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A simple model for various types of concretes and confinement conditions based on disturbed state concept

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Abstract. Concrete is widely used for many practices in Civil Engineering. Therefore, an understanding of its behavior helps engineers and researchers to perform more accurate and cost-effective analyses and designs. In this respect, several models have been proposed to predict the behaviors of concrete most of which are satisfactorily accurate. However, by increasing the accuracy of the models, their computational cost increases, too. In this study, a model with the least computational cost is proposed to predict the behaviors of various concretes and confinement conditions. This model does not require any experimental tests to determine its parameters. It was proved to be able to predict the behaviors of various concretes, including Normal-Weight Concrete (NWC) and Light-Weight Concrete (LWC), from other researchers. This model can also be applied to two- and three-dimensional problems. Moreover, the confinement conditions of concretes were considered. The predictions were in good accordance with the experimental results.

1. Introduction

Concrete is one of the most widespread materials used for construction of buildings, foundations, walls, pavements, bridges, etc. According to Mehta [1], concrete consumption is about seven billion tons a year worldwide, and it is expected to be increasing until 2050. Hence, the need to improve our understanding of the behavior of concretes is essentially a continuous one. In this respect, many models have been proposed to predict the mechanical properties of concrete. Yet, for a more economical and reliable design, prediction of the complete stress-strain curve is necessary.

Several models have been proposed to predict the complete stress-strain curve of concretes. However, they are often restricted to a specific type of concrete or loading condition. On the other hand, most of those models are mathematical models and only able to predict the behavior of concrete under uniaxial loading. More than 110 stress-strain models have been developed for predicting the axial compressive behaviors of unconfined and actively confined concretes [2]. This multiplicity of models may lead to the confusion of engineers and designers in choosing an appropriate model.

In addition to the mathematical models, there are elastic and elasto-plastic models, which can be used to model two- or three- dimensional problems. However, most of them are not capable of modeling the softening behavior of concrete and need experimental studies to determine the required parameters. For a more accurate and cost-effective design, it is necessary to consider other types of models. One of the models capable of capturing the hardening/softening behavior of materials is the Disturbed State Concept (DSC)

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model proposed by Desai [3]. A distinguishing feature of the DSC is that it takes into account the interactions between the parts of a material as affected by the microstructural self-adjustment of particles. However, its formulation does not require particle or micro-level characterization. As a result, the need of particle-level constitutive laws, which are required in the micromechanical and microcrack interaction models, is eliminated [4]. The only advantage of the mathematical models, in comparison with the DSC, is that they do not require experimental tests to determine parameters. On the other hand, the DSC can be used for various materials such as consolidation of clays [5], interfaces [6], and polymer concrete [7].

In this study, a simple relation is given for the relatively intact state of the DSC. Then, the distribution of disturbance in the DSC is simplified to eliminate the need of experimental tests to determine disturbance parameters. This disturbance distribution model is simply based on different parameters of concrete, such as ultimate strength, density, and confinement conditions, and does not require further experimental tests. Furthermore, since there is a physical justification, this disturbance distribution model can be used for two- and three- dimensional problems or in commercial packages such as Abaqus. Finally, over 50 experimental tests from other researchers were modeled using the proposed model, and the results were compared to verify the accuracy.

2. Modeling

The DSC model is based on the idea of decomposition of a material into its Relatively Intact (RI) and Fully Adjusted (FA) components [3]. The decomposition of typical concrete and its relation to the cracks of concrete are depicted in Figure 1. As can be seen, before the peak, there is almost no significant disturbance in concrete (compared to that of post peak), that is, material shows RI behavior. Although there are some cracks and the response is not entirely elastic, it is insignificant and can be considered as elastic response altogether. After the peak, as the cracks in concrete grow (propagation of disturbance), concrete loses its strength until it reaches the FA state, which has different definitions corresponding to the confinement condition, to be discussed in the following.

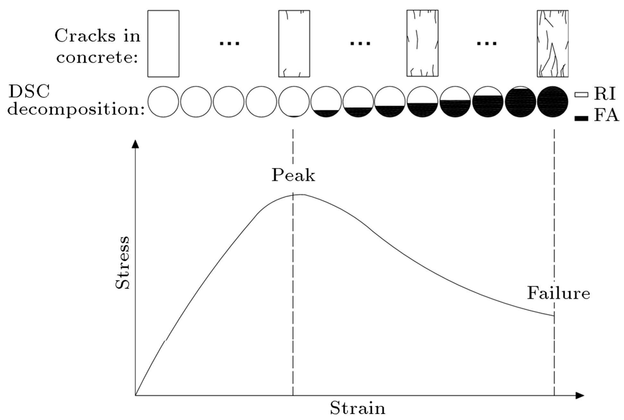

Figure 1. Typical stress-strain curve of concrete including DSC decomposition and cracks distribution.

