Chemical reaction, Soret and Dufour effects on MHD mixed convection stagnation point flow with radiation and slip condition

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**KEYWORDS**
Chemical reaction; MHD; Porous medium; Radiation; Slip; Soret/Dufour.

**Abstract.** The main aim of this study is to present the effects of Soret & Dufour on MHD mixed convection flow of a viscous fluid towards a vertical plate embedded in a porous medium in the existence of slip, radiation and chemical reaction. The numerical solutions are acquired using shooting method, and the governing equations are reformulated into ordinary differential equations by similarity transformation. The present results compared with previously presented works are found in good agreement on several distinct cases. The effects of different parameters on velocity, temperature, and concentration distributions are illustrated graphically. The variation of diverse parameters on local skin friction, rates of heat, and mass transfer is obtainable in tabular arrangement. The velocity and temperature increase on increasing the Dufour parameter, and the concentration profiles decrease when chemical reaction parameter increases.

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1. Introduction

Chemical and metallurgical industries, such as drying devices and energy-related engineering problems, have extensively encountered mixed convection flow over a vertical plate of an incompressible fluid embedded in a porous medium. The stagnation area meets the zero velocity, the highest pressure, maximum rate of mass deposition, and the highest heat transfer. In general, many researchers show interest in computational numerical studies with heat transfer of stagnation flow in the existence of chemical reaction and magnetic field areas. A clear review by Nield & Bejan [1] and Eftekhari [2] provides physical aspects of convective heat transfer and fluid flow in a porous medium under various conditions. The study of magneto-hydrodynamics (MHD) has been widespread in engineering applications, such as cooling system designing and MHD generators. Mozayeni and Rahimi [3] analytically explored the unsteady mixed convection flow near a stagnation point over a plate inserted in a porous medium. Singh et al. [4] explored the steady, laminar, mixed convection of a stagnation-point flow in a porous medium with heat absorption/generation. The velocity and temperature distributions increase upon the increase of the heat generation parameter. Jafar et al. [5] presented the two-dimensional boundary layer flow of a fluid near a stagnation-point over a shrinking/stretching sheet in the presence of magnetic field. Saleh et al. [6] numerically studied mixed convection boundary layer flow of a viscous fluid through a porous medium over a vertical sheet at a stagnation-point. The temperature decreases upon the increase of the velocity proportion parameter. Ferdows et al. [7] studied the natural convection flow of an incompressible fluid along a semi-infinite plate in a porous medium. The authors noticed
that the temperature increases and velocity decreases upon the increase of the porosity parameter.

Thermal radiation effect on fluid flow plays a vital role in heat controlling factor over a vertical plate, such as polymer processing industries, design of the pertinent equipment, and several progressions in engineering areas arising at high temperature. Hosain et al. [8] examined the fluid flow along a porous plate with variable viscosity in the incidence of thermal radiation. They detected that the temperature and local Nusselt number increase with rising radiation and suction parameters. Pop et al. [9], Aydn and Kaya [10], and Hayat et al. [11] theoretically presented the stagnation-point flow over an extending sheet with radiation. They noticed that the local skin friction decreases and local heat transfer increases upon the increase of the radiation parameter. Lee et al. [12] and Bhuvaneswari et al. [13] also explored free convection and heat transfer of an incompressible fluid with radiation effect in a porous medium. The combined MHD convection of an incompressible fluid with heat generation and radiation at a stagnation-point was studied by Makinde [14] and Hayat [15].

