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Research on the inventory control of the remanufacturing reverse logistics based on the quantitative examination

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Abstract. Based on the remanufacturing reverse logistics system, this paper studies the inventory control problem of the entire supply chain. Considering a variety of products and raw materials, we established a multi-product multi-echelon inventory control model of the remanufacturing reverse logistics based on the quantitative examination. Also, a numerical simulation is performed, which shows that the model can reduce the inventory cost of remanufacturing reverse logistics, and provides a theoretical basis for determining the production batch and the processing batch for manufacturer and the recycling center. Then, using sensitivity analysis, it shows that the recovery and the remanufacturing rates of the recycled products have a great influence on the inventory cost of the reverse logistics, production, and inventory of the manufacturer and the recycling center.

1. Introduction

Learning from the EOQ model and solution method of traditional inventory theory, the researchers have established many kinds of inventory control models of reverse logistics. These models can be divided into two main classes: models of definite demand and models of random demand.

The earliest inventory control model of definite demand was put forward by Schrady [1] in 1967. Mabini et al. [2] and Richter [3,4] analyzed the inventory control problem of reverse logistics using the traditional EOQ model. Teunter [5] and Minner [6] extended the model to consider the inventory of production and recycled production. Dobos [7] represented the inventory control model as an optimal control problem with two state variables (inventory status in the first and second stores) and with three control variables (rate of manufacturing, remanufacturing, and disposal), assuming that demand is a known continuous function in a given planning horizon, and return rate of the used items is a given function. Çorbacıoğlu and van der Laan [8] analyzed a two-product system with joint manufacturing and remanufacturing. El Saadany and Jaber [9,10] considered a production-remanufacturing inventory model for a single product, where the constant demand is satisfied by the inventory of newly produced and remanufactured items.

The earliest inventory control model of random demand was put forward by Heyman [11] in 1977. Simpson [12] and Inderforth [13] established inventory control model of periodic check. Muckstadt and Isasc [14] and van der Lannera et al. [15] established an (s, Q) inventory model, assuming that the demand and return follow Poisson distribution, and the remanufacturing time is random. Kleber et al. [16] considered a deterministic model with dynamic

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demands and returns. Kiesmüller [17] provided a new approach for the control of a stochastic hybrid manufacturing/remanufacturing system, basing the production and remanufacturing decision on two different inventory positions. Mahadevan et al. [18] studied a remanufacturing facility that receives a stream of returned products according to a Poisson process. The demand also follows a Poisson process. Zikopoulos and Tagaras [19] examined a reverse supply chain consisting of two collection sites and a refurbishing site, which faces stochastic demand for refurbished products in a single-period setting. Chung et al. [20] analyzed an inventory system with traditional forward-oriented material flow as well as a reverse material flow supply chain. Moreover, in the reverse material flow, the used products are returned, remanufactured, and shipped to the retailer for resale. Mitra [21] developed a deterministic model as well as a stochastic model under continuous review for the system and provided numerical examples as illustrations. Wei et al. [22] considered an inventory and production planning problem with uncertain demand and returns, in which the product return process is integrated into the manufacturing process over a finite planning horizon. Hsueh [23] investigated inventory control policies in a manufacturing/remanufacturing system during the product life cycle, which consists of four phases: introduction, growth, maturity, and decline. Both the demand and return rates of products are random variables with normal distribution. Flapper et al. [24] considered a production-inventory system with product returns that are announced in advance by the customers. Demands and announcements of returns occur according to independent Poisson processes. Mitra [25] addressed the inventory management issue in closed-loop supply chains, and developed deterministic and stochastic models for a two-echelon system with correlated demands and returns under generalized cost structures. Li [26] presented a Markov decision process to manage inventory systems with Markovian customer demand and Markovian product returns. Parvini et al. [27] developed a two-echelon inventory model for a single reusable product in which the return rates explicitly depend on demand streams. Zolfagharinia et al. [28] developed an inventory control model for a reverse supply chain with separate serviceable and remanufacturable inventory stock points. In this model, the return rate is expressed stochastically as a function of product demand.

