Scientia Iranica E (2017) 24(2), 808–822

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
www.scientiairanica.com

Reliability analysis of a warm standby repairable system with two cases of imperfect switching mechanism

M. Sadeghi and E. Roghanian*

Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran.

Received 6 October 2015; received in revised form 29 February 2016; accepted 18 April 2016

KEYWORDS
Markov process;
Switching mechanisms;
Reliability;
MTTF;
Steady-state availability.

Abstract. This paper studies a warm standby repairable system including two dissimilar units: one repairman and imperfect switching mechanism. Times to failure and times to repair of active and standby units are assumed to be exponentially distributed. Two cases of unreliable switching mechanism are considered. In case one, the failed active unit will be replaced by the available warm standby unit with coverage probability c. However, in case two, the switching mechanism is repairable, and its failure time and repair time are also exponentially distributed. Using Markov process and Laplace transforms, the explicit expressions of the mean time to failure, MTTF, and the steady-state availability of the two systems are derived analytically. Finally, by solving a numerical example, the two systems are compared based on various reliability and availability characteristics. Moreover, sensitivity analyses of the reliability and availability indexes are accomplished with respect to the model parameters.

1. Introduction

Reliability and availability are essential factors for evaluating the performance of engineering systems. Improving system reliability and availability is a fundamental requisite in power plants, production, manufacturing, and industrial systems. Enhancing system reliability and availability can be achieved by either increasing the reliability of each component in the system or adding redundant units. In general, redundancy strategy can be in two forms: active and standby. In active redundancy, all units operate simultaneously, whereas in standby redundancy, one of the redundant units will be put into operation when the active one fails. There are two categories of standby redundancy: cold and warm. In cold standby, the redundant units do not fail before being put into full operation, whereas, in warm standby, the inactive units may fail while they are in standby state. In standby redundancy, a switching mechanism is required to detect the failed active unit and replace it by the standby unit if one is available. This switching mechanism can be perfect or imperfect. In the case of perfect switching mechanism, two-unit standby repairable systems under different assumptions were extensively studied in the past [1-6]. Lewis [7] introduced the concept of imperfect or unreliable switching in standby systems. Coit [8,9] explained two cases of imperfect switching mechanism. In one of the cases, switching failure can only occur in response to an active component failure with probability, $1-c$. Therefore, the probability of successful detection and replacement is denoted by parameter c which is known as the coverage factor (see Trivedi [10]). In the other case, the switching mechanism continuously monitors active components functionality to detect a failure. In this case, the switching mechanism may fail at

*. *Corresponding author. Tel.: +98 21 84063368;*
Fax: +98 21 88674858
E-mail address: E_Roghanian@kntu.ac.ir (E. Roghanian)

any time, and its time to failure is often presented by a probability distribution function. As long as the switching mechanism is in working state, the active unit failure is successfully detected and immediately replaced by the available standby unit. Switching mechanism failure does not necessarily lead to the system failure because no switching may be required during the remainder of the system mission time.

By considering imperfect switching, Soltani et al. [11] and Sadjadi and Soltani [12] presented a nonlinear redundancy allocation model with the choice of redundancy strategy and component type in serial parallel systems in which component's time to failure follows an Erlang distribution with imprecise scale parameter. Wang and Chiu [13] performed the cost benefit analysis of three availability systems with warm standby units, imperfect coverage, exponentially distributed failure time, and general repair time distribution. Using the supplementary variable technique, a recursive method was presented to derive the explicit expressions of the steady state availability of three models for various repair time distributions, such as exponential, k-stage Erlang, and deterministic. Wang et al. [14] performed a comparison of the MTTF and the steady-state availability between four configurations with warm standby units, unreliable switching, and exponentially distributed failure and repair times. Trivedi [10] introduced the concept of reboot delay for repairable systems. Ke et al. [15] studied the reliability measures of a warm standby repairable system with coverage probability and reboot delay. In their analysis, times to failure, times to repair, and reboot time were assumed to be exponentially distributed. They provided a Laplace transform method for deriving the system reliability characteristics. In addition, a sensitivity analysis for the system reliability and MTTF along with changes in the system parameters was performed.

Wang and Chen [16] compared the steady-state availability of three different systems with warm standby units, standby switching failure, reboot delay, exponentially distributed failure time, and general repair time distribution. The explicit expressions of the steady-state availability of three configurations were derived, and comparative analysis for three various repair time distributions, such as exponential, gamma, and uniform, was also performed. Ke et al. [17,18] considered a repairable system with two primary units: one standby, imperfect coverage and reboot delay. In their works, times to failure, times to repair, and reboot delays were assumed to follow exponential distribution with fuzzy parameters. Using parametric nonlinear programming approach, the membership function of the system MTTF and the system steady-state availability were obtained analytically. Ke and Liu [19] studied a warm standby repairable system with imperfect coverage, reboot delay, exponentially distributed time to failure, and general repair time distribution. They constructed an efficient algorithm to compute the steady-state availability and provided numerical examples for various repair time distributions, such as exponential, gamma, log-normal, and Weibull. Wang et al. [20] examined the reliability and sensitivity analysis of a repairable system with imperfect coverage under a service pressure condition. Times to failure, times to repair, and reboot delay were assumed to be exponentially distributed in this work. Moreover, they assumed that under the pressure of long queue, the repair rate increases to reduce the queue length. The explicit expressions for the system reliability and MTTF were derived, and sensitivity analysis of reliability characteristics with respect to the system parameters was performed.

Hsu et al. [21] statistically investigated an availability system consisting of two active components and one warm standby with reboot delay, standby switching failures, and an unreliable repair facility. Times to failure and the reboot time were assumed to follow exponential distribution, whereas times to repair of failed components and repair time of the service station were assumed to be generally distributed. They developed a consistent and asymptotically normal estimator of the availability. Wang et al. [22] analyzed the warm standby M/M/R machine repair problem with multiple imperfect coverage and service pressure coefficients. Using a recursive method to develop the steady-state analytical solutions, various system performance measures, such as the expected number of failed machines, the expected number of idle servers, machine availability, and operative utilization, were calculated. Moreover, Quasi-Newton method and PSO algorithm were implemented to determine the optimal number of servers, the optimal number of warm standbys, and the optimal service rate at the maximum profit. Ke et al. [23] considered a machine repair problem with warm standbys, imperfect coverage, service pressure coefficient, and R non-reliable servers. Developing a Markov chain model, the stationary distribution was obtained through matrix recursive method, and various system performance measures were computed. In addition, the Quasi-Newton and probabilistic global search Lausanne methods were used to search for the global optimal system parameters. Hsu et al. [24] studied an M/M/R machine repair problem with warm standbys, switching failures, reboot delays, and repair pressure coefficient. Adopting the matrix-analytic method, the steady-state analytic solutions were obtained to calculate system performance measures. Moreover, probabilistic global search Lausanne method was utilized to determine the optimal values which maximize the function of expected profit per unit time. Kuo and Ke [25] studied the steady-state availability of a repairable system with standby switching failure and an unreliable server. Time to failure of the components and time to breakdown of the server were assumed to follow exponential distribution, while the repair time of the failed components and the repair time of the breakdown server were assumed to be generally distributed. Using supplementary variable method and integro-differential equations, they derived the steady-state availability for three different system configurations.

Recently, two-unit standby repairable systems under different assumptions have been studied. Kakkar et al. [26] investigated a parallel framework consisting of two dissimilar units under the presumption that a framework's unit may also fail during the preventive maintenance. Kakkar et al. [27] also studied this system under the assumption that active unit cannot fail after post-repair inspection and replacement. Considering the concepts of preventive maintenance, priority and maximum repair time, Kumar et al. [28] developed a stochastic model for a two-unit cold standby system. Moreover, Kumar and Malik [29] analyzed a computer system consisting of two identical units with independent hardware and software component failures.

