.0$, $\beta = 1$, $\chi = 1.0$, and $v = 0.2$. This set of parameter values, which satisfy the constraint relations, is chosen in order to perform numerical simulation.

3.2. Dark 1-soliton

The dark solitons are also known as topological solitons or simply topological defects. The dark soliton is characterized by a localized drop of intensity related to a more intense continuous wave background. The form of the solution in this case is assumed as:

$$U(x,y,z,t) = A\tanh^p[B_1x + B_2y + B_3z - vt)]e^{i(-k_1x-k_2y-k_3z+\omega t+\theta)}, \tag{14}$$

where A and B_i are free parameters and v is the velocity of the dark wave soliton, whose values are to be determined.

The soliton solution exists for the exponent $p > 0$. From Eq. (14), we derive:

$$U_t = [pvA\{\tanh^{p+1}(\eta) - \tanh^{p-1}(\eta)\} + i\omega A\tanh^p(\eta)]e^{i\phi}, \tag{15}$$

$$\begin{aligned} U_{xx} = & [p(p-1)AB_1^2\{\tanh^{p-2}(\eta) - \tanh^p(\eta)\} \\ & + p(p+1)AB_1^2\{\tanh^{p+2}(\eta) - \tanh^p(\eta)\} \\ & + 2ipk_1AB_1\{\tanh^{p+1}(\eta) - \tanh^{p-1}(\eta)\} \\ & - k_1^2A\tanh^p(\eta)]e^{i\phi}, \end{aligned} \tag{16}$$

$$\begin{aligned} U_{yy} = & [p(p-1)AB_2^2\{\tanh^{p-2}(\eta) - \tanh^p(\eta)\} \\ & + p(p+1)AB_2^2\{\tanh^{p+2}(\eta) - \tanh^p(\eta)\} \\ & + 2ipk_2AB_2\{\tanh^{p+1}(\eta) - \tanh^{p-1}(\eta)\} \\ & - k_2^2A\tanh^p(\eta)]e^{i\phi}, \end{aligned} \tag{17}$$

$$\begin{aligned} U_{zz} = & [p(p-1)AB_3^2\{\tanh^{p-2}(\eta) - \tanh^p(\eta)\} \\ & + p(p+1)AB_3^2\{\tanh^{p+2}(\eta) - \tanh^p(\eta)\} \\ & + 2ipk_3AB_3\{\tanh^{p+1}(\eta) - \tanh^{p-1}(\eta)\} \\ & - k_3^2A\tanh^p(\eta)]e^{i\phi}, \end{aligned} \tag{18}$$

where $\eta = B_1x + B_2y + B_3z - vt$ and $\phi = -k_1x - k_2y - k_3z + \omega t + \theta$.

Substituting Eqs. (15)-(18) into Eq. (1) and equating the real and imaginary parts equal to zero yield the following set of equations:

$$\begin{aligned} & -\omega A\tanh^p(\eta) + \frac{\beta}{2}[p(p+1)A(B_1^2 + B_2^2 + B_3^2) \\ & \{\tanh^{p+2}(\eta) - \tanh^p(\eta)\} \\ & - A(k_1^2 + k_2^2 + k_3^2)\tanh^p(\eta) \\ & + p(p-1)A(B_1^2 + B_2^2 + B_3^2) \\ & \{\tanh^{p-2}(\eta) - \tanh^p(\eta)\}] \\ & + \chi A^3\tanh^{3p}(\eta) + \alpha r^2A\tanh^p(\eta) = 0, \end{aligned} \tag{19}$$

$$\begin{aligned} & pvA\{\tanh^{p+1}(\eta) - \tanh^{p-1}(\eta)\} \\ & + \frac{\beta}{2}[2ipA(k_1B_1 + k_2B_2 + k_3B_3)\{\tanh^{p+1}(\eta) \\ & - \tanh^{p-1}(\eta)\}] = 0. \end{aligned} \tag{20}$$

Now, from Eq. (19), equating the exponents $p+2$ and $3p$ leads to $p = 1$. Setting coefficients of linearly independent functions $\tanh^{p+j}(\eta)$ to zero for $j = 0, \pm 1, \pm 2$ in Eqs. (19) and (20) gives the expressions for free parameter, A, wave number, ω, and dark soliton velocity, v. It is to be noted that the coefficients of the linearly independent functions $\tanh^{p-2}(\eta)$ in Eq. (19) are spontaneously zero for $p = 1$.