The inventory control models of reverse logistics, mentioned above, all regarded manufacturers or distributors as the research object, and only considered the production inventory and the recycled production inventory in manufacturers and distributors, lacking consideration for the inventory control problem of the whole supply chain. There is only one paper we are aware of that considers the inventory control problem of the whole supply chain. Jonrinaldi and Zhang [29] proposed a model and solution method to coordinate integrated production and inventory cycles in a whole manufacturing supply chain involving reverse logistics and consisting of multiple tier-2 suppliers, multiple tier-1 suppliers, a manufacturer, multiple distributors, multiple retailers, and a third party collecting the used finished products from end customers to be returned to the system for reuse. The basic difference between our model and that of Jonrinaldi and Zhang [29] is that shortages are allowed in the manufacturer and distributors in our model. In addition, our model is based on the quantitative examination.

The remainder of this paper is organized as follows: Section 2 provides an illustration of the model studied, lists the assumptions and notations, and builds the multi-product multi-echelon inventory control model of the remanufacturing reverse logistics based on the quantitative examination. In Section 3, a numerical example is used to demonstrate the real problem, and some results of analysis are discussed. In Section 4, conclusions and limitations in this research are presented.

2. Mathematical model

2.1. The inventory control model description

According to the multi-echelon inventory problem of supply chain taking the manufacturers as the center, this paper establishes an inventory control model of the remanufacturing reverse logistics, as shown in Figure 1. For the multi-echelon inventory control management, this paper uses the inventory control management dominated by recycled products inventory which can effectively reduce the holding cost of recycled products by the timely processing of recycled products.

The inventory of the manufacturer can be divided into two kinds including the inventory of new products and the inventory of remanufacturing products. Under the condition of the remanufacturing reverse logistics, the manufacturer develops the inventory control strategy of new products and remanufacturing products by determining the production batch of new products and the processing batch of recycled products. Regarding the view of the different recycled products, the manufacturer determines the different processing cycle and processing batch of each recycled product. In each cycle, the inventory of serviceable products is replenished by remanufacturing products first. After the recycled products are processed, the inventory of serviceable products is replenished by new products, and the inventory of raw material is reduced. The control objective of the system is to minimize the total inventory cost of supply chain.

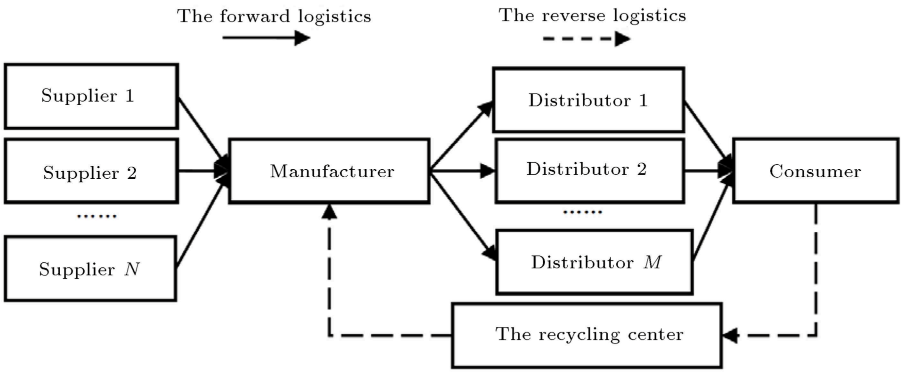

Figure 1. The inventory control model of the remanufacturing reverse logistics.

2.2. Assumptions

The assumptions which are defined are as below:

1. There is only one manufacturer. The manufacturer is not only responsible for the manufacturing of new products, but also is responsible for the remanufacturing of recycled products. The manufacturer produces M kinds of products and needs N kinds of raw materials;
2. One supplier only provides one kind of raw material. There are N suppliers;
3. One distributor only sells one kind of product. There are M distributors;
4. There is only one recycling center that is responsible for the product recycling;
5. Shortages are allowed in the manufacturer and distributors;
6. A part of the recycled products which is remanufactured has the same performance as that of new products and can be sold as new products. The remainder of the recycled products is disposed by the recycling center;
7. The suppliers are to maintain a certain inventory level. When the suppliers provide raw materials to the manufacturer, they will replenish the inventory to that level in the next cycle. Shortages are not allowed in suppliers.

