A new optimization algorithm for parameter optimization of nano-finishing processes

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1. Introduction

Manufacturing industries are experiencing a profound need to manufacture products using materials with extraordinary properties, stringent design requirements, complex geometries, miniature features, improved quality and control, reduced loss of power due to friction, and increased longevity of the product by reducing the wear in sliding components. These requirements are taxing the engineers to manufacture parts with micro and nano-level surface finish. It is evident that the performance of any machining process is greatly influenced by its process parameters. Thus, researchers have recognized the need to investigate the effect of process parameters of the finishing processes on performance measures such as surface roughness, material removal rate, cutting forces, etc. and to keep abreast of the environmental footprint and sustainability of the process.

It is observed from the literature review [1-13] that researchers had proposed theoretical and empirical models for predicting the performance of nano-finishing processes. However, the combination of process parameters recommended by the previous researchers for the best performance of different nano-finishing processes was based on either experimental observations or statistical analysis of the experimental data. Moreover, predictive models developed by previous researchers are nonlinear and complex in nature. Therefore, there is a need to apply advanced optimization algorithms to solve the predictive models in order to obtain the optimal process parameter settings for nano-finishing processes.

Many population-based advanced optimization algorithms have been developed by researchers in the past two decades. Researchers have widely applied these heuristic algorithms to solve complex engineering optimization problems of continuous and discrete nature. However, these algorithms require common control parameters, like population size, number of generations, etc., for their working. Besides, the common control parameters, different algorithms require their own algorithm-specific parameters. For example, Genetic Algorithm (GA) uses mutation rate and crossover rate; Differential Evolution (DE) uses scaling factor...
and cross-over rate; and Particle Swarm Optimization (PSO) algorithm uses inertia weight, social cognitive parameters, maximum velocity, etc. Improper tuning of algorithm-specific parameters either increases the computational effort or yields local optimal solution. In addition to the tuning of algorithm-specific parameters, the common control parameters need to be tuned which further enhances the effort.

Rao et al. [14] introduced the Teaching-Learning-Based Optimization (TLBO) algorithm which does not require any algorithm-specific parameters. The TLBO algorithm has gained wide acceptance among the optimization researchers [15,16]. Keeping in view of the success of the TLBO algorithm, another algorithm-specific parameterless algorithm has been proposed very recently by Rao [17]. However, the proposed new algorithm has only one phase, and it is comparatively simpler to apply. The algorithm is named as “Jaya algorithm”, and it has proved its effectiveness in solving a number of constrained and unconstrained benchmark functions [17]. The Jaya algorithm is described in the following section.

2. The Jaya algorithm

Let \( f(x) \) be the objective function to be minimized (or maximized). At any iteration, \( i \), assume that there are \( m \) number of design variables, and \( n \) number of candidate solutions (i.e., population size, \( k = 1, 2, ..., n \)). Let the best candidate \( \text{best} \) obtain the best value of \( f(x) \) (i.e., \( f(x)_{\text{best}} \)) in the entire candidate solutions, and let the worst candidate \( \text{worst} \) obtain the worst value of \( f(x) \) (i.e., \( f(x)_{\text{worst}} \)) in the entire candidate solutions. If \( X_{j,k,i} \) is the value of \( j \)th variable for \( i \)th candidate during \( i \)th iteration, then this value is modified as per the following equation:

\[
X'_{j,k,i} = X_{j,k,i} + r_1 \cdot j \cdot (X_{j,\text{best},i} - |X_{j,k,i}|) - r_2 \cdot j \cdot (X_{j,\text{worst},i} - |X_{j,k,i}|),
\]

(1)

where \( X_{j,\text{best},i} \) is the value of the variable \( j \) for the best candidate, and \( X_{j,\text{worst},i} \) is the value of the variable \( j \) for the worst candidate. \( X'_{j,k,i} \) is the updated value of \( X_{j,k,i} \); \( r_1 \) and \( r_2 \) are the two random numbers for the \( j \)th variable during \( i \)th iteration in the range \([0,1] \). The term \( \cdot (X_{j,\text{best},i} - |X_{j,k,i}|) \) indicates the tendency of the solution to move closer to the best solution; the term \( \cdot (X_{j,\text{worst},i} - |X_{j,k,i}|) \) indicates the tendency of the solution to avoid the worst solution. \( X'_{j,k,i} \) is accepted if it gives better function value. All the accepted function values at the end of iteration are maintained, and these values become the input to the next iteration. The random numbers, \( r_1 \) and \( r_2 \), ensure good exploration of the search space. The absolute value of the candidate solution (\( |X_{j,k,i}| \)) considered in Eq. (1) further enhances the exploration ability of the algorithm. Figure 1 shows the flow chart of the Jaya algorithm. More details of Jaya algorithm are available at https://sites.google.com/site/jayaalgorithm/.