All of the above-described studies about imperfect switching in repairable systems were concerned with the case related to coverage factor, whereas the other case, in which times to failure of switching mechanism follows a specified distribution, was rarely addressed. In this regard, Yuan and Meng [30] considered a warm standby repairable system with two dissimilar units: one repairman and unreliable switching mechanism. Working time and repair time of primary and standby units were assumed to follow exponential distribution, and unit one had priority in use. Moreover, time to failure and time to repair of switching mechanism were also exponentially distributed. They assumed that the switch failure leads to the failure of the whole system immediately, whereas in many real-world engineering system examples, switching mechanism failure does not necessarily lead to the system failure, because no switching may be required during the remainder of the system mission time, or the switching mechanism may be repaired before the time of the active unit failure. This issue motivates us to release the assumption of failing the whole system due to the switching mechanism failure. Moreover, imperfect coverage is also considered for the repairable system.

A practical example related to smart grid is presented for illustrative purposes. In power industry, improving the efficiency, reliability, economics, and safety of grids is a fundamental requisite. Therefore, in recent years, smart grid as an effective solution, in which upgrading and automating the generation, transmission, distribution, and management of electricity are performed through incorporating advanced computing and communication technologies, has been extensively deployed worldwide [31]. Fan and Gong [31] illustrated the architecture of a typical smart grid metering and control system consisting of utility company, substation/data concentrator network, Home Area Network (HAN), smart meter and third party. Herein, we consider substation/data concentrator network as a subsystem of smart grid, which consists of a number of smart meters and data concentrators employed in a specific area. Different communication technologies, such as Wi-Fi, Zigbee, Power Line Career (PLC), etc., can be used to establish a link between smart meters and data concentrators. The smart meter readings are forwarded to the data concentrator, and then the accumulative data are transmitted to the utility company through data concentrators. Ancilloti et al. [32] identified the basic requirements that should be satisfied by smart grid communication infrastructures, in which reliability is one of the key communication requirements. To ensure achieving a satisfactory level of reliability, redundancy is known as one of the most efficient approaches.

The mentioned subsystem can be composed of two dissimilar and repairable data concentrators. The active data concentrator is called the primary concentrator; the backup one is called the spare concentrator. At the beginning, the primary data concentrator is in working state, and the spare data concentrator is in warm standby state. The primary and spare data concentrators may fail due to the environmental influences. The subsystem also consists of an imperfect switching mechanism. There are two cases for switching mechanism. In one case, when the active data concentrator fails, it can be immediately replaced by the available warm standby one with coverage probability; in the other one, there is a repairable switch. As long as the switch is in working state, the failed active data concentrator is immediately replaced by the available warm standby data concentrator. The switch failure does not lead to the subsystem failure. In this paper, we assume that the working time and repair time of the active and the warm standby data concentrators are exponentially distributed. In addition, working time distribution and repair time distribution of the case two of switching mechanism are exponential.

The rest of this paper is organized as follows: Section 2 describes the model assumptions. Using Markov process and Laplace transforms, the explicit expressions of the MTTF and the steady-state availability for the system with the two cases of imperfect switching mechanism are obtained in Sections 3 and 4, respectively. In Section 5, a numerical example is provided. The influence of system parameters on the reliability and availability characteristics of the system with the two cases of imperfect switching mechanism is studied. Moreover, a sensitivity analysis is conducted to compare the two cases. Conclusions are drawn in Section 6.

2. Model assumptions

A redundant repairable system, consisting of one active unit, a warm standby unit, a repairman, and an imperfect switching mechanism, is studied by considering the following assumptions.

Assumption 1. Active and warm standby units are dissimilar. At first, unit one is in working state, whereas unit two is in warm standby state;

Assumption 2. Operating unit, i ($i = 1, 2$), has an exponential time to failure distribution with parameter λ_i. Warm standby unit i may fail before being put into full operation and has an exponential time to failure distribution with parameter ε_i ($0 < \varepsilon_i < \lambda_i$). Moreover, the repair time of unit i is exponentially distributed with parameter α_i;

Assumption 3. Two cases of imperfect switching mechanism are considered as follows.

Mechanism 1. When the active unit fails, it can be immediately replaced by the available warm standby unit with coverage probability c;

Mechanism 2. The switching mechanism has an exponential time to failure distribution with parameter β. This switching mechanism is repairable and the repair time is also exponentially distributed with parameter μ. As long as the switching mechanism is in working state, the failed active unit is immediately replaced by the available warm standby unit. Switching mechanism failure does not necessarily lead to the system failure, because no switching may be required during the remainder of the system mission time, or the switching mechanism may be repaired before the time of the active unit failure.

Assumption 4. When one of the units or Mechanism 2 of switching fails, it is immediately repaired if the repairman is idle. For the active and the warm standby units, repair is performed based on the first come and first repaired discipline. But, Mechanism 2 of switching has priority in repair than the units;

Assumption 5. When the standby unit successfully replaces the failed one, its failure characteristics turn into those of an active unit. Moreover, the failed units will be in warm standby state after performing repair;

Assumption 6. A fault-detecting device continuously monitors the warm standby unit to identify its failure;

Assumption 7. For the system consisting of Mechanism 1 of switching, reboot delay, which is exponentially distributed with parameter θ, occurs after an unsuccessful switching. The other events cannot take place during a reboot;

Assumption 8. Time to failure and repair time of units and Mechanism 2 of switching are independent from each other.

Henceforth, the systems, consisting of Mechanisms 1 and 2 of switching, are called as System 1 and System 2, respectively.

3. The reliability function and MTTF

Let $N(t)$ indicate the state of the repairable systems at time t. Then, for reliability models, $\{N(t); t \geq 0\}$ is a continuous-time Markov process whose states corresponding to Systems 1 and 2 are presented in Tables 1 and 2, respectively.

The state transition diagrams of two systems are also depicted in Figure 1(a) and (b). In accordance with these figures, the transition rate matrices of Systems 1 and 2 can be respectively obtained as follows:

$$Q_1 = \begin{bmatrix} A_1 & B_1 \\ O_1 & 0 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} A_2 & B_2 \\ O_2 & 0 \end{bmatrix},$$

in which O_1 and O_2 are zero row vectors, and the other matrices are defined as shown in Box I.

Herein, $\Pr(N(t) = j)$ is represented by $P_j(t)$ for $j = 0, 1, 2, 3, F$ in the case of System 1 and $j = 0, 1, 2, 3, 4, 5, 6, 7, F$ in the case of System 2. The Kolmogorov forward equations (see [33]) of Systems 1 and 2, respectively, are given by:

Table 1. The states of Markov process of reliability models related to System 1.

State index	Unit 1 status	Unit 2 status
0	Working	Warm standby
1	Warm standby	Working
2	Working	Under repair
3	Under repair	Working
F	None of the two units is in working status.	

Table 2. The states of Markov process of reliability models related to System 2.

State index	Unit 1 status	Unit 2 status	Switch status
0	Working	Warm standby	Good
1	Warm standby	Working	Good
2	Working	Under repair	Good
3	Under repair	Working	Good
4	Working	Warm standby	Under repair
5	Warm standby	Working	Under repair
6	Working	Waiting for repair	Under repair
7	Waiting for repair	Working	Under repair
F	None of the two units is in working status.		

$$A_1=\begin{bmatrix}-(\varepsilon_2+\lambda_1) & 0 & \varepsilon_2 & c\lambda_1\\ 0 & -(\varepsilon_1+\lambda_2) & c\lambda_2 & \varepsilon_1\\ \alpha_2 & 0 & -(\alpha_2+\lambda_1) & 0\\ 0 & \alpha_1 & 0 & -(\alpha_1+\lambda_2)\end{bmatrix},\qquad B_1=\begin{bmatrix}(1-c)\lambda_1\\(1-c)\lambda_2\\ \lambda_1\\ \lambda_2\end{bmatrix},$$

$$A_2=\begin{bmatrix}-(\lambda_1+\varepsilon_2+\beta) & 0 & \varepsilon_2 & \lambda_1 & \beta & 0 & 0 & 0\\ 0 & -(\lambda_2+\varepsilon_1+\beta) & \lambda_2 & \varepsilon_1 & 0 & \beta & 0 & 0\\ \alpha_2 & 0 & -(\alpha_2+\beta+\lambda_1) & 0 & 0 & 0 & \beta & 0\\ 0 & \alpha_1 & 0 & -(\alpha_1+\beta+\lambda_2) & 0 & 0 & 0 & \beta\\ \mu & 0 & 0 & 0 & -(\mu+\varepsilon_2+\lambda_1) & 0 & \varepsilon_2 & 0\\ 0 & \mu & 0 & 0 & 0 & -(\mu+\varepsilon_1+\lambda_2) & 0 & \varepsilon_1\\ 0 & 0 & \mu & 0 & 0 & 0 & -(\mu+\lambda_1) & 0\\ 0 & 0 & 0 & \mu & 0 & 0 & 0 & -(\mu+\lambda_2)\end{bmatrix},$$

$$B_2=\begin{bmatrix}0 & 0 & \lambda_1 & \lambda_2 & \lambda_1 & \lambda_2 & \lambda_1 & \lambda_2\end{bmatrix}^T.$$