Equating the coefficients of $\tanh^{3p}(\eta)$ and $\tanh^{p+2}(\eta)$ in Eq. (19) gives:

$$A = \sqrt{-\beta\sum_{i=1}^{3}(B_i^2)/\chi}. \tag{21}$$

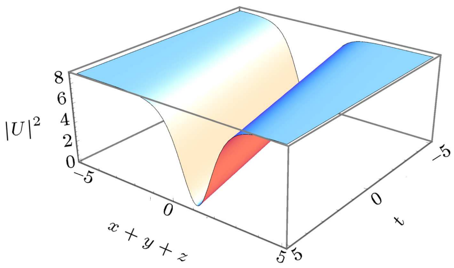
Eq. (21) leads to the constraint condition $\beta\sum_{i=1}^{3}(B_i^2)/\chi < 0$, which must be valid for the existence of 1-dark solitary waves in Eq. (1). Finally, equating the coefficients of $\tanh^p(\eta)$ in Eq. (19) yields:

$$\omega = -\beta\sum_{i=1}^{3}(B_i^2 + k_i^2) + \alpha(x^2 + y^2 + z^2). \tag{22}$$

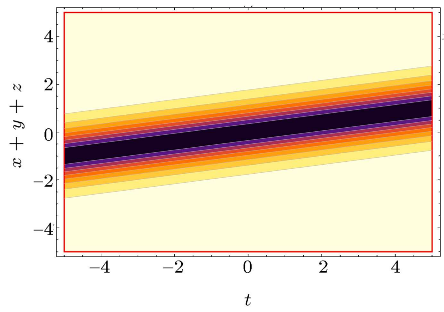
On equating the coefficients of $\tanh^p(\eta)$ in Eq. (20), we obtain:

$$v = -\beta\sum_{i=1}^{3}(k_iB_i). \tag{23}$$

From Eqs. (11) and (23), we conclude that the bright and dark solitary waves move in the same direction with equal velocity. Again, equating the coefficients of $\tanh^{p+1}(\eta)$ in Eq. (20) results in the same value for v



(a)



(b)

Figure 2. Profile of the 1-dark solitary wave solution.

as given by Eq. (23). Hence, the one topological soliton solution to the GPE in $1+3$ dimensions is given by:

$$U(x,y,z,t)=A\tanh^p[B_1x+B_2y+B_3z - vt]e^{i(-k_1x-k_2y-k_3z+\omega t+\theta)}, \quad (24)$$

where the free parameter A is related to the parameter B_i in the x, y, and z directions as given by Eq. (21); the velocity of the dark 1-soliton is given by Eq. (23); and, finally, the wave number is given by Eq. (22).

The corresponding intensity of the dark soliton profiles takes the form:

$$|U(x,y,z,t)|^2=\frac{-\beta\sum_{i=1}^{3}(B_i^2)}{\chi}\tanh^2[B_1x+B_2y + B_3z-vt]. \quad (25)$$

Figure 2 shows the intensity profile of 1-dark solitary wave solution (Eq. (25)) for different model coefficients, which satisfy the constrained condition. The parameters for numerical simulation of the dark soliton solution are chosen by $B_1=B_2=B_3=1.0$, $\beta=1$, $\chi=1.0$, and $v=0.2$.

It is worth mentioning that the existence of bright and dark soliton solutions given by Eqs. (12) and (24) depends on the specific nonlinear and dispersive features of the medium, which have to satisfy the parametric constrained conditions. Also, from Eqs. (12) and (24), we note that the formation conditions of the bright and dark solitary waves are opposite to each other.

4. Integrals of motion

The GPE in $1+3$ dimensions, with cubic nonlinearity and quadratic potential term, has at least two integrals of motion. They are power (P) and linear momentum (M) respectively given by:

$$P=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|U|^2dxdydz, \quad (26)$$

$$M=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{(UU_x^*-U^*U_x)+(UU_y^*-U^*U_y) + (UU_z^*-U^*U_z)\}dxdydz. \quad (27)$$

In order to calculate the conserved quantities, the 1-soliton solutions given by Eqs. (12) and (24) can be used.

5. Conclusions

In this paper, the soliton ansatz is used to obtain the topological and non-topological 1-soliton solutions to the $1+3$-dimensional GPE in presence of quadratic potential term. The physical parameters in the soliton solutions are obtained as a function of the dependent model coefficients. Parametric conditions for the existence of envelope solitons have also been reported. The method used is far less complex than the standard techniques, which are used to study this kind of problems. The solitary wave ansatz is used to carry out the integration of this equation since, as it was pointed out, the classical method of IST will not work in this case. In future, this method will be extended to obtain the topological and non-topological 1-soliton solutions to the governing equation with time-dependent coefficients.

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