Now, in order to distinguish the working and to highlight the merits of Jaya algorithm as compared

![Figure 1. Flow chart for the Jaya algorithm.](image)
to other well-known optimization algorithms, such as DE, ABC, and PSO, a brief discussion is provided as follows.

In DE algorithm, a solution is modified in two phases, i.e. the mutation and the crossover phases. However, the Jaya algorithm involves only one phase which makes it simpler to apply compared to DE. The working of DE is governed by two important parameters, i.e. scaling factor and cross-over rate. However, the Jaya algorithm requires no such algorithm-specific parameters. In the mutation phase of DE, a vector difference of randomly chosen vectors (solution) is added to a third vector in order to generate a new vector (solution). However, in the case of Jaya algorithm, difference between the absolute values of a solution and the best and the worst solutions is found and scaled using a random number in the range [0,1], and the obtained value is added to the old value of the solution in order to generate the new solution.

ABC algorithm is inspired by the foraging behavioral patterns of honeybees. The ABC algorithm involves three phases, i.e. employed bee phase, onlooker bee, and abandoned food source phases. However, the Jaya algorithm requires only one phase, making it much simpler to apply than the ABC algorithm. In the employed bee phase and the onlooker bee phase, a candidate solution is updated by adding the scaled difference between the candidate solution and its neighbor to the initial value of the candidate solution. A random number in the range [-1,1] is used as the scaling factor. However, in the case of Jaya algorithm, difference between a candidate solution and the best and the worst solutions is found and scaled using a random number in the range [0,1], and the obtained value is added to the old value of the solution in order to generate the new solution. Furthermore, the best and the worst may not be necessarily the neighbors of the candidate solution to be updated.

The PSO algorithm simulates the social behavior of organisms by using the physical movements of the individuals in the swarm. The velocity of a particle (candidate solution) in the swarm is updated based on the personal best (pbest) solution of a particle and the global best (gbest) solution of the whole swarm, i.e. the best solution found so far in all the iterations. In addition to pbest and gbest, the velocity updating depends upon tuning of algorithm-specific parameters such as inertia weight and learning factors $c_1$ and $c_2$. The updated velocity is then added to the initial position of the particle in the swarm in order to obtain the new position of the particle in the swarm. Therefore, updating a solution requires execution of two separate equations in PSO. On the other hand, the Jaya algorithm does not require tuning of any algorithm-specific parameters for its working, and the solution is updated using a single equation based on the best solution ($\text{best}$) and the worst solution ($\text{worst}$) found in the current iteration. The significant difference between the working of PSO and Jaya algorithm is that the PSO algorithm does not consider the effect of the worst solution while updating a solution. Also, it is worthy noting that the best in the case of Jaya algorithm is not the best solution found so far in all the iterations, rather it is the best solution found only in the current iteration.

Therefore, it can be concluded from the above discussion that the Jaya algorithm is a new optimization algorithm. It is a simple, free from tuning of algorithm specific parameters and is a powerful algorithm for solving the engineering optimization problems.

3. Examples

3.1. Optimization of process parameters of AFM

Abrasive Flow Machining (AFM) process is used to finish difficult-to-reach surfaces by flowing abrasive-laden viscoelastic polymer over them. The objective of this work is to maximize the Material Removal Rate (MRR) (mg/min) in AFM process while obeying the surface roughness constraint.

The optimization problem formulated in this work is based on the empirical models developed by Jain and Jain [2] for MRR and $Ra$ in AFM process. The process parameters considered are: media flow ($v$) (cm/min), percentage concentration of abrasives ($c$), abrasive mesh size ($d$), and number of cycles ($n$). The objective functions, process parameters, and process parameter bounds considered in this work are same as those considered by Jain and Jain [2].