2.1. Disturbance

In this study, disturbance expression is similar to the one used conventionally [8], and the definition of disturbance is based on the concept proposed by Desai [9] as an extension to the uniaxial model proposed by Kachanov [8] and Rabotnov [10]. Thus, the disturbance parameter, D, is [11,12]:

$$D = D_u(1 - e^{-A\zeta_D^Z}), \tag{1}$$

where D_u is the ultimate value of D; A and Z are material parameters; and ζ_D is the trajectory of deviatoric plastic strain:

$$\zeta_D = \int (de_{ij}^p de_{ij}^p)^{0.5}, \tag{2}$$

where e_{ij} is the deviatoric strain tensor of total strain tensor ε_{ij}.

As proposed by Desai [13] and Ma and Desai [14], the disturbance based on the stress-strain response can be expressed generally as follows:

$$D = \frac{\sigma^{\mathrm{RI}} - \sigma^{\mathrm{obs}}}{\sigma^{\mathrm{RI}} - \sigma^{\mathrm{FA}}}, \tag{3}$$

where σ^{RI}, σ^{obs}, and σ^{FA} are relatively intact, observed, and fully adjusted stresses, respectively. Equating Eq. (1) with Eq. (3) for two arbitrary points of the experimental stress-strain curve yields A and Z values. After calculating material parameters, the stress-strain curve can be obtained through the following incremental equation:

$$d\sigma_{ij} = (1 - D)d\sigma_{ij}^{\mathrm{RI}} + Dd\sigma_{ij}^{\mathrm{FA}} + dD\left(\sigma_{ij}^{\mathrm{RI}} - \sigma_{ij}^{\mathrm{FA}}\right). \tag{4}$$

At the pre-failure stage ($D = dD = 0$), Eq. (4) yields the RI stress-strain response.

2.2. RI response

In this study, the RI response was assumed to be a "nonlinear elastic-perfectly plastic" response to one-dimensional behavior, which can be achieved easily in two- and three-dimensional problems using Mohr-Coulomb yield function with only a slight difference (elastic-perfectly plastic). Although sophisticated models can also be used for the RI behavior, the goal of this paper is to propose a model with the least computational costs.

For one-dimensional analysis, the RI state should satisfy the following conditions:

1. The value of 28-day compressive strength of concrete (stress at peak for NWC) can be obtained

from the following empirical expression [2]:

$$f'_c = \left(\frac{21}{w/c} + 32\sqrt{sf/c}\right)\left(\frac{\rho}{2400}\right)^{1.6}, \tag{5}$$

where w/c is the water-cement ratio, ρ is the density of concrete, and sf/c is the silica fume-cement ratio;

2. Initial slope of the stress-strain curve is equal to the initial modulus of elasticity of concrete which can be determined, based on the empirical relation derived from Ozbakkaloglu and Lim [2]:

$$E_i = 4400\sqrt{f'_c}\left(\frac{\rho}{2400}\right)^{1.4}, \tag{6}$$

where f'_c is the 28-day compressive strength of concrete from Eq. (5), and ρ is the density of concrete;

3. Slope of the curve at peak is equal to zero due to the perfectly plastic nature of the post peak response;
4. The initial value of stress is equal to zero, where the strain is equal to zero.

Having these four conditions for one-dimensional behavior, a third-order polynomial can perfectly satisfy the pre-peak response as follows:

$$f = \alpha\varepsilon^3 + \beta\varepsilon^2 + \gamma\varepsilon + \lambda. \tag{7}$$

Substituting the conditions mentioned above into Eq. (7), the coefficients of the RI response are:

$$\alpha = \frac{-2f_{\text{peak}} + \gamma\varepsilon_{\text{peak}}}{\varepsilon^3_{\text{peak}}}, \tag{8a}$$

$$\beta = \frac{3f_{\text{peak}} - 2\gamma\varepsilon_{\text{peak}}}{\varepsilon^2_{\text{peak}}}, \tag{8b}$$

$$\gamma = E_i, \tag{8c}$$

$$\lambda = 0. \tag{8d}$$

For the post-peak response to the RI behavior, it is assumed that concrete can no longer undertake additional stresses; therefore, stress is constant due to the lack of disturbance in the RI behavior (perfectly plastic).

