A New Method for Three Dimensional Stability Analysis of Earth Slopes

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Abstract

In this research, a new three-dimensional limit equilibrium method was developed. In the proposed method, the slip surface was assumed to be spherical and slices were assumed along the slip surface radius converging to a point. Moreover, force and moment equilibrium equations were used. In order to calculate factor of safety (*FS*) with the proposed method, a code was developed for two- and three-dimensional state. The model uses the iteration (trial and error) method to simultaneously satisfy force and moment equilibrium equations. In order to ensure convergence of the numerical model developed in the three dimensional state, the under relaxation method used in the trial and error operations. In examples, the proposed method was obtained the value of FS, 2.5 to 12 percent higher than other methods. In order to validate the proposed method and assess the functionality of the three dimensional numerical model a few examples were solved under different circumstances and the results were properly compliant with the results of other methods.

Keywords: Limit equilibrium; Factor of safety; 3D analysis; Earth slope; 3D slice.

1. Introduction

Stability analysis of earth slopes is an important concern in civil engineering and one of the substantial concerns in the design of earth slopes, routes, canals, and embankments. Slope stability analysis is the process of controlling the safety of a natural or artificial slope [1]. The slope might be the result of excavation or embanking. The control involves calculation of shear stresses resulted along the most critical and probable slip surface and comparison of the shear stress values with the shear strength of soil. Finding the critical slip surface of slopes and calculating the *FS* corresponding to this surface are two of the fundamental concerns in the stability analysis of earth slopes. It is important to find the most critical slip surface because calculation of the minimum *FS*, which is obtained at this surface, is usually the objective of stability analysis. Limit equilibrium (LE) methods are among the oldest means of determining critical slip surfaces and the minimum *FS*. In the LE methods, *FS* is calculated by assuming a failure surface and satisfying the static equilibrium equations for that surface. Moreover, in these methods the number of equilibrium equations is lower than the number of unknowns and therefore usually additional hypotheses are required to turn the problem at hand into a determinate one.

In the Fellenius method [2], the inter-slice forces were neglected and FS was calculated based on the moment equilibrium around the center of the failure surface. In the Bishop method [3] FS is calculated by assuming a circular failure surface and satisfaction of moment and vertical force equilibrium equations. The methods by Lowe and Karafiath [4] and the modified Janbu method [5] as well as U.S. Army Corps of Engineers [6] also calculate FS by satisfying the force equilibrium equations. The difference in different methods, which obtain FS by satisfying the force equilibrium equations, lies in their hypotheses on the angle of the forces between the slices. Therefore, every method considers different force angles. The methods by Spencer [7], Morgenstern-Price [8] and Sarma [9] calculate FS based on a few hypotheses and satisfaction of the force and moment equilibrium equations. Cheng [10] calculated FS using the double QR method and indicated that the new method performs better in converging the results. Cheng and Zhu [11] proposed a single relation for two-dimensional calculation of FS using the LE method. In addition, in order to calculate FS they used the Gauss-Newton linear approximation. Zhu et al. [12] proposed a simple algorithm for calculating FS using the Morgenstern-

Price method [8]. In this algorithm, the initial values of FS and the scale factor λ are used in an iterative process to calculate the converged FS and λ. Hajiazizi and Mazaheri [13] presented the line segments slip surface for optimized design of piles in stabilization of the earth slopes using limit equilibrium method for calculating FS. Xiao et al. [14] combined the numerical method and LE method to define the critical failure surface for earth slopes. In this method, numerical analysis is used to determine the assumptive failure surface and the LE method is used to calculate *FS*. Deng and Li used radiation slices method for arbitrary curved slip surface in 2D state [15].

Hajiazizi and Tavana [16] used the TDAVLG optimization method and LE method to calculate the minimum *FS* corresponding to the non-spherical slip surface in three dimensions. Cheng et al. [17] used the LE method for stability analysis of slopes and faced a convergence problem. In order to solve that problem, they proposed a simple method and assessed it. Stability analysis of earth slopes mainly takes place in two dimensions by assuming a planar strain. However, in many cases three-dimensional analysis is necessary as a two-dimensional analysis would be unrealistic. These cases usually include slopes under concentrated loads, slopes with small length to height ratios, and slopes with curved plans. Three-dimensional stability analyses of slopes, which have been carried out rarely, are mostly based on the LE method. Many of the three-dimensional analysis methods were proposed by extending two-dimensional methods to the three-dimensional space. The three-dimensional method by Hovland [18] was an extension of the simple slice method which is a two-dimensional method. Hungr [19] also proposed a three-dimensional analysis method which is a direct extension of the hypotheses used in the simplified Bishop method. The three-dimensional method by Ugai and Hosobori [20] is also an extension of Spencer's two dimensional method [7], which calculates the three-dimensional *FS* using the force-moment equilibrium. Cheng and Lau [21] proposed the asymmetric slope stability models based on extensions of the Bishop [3], Janbu [5] and Morgenstern-Price methods [8] in three dimensions. Much progress presented in 2D and 3D methods of stability of slopes in recent years [22-24]. Zhou and Cheng presented [25] the rigorous limit equilibrium column method, in which inter-column forces are taken into account, based on six equilibrium conditions which include three direction force equilibrium conditions along coordinate axes and three direction moment equilibrium conditions around three coordinate axes. Zhu and Qian presented [26] the rigorous and quasi-rigorous limit equilibrium solutions to 3D slope stability, which satisfy six and five equilibrium conditions. Cheng and Zhou presented [27] a novel displacement-based rigorous limit equilibrium method to investigate the displacements and stabilities of three-dimensional landslides. Zhou and Cheng [28] presented a novel displacement-based rigorous limit equilibrium method to analyze the stability and the displacement of three-dimensional (3D) creeping slopes.

Three-dimensional slip surfaces are closer to reality than two-dimensional slip surfaces. Failure surfaces are usually spherical in homogeneous clay soils. In this paper a new limit equilibrium method has been proposed for these surfaces. Although, in multi-layered soils, consideration of spherical slip surface is a simplifying assumption, but in homogeneous clay soils it can be a reasonably assumption. It is noteworthy that in multi-layered slopes, failure surface is non-aspherical which are studied by [16]. In soils with zero internal friction angle, slip surface is circular in two-dimensional space and spherical in three-dimensional space [29].