3.1.1. Objective functions

The objective function is expressed by Eq. (2):

$$
\text{Maximize } \text{MRR} = 5.285E - 7v^{1.649}d^{0.0796} \times c^{0.1517} n^{0.1861}. 
$$

3.1.2. Constraint

The constraint on surface roughness is expressed as:

$$
Ra \leq Ra_{\text{max}}. 
$$

where $Ra$ is the surface roughness in $\mu m$ given by Eq. (4), and $Ra_{\text{max}}$ is the maximum allowable value of surface roughness:

$$
Ra = 282751.0v^{-1.8222}c^{-1.8222}d^{0.1068} n^{-0.2228}. 
$$

3.1.3. Parameter bounds

The bounds on the process parameters are expressed as:

$$
40 \leq v \leq 85, 
$$

$$
0 < c < 1, 
$$

$$
0 < d < 1. 
$$
33 \leq v \leq 45, \quad (6) \\
100 \leq d \leq 240, \quad (7) \\
20 \leq n \leq 120. \quad (8)

Jain and Jain [2] applied GA to solve the optimization problem considering a population size equal to 50, maximum number of generations equal to 200 (i.e., maximum number of function evaluations equal to 10000), total string length equal to 40, crossover probability equal to 0.8, and mutation probability equal to 0.01. Now, the same problem is solved using the Jaya algorithm in order to see whether or not improvement in the results can be achieved. For the purpose of fair comparison of results, the maximum number of function evaluations considered by Jaya algorithm is maintained as 10000. For this purpose, a population size of 10 and number of generations equal to 1000 are chosen for the Jaya algorithm after conducting several trials with different values of population sizes.

A computer code for Jaya algorithm is developed in MATLAB v2009a. A computer system with a 2.3GHz processor and 4 GB random access memory is used for execution of the program.

The results obtained using Jaya algorithm for different values of maximum allowable surface roughness (i.e., $R_{a_{\text{max}}} = 0.7, 0.6, 0.5, \text{and } 0.4$) are reported in Table 1. The comparison of results obtained using Jaya algorithm and GA is shown in Table 2. The values of MRR provided by Jaya algorithm are 6.41%, 6.27%, and 5.68% which are higher than the values of MRR provided by GA for $R_{a_{\text{max}}} = 0.7, 0.6, \text{and } 0.5$, respectively. It can be observed from Figure 2(a)-(d) that the convergence graph for Jaya algorithm rises continuously until it reaches the maximum value of MRR, and then remains stable. This shows that the Jaya algorithm is robust and does not get trapped in local optima. The Jaya algorithm has shown a better performance in terms of convergence rate and objective function value as compared to GA.

3.2. Optimization of process parameters of R-AFF process

In the Rotational Abrasive Flow Finishing (R-AFF) process, in addition to the back and forth motions of the abrasive medium, a rotary motion is given to the workpiece in order to enhance the performance of the process. The objective of this work is to maximize the improvement in surface roughness ($\Delta R_{a}$ (µm)) in R-AFF process.

The optimization problem formulated in this work is based on the empirical models developed by Sankar et al. [6] for $\Delta R_{a}$ in R-AFF process. The process parameters considered are process oil %wt in the medium, $M$, extrusion pressure, $P$ (MPa), number of cycles, $N$, and rotational speed, $R$ (rpm). Separate mathematical models for $\Delta R_{a}$ were developed considering three different work-piece materials such as Al alloy, Al alloy/SiC (10%), and AlAlloy/SiC (15%). The objective functions, process parameters, and process parameter bounds considered in this work are same as those considered by Sankar et al. [6].

3.2.1. Objective functions

The objective functions are expressed by Eqs. (9)-(11):

Maximize $\Delta R_{a_{\text{Al alloy}}}$ = $0.098M + 0.875P + 0.002N$

$+ 0.05R - 0.006M^{2} - 0.06SP^{2} - 9.6E$

$- 7N^{2} - 0.002R^{2}. \quad (9)$

Table 1. Optimum combination of process parameters for AFM process obtained using Jaya algorithm.

<table>
<thead>
<tr>
<th>Sr. no.</th>
<th>$R_{a_{\text{max}}}$ (µm)</th>
<th>$v$</th>
<th>$c$</th>
<th>$D$</th>
<th>$n$</th>
<th>$R_{a}$ (µm)</th>
<th>MRR (mg/min)</th>
<th>Computational time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7</td>
<td>85</td>
<td>45</td>
<td>100</td>
<td>20</td>
<td>0.5367</td>
<td>0.738</td>
<td>0.160</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>85</td>
<td>45</td>
<td>100</td>
<td>20</td>
<td>0.5367</td>
<td>0.738</td>
<td>0.158</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>85</td>
<td>45</td>
<td>100</td>
<td>27.3758</td>
<td>0.5</td>
<td>0.6954</td>
<td>0.241</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>85</td>
<td>45</td>
<td>100</td>
<td>73.5436</td>
<td>0.4</td>
<td>0.5768</td>
<td>0.206</td>
</tr>
</tbody>
</table>

Table 2. Comparison of results obtained using Jaya algorithm and GA [2] for AFM process.