2.3. FA response

The Fully Adjusted (FA) state is directly related to the confinement condition. With regard to the behavior of concretes, if the tests are unconfined, zero stress condition represents the fully adjusted state, i.e. the concrete specimen can no longer bear any stresses at the final stage. On the other hand, if the tests are confined, then concrete has residual stress at the steady-state response which is the fully adjusted state.

2.4. Disturbance simplification

In order to simplify the disturbance distribution, it is assumed that concrete fails when it completely reaches the fully adjusted state, which means that $D_u = 1$. Moreover, based on empirical observations, taking $Z = 1.622$ (constant number) allows us to obtain different types of distribution by changing parameter A; therefore, the only remaining variable is A. Disturbance distribution with different values of A is shown in Figure 2.

For evaluating the variations of disturbance distribution with changes of parameter A, more than 50 stress-strain curves of the behavior of Light-Weight Concrete (LWC) and Normal-Weight Concrete (NWC) were modeled using $D_u = 1.0$, $Z = 1.622$, and the best possible value of A. The results are shown in Figure 3.

The relation between parameter A and peak and

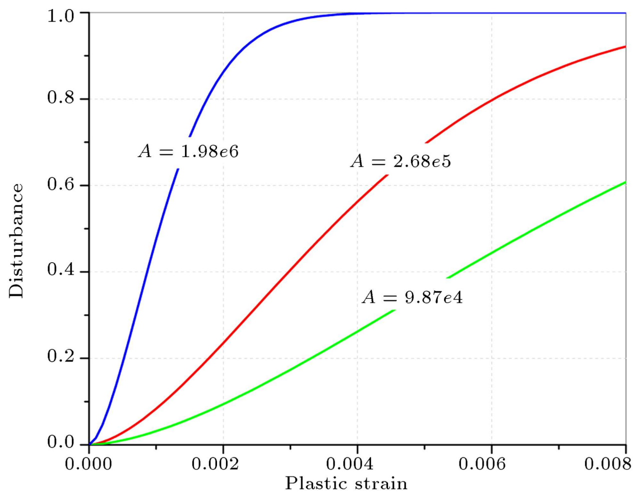

Figure 2. Disturbance distribution with different values of A.

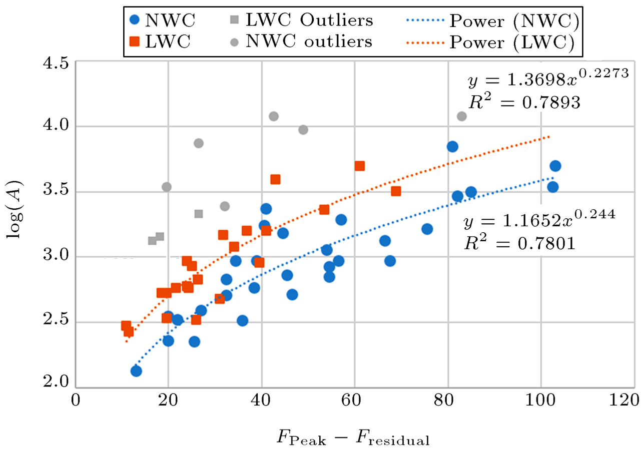

Figure 3. $\log(A)$ versus $(f_{\text{peak}} - f_{\text{residual}})$.