2.3. Notations

To model the problem the following notations are used:

Q_m	Production batch of new product m
Q_{rm}	Processing batch of recycled product m
λ_m	Recovery rate of recycled product m
T_m	Examination cycle of recycled product m, $T_m = \frac{Q_{rm}}{\lambda_m}$
T	The least common multiple of $T_1, T_2, \cdots, T_m$
α	Remanufacturing rate of the recycled products
β_n	The coefficients of raw materials constituted in the new products, that is, it needs β_n and raw material n to produce a new product
d_{sm}	Distributors' demand rate of product m
d_{xm}	Consumers' demand rate of product m
C_{sm}	Inventory cost per unit for serviceable product m
C_{rm}	Inventory cost per unit for remanufacturing recycled product m
C_{hrm}	Inventory cost per unit for recycled product m
C_{pm}	Manufacturing cost per unit for product m
C_{rrm}	Remanufacturing cost per unit for remanufacturing recycled product m
C_{xm}	Inventory cost per unit for product m in the distributors
C_{cm}	Shortage cost per unit for product m in the manufacturer
C_{xcm}	Shortage cost per unit for product m in the distributors
C_{gn}	Inventory cost per unit for raw material n
C_t	Dismantling cost per unit for the recycled products
C_h	Waste disposal cost per unit for the waste products
Q_{gn}	The inventory level of raw material n in the suppliers
Q_{sm}	The order quantity of product m, $Q_{sm} = d_{sm}T_m$

2.4. The inventory control model

This paper establishes the multi-echelon inventory relationship, as shown in Figure 2. The inventory cost of the remanufacturing reverse logistics involves the

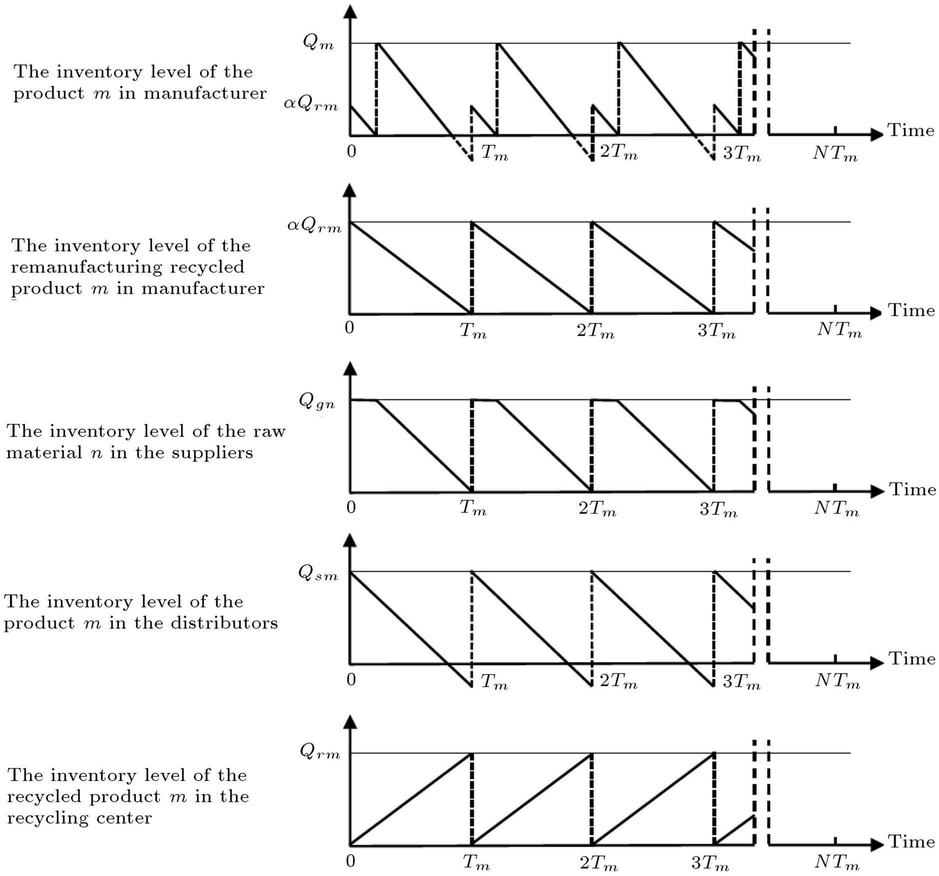

Figure 2. Behaviors of the inventory of remanufacturing reverse logistics based on the quantitative examination.

inventory cost of manufacturer, suppliers, distributors, and the recycling center.