<table>
<thead>
<tr>
<th>Sr. no.</th>
<th>$R_{a_{\text{max}}}$ (µm)</th>
<th>GA [2]</th>
<th>Jaya algorithm</th>
<th>% Increase in MRR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_{a}$ (µm)</td>
<td>MRR</td>
<td>$R_{a}$ (µm)</td>
<td>MRR</td>
</tr>
<tr>
<td>1</td>
<td>0.5433</td>
<td>0.6935</td>
<td>0.5367</td>
<td>0.738</td>
</tr>
<tr>
<td>2</td>
<td>0.5113</td>
<td>0.6944</td>
<td>0.5367</td>
<td>0.738</td>
</tr>
<tr>
<td>3</td>
<td>0.4812</td>
<td>0.6860</td>
<td>0.5</td>
<td>0.6954</td>
</tr>
<tr>
<td>4</td>
<td>0.4171*</td>
<td>0.5803</td>
<td>0.4</td>
<td>0.5768</td>
</tr>
</tbody>
</table>

* Constraint violated by GA [2].
Figure 2. (a) Convergence graph of Jaya algorithm for AFM ($R_{a_{\max}} < 0.7$), (b) Convergence graph of Jaya algorithm for AFM ($R_{a_{\max}} < 0.6$), (c) Convergence graph of Jaya algorithm for AFM ($R_{a_{\max}} < 0.5$). (d) Convergence graph of Jaya algorithm for AFM ($R_{a_{\max}} < 0.4$).

Table 3. Optimum combination of process parameters for maximization of $\Delta R_{a_{Al\ alloy}}$, $\Delta R_{a_{Al}/SIC(10\%)}$, $\Delta R_{a_{Al}/SIC(15\%)}$ obtained using the Jaya algorithm for R-AFF process.

<table>
<thead>
<tr>
<th>Sr. no.</th>
<th>$M$</th>
<th>$P$</th>
<th>$N$</th>
<th>$R$</th>
<th>$\Delta R_{a_{Al\ alloy}}$</th>
<th>$\Delta R_{a_{Al\ alloy}/SIC(10%)}$</th>
<th>$\Delta R_{a_{Al\ alloy}/SIC(15%)}$</th>
<th>Computational time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.1671</td>
<td>6.434</td>
<td>727.998</td>
<td>10</td>
<td>3.5281</td>
<td>—</td>
<td>—</td>
<td>0.697</td>
</tr>
<tr>
<td>2</td>
<td>9.833</td>
<td>6.209</td>
<td>727.997</td>
<td>7.75</td>
<td>3.2835</td>
<td>—</td>
<td>—</td>
<td>0.51471</td>
</tr>
<tr>
<td>3</td>
<td>10.978</td>
<td>6.7163</td>
<td>728.000</td>
<td>10.0</td>
<td>—</td>
<td>—</td>
<td>3.3732</td>
<td>0.54269</td>
</tr>
</tbody>
</table>

Maximize $\Delta R_{a_{Al\ alloy}/SIC(10\%)} = 0.118M + 0.831P$

\[+ 0.001N + 0.031R - 0.0006M^2\]

\[= 0.067P^2 - 1.2E - 6N^2\]

\[- 0.002R^2.\]  \hspace{1cm} (10)

Maximize $\Delta R_{a_{Al\ alloy}/SIC(15\%)} = 0.101M + 0.767P$

\[+ 0.002N + 0.043R - 0.0046M^2\]

\[= 0.0571P^2 - 8.28E - 7N^2\]

\[- 0.002R^2.\]  \hspace{1cm} (11)

\[7 \leq M \leq 13,\]  \hspace{1cm} (12)

\[5.35 \leq P \leq 7.15,\]  \hspace{1cm} (13)

\[372 \leq N \leq 728,\]  \hspace{1cm} (14)

\[2 \leq R \leq 10.\]  \hspace{1cm} (15)

3.2.2. Parameter bounds

The bounds on the process parameters are expressed as:
pressure \( P \) (bar), number of finishing cycles \( N \), rotational speed of magnet \( S \) (RPM), and volume ratio of CIP/SIC \( R \). The objective functions, process parameters, and process parameter bounds considered in this work are same as those considered by Das et al. [8].