According to the relations proposed by the aforementioned methods, along of slip is similarly obtained in all methods based on the force-moment equilibrium in three-dimensions. In this research, all of the forces acting on slices were calculated in the three-dimensional state using the LE method based on given slice shapes. The moment-force equilibrium equations were also applied. Afterwards, after satisfying the equilibrium equations (moment and force equilibrium), *FS* was calculated more precisely and the result was larger than the result of other LE methods.

The considered assumptions were as follows: (i) Morgenstern-Price equation is valid. (ii) Forces are applied at the center of slices surfaces.

2. Limit Equilibrium Equations for the Method Proposed for Two Dimensions

Fig. 1 shows the slip wedge in an earth slope in two dimensions. The slip wedge is divided into several slices along the semidiameter. Fig. 2 shows the free diagram for forces acting on a slice. The forces include the slice weight (W_i), inter-slice forces (including the shear component, X_i, and the normal component, E_i), shear strength (S_i) and normal force (N_i) that act on the slice floor. By writing the force equilibrium equations along the X_{i+1} and E_{i+1} directions and satisfying the moment equilibrium conditions around the failure surface center (point o) it is possible to obtain the force-moment equilibrium equations.

2.1. Force Equilibrium Equations

Eqs. 1 and 2 are obtained based on Fig. 2 and the force equilibrium equations that were written along the X_{i+1} and E_{i+1} directions.

$$X_{i+1} + W\cos\left(\propto_i - \theta/2\right) - N\cos\theta/2 - S\sin\theta/2 + E_i\sin\theta - X_i\cos\theta = 0 \quad (1)$$

$$E_{i+1} - W\sin\left(\alpha_i - \theta/2\right) - N\sin\theta/2 + S\cos\theta/2 - E_i\cos\theta - X_i\sin\theta = 0 \quad (2)$$

where w is weight of slice $X_i, X_{i+1}, E_i, E_{i+1}, S, N$ are forces acting on a slice (Fig. 2).

By multiplying the expression on both sides of Eqs. 1 and 2 by *cosθ/2* and *sinθ/2* Eqs. 3 and 4 are obtained respectively.

$$X_{i+1}\cos\theta/2 + W\cos\left(\alpha_i - \theta/2\right)\cos\theta/2 - N\cos^2\theta/2 - S\sin\theta/2\cos\theta/2 + E_i\sin\theta\cos\theta/2 - X_i\cos\theta\cos\theta/2 = 0 \quad (3)$$

$$E_{i+1}\sin\theta/2 - W\sin\left(\alpha_i - \theta/2\right)\sin\theta/2 - N\sin^2\theta/2 + S\cos\theta/2\sin\theta/2 - E_i\cos\theta\sin\theta/2 - X_i\sin\theta\sin\theta/2 = 0 \quad (4)$$

By summing up Eqs. 3 and 4 it is possible to calculate the N force as follows.

$$N = X_{i+1}\cos\theta/2 + E_{i+1}\sin\theta/2 + W\cos\alpha_i + E_i\sin\theta/2 - X_i\cos\theta/2 \quad (5)$$

Shear strength S is calculated by multiplying both sides of Eqs. 1 and 2 by *sinθ/2 and -cosθ/2*.

$$X_{i+1}\sin\theta/2 + Ws\left(\alpha_i - \theta/2\right)\sin\theta/2 - N\cos\theta/2\sin\theta/2 - S\sin^2\theta/2 + E_i\sin\theta\sin\theta/2 - X_i\cos\theta\sin\theta/2 = 0 \quad (6)$$

$$-E_{i+1}\cos\theta/2 + W\sin\left(\alpha_i - \theta/2\right)\cos\theta/2 + N\sin\theta/2\cos\theta/2 - S\cos^2\theta/2 + E_i\cos\theta\cos\theta/2 + X_i\sin\theta\cos\theta/2 = 0 \quad (7)$$

By summing up Eqs. 6 and 7, is obtained Eq. 8.

$$S = X_{i+1}\sin\theta/2 - E_{i+1}\cos\theta/2 + X_i\sin\theta/2 + E_i\cos\theta/2 + W\sin\alpha_i \quad (8)$$

On the other hand, Eq. 8 is rewritten as Eq. 10 based on Eq. 9.

$$S = \frac{1}{F}\left(C'\Delta L + \left(N - U\Delta L\right)\tan\varphi'\right) \tag{9}$$

$$\begin{aligned} & X_{i+1}\sin\theta/2 - E_{i+1}\cos\theta/2 + X_i\sin\theta/2 + E_i\cos\theta/2 + W\sin\alpha_i - \frac{C'\Delta L}{F} \\ & - \frac{\tan\varphi'}{F}\begin{pmatrix} X_{i+1}\cos\theta/2 + E_{i+1}\sin\theta/2 + W\cos\alpha_i \\ + E_i\sin\theta/2 - X_i\cos\theta/2 \end{pmatrix} + \frac{U\Delta L}{F}\tan\varphi' = 0 \end{aligned} \tag{10}$$

where *FS is* safety factor, C' and ϕ' are effective cohesion and effective friction angle of soil, respectively. *U* is the pore water pressure and *ΔL* the length of each slice floor. The force equilibrium equation is obtained as follows,

$$E_{i+1} = \frac{\frac{C'\Delta L}{F} - \frac{U\Delta L}{F}\tan\varphi' + W\left(\frac{\tan\varphi'}{F}\cos\alpha_i - \sin\alpha_i\right)}{\lambda f_{(x)}\sin\theta/2 - \cos\theta/2 - \frac{\tan\varphi'}{F}\left(\lambda f_{(x)}\cos\theta/2 + \sin\theta/2\right)} + \frac{\left(-\lambda f_{(x)}\sin\theta/2 - \cos\theta/2 + \frac{\tan\varphi'}{F}\left(\sin\theta/2 - \lambda f_{(x)}\cos\theta/2\right)\right)E_i}{\lambda f_{(x)}\sin\theta/2 - \cos\theta/2 - \frac{\tan\varphi'}{F}\left(\lambda f_{(x)}\cos\theta/2 + \sin\theta/2\right)} \tag{11}$$

where α_i is the angle of each slice floor to the horizon line while *f(x)* is a hypothetical function showing the relationship between the normal and shear forces in each slice. λ is the scale factor, θ shows the central angle in each slice. Concerning Eq. 11, which was obtained based on the force equilibrium, the following points shall be taken into account: (1) There are two unknowns in Eq. 11: the factor of safety and the scale factor (λ); (2) The inter-slice force (E_{i+1}) in each phase depends on the inter-slice force (E_i) in the previous step (recursive relation); (3) The boundary conditions for the equation are E_i=0 and E_{i+1}=0 because there is no slice before the first slice and after the last one.