1. *The inventory cost of manufacturer:* In cycle T_m, the inventory and shortage costs of serviceable product m are defined as follows:

$$C_m^{cs}=\begin{cases}\frac{1}{2}\frac{C_{sm}}{d_{sm}}(Q_m-\alpha Q_{rm})^2 \\ \quad +\frac{1}{2}C_{cm}d_{sm}T_m^2 & \frac{\alpha Q_{rm}+Q_m}{d_{sm}}<T_m \\ \quad -\frac{1}{2}\frac{C_{cm}}{d_{sm}}(Q_m+\alpha Q_{rm})^2 \\ \frac{C_{sm}}{d_{sm}}(\alpha^2 Q_{rm}^2-\alpha Q_{rm}Q_m) \\ \quad +C_{sm}Q_mT_m & \frac{\alpha Q_{rm}+Q_m}{d_{sm}}\geq T_m \\ \quad -\frac{1}{2}C_{sm}d_{sm}T_m^2\end{cases} \tag{1}$$

In cycle T_m, the manufacturing cost of product m is defined as follows:

$$C_m^{pc}=C_{pm}Q_m. \tag{2}$$

In the cycle T_m, the inventory cost of remanufacturing recycled product m is defined as follows:

$$C_m^{rc}=\frac{1}{2\lambda_m}C_{rm}\alpha Q_{rm}^2. \tag{3}$$

In cycle T_m, the remanufacturing cost of remanufacturing recycled product m is defined as follows:

$$C_m^{rz}=C_{rrm}\alpha Q_{rm}. \tag{4}$$

Therefore, in cycle T, the total inventory cost of manufacturer is defined as follows:

$$C^{zc}=\sum_{m=1}^{M}\frac{T}{T_m}(C_m^{cs}+C_m^{pc}+C_m^{rc}+C_m^{rz}). \tag{5}$$

2. *The inventory cost of suppliers:* In cycle T_m, the quantity demanded for the raw material n to produce production m is defined as follows:

$$Q_{nm}=\beta_n Q_m. \tag{6}$$

In cycle T_m, the total quantity demanded for raw material n is defined as follows:

$$Q_n=\sum_{m=1}^{M}\frac{T}{T_m}Q_{nm}=\sum_{m=1}^{M}\frac{T}{T_m}\beta_n Q_m. \tag{7}$$

Therefore, in cycle T, the total inventory cost of suppliers is defined as follows:

$$C^{gc}=\sum_{n=1}^{N}C_n^{gc}=\sum_{n=1}^{N}C_{gn}(Q_{gn}-Q_n)$$
$$=\sum_{n=1}^{N}C_{gn}\left(Q_{gn}-\sum_{m=1}^{M}\frac{T}{T_m}\beta_nQ_m\right). \tag{8}$$

3. *The inventory cost of distributors:* In cycle T_m, the inventory cost of the product m is defined as follows:

$$C_m^{xc}=\begin{cases}\frac{1}{2}\frac{C_{xm}}{d_{xm}}(2Q_{sm}d_{sm}T_m-d_{sm}^2T_m^2) & \\ +\frac{1}{2}\frac{C_{xcm}}{d_{xm}}(d_{xm}^2-d_{sm}^2)T_m^2 & d_{sm}<d_{xm}\\ C_{xm}(Q_{sm}T_m-\frac{1}{2}d_{xm}T_m^2) & d_{sm}\geq d_{xm}\end{cases} \tag{9}$$

Therefore, in cycle T, the total inventory cost of distributors is defined as follows:

$$C^{xc}=\sum_{m=1}^{M}\frac{T}{T_m}C_m^{xc}. \tag{10}$$

4. *The inventory cost of the recycling center:* In cycle T_m, the inventory cost of recycled product m is defined as follows:

$$C_m^{hrc}=\frac{1}{2}C_{hrm}\lambda_mT_m^2. \tag{11}$$

In cycle T_m, the dismantling cost of recycled product m is defined as follows:

$$C_m^{tc}=C_tQ_{rm}. \tag{12}$$

In cycle T_m, the cost of processing waste product m is defined as follows:

$$C_m^{dc}=C_hQ_{rm}(1-\alpha). \tag{13}$$

Therefore, in cycle T, the total inventory cost of the recycling center is defined as follows:

$$C^{hc}=\sum_{m=1}^{M}\frac{T}{T_m}(C_m^{hrc}+C_m^{tc}+C_m^{dc}). \tag{14}$$

In conclusion, in cycle T, we develop the inventory control model of the remanufacturing reverse logistics as follows:

$$\min C=\min(C^{zc}+C^{gc}+C^{xc}+C^{hc}), \tag{15}$$

where $m=1,2,\cdots,M$ and $n=1,2,\cdots,N$.