Box I

$$\frac{dP(t)}{dt}=P(t)Q_1,\qquad \frac{dP(t)}{dt}=P(t)Q_2,$$

where:

$$P(t)=\begin{bmatrix}P_0(t) & \dots & P_F(t)\end{bmatrix}.$$

If we assume that the process is initially in state 0, one can have $P_0(0)=1$ and $P_j(0)=0$, $j\neq 0$; hence, the system differential equations, employing Laplace transforms for Systems 1 and 2 are attained by Eqs. (1) and (2) respectively:

$$s\tilde{P}_0(s)-1=-(\lambda_1+\varepsilon_2)\tilde{P}_0(s)+\alpha_2\tilde{P}_2(s), \tag{1a}$$

$$s\tilde{P}_1(s)=-(\lambda_2+\varepsilon_1)\tilde{P}_1(s)+\alpha_1\tilde{P}_3(s), \tag{1b}$$

$$s\tilde{P}_2(s)=\varepsilon_2\tilde{P}_0(s)+c\lambda_2\tilde{P}_1(s)-(\lambda_1+\alpha_2)\tilde{P}_2(s), \tag{1c}$$

$$s\tilde{P}_3(s)=c\lambda_1\tilde{P}_0(s)+\varepsilon_1\tilde{P}_1(s)-(\lambda_2+\alpha_1)\tilde{P}_3(s), \tag{1d}$$

$$s\tilde{P}_F(s)=(1-c)\lambda_1\tilde{P}_0(s)+(1-c)\lambda_2\tilde{P}_1(s)+\lambda_1\tilde{P}_2(s)+\lambda_2\tilde{P}_3(s), \tag{1e}$$

$$s\tilde{P}_0(s)-1=-(\lambda_1+\varepsilon_2+\beta)\tilde{P}_0(s)+\alpha_2\tilde{P}_2(s)+\mu\tilde{P}_4(s), \tag{2a}$$

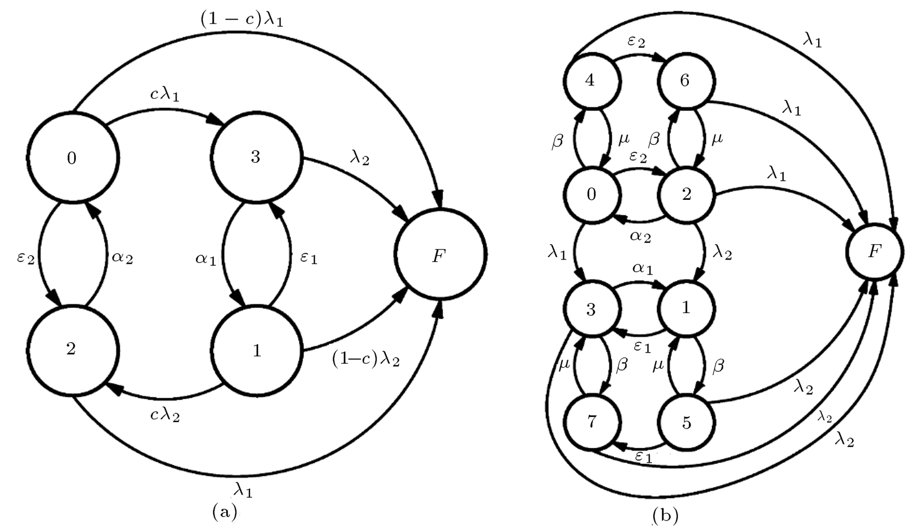

Figure 1. State transition diagram for reliability models related to: (a) System 1 and (b) System 2.

$$s\tilde{P}_1(s)=-(\lambda_2+\varepsilon_1+\beta)\tilde{P}_1(s)+\alpha_1\tilde{P}_3(s)+\mu\tilde{P}_5(s), \quad (2b)$$

$$s\tilde{P}_2(s)=\varepsilon_2\tilde{P}_0(s)+\lambda_2\tilde{P}_1(s)-(\alpha_2+\beta+\lambda_1)\tilde{P}_2(s)+\mu\tilde{P}_6(s), \quad (2c)$$

$$s\tilde{P}_3(s)=\lambda_1\tilde{P}_0(s)+\varepsilon_1\tilde{P}_1(s)-(\alpha_1+\beta+\lambda_2)\tilde{P}_3(s)+\mu\tilde{P}_7(s), \quad (2d)$$

$$s\tilde{P}_4(s)=\beta\tilde{P}_0(s)-(\mu+\varepsilon_2+\lambda_1)\tilde{P}_4(s), \quad (2e)$$

$$s\tilde{P}_5(s)=\beta\tilde{P}_1(s)-(\mu+\varepsilon_1+\lambda_2)\tilde{P}_5(s), \quad (2f)$$

$$s\tilde{P}_6(s)=\beta\tilde{P}_2(s)+\varepsilon_2\tilde{P}_4(s)-(\mu+\lambda_1)\tilde{P}_6(s), \quad (2g)$$

$$s\tilde{P}_7(s)=\beta\tilde{P}_3(s)+\varepsilon_1\tilde{P}_5(s)-(\mu+\lambda_2)\tilde{P}_7(s), \quad (2h)$$

$$s\tilde{P}_F(s)=\lambda_1\tilde{P}_2(s)+\lambda_2\tilde{P}_3(s)+\lambda_1\tilde{P}_4(s)+\lambda_2\tilde{P}_5(s)+\lambda_1\tilde{P}_6(s)+\lambda_2\tilde{P}_7(s). \quad (2i)$$

Solving these two systems of equations leads to $\tilde{P}_j(s)$, which is not shown here due to the spacious forms. The Laplace transform of the system reliability is expressed by:

$$\tilde{R}(s)=\sum_{j\neq F}\tilde{P}_j(s). \quad (3)$$

The mean time to failure of the system can be achieved by evaluating $\tilde{R}(s)$ at $s=0$.

$$\text{MTTF}=\int_0^\infty R(t)dt=\tilde{R}(0). \quad (4)$$

Therefore, the MTTF of two systems is obtained as:

$$\text{MTTF}_1=\frac{f_1(\lambda_1,\lambda_2,\varepsilon_1,\varepsilon_2,\alpha_1,\alpha_2,c)}{g_1(\lambda_1,\lambda_2,\varepsilon_1,\varepsilon_2,\alpha_1,\alpha_2,c)}, \quad (5)$$

$$\text{MTTF}_2=\frac{f_2(\lambda_1,\lambda_2,\varepsilon_1,\varepsilon_2,\alpha_1,\alpha_2,\beta,\mu)}{g_2(\lambda_1,\lambda_2,\varepsilon_1,\varepsilon_2,\alpha_1,\alpha_2,\beta,\mu)}, \quad (6)$$

in which functions f_1 and g_1 are defined in the Appendix and the functions f_2 and g_2 are too spacious to be shown.

4. The steady-state availability

Considering $N(t)$ be the state of the repairable system at time t, then $\{N(t); t \geq 0\}$ is a continuous-time Markov process for availability models with the states described in Tables 3 and 4 for Systems 1 and 2, respectively.