3.3.1. Objective Functions

The objective functions are expressed by Eqs. (16) and (17):

Maximize \[ \% \Delta Ra_{SS} = -403 + 17.66P + 0.2N \\
+ 0.83S + 10.38R + 1.89E - 4PN \\
- 1.67E - 3PS - 3.56E - 3PR \\
- 6.53E - 5NS - 7E - 3NR + 8.46E \\
- 3SR - 0.23P^2 - 1.39E - 4N^2 \\
- 5.56E - 3S^2 - 1.35R^2. \] (16)

Maximize \[ \% \Delta Ra_{BR} = -912.47 + 39.27P + 0.41N \\
+ 1.67S + 28.49R - 4.02E - 3PN \\
- 0.01PS - 0.08PR + 3.93E - 4NS \\
- 2.76E - 3NR - 0.10SR - 0.46P^2 \\
- 2.07E - 4N^2 - 9.26E - 3S^2 - 3.83R^2. \] (17)

3.3.2. Process Parameters

The bounds on the process parameters are expressed as:

\[ 32.5 \leq P \leq 42.5, \] (18)

\[ 400 \leq N \leq 800, \] (19)

\[ 20 \leq S \leq 100, \] (20)

\[ 0.34 \leq R \leq 4. \] (21)

Das et al. [8] applied desirability function approach to determine the optimum combination of process parameters for R-MRAFF process. The maximum value of \( \% \Delta Ra_{SS} \) and \( \% \Delta Ra_{BR} \) obtained by Das et al. [8] using desirability function approach is reported in Tables 4 and 5, respectively. Now, the same problem is solved using Jaya algorithm by considering a population size of 10 and maximum number of function evaluations as 1000. The optimum value of \( \% \Delta Ra_{SS} \) and \( \% \Delta Ra_{BR} \) obtained by Jaya algorithm along with the optimum combination of process parameters of R-MAFF process is also reported in Tables 4 and 5, respectively. The Jaya algorithm achieved a better value of \( \% \Delta Ra_{SS} \).
and $\%\Delta Ra_{SS}$ in 310 and 210 function evaluations, respectively, as compared to the values of $\%\Delta Ra_{SS}$ and $\%\Delta Ra_{BR}$ obtained using the desirability approach.

All the optimization problems formulated in this work are based on the mathematical models developed by previous researchers based on experimentation. The confirmation experiments for the developed mathematical models were also conducted by the previous researchers for processes such as AFM [2], R-AFF [6], and R-MRAFF [8]. In addition, the previous researchers had solved the optimization problems using GA and desirability function approach. Now, the same mathematical models are solved using Jaya algorithm, and the results obtained using Jaya algorithm are compared with the results obtained by the previous researchers. The previous researchers had considered the process parameters in their continuous form. Therefore, all the process parameters considered in this work are in their continuous form only. However, in actual practice, the values allowed by the machining process which are closer to the suggested optimum values may be considered.

### 4. Conclusions

- In the present work, the optimization problems of the three advanced finishing processes, i.e., AFM, R-AFF, and R-MRAFF, are solved using the newly proposed Jaya algorithm;
- The performance of the Jaya algorithm is studied in terms of convergence rate and accuracy of the solution. Compared to other advanced optimization methods, the Jaya algorithm does not require selection of algorithm-specific parameters, and this feature makes the Jaya algorithm applicable to real-life optimization problems, easily and effectively;
- In the case of AFM process, maximization of MRR is considered as the objective function, while the constraint is on the allowable value of surface roughness. The process parameter combination, as suggested by Jaya algorithm, increases the MRR by 6.41%, 6.27%, and 5.68% as compared to the MRR provided by GA for $Ra_{max} = 0.7, 0.6$, and 0.5, respectively;
- In the case of R-AFF process, maximization of improvement in surface roughness is considered as the objective function. The Jaya algorithm obtained a maximum value of $\Delta Ra_{Alloy}, \Delta Ra_{Alloy/SiC(10\%)}$, and $\Delta Ra_{Alloy/SiC(15\%)}$ in 120, 170, and 210 function evaluations, respectively, as compared to the values of $\%\Delta Ra_{SS}$ and $\%\Delta Ra_{BR}$ obtained by using the desirability approach;
- The results reported in this work show that the convergence accuracy and its speed are very high. The results obtained by Jaya algorithm are found to be better in terms of objective function values as compared to those obtained by using GA and desirability function approach and have also demonstrated the ability of Jaya algorithm to handle the constraints.

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References


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