3. Simulation and analysis

3.1. Simulation model

As an illustration, we develop a multi-echelon inventory control model which has one manufacturer, one recycling center, one consumer, three suppliers, and two distributors (Figure 3). The manufacturer produces two products and needs three raw materials.

Cycle T is not a fixed value, so we determine the inventory cost per unit time as the evaluation target shown as follows:

$$\min C_{\text{average}}=\min\frac{C}{T}$$
$$=\min\frac{1}{T}(C^{zc}+C^{gc}+C^{xc}+C^{hc}), \tag{16}$$

s.t.:

T : The least common multiple of T_1 and T_2,

$m=1,2, \quad n=1,2,3,$

$\beta_1=1, \quad \beta_2=2, \quad \beta_3=4.$

3.2. Simulation

The initial values of the parameters are shown in Table 1. Q_1, Q_2, Q_{r1}, and Q_{r2} are the decision variables.

We simulate the inventory control model using Matlab software, and the simulation results are shown in Figure 4 and Table 2, respectively.

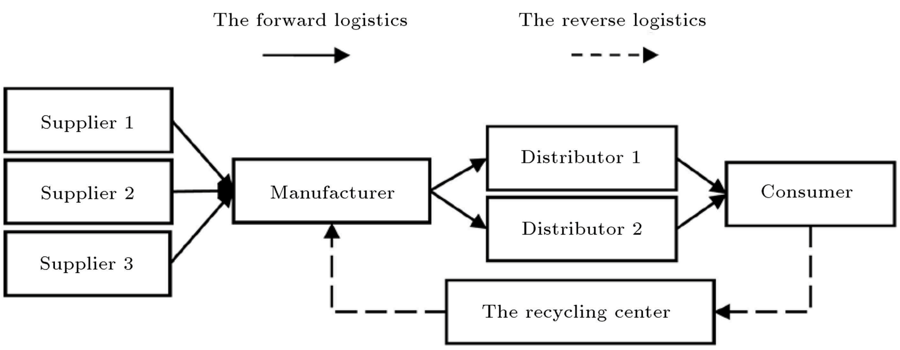

Figure 3. The structure of simulation model.

Table 1. The initial values of the parameters.

Manufacturer	Parameters	α	C_{s1}	C_{s2}	C_{r1}	C_{r2}	C_{p1}	C_{p2}	C_{rr1}	C_{rr2}	C_{c1}	C_{c2}
	values	0.5	\$2	\$3	\$2	\$3	\$50	\$60	\$30	\$35	\$5	\$5
Suppliers	Parameters	β_1	β_2	β_3	C_{g1}	C_{g2}	C_{g3}	Q_{g1}	Q_{g2}	Q_{g3}		
	values	1	2	4	\$3	\$2	\$1	1000	1000	1000		
Distributors	Parameters	d_{s1}	d_{s2}	C_{x1}	C_{x2}	C_{xc1}	C_{xc2}					
	values	15	18	\$1	\$2	\$5	\$5					
Recycling center	Parameters	λ_1	λ_2	C_{hr1}	C_{hr2}	C_t	C_h					
	values	0.5	1	\$2	\$3	\$10	\$5					
Consumer	Parameters	d_{x1}	d_{x2}									
	values	15	20									

Table 2. The simulation results.