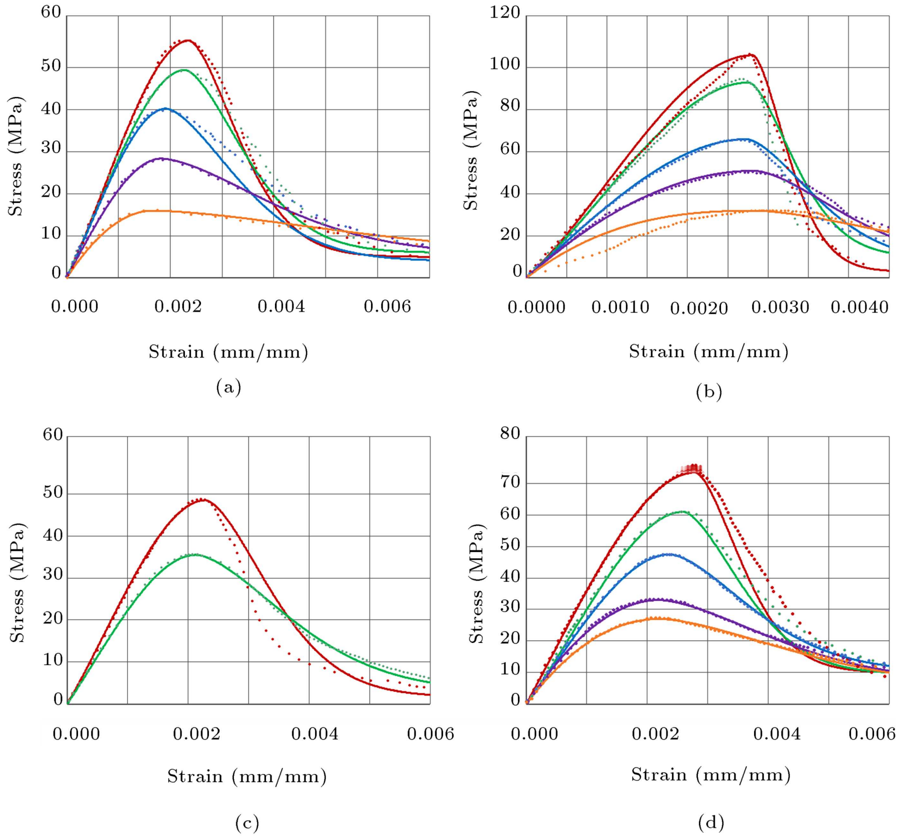

Figure 4. Comparison of the predicted curves (continuous line) with experimental unconfined results (dotted line) of different NWCs from: (a) Wischers [16], (b) Dahl [23], (c) Taerwe [24], and (d) Ahmad and Shah [20].

residual stresses is given in Eq. (9) based on the trend line in Figure 3 for NWC. For the case of LWC, the relation can be obtained simply using an additional constant term, while it was observed that the second term in Eq. (9) yields more accurate results:

$$\log(A) = 1.1652\,(f_{\text{peak}} - f_{\text{residual}})^{0.244} + \left(1 - \frac{\rho}{2400}\right)^{0.65} - (f_{\text{residual}})^{0.08}, \qquad (9)$$

where the value of the stress at peak (f_{peak}) for NWC can be obtained using Eq. (5), while this value for LWC is as follows [2]:

$$f_{\text{peak}} = f'_c + 5.2 f'^{0.91}_c \left(\frac{f^*_l}{f'_c}\right)^a, \qquad (10)$$

where $\frac{f^*_l}{f'_c}$ is the residual stress ratio, $a = f'^{-0.06}_c$, and residual stress (f_{residual}) can be obtained using the equation below [2]:

$$f_{\text{residual}} = 1.6 f_{\text{peak}} \left(\frac{f^{*0.24}_l}{f'^{0.32}_c}\right) \leq f_{\text{peak}} - 0.15 f'_c. \qquad (11)$$

3. Results

Experimental data of more than 50 tests in [15-30] were predicted using Eqs. (1), (4), (6), and (11). The results of normal-weight concrete are shown in Figures 4 and 5; the results of light-weight concrete are shown in Figures 6 and 7; the results of actively confined concrete are shown in Figures 8 and 9.

4. Conclusion

In this study, a model with the least computational cost was presented to predict the behaviors of various concretes. In this respect, a simple relatively intact state was presented for the disturbed state concept. Then, the number of parameters used in the disturbance was reduced from three to one to simplify the parameter determination procedure. An empirical equation was then given to calculate parameter A based on different parameters of concrete such as ultimate strength, density, and confinement conditions.