In order to obtain the moment equilibrium equation, first the shear strength relation is calculated based on inter-slice forces. Next, by satisfying the moment equilibrium around the failure surface center the moment equilibrium relation is obtained. The shear strength relation is also expressed as follows based on the inter-slice forces.

$$N = E_{i+1}\left(\sin\theta/2 + \lambda f_{(x)}\cos\theta/2\right) + E_i\left(\sin\theta/2 - \lambda f_{(x)}\cos\theta/2\right) + W\cos\alpha_i \tag{12}$$

$$S = \frac{1}{F}\left(C'\Delta L + \left[E_{i+1}\left(\sin\theta/2 + \lambda f_{(x)}\cos\theta/2\right) + E_i\left(\sin\theta/2 - \lambda f_{(x)}\cos\theta/2\right) + W\cos\alpha_i - U\Delta L\right]\tan\varphi'\right) \tag{13}$$

The moment equilibrium equation is also written as follows:

$$\sum W_i \sin\alpha_i \beta_i - Sr = 0 \tag{14}$$

By putting S_i into Eq. 14 and simplifying it is possible to achieve the moment equilibrium equation as follows,

$$F=\frac{\sum\left(C'\Delta Lr+\left[\left(\sin\theta/2+\lambda f_{(x)}\cos\theta/2\right)E_{i+1}\right]r\tan\varphi'\right)}{\sum\left(W_i\sin\alpha_i\beta_i\right)}+\frac{\sum\left(\left[\left(\sin\theta/2-\lambda f_{(x)}\cos\theta/2\right)E_i+W\cos\alpha_i-U\Delta L\right]r\tan\varphi'\right)}{\sum\left(W_i\sin\alpha_i\beta_i\right)} \quad (15)$$

Where, r is the failure semidiameter and β is the moment arm in each slice.

2.2. Calculation of *FS* in Two Dimensions

Eqs. 11 and 15 are the basic equations resulted from the force and moment equilibrium equations. Simultaneous satisfaction of these equations leads to *FS*. In this research, *FS* was calculated using the iterative method and *FS* was obtained by satisfying the force and moment equilibrium equations. Methods that satisfy all equilibrium equations and employ fewer simplifying assumptions lead to less error compared to other methods. Hence, Eqs. 11 and 15 are expected to determine *FS* more precisely.

2.3. Developing a Numerical Model for Calculation of *FS*

In order to calculate *FS* with the method proposed in this research, a code was developed in FORTRAN. In this model, the specifications of the earth slope (including the slope geometry, soil resistance properties, and water level) are taken as inputs. Next, the slip wedge is introduced and based on the number of divisions defined by the user, the slip wedge is sliced. Afterwards, the coordinates of each node on these slices are determined. After determining the coordinates of nodes on each slice it is possible to calculate the area, weight, the floor-horizon angle, surface center, moment arm and pore water pressure on each slice.

Based on the coordinates of all slices it is then possible to calculate *FS*. In order to calculate *FS* it is necessary to go through a process that simultaneously satisfies force and moment equilibrium equations (Eqs. 11 and 15). Therefore, the force equilibrium equation is satisfied using an initial *FS*. To this end, the inter-slice forces acting on other slices are calculated using a scale factor λ and a zero inter-slice force for first slice (E_1=0). When the inter-slice force acting on the last slice is zero, the force equilibrium equation is satisfied; otherwise, λ changes in order for the last boundary condition ($E_{n+1}\approx 0$) to be met. By putting λ and inter-slice forces resulted from satisfaction of the force equilibrium into the moment equilibrium equation it is possible to calculate the *FS* resulting from the moment equation.

If the difference between the initial *FS* and the *FS* resulting from the moment equilibrium is slight (smaller than 10^{-4}), the resulting *FS* is acceptable; otherwise, the initial *FS* is replaced by the *FS* resulting from the moment equilibrium and the previous process is iterated until the difference between the two consecutive *FS* values becomes slight. This way the final *FS* is obtained.

3. Three-Dimensional Limit Equilibrium Equations in the Proposed Method

Fig. 3 shows a three-dimensional spherical slip surface. In order to divide this slip surface into several slices, the slip wedge is divided into sectors with equal intervals. By sectoring the slip surface along the radius it is possible to calculate the coordinates of points on each slice. Fig. 4 shows the free diagram of forces acting on a three-dimensional slice.

In this research, in order to obtain the three-dimensional equilibrium equations, the following hypotheses were developed: (1) the soil was homogenous; (2) the Mohr-Coulomb failure criterion was valid; (3) The three-dimensional failure surface was spherical; (4) all of the slices move in one direction. Based on these hypotheses and Fig. 4, the limit equilibrium equations for the three-dimensional state were written.