The diffusion of species of boundary layer fluid flow to chemical reaction has various uses in water and air pollutants, atmospheric flows, fibrous separation, other chemical engineering, and technological processes. Bhattacharyya and Layek [16] and Bhuvaneswari et al. [17] analyzed effect of the chemical reaction on MHD flow over a plate. Niranjan et al. [18] and Raju et al. [19] investigated the MHD flow over a vertical surface embedded in porous medium with chemical reaction and thermal radiation. The velocity and concentration increase upon the increase of the chemical reaction parameter values in the case of generative reaction, and they are decreasing in the case of destructive reaction. Kasmani et al. [20] studied effect of the chemical reaction on boundary layer flow of a nanofluid beside a wedge with suction and heat generation/absorption. Bhattacharyya et al. [21] and Abbas et al. [22] deliberated the effects of slip on heat transfer, boundary layer flow near a stagnation-point towards a shrinking sheet. They reported that the heat transfer decreases upon the increase of the slip and velocity ratio parameters. Turkyilmazoglu [23] analytically studied heat transfer and MHD viscos flow with thermal slip and radiation over a stretching sheet. They observed from the outcomes that the skin friction increases at the wall upon the increase of the magnetic field, and the shear stress near the wall decreases as the slip parameter increases.

The effects of thermal-diffusion and diffusion-thermo are normally ignored due to negligible order of intensities rather than the effects defined by Fick’s law of diffusion and Fourier’s law of conduction. Pal and Mondal [24] and Postelnicu [25] studied the effects of magnetic field and thermal diffusion on flow over a stretching surface in a porous medium in the presence of Dufour and Soret effects. Vedavathi et al. [26] considered the effects of Soret & Dufour on unsteady two-dimensional flow of MHD over a vertical porous plate with thermal radiation.

Motivated by the aforementioned literature review, the authors explored the Soret and Dufour effects on MHD stagnation point flow over a surface in a porous medium in the presence of chemical reaction, slip, and thermal radiation. No effort has been made until now to study this problem. Hence, we propose to analyze this problem numerically.

2. Mathematical model

The steady, laminar, 2-D mixed convective boundary layer flow, heat & mass transfer of an electrically conducting fluid over a vertical plate in a porous medium near the stagnation point in the existence of magnetic field, radiation, and slip effects are considered. In addition, chemical reaction with first order, Soret & Dufour effects are deliberated. Figure 1 describes the coordinate system and physical model of the problem. The $x$-axis along the surface and $y$-axis perpendicular to the plate is considered. Then, $u$ and $v$ are taken as the velocity components along $x$ and $y$ directions, respectively. It is supposed that the distribution of velocity of the potential flow is occupied as $U_\infty = cx$ along the $y$-axis at time $t = 0$, where $c$ is a constant. The magnetic field is executed along the $y$ direction. We have neglected induced magnetic field owing to assumption of insignificant magnetic Reynolds number of the flow. The governing equations are given by:

$$u_x + u_y = 0,$$

![Figure 1. Coordinate system of the problem.](image-url)
\[ uu_x + \nu u_y = v u_{yy} + U_\infty (U_\infty)_x \]
\[ - \left( \frac{\sigma E_0^2 \rho}{\rho} + \frac{v}{K} \right) (u - U_\infty) + g \beta (T - T_\infty) \]
\[ + g \beta^2 (C - C_\infty). \]  

(2)

\[ uT_x + \nu T_y = \alpha (T_{yy} + \frac{Q}{\rho c_p} (T - T_\infty) - \frac{\alpha}{K} (q_x)_y \]
\[ + \frac{D_m K_T}{c_\epsilon c_p} C_{yy}. \]  

(3)

\[ uC_x + \nu C_y = D C_{yy} - \frac{G_0 (C - C_\infty)}{T_m} + \frac{D_m K_T}{T_m} T_{yy}. \]  

(4)

subject to the boundary conditions:
\[ u = N_1 u_y, \quad \nu = 0, \quad C = C_w, \quad T = T_w \]
\[ y = 0, \]
\[ u \rightarrow U_\infty = cx, \quad C = C_\infty, \quad T \rightarrow T_\infty \]
\[ y \rightarrow \infty. \]  

(5)

Thus, the heat flux \( (q_x) \) radiative term in the energy equation is simplified by employing the approximation of Rosendal diffusion:
\[ q_x = -\frac{4\sigma^*}{3K^2} (T^4)_y. \]  

(6)