According to the state transition diagrams shown in Figure 2(a) and (b), the transition rate matrices are respectively attained as follows:

$$Q_3=\begin{bmatrix} A_1 & C_1 \\ D_1 & E_1 \end{bmatrix}, \qquad Q_4=\begin{bmatrix} A_2 & C_2 \\ D_2 & E_2 \end{bmatrix},$$

Table 3. The states of Markov process of steady-state availability models related to system one.

State index	Unit 1 status	Unit 2 status
0	Working	Warm standby
1	Warm standby	Working
2	Working	Under repair
3	Under repair	Working
4	Warm standby	Under repair
5	Under repair	Warm standby
6	Under repair	Waiting for repair
7	Waiting for repair	Under repair

Table 4. The states of Markov process of steady-state availability models related to System 2.

State index	Unit 1 status	Unit 2 status	Switch status
0	Working	Warm standby	Good
1	Warm standby	Working	Good
2	Working	Under repair	Good
3	Under repair	Working	Good
4	Working	Warm standby	Under repair
5	Warm standby	Working	Under repair
6	Working	Waiting for repair	Under repair
7	Waiting for repair	Working	Under repair
8	Under repair	Waiting for repair	Good
9	Waiting for repair	Under repair	Good
10	Warm standby	Waiting for repair	Under repair
11	Waiting for repair	Warm standby	Under repair
12	Waiting for repair firstly	Waiting for repair after unit 1	Under repair
13	Waiting for repair after unit 2	Waiting for repair firstly	Under repair

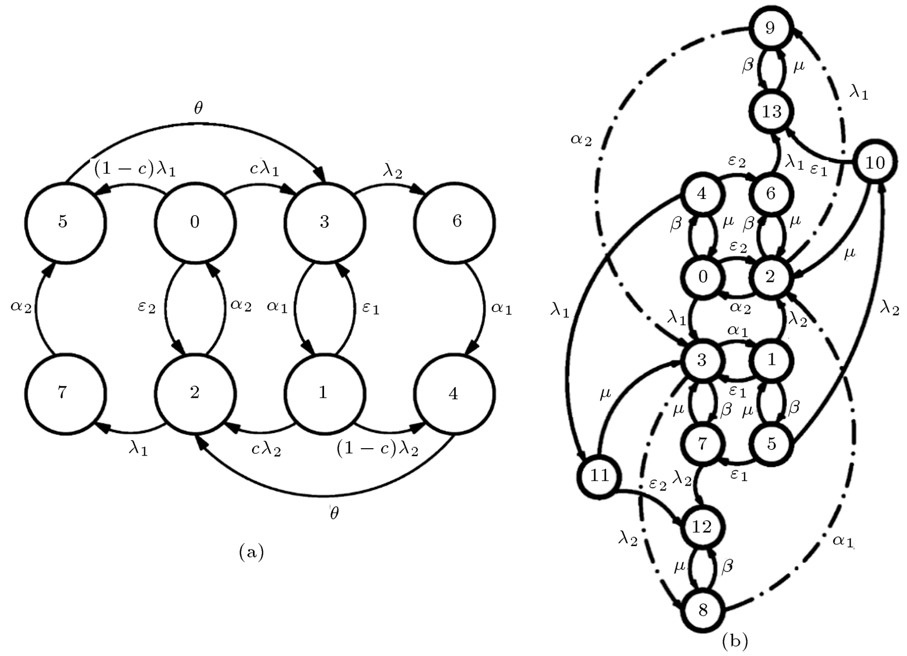

Figure 2. State transition diagram for steady-state availability models related to: (a) System 1 and (b) System 2.

in which:

$$C_1 = \begin{bmatrix} 0 & (1-c)\lambda_1 & 0 & 0 \\ (1-c)\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & \lambda_2 & 0 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} -\theta & 0 & 0 & 0 \\ 0 & -\theta & 0 & 0 \\ \alpha_1 & 0 & -\alpha_1 & 0 \\ 0 & \alpha_2 & 0 & -\alpha_2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} -(\alpha_1+\beta) & 0 & 0 & 0 & \beta & 0 \\ 0 & -(\alpha_2+\beta) & 0 & 0 & 0 & \beta \\ 0 & 0 & -(\mu+\varepsilon_1) & 0 & 0 & \varepsilon_1 \\ 0 & 0 & 0 & -(\mu+\varepsilon_2) & \varepsilon_2 & 0 \\ \mu & 0 & 0 & 0 & -\mu & 0 \\ 0 & \mu & 0 & 0 & 0 & -\mu \end{bmatrix}.$$

The corresponding balanced equations are given by the two following systems of equations:

$$\begin{bmatrix}\pi_0 & \dots & \pi_7\end{bmatrix} Q_3 = \begin{bmatrix}0 & \dots & 0\end{bmatrix}$$

$$\sum_{j=0}^{7} \pi_j = 1, \tag{7}$$

$$\begin{bmatrix}\pi_0 & \dots & \pi_{13}\end{bmatrix} Q_4 = \begin{bmatrix}0 & \dots & 0\end{bmatrix},$$

$$\sum_{j=0}^{13} \pi_j = 1. \tag{8}$$

The steady-state availability of System 1 is defined as:

$$A_1 = \sum_{j=0}^{3} \pi_j. \tag{9}$$

Therefore, the expression of the steady-state availability of System 1 can be obtained explicitly through solving Eq. (7):

$$A = \frac{\theta\alpha_1\alpha_2(\lambda_1+\lambda_2)(\lambda_1+\alpha_2+\varepsilon_2)(\lambda_2+\alpha_1+\varepsilon_1)}{k_1(\lambda_1,\lambda_2,\varepsilon_1,\varepsilon_2,\alpha_1,\alpha_2,c)}, \tag{10}$$

where:

$$k_1(\lambda_1,\lambda_2,\varepsilon_1,\varepsilon_2,\alpha_1,\alpha_2,c) = \sum_{i=1}^{3} k_{1,i}, \tag{11a}$$

$$\begin{aligned} k_{1,1} = \Big(& ((2\alpha_2+\theta)\alpha_1+\theta\alpha_2)\,\lambda_2^2 \\ & + \Big(((2-c)\alpha_2+\theta)\,\alpha_1^2 + \Big((2\varepsilon_1 \\ & + \theta)\alpha_2 + \theta\varepsilon_1 \Big)\alpha_1 + \theta\alpha_2\varepsilon_1 \Big)\lambda_2 \\ & + \theta\alpha_1\alpha_2(\alpha_1+\varepsilon_1)\Big)\lambda_1^2, \end{aligned} \tag{11b}$$

$$\begin{aligned} k_{1,2} = \Big\{ & \Big(\Big((2-c)\alpha_2^2 + (2\varepsilon_2+\theta)\alpha_2 + \theta\varepsilon_2 \Big)\alpha_1 \\ & + \theta\alpha_2(\alpha_2+\varepsilon_2)\Big)\lambda_2^2 + \Big(\Big((2-2c)\alpha_2^2 \\ & + \Big(\theta + (2-c)\varepsilon_2\Big)\alpha_2 + \theta\varepsilon_2 \Big)\alpha_1^2 \\ & + \Big(\Big(\theta + (2-c)\varepsilon_1\Big)\alpha_2^2 + \Big((\varepsilon_1+\varepsilon_2)\theta \\ & + 2\varepsilon_1\varepsilon_2 \Big)\alpha_2 + \theta\varepsilon_1\varepsilon_2 \Big)\alpha_1 \\ & + \theta\alpha_2\varepsilon_1(\alpha_2+\varepsilon_2)\Big)\lambda_2 \\ & + \theta\alpha_1\alpha_2(\alpha_1+\varepsilon_1)(\alpha_2+\varepsilon_2)\Big\}\lambda_1, \end{aligned} \tag{11c}$$

$$k_{1,3} = \theta\alpha_1\alpha_2\lambda_2(\alpha_2+\varepsilon_2)(\alpha_1+\varepsilon_1+\lambda_2). \tag{11d}$$

In the case of System 2, the steady-state availability is determined by:

$$A_2 = \sum_{j=0}^{7} \pi_j. \tag{12}$$

Solving Eq. (8) and then substituting the resulting answers in Eq. (12) gives the steady-state availability of System 2 which is too ample to be shown here. However, a numerical example is presented in the following section to calculate the steady-state availability of this case.