3.1. Force Equilibrium Equations

Based on Fig. 4 and forces applied to each slice, the force equilibrium equations were written through Eqs. 16 to 18 along the *x*, *y* and *z* directions.

$$S_m cos\alpha_x + Ncos\theta_x - E_{xi}cos\gamma_{xi} + E_{xi+1}cos\gamma_{xi+1} - X_{xi+1}sin\gamma_{xi+1} + X_{xi}sin\gamma_{xi} = 0 \quad (16)$$

$$Ncos\theta_y - E_{yi}cos\gamma_{yi} + E_{yi+1}cos\gamma_{yi+1} + X_{yi}sin\gamma_{yi} - X_{yi+1}sin\gamma_{yi+1} = 0 \quad (17)$$

$$X_{xi+1}cos\gamma_{xi+1} - X_{xi}cos\gamma_{xi}) + \left(X_{yi+1}cos\gamma_{yi+1} - X_{yi}cos\gamma_{yi}\right) + \left(E_{xi+1}sin\gamma_{xi+1} - E_{xi}sin\gamma_{xi}\right)$$
$$+\left(E_{yi+1}sin\gamma_{yi+1} - E_{yi}sin\gamma_{yi}\right) - \mathrm{W} + \mathrm{N}cos\theta_z + S_m sin\alpha_x = 0 \tag{18}$$

$$E_{xi+1}cos\gamma_{xi+1} - E_{xi}cos\gamma_{xi} = \lambda_x\left(E_{xi+1}sin\gamma_{xi+1} - E_{xi}sin\gamma_{xi}\right) - Ncos\theta_x - \frac{cos\alpha_x}{F}\left(c'A + (N-U)tan\varphi'\right) \tag{19}$$

$$\mathrm{E}_{yi+1}\cos\gamma_{yi+1} - \mathrm{E}_{yi}\cos\gamma_{yi} = \lambda_y\left(\mathrm{E}_{yi+1}\sin\gamma_{yi+1} - \mathrm{E}_{yi}\sin\gamma_{yi}\right) - \mathrm{N}\cos\theta_y \tag{20}$$

$$N = \frac{1}{cos\theta_z + \frac{tan\varphi' sin\alpha_x}{F}}\left[W - \lambda_x\left(E_{xi+1}cos\gamma_{xi+1} - E_{xi}cos\gamma_{xi}\right) - \lambda_y\left(E_{yi+1}cos\gamma_{yi+1} - E_{yi}cos\gamma_{yi}\right)\right.$$
$$\left. -\left(E_{xi+1}sin\gamma_{xi+1} - E_{xi}sin\gamma_{xi}\right) - \left(E_{yi+1}sin\gamma_{yi+1} - E_{yi}sin\gamma_{yi}\right) - \frac{Ac' sin\alpha_x}{F} + \frac{Utan\varphi'}{F}sin\alpha_x\right] = 0 \tag{21}$$

As seen in Eq. 21, *N* (force) is a function of *FS*. In order to determine *N*, the *FS* of the moment equilibrium is put into Eq. 21 provided that the moment equilibrium is required. However, if the force equilibrium is required the *FS* of the force equilibrium is put into Eq. 21.

3.2. Force Equilibrium along the Slip Direction

In order to obtain the *FS* equation, a single slip mechanism was assumed in which the paths for all slices were defined along one direction. Hence, in this method, *FS* of the force equilibrium is calculated by writing the force equation along the x direction.

$$\sum S_m \cos\alpha_x + \mathrm{N}\cos\theta_x = 0 \tag{22}$$

By putting the Mohr-Coulomb relation into Eq. 22 the *FS* equation, which results from the force equilibrium, is obtained as follows.

$$F_f = \frac{\sum\left(c'A + (N-U)tan\varphi'\right)\cos\alpha_x}{\sum -Ncos\theta_x} \tag{23}$$

By calculating the moments of all the slices around a rotation axis, that is in parallel to the y-axis and passes through the center of the sphere, the relation for the *FS* resulting from the moment equilibrium (FS_m) is obtained. According to Fig. 4, the moment equilibrium equation is written as follows.

$$\sum W\ disw - S_m\ cos\alpha_x\ disz - S_m\ sin\alpha_x\ disx - Ncos\theta_x\ disz - Ncos\theta_z\ disx = 0 \tag{24}$$

where $disw$=distance of the weight force and the rotation axis; $disx$=the horizontal distance to rotation axis and $disz$= the vertical distance to rotation axis.

Moreover, by putting the Mohr-Coulomb relation into Eq. 24 the relation for calculation of the *FS* resulted from the moment equilibrium is turned into Eq. 25.

$$F_m = \frac{\sum\left[c'A + (N-U)tan\varphi'\right](cos\alpha_x\ disz + sin\alpha_x\ disx)}{\sum W\ disw - Ncos\theta_x\ disz - Ncos\theta_z\ disx} \tag{25}$$

Examination of equations for calculation of the *FS* resulted from the force and moment equilibrium (Eqs. 23 and 25) indicates that these equations are nonlinear. Moreover, the sum of inter-slice forces (*E*) can be calculated using equations for calculation of *FS*. The sum of these forces has to be almost equal to zero.

3.3. Three-Dimensional Calculation of *FS*

The three-dimensional method proposed in this article is an extension of the two-dimensional method and is obtained based on the minimum simplifying assumptions, comprehensible concepts, and adequate precision. In order to calculate *FS* the simultaneous satisfaction of force and moment equilibrium equations was employed. Eqs. 23 and 25 are basic equations resulted from the force and moment equilibrium. Simultaneous satisfaction of these equations, therefore, results in the required *FS*. In this case, in order to calculate *FS* the iterative method is used and *FS* is obtained by satisfying the force and moment equilibrium equations.

3.4. Developing a Numerical Model for 3D Calculation of *FS*

In order to calculate *FS* using the method proposed in this research, a numerical model was developed in FORTRAN. In this model, the specifications of the earth slope (including the slope geometry and soil resistance properties) are taken as inputs. Next, the radius of the sphere is set by the user and the soil sliding mass is obtained. In the next step, based on the number of divisions defined by the user, the slip wedge is classified and the coordinates of each node on these slices are also determined.

After determining the coordinates of nodes on each slice it is possible to calculate the following parameters for each slice: volume, area, weight, the floor-horizon angle, the location for application of the normal force and its angle to the axes, angle of inter-slice normal forces to the horizon, and moment arm. In order to calculate the *FS* resulting from the force and moment equilibrium equations it is necessary to estimate the initial *FS* and inter-slice forces.