Parameters	Q_1	Q_2	Q_{r1}	Q_{r2}	T_1	T_2	C_{average}
Optimum values	207	193	7	11	14	11	\$1610

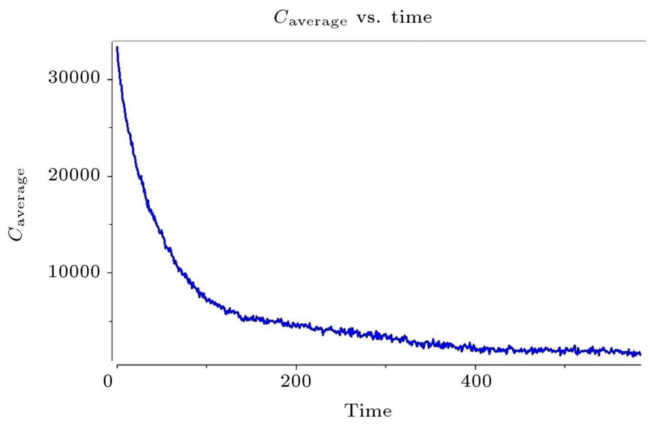

Figure 4. The simulation results.

Table 3. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, C_{average} for varying remanufacturing rate, α.

α	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.1	14	11	210	197	7	11	\$1630
0.2	14	11	209	196	7	11	\$1624.6
0.3	14	11	208	195	7	11	\$1619.2
0.4	14	11	207	194	7	11	\$1615
0.5	14	11	207	193	7	11	\$1610
0.6	14	11	206	192	7	11	\$1604.9
0.7	14	11	205	190	7	11	\$1600.1
0.8	14	11	205	189	7	11	\$1595.4
0.9	14	11	204	188	7	11	\$1590.1
1	14	11	203	187	7	11	\$1584.9

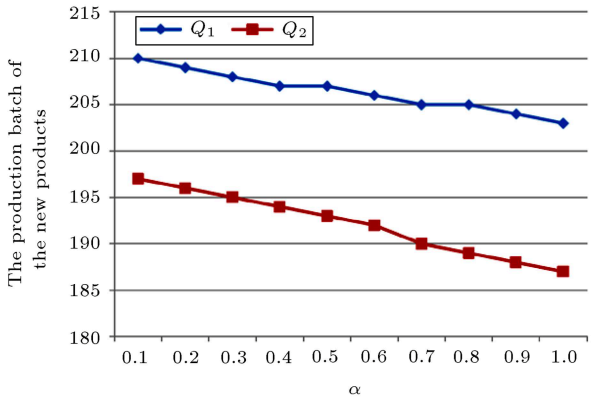

Figure 5. Behaviors of Q_1 and Q_2 for varying remanufacturing rate, α.

3.3. Sensitivity analysis

We now study the sensitivity analysis of the parameters present in this model under two conditions:

1. The behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying remanufacturing rate, α, under the condition of other parameters constant.

   When α increases from 0.1 to 1, the behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying α are shown in Table 3; the behaviors of Q_1 and Q_2 for varying α are shown in Figure 5.

   On the basis of sensitivity analysis of the parameters, the following features are observed:

   1.1 T_1 and T_2 do not vary with α. Because the inventory cost of recycled products has little proportion in the total inventory cost, α will not affect the production of new products, that is to say, T_1 and T_2 do not vary with α;

   1.2 Q_{r1} and Q_{r2} do not vary with α because T_1 and T_2 do not vary with α, and $Q_{rm} = T_m\lambda_m$, Q_{r1} and Q_{r2} do not vary with α when λ_m is constant;

   1.3 Q_1 and Q_{r2} decrease with the increase of α. Because T_1, T_2, Q_{r1}, and Q_{r2} do not vary with α, the quantity of remanufacturing product will increase with the increase of α. Therefore, under the condition of the same quantity demanded, it could reduce Q_1 and Q_2 to meet the demand;

1.4 C_{average} decreases with the increase of α. Under the condition of invariables T_1, T_2, Q_{r1}, and Q_{r2}, the quantity of remanufacturing product will increase with the increase of α, and the remanufacturing cost of the remanufacturing recycled products is less than the manufacturing cost of the new products, so the total inventory cost of the supply chain and the inventory cost per unit time will decrease;

1.5 In reality, when the quantity of the recycled products is not big, we can increase the remanufacturing rate of the recycled products in order to reduce the total inventory cost of the remanufacturing reverse logistics.

2. The values of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} vary with λ_1 and λ_2 under the condition of other parameters constant.

When λ_1 and λ_2 increase from 0.5 to 3, respectively, the behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_1 and λ_2 are shown in Tables 4 to 9.

Based on Tables 4 to 9, the behaviors of Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_1 and λ_2 are shown in Figures 6 and 7.