5. Numerical examples

One way to obtain the results of reliability characteristics is using the inverse Laplace transform which is a tedious task. On the other hand, the corresponding explicit expressions are also very spacious. Thus, an efficient numerical scheme called Runge-Kutta [34] is employed herein to investigate the effects of system parameters on the system reliability characteristics. It should be noted that the reliability of Systems 1 and 2 is calculated from the following equations:

$$R_1(t) = \sum_{j=0}^{3} P_j(t), \qquad R_2(t) = \sum_{j=0}^{7} P_j(t),$$

in which $P_j(t)$ related to Systems 1 and 2 is obtained from the numerical solutions of $\frac{dP(t)}{dt} = P(t)Q_1$ and $\frac{dP(t)}{dt} = P(t)Q_2$, respectively. Furthermore, the availability of Systems 1 and 2 is evaluated from the two preceding equations in which relevant $P_j(t)$

is determined numerically from $\frac{dP(t)}{dt} = P(t)Q_3$ and $\frac{dP(t)}{dt} = P(t)Q_4$, correspondingly.

In the following numerical examples, the values of system parameters used in [30] are considered. These parameters are $\lambda_1 = 0.01$, $\lambda_2 = 0.05$, $\varepsilon_1 = 0.002$, $\varepsilon_2 = 0.003$, $\alpha_1 = 0.2$, and $\alpha_2 = 0.5$. Furthermore, θ is set as 10.

5.1. Numerical analysis of System 1

The reliability and availability of System 1 with different values of parameter c are shown in Figure 3(a) and (b), respectively. As depicted, this parameter has considerable effect on both reliability and availability of System 1, such that they increase with increasing parameter c for a given time. Obviously, reliability vanishes and availability approaches a constant value named steady-state availability as time goes to infinite.

In Figure 4(a) and (b), the MTTF and steady-state availability of System 1 are respectively plotted with respect to parameter c. From Figure 4(a), the behavior of MTTF with parameter c is rising. When $c = 0$, there is no effective switching, so the system becomes a non-redundant system including unit one. Thus, in this case, the MTTF of system equals the expected value of unit one working time exponential distribution, i.e. $1/\lambda_1 = 100$. For $c = 1$, the switching mechanism becomes perfect and the MTTF of system equals 536.2 (days). With respect to Figure 4(b), it can be seen that steady-state availability increases from 0.9894 to 0.9909 as parameter c varies from 0 to 1.

5.2. Numerical analysis of System 2

The reliability of System 2 with different values of parameters β and μ is depicted in Figure 5(a) and (b), respectively. As seen, the system reliability is sensitive to the changes of these parameters and increases as parameters β and μ decrease and increase, respectively. As time goes to infinity, the system reliability tends to zero. The availability of System 2 with different values of parameters β and μ is correspondingly represented in Figure 6(a) and (b). From this figure, one can observe that these two parameters have significant effect on the system availability. Decreasing parameter β leads to higher values for system availability, which can be also achieved by increasing parameter μ. As time goes to infinity, the system availability tends to steady-state availability.

In Figure 7(a) and (b), the MTTF of System 2 is plotted with respect to parameters β and μ, respectively. From Figure 7(a), the behavior of MTTF with parameter β is descending. In the case of $\beta = 0$, the switching mechanism becomes reliable and the

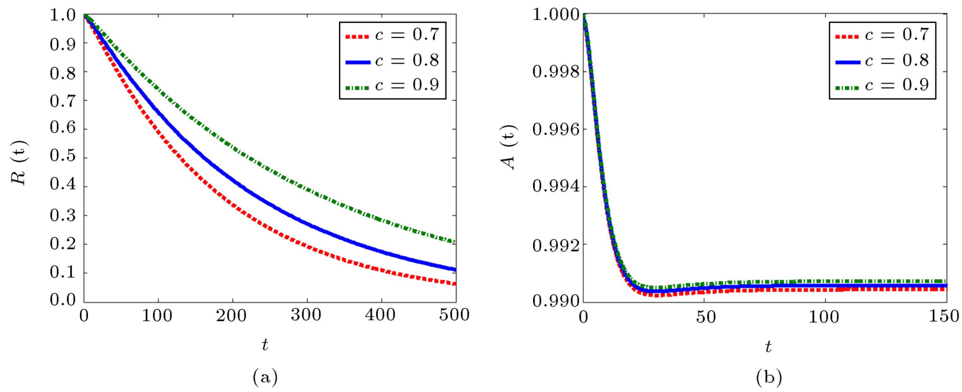

Figure 3. Effect of parameter c on (a) reliability and (b) availability of System 1.

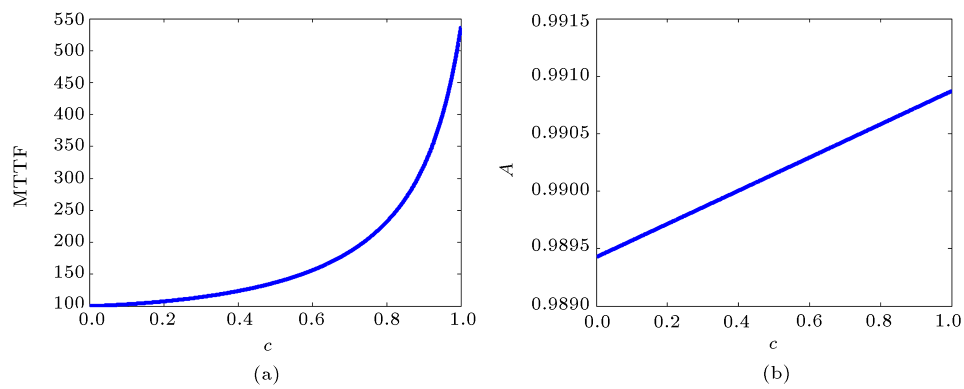

Figure 4. Variations of (a) MTTF and (b) steady-state availability of System 1 with parameter c.

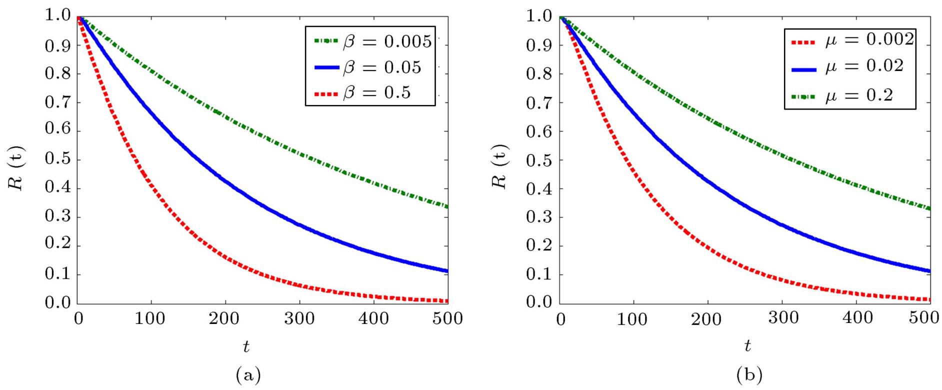

Figure 5. Effect of parameters (a) β and (b) μ on the reliability of System 2.

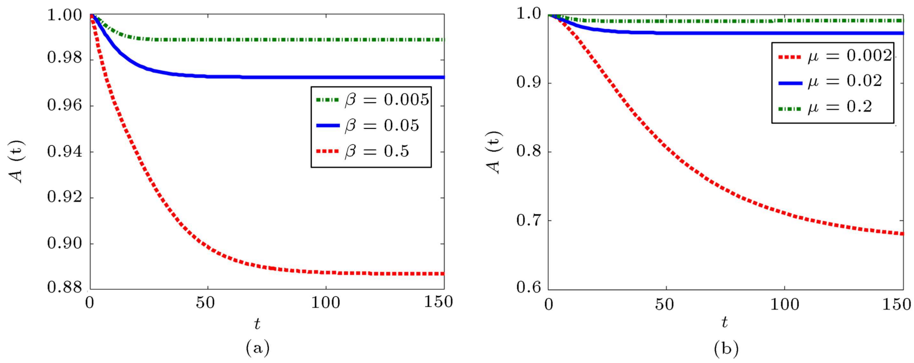

Figure 6. Effect of parameters (a) β and (b) μ on the availability of System 2.