Afterwards, by averaging the resulting factors of safety for different slices, the initial *FS* for the three-dimensional state is estimated. Moreover, the inter-slice forces calculated in the two-dimensional state are used as the basis for the three-dimensional estimation. In order to calculate *FS* it is necessary to go through a process that simultaneously satisfies force and moment equilibrium equations (Eqs. 23 and 25). To this end, using the inter-slice forces and the initial *FS* and by assuming initial λx and λy values, normal forces acting on each slice are calculated using Eq. 21. Next, using Eq. 25, the *FS* resulting from the moment equilibrium is obtained. Next, the force equilibrium equations along the x and y directions (Eqs. 19 and 20), the values of forces perpendicular to the slip surface in each slice, and the *FS* resulting from the previous phase are used to calculate the new inter-slice forces. In the next stage, the under relaxation method is implemented to avoid non-convergence. If the difference between the new inter-slice force and the initial inter-slice force is large, a percentage of the difference between these two forces is added to the initial force and the resulting force is used as the new inter-slice force.

This process is iterated until the *FS* resulting from the moment equilibrium reaches convergence. In the next phases, this process is iterated to determine the *FS* of the force equilibrium. If the difference between the resulting factors of safety is slight, the final *FS* is calculated; otherwise, values of λx and λy are changed and the process is repeated until the difference between the factors of safety resulted from the force and moment equilibrium equations becomes slight.

4. Three-dimensional limit equilibrium and numerical methods

Three-dimensional limit equilibrium methods and three-dimensional numerical methods, such as finite element method, are located in two different categories. Numerical methods, due to their punctuated equilibrium nature, are more accurate than limit equilibrium methods. Slice equilibrium is considered in limit equilibrium methods.

In the limit equilibrium analysis equilibrium may be considered either for a single free body or for individual vertical slices. Depending on the analysis procedure, complete static equilibrium may or not be satisfied. Some assumptions must be made to obtain a statically determinate solution for the factor of safety. Different procedures make different assumptions, even when they may satisfy the same equilibrium equations. The factor of safety is defined with respect to shear strength and it is computed for an assumed slip surface. In the limit equilibrium analysis, it is necessary to make sufficient assumptions regarding the stress distribution along the failure surface such that an overall equation of equilibrium, in term of stress resultants, may be written for a given problem. Therefore, this simplified approach makes it possible to solve various problems by simple statics [30].

5. Some Examples

In this section, in order to examine the proposed method in the two- and three-dimensional states, 5 examples with different conditions are provided. Results of these examples are also analyzed and compared to the results reported by other researchers.

5.1. Example 1

In this example, an earth slope with a height of 10 meters and a 3 to 2 slope ratio (Fig. 5) is studied. The slope has the unit weight, cohesion and angle of friction of 20 kN/m^3, 15 kPa, and 25 degree, respectively. Coordinates of the slip circle center and its radius are X_c=10.8m, Y_c=23.8m and r=18.8 m, respectively. The groundwater level is assumed to be 10 meters. The *FS* resulting from the proposed method and results of its comparison to the results of other methods are presented in Table 1. In this example, the lowest *FS* is resulted from the Fellenius relation by satisfying the moment equilibrium equation and overlooking inter-slice shear forces. The *FS* from the proposed method is larger than the *FS* from other methods and is the closest to the result of the Morgenstern-Price relation. The proposed method is more reliably than other methods that it represents the fewer simplifying assumptions. The fewer simplifying assumptions lead to less error compared to other methods.

5.2. Example 2

In this example, the first example without groundwater is solved for the pseudo-dynamic state with a horizontal acceleration of a_h=0.1g. Results of the *FS* from the proposed method and other methods are shown in Table 2. The presence of the earthquake force has reduced *FS* and increased the instability of the earth slope. These changes can be ascribed to the growth of the driving forces resulting from seismic forces. In this state, the *FS* resulted from the proposed method is also similar to the results of the modified Bishop and Morgenstern-Price relations [8]. Moreover, Janbu [5] and Fellenius [2] relations also yielded the lowest factors of safety.

The proposed method is more reliably than other methods that it represents the fewer simplifying assumptions. The fewer simplifying assumptions lead to less error compared to other methods.

5.3. Example 3

This example is analyzed in [31]. Table 3 presents the results of the method proposed in [31] and methods developed by other researchers for stability analysis of the earth slope in Fig. 6. The slope in this figure lacks the weak layer and groundwater parameters. Using the three-dimensional numerical model developed in this research, the *FS* for the slope shown in Fig. 6 was obtained for different number of slices (Table 4). As seen in Table 4, the numerical value of *FS* declines with the growth of the number of slices. Since there is no considerable difference between the *FS* resulted for 40 and 30 slices, a *FS*=2.234 is selected as the three-dimensional *FS* for the slope. Results of comparisons between the *FS* from the proposed method and those reported by other researchers (Table 3) reflects the adequate precision of the proposed method.

The proposed method is more reliably than other methods that it represents the fewer simplifying assumptions. The fewer simplifying assumptions lead to less error compared to other methods.

5.4. Example 4

Wei et al. [32] assessed the three-dimensional slope stability model using SRM and LE methods for different conditions. One of the cases studied by these researchers was an earth slope with a height of 6 meter, slope angle of 45 degrees, cohesion of 15 kPa, angle of friction of 10 degrees and unit weight of 20 kN/m^3. As seen in Fig. 7, in this slope, the failure mechanism resulted from the three-dimensional SRM method is similar to the two-dimensional failure mechanism while *FS* calculated using the three-dimensional shear strength reduction method is 1.17. Moreover, based on the spherical search process, the two- and three-dimensional minimum factors of safety resulted from the Spencer's method were 1.07 and 1.12, respectively.

By modeling the aforementioned slope using the two- and three-dimensional numerical model proposed in this research, the factors of safety for the two- and three-dimensional states were calculated to be 1.11 and 1.254 for 40 slices, respectively. Results of the proposed method highly comply with the results of the three-dimensional shear reduction method and Spencer's method.

The proposed method is more reliably than other methods that it represents the fewer simplifying assumptions. The fewer simplifying assumptions lead to less error compared to other methods.