On the basis of sensitivity analysis of the parameters, the following features are observed:

2.1 T_1 and T_2 do not vary with λ_1 and λ_2. Because the inventory cost of recycled products has little proportion in the total inventory cost, λ_1 and λ_2 do not affect the production of new products, that is to say, T_1 and T_2 do not vary with λ_1 and λ_2;

2.2 Q_1 and Q_2 vary with λ_1 and do not vary with λ_2. Q_2 and Q_{r2} vary with λ_2 and do not vary with λ_1. Because the production and recycling of two kinds of products in this model are independent of each other, the production batch and the processing batch of the product only vary with the recovery rate of this product.

2.3 Q_{r1} increases with the increase of λ_1, and Q_{r2} increases with the increase of λ_2. Because λ_1 and λ_2 increase, that is to say, the quantity of remanufacturing products increases, it needs to handle the recycled products as soon as possible to reduce the inventory cost under the condition of invariable α, T_1 and T_2. Therefore, Q_{r1} and Q_{r2} will increase with the increase of λ_1 and λ_2.

Table 4. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_2 when $\lambda_1 = 0.5$.

λ_2	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.5	14	10	207	195	7	5	$1613.1
1	14	11	207	193	7	11	$1610
1.5	14	11	207	190	7	17	$1609.4
2	14	11	207	187	7	22	$1609.1
2.5	14	11	207	184	7	28	$1609.5
3	14	11	207	181	7	34	$1610.5

Table 5. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_2 when $\lambda_1 = 1$.

λ_2	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.5	14	10	203	195	14	5	$1612.7
1	14	11	203	193	14	11	$1609.4
1.5	14	11	203	190	14	17	$1608.8
2	14	11	203	187	14	22	$1608.5
2.5	14	11	203	184	14	28	$1608.9
3	14	11	203	181	14	34	$1609.9

Table 6. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_2 when $\lambda_1 = 1.5$.

λ_2	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.5	14	10	199	195	22	5	$1613
1	14	11	199	193	22	11	$1609.5
1.5	14	11	199	190	22	17	$1608.9
2	14	11	199	187	22	22	$1608.6
2.5	14	11	199	184	22	28	$1609
3	14	11	199	181	22	34	$1610

Table 7. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_2 when $\lambda_1 = 2$.

λ_2	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.5	14	10	196	195	28	5	$1614.5
1	14	11	196	193	28	11	$1610.8
1.5	14	11	196	190	28	17	$1610.2
2	14	11	196	187	28	22	$1609.9
2.5	14	11	196	184	28	28	$1610.3
3	14	11	196	181	28	34	$1611.3

Table 8. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_2 when $\lambda_1 = 2.5$.

λ_2	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.5	14	10	192	195	36	5	$1615.9
1	14	11	192	193	36	11	$1612
1.5	14	11	192	190	36	17	$1611.4
2	14	11	192	187	36	22	$1611.1
2.5	14	11	192	184	36	28	$1611.5
3	14	11	192	181	36	34	$1612.5

Table 9. Behaviors of T_1, T_2, Q_1, Q_2, Q_{r1}, Q_{r2}, and C_{average} for varying λ_2 when $\lambda_1 = 3$.

λ_2	T_1	T_2	Q_1	Q_2	Q_{r1}	Q_{r2}	C_{average}
0.5	14	10	188	195	44	5	$1617.9
1	14	11	188	193	44	11	$1613.7
1.5	14	11	188	190	44	17	$1613.2
2	14	11	188	187	44	22	$1612.8
2.5	14	11	188	184	44	28	$1613.3
3	14	11	188	181	44	34	$1614.3