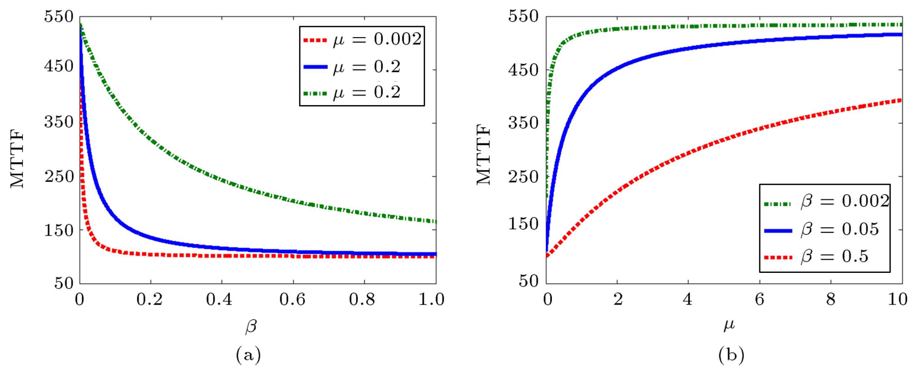

Figure 7. Variations of MTTF of System 2 with parameters (a) β and (b) μ.

MTTF of system equals 536.2 (days). With increasing parameter β, the system turns to be a non-redundant system consisting of unit one. Therefore, the MTTF of system tends to the expected value of unit one working time exponential distribution which is equal to 100 (days). In addition, for a given value of parameter β, the system MTTF intensifies as parameter μ increases. Figure 7(b) indicates that the behavior of MTTF is ascending with respect to parameter μ. When $\mu = 0$, the switching mechanism will not be repaired after the first failure. In this case, the MTTF of system equals 100.4, 109.3, and 210.4 (days) for $\beta = 0.5$, $\beta = 0.05$, and $\beta = 0.005$, respectively. With increasing parameter μ, the switching mechanism becomes reliable. Therefore, the MTTF of system tends to 536.2 (days). Moreover, for a fixed value of parameter μ, the system MTTF declines as parameter β increases.

The steady-state availability of System 2 versus parameters β and μ, respectively, is graphically illustrated in Figure 8(a) and (b). As shown, the behavior of steady-state availability is descending with parameter β and is ascending with parameter μ. In the case of $\beta = 0$, which is equivalent to the case of considerably increasing parameter μ, the switching mechanism becomes reliable, and the steady-state availability of system equals 0.9911. As previously observed, the steady-state availability of System 1 equals 0.9909 for $c = 1$. This deviation is due to the

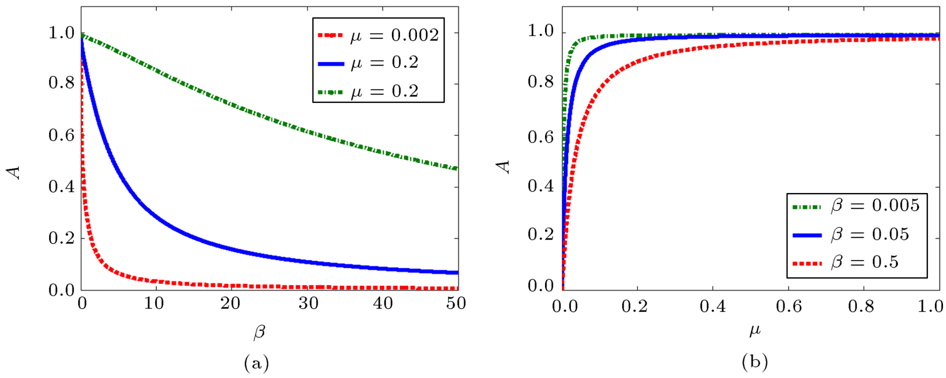

Figure 8. Variations of steady-state availability of System 2 with parameters (a) β and (b) μ.

reboot delay parameter. With increasing parameter θ, the steady-state availability of System 1 also converges to 0.9911. In the case of $\mu = 0$, which is equivalent to the case of significantly increasing parameter β, the steady-state availability of System 2 equals zero. The deviation between the steady-state availability of System 1 in the case of $c = 0$ and this case is owing to the reboot delay parameter, too. With decreasing parameter θ, the steady-state availability of System 1 vanishes. In Figure 8(a), considering a constant value for parameter β, the system steady-state availability increases with increasing parameter μ. In Figure 8(b), for a fixed value of parameter μ, the system steady-state availability declines as parameter β gets larger.

5.3. *Comparison of reliability characteristics between Systems 1 and 2*

In this section, comparisons concerning the reliability, availability, MTTF, and steady-state availability of the two systems are made. At first, parameters c, β, and μ are set as 0.8, 0.05, and 0.2, respectively, and the other parameters are fixed as mentioned above. The reliability and availability of systems are depicted in Figure 9(a) and (b), respectively. In the case of reliability, System 2 is preferred, while in the case of availability, System 1 is dominant. The MTTF and steady-state availability of systems are represented in

Table 5. MTTF and steady-state availability of System 1 and 2.

System	MTTF	A
1	231.5089	0.9906
2	233.2412	0.9725

Table 5. It can be seen that System 2 has a greater MTTF, whereas the steady-state availability of System 1 is more than that of System 2.

Then, in order to compare the MTTF and steady-state availability of two systems, the values of systems parameters are fixed as mentioned above, and in each case, one of the parameters is variable. Numerical results of MTTF and steady-state availability are presented in Tables 6 and 7, respectively.

Finally, it is assumed that the values of all parameters except c and β are fixed as mentioned above. By replacing the parameters in the equations obtained for MTTF and steady-state availability of two systems, one can arrive at equations shown in Box II.

By equating Eqs. (13) and (14) and solving them numerically, Figure 10(a) is plotted. For all of the points on the curve, MTTF of two systems is the same. In Region 1, the MTTF of System 1 is higher than that of System 2; in Region 2, the opposite behavior is observed. It can be seen that by increasing parameter

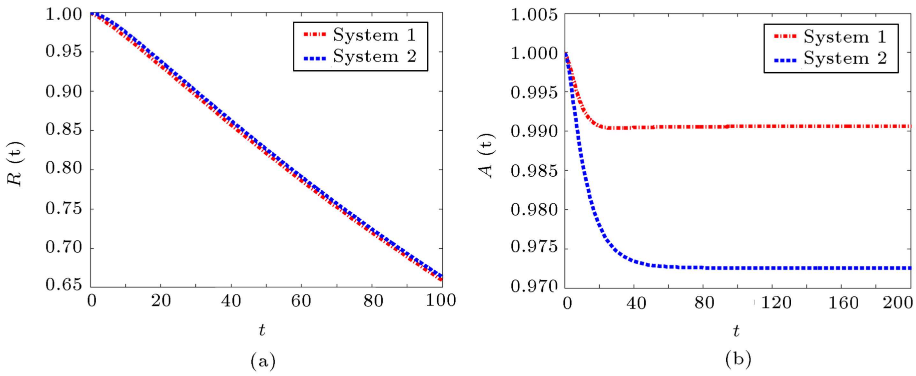

Figure 9. (a) Reliability and (b) availability of System 1 and System 2.

$$\mathrm{MTTF}_1 = \frac{2000(0.0001c^2 + 0.0012852c + 0.00646380)}{-0.1c^2 + 0.129276}, \tag{13}$$

$$\mathrm{MTTF}_2 = \frac{100(\beta + 8.39673)(\beta + 1.21525)(\beta + 0.26834)(\beta + 0.23899)}{(\beta + 8.40618)(\beta + 1.15384)(\beta + 0.50143)(\beta + 0.02509)}, \tag{14}$$

$$A_1 = \frac{2154600}{2177626 - 3175c}, \tag{15}$$

$$A_2 = \frac{3.42857(\beta + 8.38493)(\beta + 1.21120)(\beta + 0.25209)(\beta + 0.21049)}{(\beta + 1.51492)(\beta + 0.24494)(\beta + 0.20000)(\beta^2 + 9.04646\beta + 25.12183)}. \tag{16}$$

Box II

Table 6. Comparison for MTTF of the two systems.