Accordingly, more than 50 experimental tests of different types of concrete and confinement conditions from other researchers were predicted using the proposed model. In most cases, predictions show good

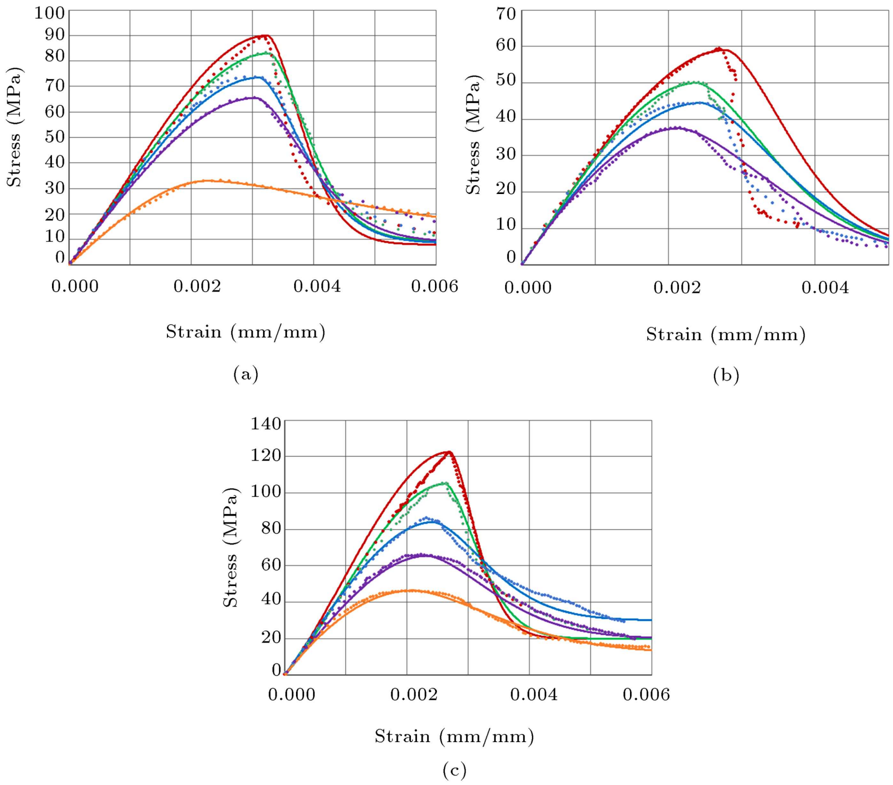

Figure 5. Comparison of the predicted curves (continuous line) with experimental unconfined results (dotted line) of different NWCs from: (a) Hsu and Hsu [25], (b) Desnerck et al. [30], and (c) Wee et al. [27].

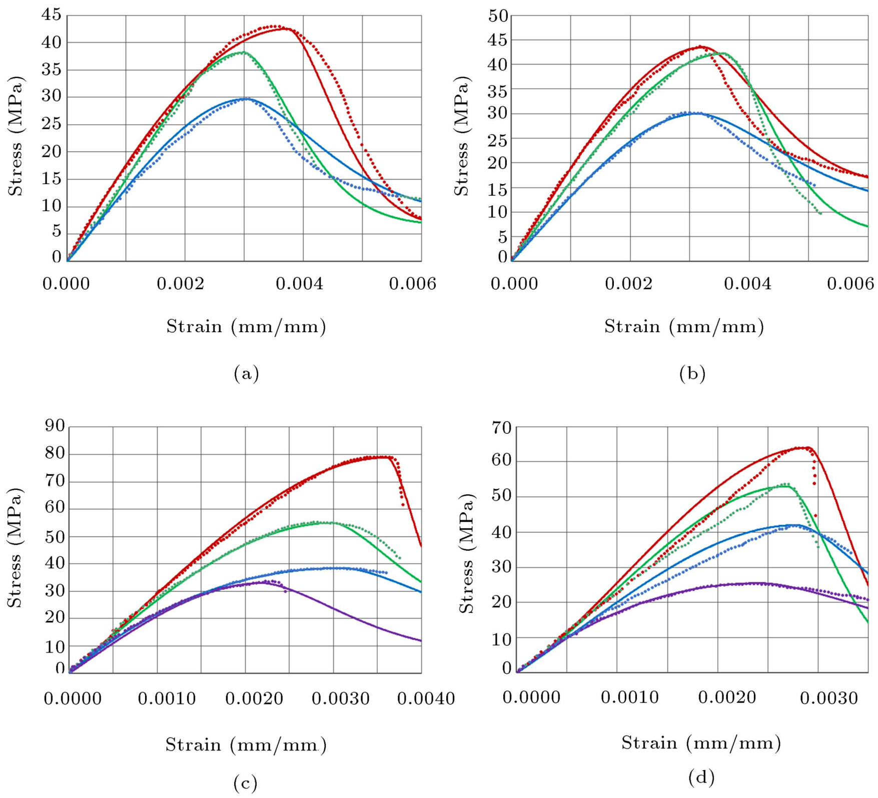

Figure 6. Comparison of the predicted curves (continuous line) with experimental unconfined results (dotted line) of different LWCs from: (a and b) Shah et al. [18], and (c and d) Kaar et al. [15].