We assume that the temperature differences are too small, and \( T^4 \) can be accepted as a linear combination of the temperature by Taylor’s series about free stream temperature and ignoring the higher order expressions, we get \( T^4 \approx 4\sigma^* T^2 - \frac{3}{3} T^4 \), then Eq. (6) reduces to:
\[ q_x = -\frac{16\sigma^* T^2}{3K^2} T_y. \]  

(7)

Eq. (7) is substituted into Eq. (3) for temperature. The variables of non-dimensional are introduced as follows:
\[ \eta = y \sqrt{\frac{c}{v}}, \quad \psi(x, y) = \sqrt{cv} f(\eta), \]
\[ b = N_1 \left( \frac{c}{v} \right)^{1/2}, \quad Cr = \frac{\Gamma_0}{c}, \]
\[ Df = \frac{D_m K_T (C_w - C_\infty)}{c_\epsilon c_p (T_w - T_\infty) v^2}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \]
\[ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad Gr_T = g \beta (T_w - T_\infty) x^3, \]
\[ Gr_c = g \beta^2 (C_w - C_\infty) x^3, \quad Ri_T = \frac{Gr_T}{Re^2}, \]
\[ Ri_C = \frac{Gr_c}{Re^3}, \quad Re_x = \frac{U_\infty x}{v}, \quad Rd = \frac{4\sigma^* T^3}{kK^2}, \]
\[ K = \frac{v}{cK}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}. \]
\[ M = \frac{\sigma E_0^2}{c\rho}, \quad S = \frac{Qv}{\rho c_p}, \quad Sr = \frac{D_m K_T (T_w - T_\infty)}{T_m \alpha (C_w - C_\infty)}. \]  

(8)

where the stream function is represented as \( \psi \), well-defined as \( u = \partial \psi / \partial y \), and \( \nu = -\partial \psi / \partial x \). Substituting Eqs. (7) and (8) into Eqs. (1)-(5), we obtain the ensuing coupled ordinary differential equations with boundary conditions:
\[ f'' + f f' - f^2 - (K + M)(f' - 1) + Ri_T \theta = 0, \]  

(9)

\[ \left( 1 + \frac{4}{3} \text{Rd} \right) \theta'' + S \theta + Pr f \theta' + D f \phi'' = 0, \]  

(10)

\[ \phi'' + Sc f \phi' + Sr Sc \theta' - Sc Cr \phi = 0, \]  

(11)

\[ f = f = b f(0), \quad \phi = 1, \quad \theta = 1, \]  

(12)

at \( \eta = 0, \)
\[ f' = 1, \quad \phi = 0, \quad \theta = 0, \]  

(12)

at \( \eta \rightarrow \infty. \)

The coupled Eqs. (9)-(11) cannot get the closed loop solution. Therefore, the authors numerically solved the problem using shooting method along with Runge-Kutta 4th order integration. From the process of numerical computation, the physical quantities describe the local skin-friction coefficient, local Nusselt number, and local Sherwood number, which are interesting of the present problem, and are designated for physical wall shear stress, rates of heat transfer, and mass transfer. They are given by the expressions:
\[ C_f = \frac{2\tau_w}{\rho U_\infty^2}, \quad Nu = \frac{x q_w}{k(T_w - T_\infty)} \]
\[ Sh = \frac{x q_m}{D(C_w - C_\infty)}. \]  

(13)

where:
\[ \tau_w = \mu \left. u_y \right|_{y=0}, \quad q_w = -k T_y \left. \right|_{y=0} - \frac{4\sigma^* T^4}{3K^2} \left. T_y \right|_{y=0}, \]
\[ q_m = -DC \left. g_y \right|_{y=0}. \]

Finally, we get the non-dimensional local skin friction \( (C_f) \), Nusselt \( (Nu) \), and Sherwood \( (Sh) \) numbers as follows:
\[ Re \left( \frac{1}{2} \right) C_f = f''(0), \quad \frac{Re^{-1/2} Nu}{1 + \frac{4}{3} \text{Rd}} = -\theta'(0), \]
\[ Re^{-1/2} Sh = -\phi'(0). \]  

(14)

The authenticity of our numerical procedure is compared with results of Singh [4] and Makinde [14].
Table 1. Computations showing the comparison with Singh et al. [4] and Makinde [14] for different values of \( S \) when \( R_i = 1, R_k = 0.3, \Pr = 1, Sc = 0.5, M = 0, K = 0, Cr = 0 \), and \( Rd = 0 \).