	Comparison result
Range of λ_1	
$0 < \lambda_1 < 0.0010218$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
$\lambda_1 > 0.0010218$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of λ_2	
$\lambda_2 > 0$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of ε_1	
$0 < \varepsilon_1 < 0.0236240$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
$0.0236240 < \varepsilon_1 < 0.3892404$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
$\varepsilon_1 > 0.3892404$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of ε_2	
$0 < \varepsilon_2 < 0.0211665$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
$0.0211665 < \varepsilon_2 < 2.700003$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
$\varepsilon_2 > 2.700003$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of α_1	
$0 < \alpha_1 < 0.0007722$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
$0.0007722 < \alpha_1 < 0.1584121$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
$\alpha_1 > 0.1584121$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of α_2	
$0 < \alpha_2 < 0.0012769$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
$0.0012769 < \alpha_2 < 0.1583571$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
$\alpha_2 > 0.1583571$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of β	
$0 < \beta < 0.0508775$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
$\beta > 0.0508775$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
Range of μ	
$0 < \mu < 0.1960735$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$
$\mu > 0.1960735$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
Range of c	
$0 < c < 0.8028090$	$\mathrm{MTTF}_2 > \mathrm{MTTF}_1$
$0.8028090 < c < 1$	$\mathrm{MTTF}_1 > \mathrm{MTTF}_2$

c, the interval for parameter β, in which System 2 is dominant, decreases. Similarly, by equating Eqs. (15) and (16) and solving them numerically, Figure 10(b) is graphed. For all of the points on the curve, two systems have equal steady-state availability. In Region 1, System 1 is preferred, while System 2 is dominant in Region 2.

Table 7. Comparison for steady-state availability of the two systems.

	Comparison result
Range of λ_1	
$\lambda_1 > 0$	$A_1 > A_2$
Range of λ_2	
$\lambda_2 > 0$	$A_1 > A_2$
Range of ε_1	
$\varepsilon_1 > 0$	$A_1 > A_2$
Range of ε_2	
$\varepsilon_2 > 0$	$A_1 > A_2$
Range of α_1	
$\alpha_1 > 0$	$A_1 > A_2$
Range of α_2	
$\alpha_2 > 0$	$A_1 > A_2$
Range of β	
$0 < \beta < 0.0011017$	$A_2 > A_1$
$\beta > 0.0011017$	$A_1 > A_2$
Range of μ	
$0 < \mu < 2.3710927$	$A_1 > A_2$
$\mu > 2.3710927$	$A_2 > A_1$
Range of c	
$0 < c < 1$	$A_1 > A_2$
Range of θ	
$0 < \theta < 0.2539104$	$A_2 > A_1$
$\theta > 0.2539104$	$A_1 > A_2$

Based on the performed study, some significant corollaries of this research can be attained as special cases listed below:

- Case 1: When $\varepsilon_1 = \varepsilon_2 = 0$, Systems 1 and 2 turn to be cold standby repairable systems consisting of two dissimilar units and an imperfect switching mechanism;
- Case 2: When $\varepsilon_1 = \varepsilon_2 = 0$, $c = 1$, and $\beta = 0$, the systems turn to be cold standby repairable systems

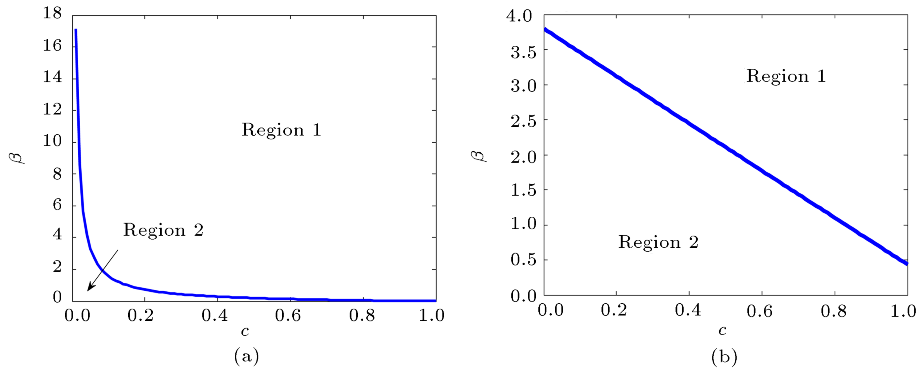

Figure 10. Comparison for (a) MTTF and (b) steady-state availability of two systems.

consisting of two dissimilar units and a perfect switching mechanism;

- Case 3: When $\varepsilon_1 = \varepsilon_2 = 0$, $\alpha_1 = \alpha_2 = 0$, and $\mu = 0$, the systems turn to be cold standby non-repairable systems consisting of two dissimilar units and an imperfect switching mechanism;
- Case 4: When $\varepsilon_1 = \varepsilon_2 = 0$, $\alpha_1 = \alpha_2 = 0$, $\mu = 0$, $c = 1$, and $\beta = 0$, the systems turn to be cold standby non-repairable systems consisting of two dissimilar units and a perfect switching mechanism;
- Case 5: When $\varepsilon_1 = \varepsilon_2 = 0$, $\lambda_1 = \lambda_2 = \lambda$, and $\alpha_1 = \alpha_2 = \alpha$, the systems turn to be cold standby repairable systems consisting of two identical units and an imperfect switching mechanism.

6. Conclusion

In this study, two cases of imperfect switching mechanism were investigated for a warm standby repairable system consisting of two dissimilar units and one repairman. In case 1, it was assumed that in case the active unit fails, it can be immediately replaced by the available warm standby unit with coverage probability c. In case 2, however, it was assumed that the failure time and the repair time of the switching mechanism are exponentially distributed. In this case, so long as the switching mechanism is in working state, the failed active unit is immediately replaced by the available warm standby unit. Switching mechanism failure does not necessarily lead to the system failure, because no switching may be required during the remainder of the system mission time, or the switching mechanism may be repaired before the time of the active unit failure. This case of switching mechanism has priority in repair than the units. In addition, times to failure and times to repair of active and standby units were assumed to be exponentially distributed. Using Markov process theory and Laplace transforms, the explicit expressions of the MTTF and the steady-state availability of two systems were derived analytically. Sensitivity analyses of the reliability and availability characteristics of each system with respect to the switching mechanism parameters were performed through solving a numerical example. It was illustrated that the reliability and availability indexes are very sensitive to the changes of the switching mechanism parameters. Moreover, two systems were compared based on different reliability and availability characteristics.

Assuming exponential assumption for time to failure and time to repair is often too restrictive to most actual engineering systems consisting of units with increasing hazard rate functions. Erlang distribution has superior characteristics than those of exponential distribution by offering a wide variety of different increasing hazard functions. Therefore, for future research, we can consider Erlang distribution for time to failure and time to repair of units and case two of switching mechanism. Moreover, in real-world applications, estimating the model parameters is usually accompanied with uncertainty. Therefore, various forms of uncertainty, such as fuzzy and interval uncertainties, can be incorporated into the system parameters. This subject will be conducted in our future work.

References

1. Kumar, A. and Agarwal, M. "A review of standby redundant systems", *IEEE Trans. Rel.*, **29**(4), pp. 290-294 (1980).
2. Yearout, R.D., Reddy, P. and Grosh, D.L. "Standby redundancy in reliability - a review", *IEEE Trans. Rel.*, **3**(35), pp. 285-292 (1986).
3. De Almeida, A.T. and De Souza, F.M.C. "Decision theory in maintenance strategy for a 2-unit redundant standby system", *IEEE Trans. Rel.*, **42**(3), pp. 401-407 (1993).
4. Subramanian, R. and Anantharaman, V. "Reliability analysis of a complex standby redundant systems", *Reliab. Eng. Syst. Saf.*, **48**(1), pp. 57-70 (1995).
5. Rajamanickam, S.P. and Chandrasekar, B. "Reliability measures for two-unit systems with a dependent

structure for failure and repair times" *Microelectron. Reliab.*, **37**(5), pp. 829-833 (1997).