5.5. Example 5

In this example, an earth slope with a slope angle of 45 degrees, cohesion of 20 kN/m^2, friction of 25 degrees, and unit weight of 18 kN/m^2 is studied. This example was also studied in [16] (Fig. 8). The factor of safety for the most critical spherical slip surface obtained from Spencer's LE method in [16] was 1.99. In the present study, the *FS* calculated for 40 slices was 2.04, which highly complies with the *FS* reported in [16]. Also, the fewer simplifying assumption in this example lead to more precision.

6.5. Example 6

In 2005 a landslide occurred in Saein-Ardabil, Iran (Fig. 9). Fig. 10 illustrates its geological cross-section. This figure indicates that sliding wedge mass with tensile cracks was geologically possible and the slip depth was about 50 to 60 meters. This landslide was analyzed using Geo-Slope program and the safety factor was equal to 0.904 (Fig. 11) [33].

This landslide was analyzed using the proposed method and the safety factor of 0.955 was calculated. The Geo-Slope program considers circular slip surfaces but the proposed method is capable to consider non-circular slip surfaces as well. Consequently, the proposed method can obtain more critical slip surfaces with less safety factors.

6. Conclusion

In this paper, a new three-dimensional LE method proposed that uses the force and moment equilibrium equations and 3D slices along the slip surface radius converging to one point. With simultaneous satisfaction of the aforementioned equations it is possible to find the FS satisfying the forces and moment equilibriums. For this purposes, the method proposed in this paper was used for the two- and three-dimensional states and a numerical model was developed for the calculation of FS using the iterative method (trial and error). The method proposed in this paper was limited to spherical slip surfaces in three-dimensional case. Spherical slip surfaces are consistent with the failure surfaces in homogeneous clay soils, but in multilayer soils they are not necessarily the appropriate slip surfaces. The proposed method is a subset of limit equilibrium method that takes into account the balance of elements. This method is less accurate than the finite element method. The finite element method considers the punctuated equilibrium that is much more accurate than slice equilibrium.

Results of the proposed method were compared to the results of other methods and an adequate level of conformity was observed between the results. In sum, the proposed method yields a larger *FS* which reflects the higher precision of this method compared to other LE methods. Results of the three-dimensional method proposed in this paper with the results of other three-dimensional methods reflect the efficiency and precision of this method. Moreover, the *FS* resulted from the proposed method highly complies with the factors of safety resulted from other methods such as SRM and 3D Spencer methods. The proposed method was obtained the value of FS, 2.5 to 12 percent higher than other methods and it has reduced the conservative approach of the LE method. According to the shape intended for the elements, without simplifying assumptions, forces tangent to the lateral surfaces pass through the center of rotation and are removed from the equilibrium equation. But in the other limit equilibrium methods, with vertical slices, some unknown forces are removed until the number of equations become equal to the number of unknowns. So it is expected that results obtained from the method proposed in this paper be closer to reality. In all of the solved examples, the safety factor calculated from the proposed method was greater than other methods. This value varies from 2.5 to 12%. According to the tips, it can be concluded that the conventional limit equilibrium methods are conservative. In other words, simplifying assumptions lead to safety factor less than the actual value. But in the proposed method, where no simplifying assumption was considered, the safety factor was much closer to reality.

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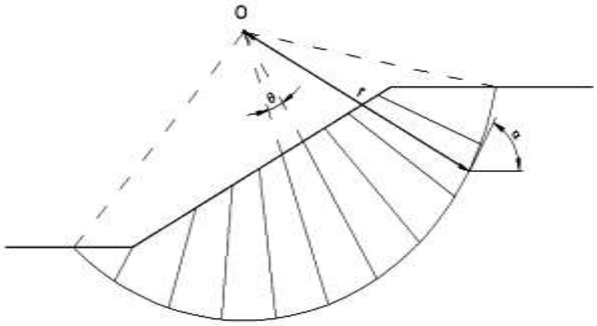
FIGURES



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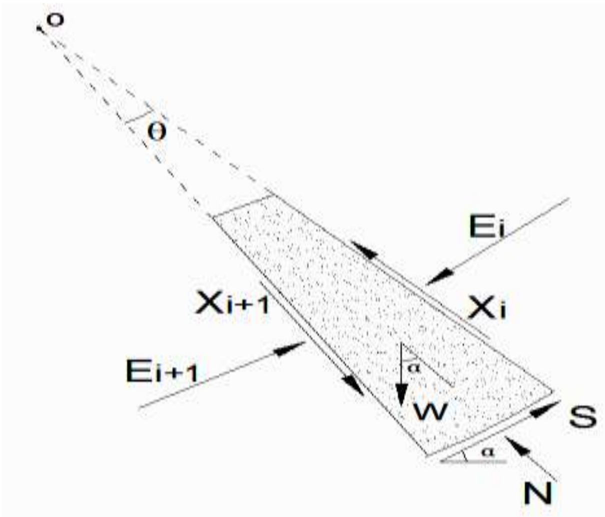

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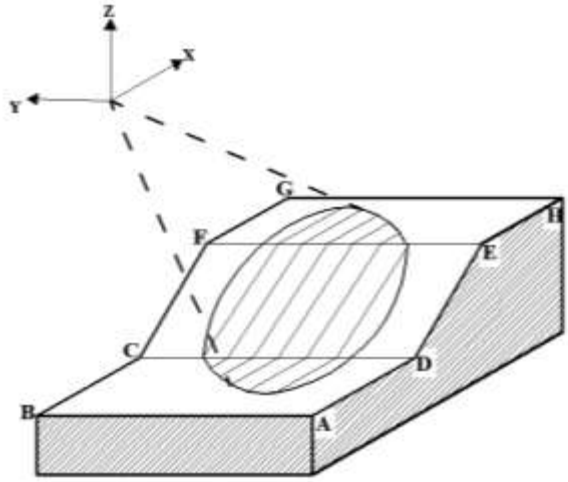