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for various values. Table 1 displays the agreement among the results. This gives assurance on the present numerical calculation to be testified here.

3. Results and discussion

The authors managed to understand the effects of Soret & Dufour on MHD heat transfer and mixed convective flow characteristics of viscous fluid near a stagnation-point flow for various values of the pertinent parameter elaborated in the study. The local skin friction coefficient dissimilarities, the rates of heat, and mass transfer are presented in Table 2 for various values of the parameters \( S, Rd, Cr, Df, Sr, \) and \( b \) with fixed values of \( R_i = 1, R_k = 1, \Pr = 0.7, \Sc = 0.5, K = 2, M = 2 \). The skin friction increases at the plate upon the increase of the values of the parameters \( S, Rd, \) and \( Df \). The rate of heat transfer decreases upon the increase of \( S, Cr, Df, \) and \( Sr \) parameters. The heat transfer rate increases as the thermal radiation and slip parameters increase. By increasing \( S, Cr, Df, Sr, \) and \( b \) parameters, the rate of mass transfer increases.

Figure 2(a)-(c) shows the distributions of velocity, temperature, and concentration for numerous values of heat generation parameter. Figure 2(a) illustrates that the fluid gets more heat energy due to internal heat generation/absorption, and it results in higher buoyancy force. Hence, the velocity of fluid particle is increasing inside the boundary layer upon the increase of heat generation parameter. The velocity profile overshoots heat generation \( (S > 0) \) case. The velocity of the fluid increases upon the increase of the internal heat generation parameter \( (S) \). There is less effect observed on the case of heat absorption \( (S < 0) \). Figure 2(b) depicts the different values of temperature for \( S \) with constant parameters of \( K, M, Rd, Df, Sr, Cr, \) and \( b \). The temperature profiles overshoots on increasing the heat generation \( (S > 0.5) \) parameter. This effect occurs because buoyancy force is larger when fluid gets large quantity of heat energy due to internal heat energy. It is clearly detected that the profiles of temperature increase as the heat generation parameter increases. When \( S < 1 \), the influence of heat generation is less on temperature distribution inside the thermal boundary layer. Figure 2(c) shows the profiles of concentration with constant values of \( K, M, Rd, Df, Sr, Cr, \) and \( b \). The concentration decreases upon the increase of the \( S \). Figure 3(a) and (b) show the different values of the Radiation Parameter (Rd) on dimensionless profiles of velocity and temperature. The radiation parameter increases on increasing the temperature of the fluid and decreasing the value of mean absorption.
Figure 2. (a) Velocity, (b) temperature, and (c) concentration profiles for different heat generation parameters, \( S \), with \( K = 2, M = 2, \text{Rd} = 0.5, \text{DF} = 0.5, \text{Sr} = 0.5, \text{Cr} = 0.5, \) and \( b = 0.5 \).

The radiation parameter increases, then the boundary layer thickness decreases. The momentum and thermal boundary layer thicknesses increase upon increasing the radiation parameter.

Figure 4(a)-(c) illustrate the profiles of velocity, temperature, and concentration for different values of Dufour parameter with fixed values of \( K, M, \text{Rd}, S, \text{Sr}, \text{Cr}, \) and \( b \). It is perceived that the velocity of the fluid increases as the Dufour number increases. Since the Dufour parameter is in energy equation, it affects mainly the temperature distribution. The higher temperature difference reduces the value of Dufour effect on thermal flow. Figure 4(b) displays that the temperature of the fluid near the plate surface overshoots on increasing the Dufour number values. The thickness of thermal boundary layer increases as the Dufour number increases. It is detected from Figure 4(c) that the fluid concentration profile decreases upon the increase of the Dufour number.