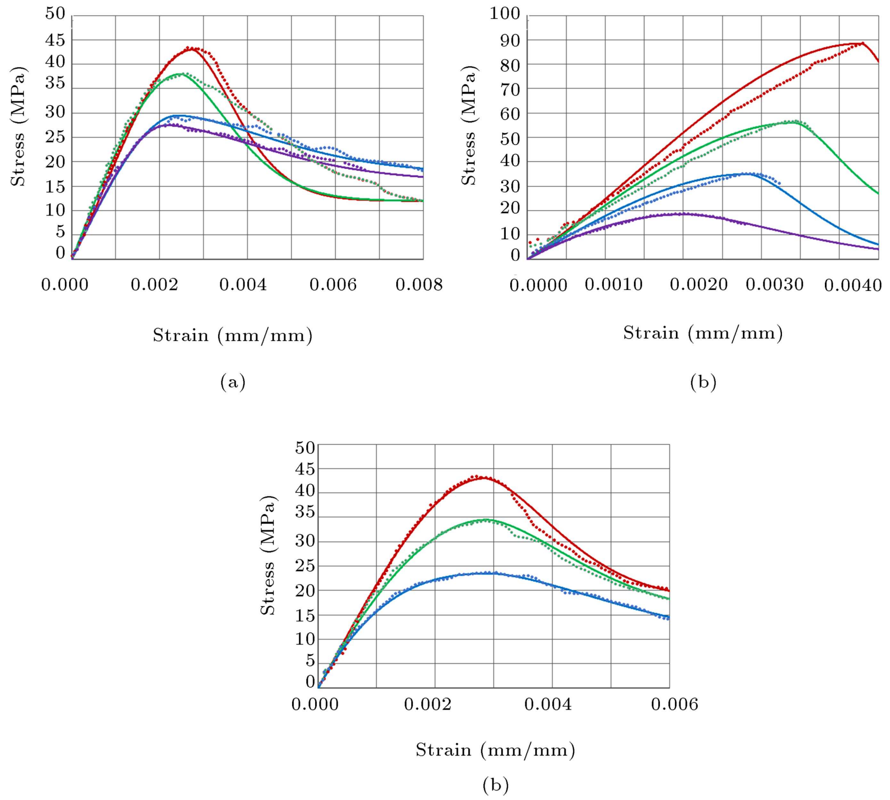

Figure 7. Comparison of the predicted curves (continuous line) with experimental unconfined results (dotted line) of different LWCs from: (a and c) Shannag [29], and (b) Zhang and Gjorv [21].