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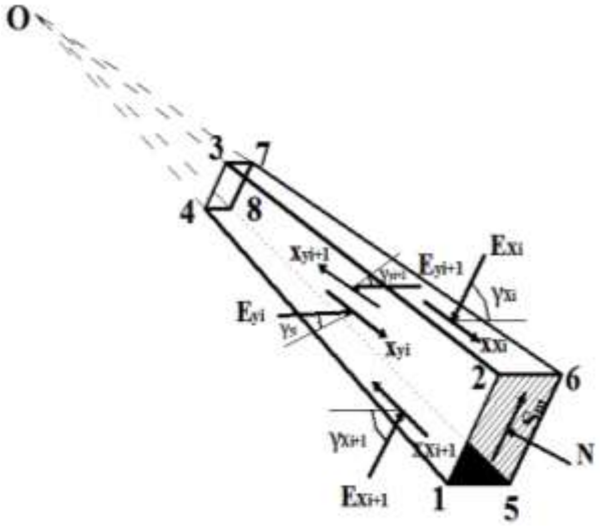

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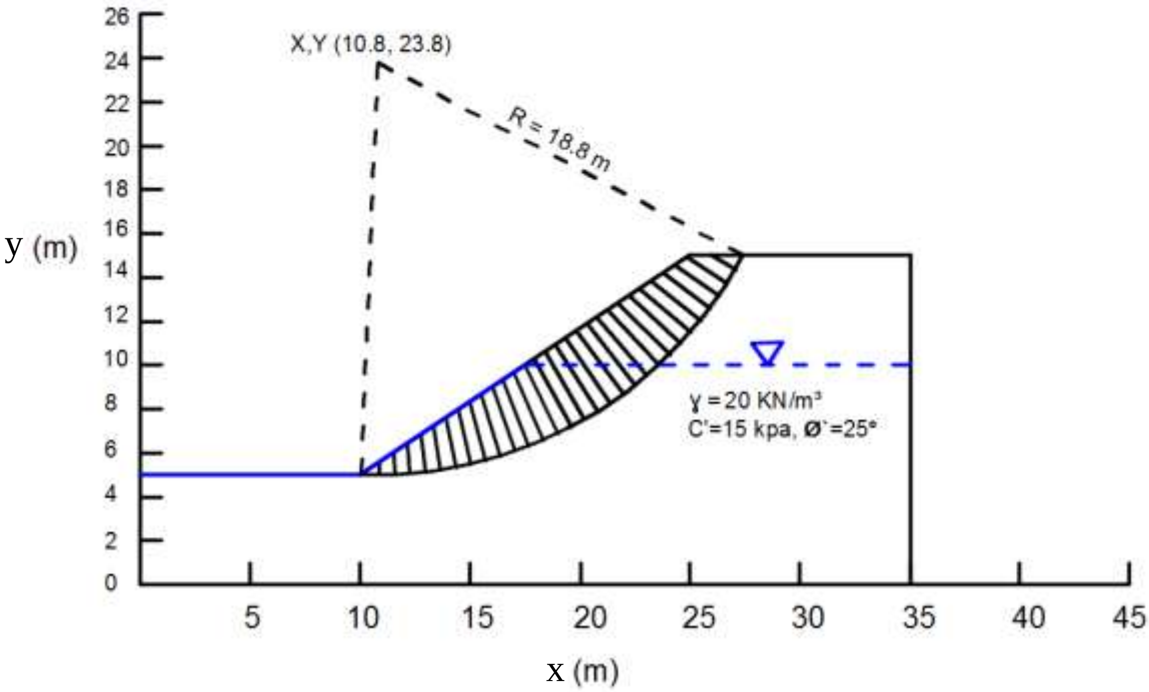

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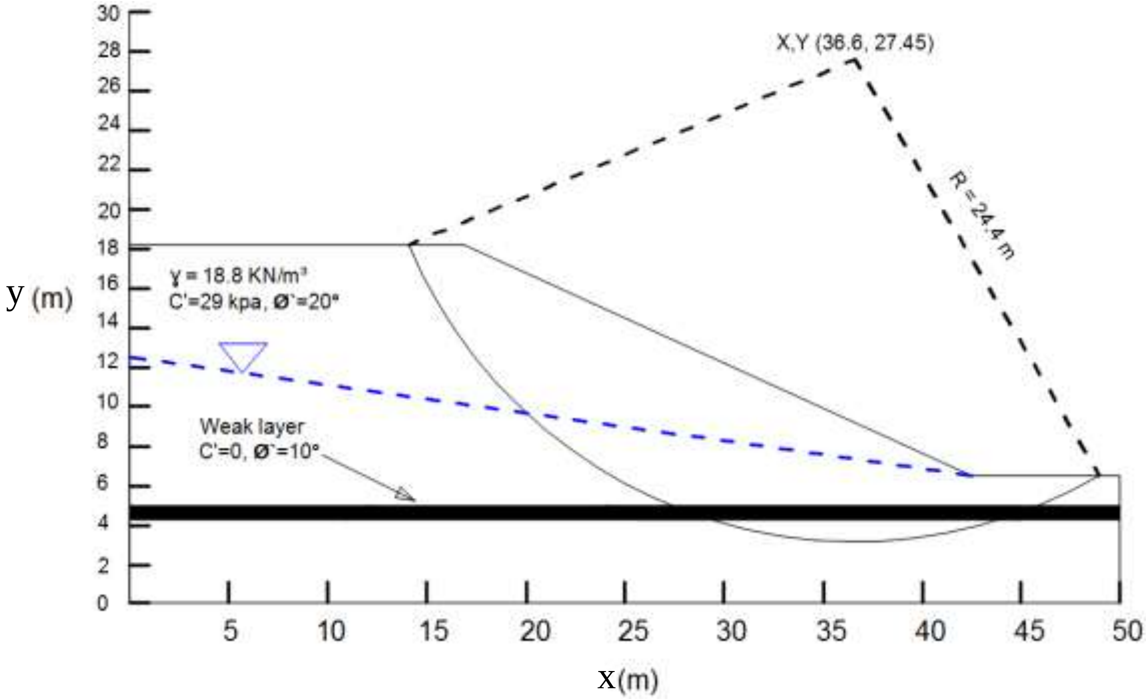