6. Yadavalli, V.S.S., Botha, M. and Bekker, A. "Asymptotic confidence limits for the steady state availability of a two-unit parallel system with preparation time for the repair facility", *Asia-Pacific J. Oper. Res.*, **19**(2), pp. 249-256 (2002).

7. Lewis, E.E., *Introduction to Reliability Engineering*, Second Ed., Wiley, New York (1996).

8. Coit, D.W. "Cold-standby redundancy optimization for nonrepairable systems", *IIE Transactions*, **33**(6), pp. 471-478 (2001).

9. Coit, D.W. "Maximization of system reliability with a choice of redundancy strategies", *IIE transactions*, **35**(6), pp. 535-543 (2003).

10. Trivedi, K.S., *Probability & Statistics with Reliability, Queuing and Computer Science Applications*, John Wiley & Sons (2008).

11. Soltani, R., Sadjadi, S.J. and Tavakkoli-Moghaddam, R. "Interval programming for the redundancy allocation with choices of redundancy strategy and component type under uncertainty: Erlang time to failure distribution", *Appl. Math. Comput.*, **244**, pp. 413-421 (2014).

12. Sadjadi, S.J. and Soltani, R. "Minimum-maximum regret redundancy allocation with the choice of redundancy strategy and multiple choice of component type under uncertainty", *Comput. Ind. Eng.*, **79**, pp. 204-213 (2015).

13. Wang, K.H. and Chiu, L.W. "Cost benefit analysis of availability systems with warm standby units and imperfect coverage", *Appl. Math. Comput.*, **172**(2), pp. 1239-1256 (2006).

14. Wang, K.H., Dong, W.L. and Ke, J.B. "Comparison of reliability and the availability between four systems with warm standby components and standby switching failures", *Appl. Math. Comput.*, **183**(2), pp. 1310-1322 (2006).

15. Ke, J.B., Chen, J.W., and Wang, K.H. "Reliability measures of a repairable system with standby switching failures and reboot delay", *Qual. Technol. & Quant. Manage.*, **8**(1), pp. 15-26 (2011).

16. Wang, K.H. and Chen, Y.J. "Comparative analysis of availability between three systems with general repair times, reboot delay and switching failures", *Appl. Math. Comput.*, **215**(1), pp. 384-394 (2009).

17. Ke, J.C., Huang, H.I. and Lin, C.H. "A redundant repairable system with imperfect coverage and fuzzy parameters", *Appl. Math. Modell.*, **32**(12), pp. 2839-2850 (2008).

18. Ke, J.C., Huang, H.I. and Lin, C.H. "Parametric programming approach for a two-unit repairable system with imperfect coverage, reboot and fuzzy parameters", *IEEE Trans. Rel.*, **57**(3), pp. 498-506 (2008).

19. Ke, J.C. and Liu, T.H. "A repairable system with imperfect coverage and reboot", *Appl. Math. Comput.*, **246**, pp. 148-158 (2014).

20. Wang, K.H., Yen, T.C. and Jian, J.J. "Reliability and sensitivity analysis of a repairable system with imperfect coverage under service pressure condition", *J. Manuf. Syst.*, **32**(2), pp. 357-363 (2013).

21. Hsu, Y.L., Ke, J.C. and Liu, T.H. "Standby system with general repair, reboot delay, switching failure and unreliable repair facility - A statistical standpoint", *Math. Comput. Simul.*, **81**(11), pp. 2400-2413 (2011).

22. Wang, K.H., Liou, C.D. and Lin, Y.H. "Comparative analysis of the machine repair problem with imperfect coverage and service pressure condition", *Appl. Math. Modell.*, **37**(5), pp. 2870-2880 (2013).

23. Ke, J.C., Hsu, Y.L., Liu, T.H. and Zhang, Z.G. "Computational analysis of machine repair problem with unreliable multi-repairmen", *Comput. & Oper. Res.*, **40**(3), pp. 848-855 (2013).

24. Hsu, Y.L., Ke, J.C., Liu, T.H. and Wu, C.H. "Modeling of multi-server repair problem with switching failure and reboot delay and related profit analysis", *Comput. Ind. Eng.*, **69**, pp. 21-28 (2014).

25. Kuo, C.C. and Ke, J.C. "Comparative analysis of standby systems with unreliable server and switching failure", *Reliab. Eng. Syst. Saf.*, **145**, pp. 74-82 (2016).

26. Kakkar, M., Chitkara, A. and Bhatti, J. "Reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance", *Manage. Sci. Lett.*, **6**(4), pp. 285-296 (2016).

27. Kakkar, M., Chitkara, A. and Bhatti, J. "Reliability analysis of two unit parallel repairable industrial system", *Decision Sci. Lett.*, **4**(4), pp. 525-536 (2015).

28. Kumar, A., Baweja, S. and Barak, M. "Stochastic behavior of a cold standby system with maximum repair time", *Decision Sci. Lett.*, **4**(4), pp. 569-578 (2015).

29. Kumar, A. and Malik, S. "Reliability measures of a computer system with priority to PM over the H/W repair activities subject to MOT and MRT", *Manage. Sci. Lett.*, **5**(1), pp. 29-38 (2015).

30. Yuan, L. and Meng, X.Y. "Reliability analysis of a warm standby repairable system with priority in use", *Appl. Math. Modell.*, **35**(9), pp. 4295-4303 (2011).

31. Fan, X. and Gong, G. "Security challenges in smart-grid metering and control systems", *Technol. Innovation Manage. Rev.*, **3**(7), p. 42 (2013).

32. Ancillotti, E., Bruno, R. and Conti, M. "The role of communication systems in smart grids: Architectures, technical solutions and research challenges", *Comput. Commun.*, **36**(17), pp. 1665-1697 (2013).

33. Ross, S.M., *Introduction to Probability Models*, Academic press (2014).

34. Burden, R., Faires, J. and Burden, A., *Numerical Analysis*, Nelson Education (2015).

Appendix

The definitions of f_1 and g_1 are given below.

$$f_1(\lambda_1, \lambda_2, \varepsilon_1, \varepsilon_2, \alpha_1, \alpha_2, c) = (\lambda_1 + \alpha_2 + \varepsilon_2)\lambda_2^2 + \Big(c\lambda_1^2 + \big((1+c^2)\alpha_1 + c\alpha_2 + \varepsilon_1\big)\lambda_1 + (\alpha_2 + \varepsilon_2)(\alpha_1 + \varepsilon_1)\Big)\lambda_2 + c\lambda_1(\lambda_1 + \alpha_2)(\varepsilon_1 + \alpha_1), \tag{A.1}$$

$$g_1(\lambda_1, \lambda_2, \varepsilon_1, \varepsilon_2, \alpha_1, \alpha_2, c) = \lambda_1\Big((\lambda_1 + \alpha_2 + \varepsilon_2)\lambda_2 + (\alpha_1 + \varepsilon_1)\lambda_1 + \big((1-c^2)\alpha_2 + \varepsilon_2\big)\alpha_1 + \varepsilon_1(\alpha_2 + \varepsilon_2)\Big)\lambda_2. \tag{A.2}$$

Biographies

Meisam Sadeghi received his BSc and MSc degrees in Industrial Engineering from Iran University of Science and Technology, Tehran, Iran, in 2003 and 2006, respectively. Currently, he is a PhD candidate in Industrial Engineering at K.N. Toosi University of Technology. His research background and interests mainly include reliability engineering, probability models, and applied operations research.

Emad Roghanian received his PhD degree from Iran University of Science and Technology, Tehran, Iran, in 2006. He is currently an Associate Professor in the Faculty of Industrial Engineering at K.N. Toosi University of Technology. His research interests include supply chain management, project management, probability models and applied operations research. He has authored numerous papers presented at conferences and published in journals, including Applied Mathematical Modelling, Computers & Industrial Engineering and Applied Mathematics and Computation.