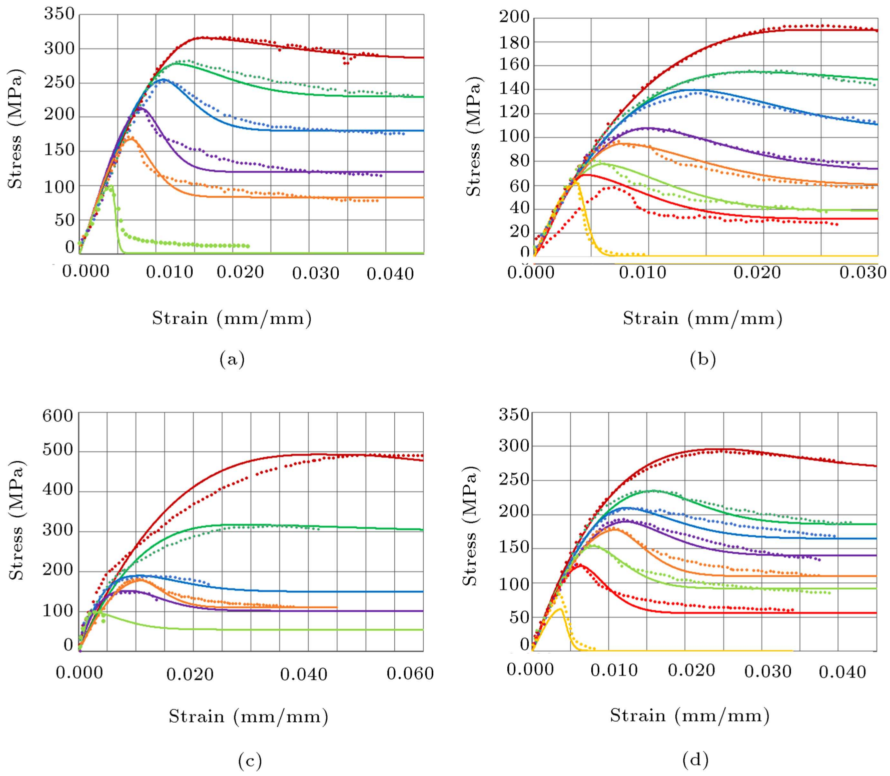

Figure 8. Comparison of the predicted curves (continuous line) with experimental confined results (dotted line) of different concretes from: (a, b, and c) Xie et al. [26], and (d) Newman [17].

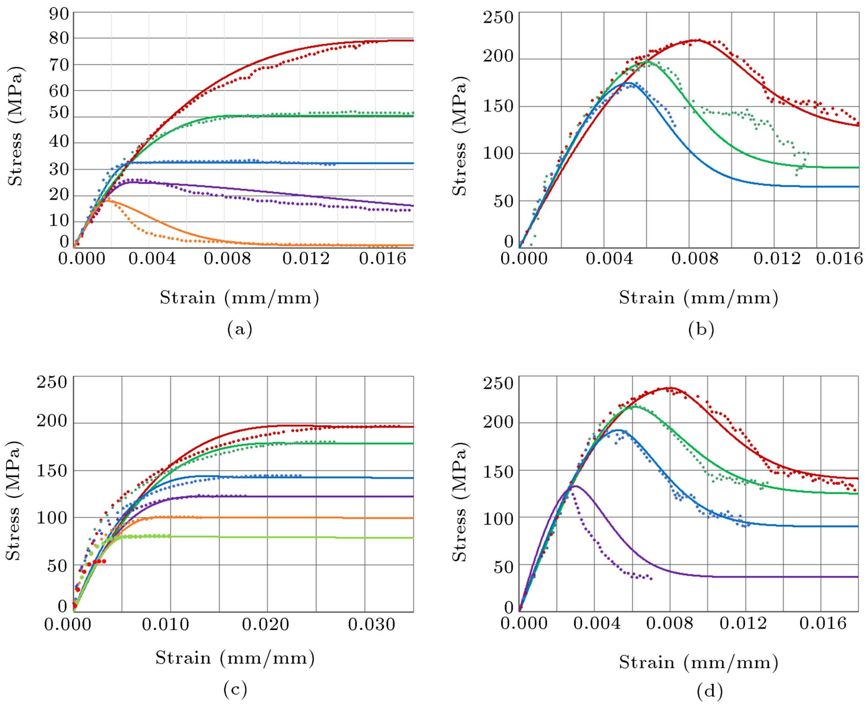

Figure 9. Comparison of predicted curves (continuous line) with experimental confined results (dotted line) of different concretes from: (a) Hurlbut [19], (b and d) Attard and Setunge [28], and (c) Belloti and Rossi [22].

agreement with experimental results. The proposed model has the least computational cost and does not require experimental tests. Moreover, since there is a physical justification for the disturbed state concept, it can be used also for two- and three-dimensional problems.

Nomenclature

A	Material parameter
C^{ep}	Elasto-plastic constitutive matrix
D	Disturbance parameter
D_U	Ultimate value of D
E_i	Modulus of elasticity
Z	Material parameter
α	Coefficient in the RI response
β	Coefficient in the RI response
e_{ij}	Deviatoric strain tensor
f'_c	28-day compressive strength of concrete
f_{peak}	Stress at peak
$\left(\frac{f_l^*}{f'_c}\right)$	Residual stress ratio
sf/c	Silica fume-concrete ratio
w/c	Water-cement ratio
δ_{ij}	Kronecker delta
ε_{ij}	Strain tensor
$\varepsilon_{\text{peak}}$	Strain at peak
ρ	Density of concrete
σ^{RI}	Relatively intact stress
σ^{obs}	Observed stress
σ^{FA}	Fully adjusted stress
σ_{ij}	Stress tensor
λ	Coefficient in the RI response
γ	Coefficient in the RI response
ξ_D	Trajectory of deviatoric plastic strain

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