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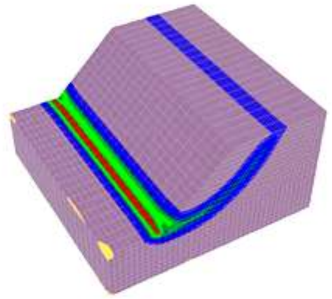


homogeneous, FOS=1.17

Fig. 7

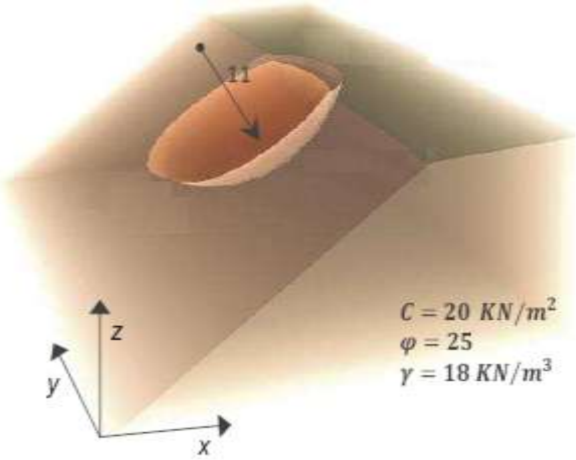


Fig. 8

Fig. 9

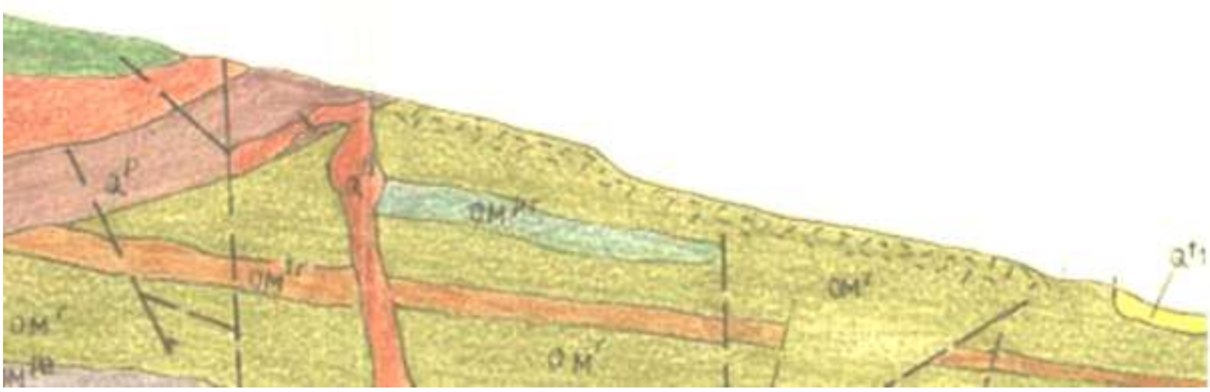
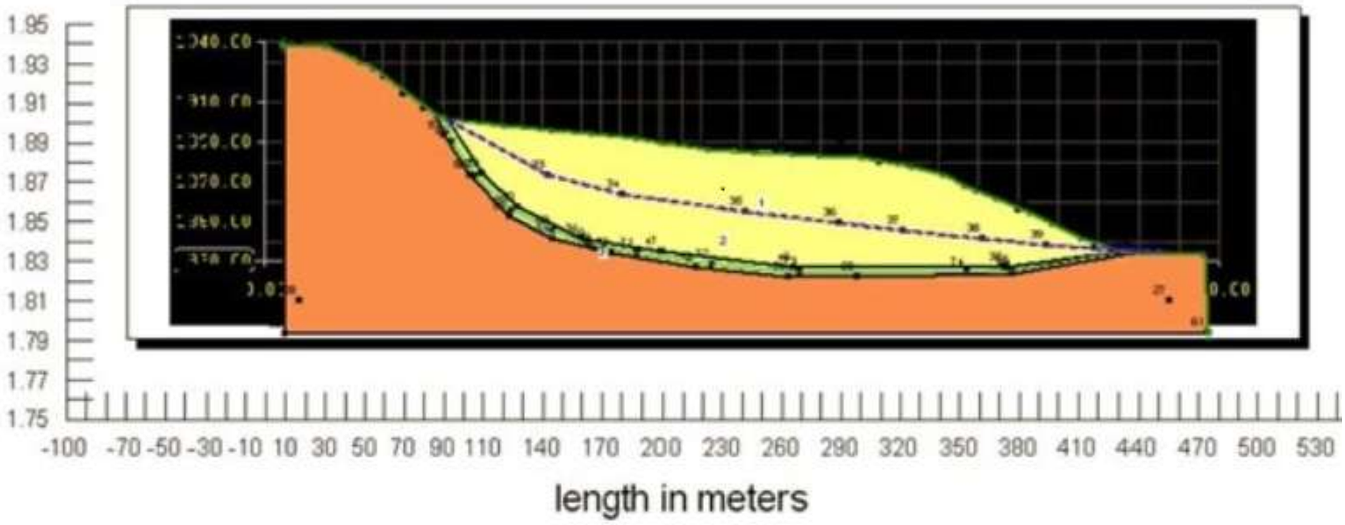


Fig. 10


1.95
1.93
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1.83
1.81
1.79
1.77
1.75
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length in meters

Fig. 11

TABLES

Table 1

Fellenius	Bishop	Janbu	Morgenstern-Price	proposed method
1.12	1.22	1.14	1.22	1.25

Table 2

Fellenius	Bishop	Janbu	Morgenstern-Price	proposed method
1.25	1.31	1.22	1.30	1.31

Table 3

Methods	Safety factor
Average of 2D methods by Fredlund and Krahn (1977)	2.034
3D LEM by Zhang (1988)	2.122
3D UB-LAM by Chen et al. (2001)	2.262
3D-LEM by Chen et al. (2003b)	2.187
3D-LEM CLARA (software package) by Hungr (1987)	2.167
3D-FEM by Griffiths and Marquez (2007)	2.17
Nian et al. (2012)	2.15
Proposed method	2.235

Table 4

Number of slices	40	30	20	10
Safety factor	2.234	2.235	2.243	2.284

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