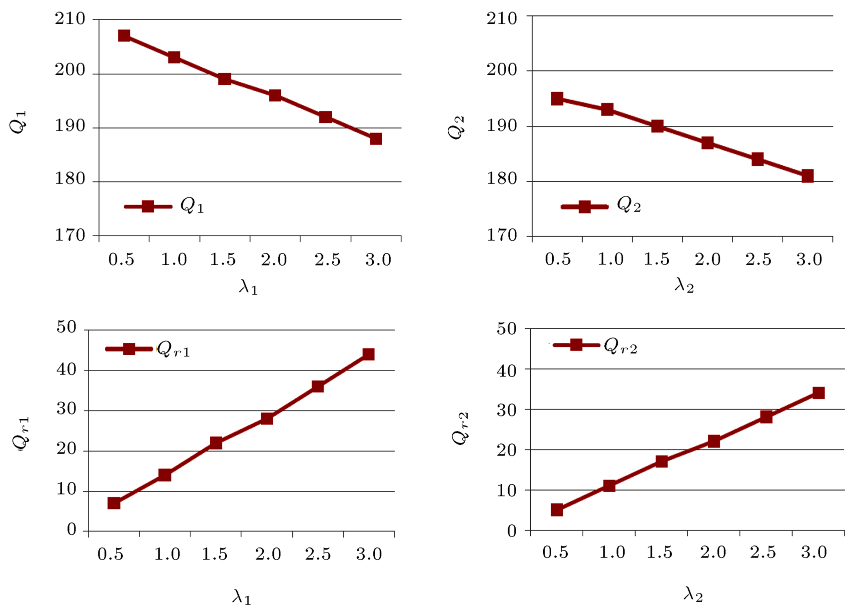

Figure 6. Behaviors of Q_1, Q_2, Q_{r1}, and Q_{r2} for varying λ_1 and λ_2.

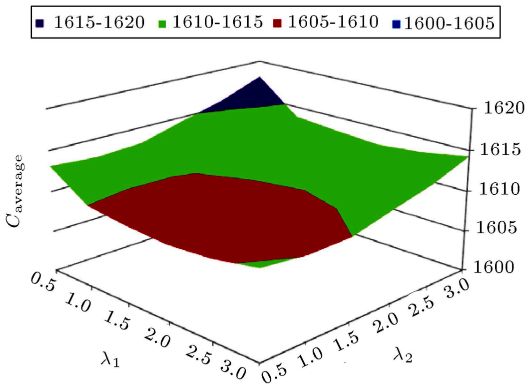

Figure 7. Behaviors of C_{average} for varying λ_1 and λ_2.

2.4 Q_1 decreases with the increase of λ_1, and Q_2 decreases with the increase of λ_2. Because Q_{r1} and Q_{r2} increase with the increase of λ_1 and λ_2, respectively, that is to say, the quantity of remanufacturing products increases with the increase of λ_1 and λ_2, respectively; it could reduce Q_1 and Q_2 to meet the demand under the condition of the same quantity demanded. Therefore, Q_1 and Q_2 decrease with the increase of λ_1 and λ_2, respectively.

2.5 When $\lambda_1 = 1$ and $\lambda_2 = 2$, C_{average} is the lowest ($C_{\text{average}} = 1608.5$). If λ_1 and λ_2 are too low, more recycled products need to be stored and the inventory cost of the recycled products must be higher. If λ_1 and λ_2 are too high, the dismantling cost of the recycled product and the cost of processing the waste product must be higher. So, regardless of λ_1 and λ_2 being too low or too higher, C_{average} will be higher.

2.6 In reality, in order to make sure that the total inventory cost of the remanufacturing reverse logistics is the lowest, we must determine the recovery rate of the recycled product according to the circumstance of the remanufacturing reverse logistics, and the recovery rate will not be too low or too high.

4. Conclusion

The problem of the inventory control of the remanufacturing reverse logistics is becoming more important. Aiming at this problem, we build a multi-product multi-echelon inventory control model of the remanufacturing reverse logistics based on the quantitative examination. A numerical simulation using the Matlab software is performed. The simulation can show that this model can reduce the inventory cost of remanufacturing reverse logistics and can provide a theoretical basis to determine the production batch and the processing batch for manufacturer and recycling center. Then, using sensitivity analysis, it shows that the recovery and remanufacturing rates of the recycled products have great influence on the inventory cost of the reverse logistics, production and inventory of the manufacturer, and the recycling center. Therefore, we need to give more and more attention to how to recoup and use the recycled products effectively.

There are some limitations in this research. First, we do not consider the lead time in the model. In practice, however, it has lead time and the lead time is usually a random variable. Second, we assume that the recycled products after being remanufactured can be sold as new products. In reality, the consumers often have a scruple to the remanufactured products, and this will affect the sales market of the remanufactured products. Thus, it will affect the remanufacturing process of the recycling products and affect the inventory control of the reverse logistics. These will be done in our future research.

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