The Soret effect on the velocity and concentration profiles is illustrated in Figure 5(a) and (b). Figure 5(a) shows that the velocity increases as the Soret number increases. The Soret number increases as the temperature of the fluid increases. The larger Soret number is viewed for a larger temperature variance and hasty gradient. Thus, the fluid velocity increases due to greater Soret effect. The effects of Soret number are clearly playing a significant role in concentration profile; see Figure 5(b). The concentration increases on rising the Soret number. Therefore, we can understand that the effects of Soret and Dufour are generally more enthusiastic in the study of mixed convection problems. The effects of Dufour and Soret are apparently playing a vital role in combined convection in a porous medium in the occurrence of radiation, chemical reaction, and slip parameter. So, we can appreciate that the effect of diffusion-thermal greatly genuine on the mixed convection problems of the study.

Figure 6(a)-(c) displays the effect of chemical reaction \((\text{Cr})\) on various boundary layer profiles. It is
Figure 4. (a) Velocity, (b) temperature, and (c) concentration profiles for different Dufour parameters, $D_f$, with $K = 2$, $M = 2$, $R_d = 0.5$, $S = 0.5$, $S_r = 0.5$, $C_r = 0.5$, and $b = 0.5$.

Figure 5. (a) Velocity, and (b) concentration profiles for different Soret parameters, $S_r$, with $K = 2$, $M = 2$, $R_d = 0.5$, $S = 0.5$, $D_f = 0.5$, $C_r = 0.5$, and $b = 0.5$.

boundary layer thickness is very less in destructive case ($Cr > 0$). Figure 7(a) and (b) illustrate effect of the slip parameter on the profiles of velocity and temperature. It is perceived from Figure 7(a) that the profiles of velocity increases on increasing the slip parameter. Figure 7(b) shows that the distribution of temperature increases on decreasing the slip parameter. It is readily seen that slip parameter has substantial effect.

4. Conclusion

The authors investigated the effects of diffusion-thermo and thermal-diffusion on steady magneto-convection flow of an incompressible viscous fluid in the presence of slip, chemical reaction, and radiation over a vertical plate nearby a stagnation-point. The governing equations are altered into non-linear coupled ordinary differential equations by similarity transformations. The numerical solutions are attained by applying shooting method using Runge-Kutta 4th-order integration. From the present study, we observe the following:

- The distributions of velocity and temperature of the fluid increase with Dufour number increases and the
fluid concentration drops on increasing the Dufour number. On increasing the Soret number, the velocity and concentration distributions increase;

- It is noticed that the profiles of velocity and concentration increase on decreasing the chemical reaction parameter. The temperature rises on increasing the chemical reaction;
- From the present analysis, the velocity increases and temperature decreases on increasing the slip parameter;
- The skin friction increases on increasing the $K$, $M$, $S$, Rd, and Df, and it decreases on increasing the values of Cr, Sf, and $b$;
- Finally, the rate of heat transfer increases on increasing the radiation and slip parameters. The rate of local mass transfer increases on increasing $S$, $Cr$, Df, Sf, and $b$ parameters.

**Nomenclature**

- $a$: Constant
- $b$: Slip parameter
- $B_0$: Strength of magnetic field
- $C$: Species concentration
- $Cr$: Chemical reaction parameter
- $D$: Diffusion coefficient
- $Df$: Dufour number
- $f$: Dimensionless stream function
- $g$: Gravitational acceleration
Gr$_c$ Solutal Grashof number
Gr$_T$ Thermal Grashof number
h Convective heat transfer coefficient
k Thermal conductivity
K Permeability parameter
K' Porous medium permeability
K'' Mean absorption coefficient
M Magnetic field parameter
N$_s$ Navier slip coefficient
Nu Nusselt number
Pr Prandtl number
Q Heat generation/absorption
Rd Thermal radiation parameter
Re$_x$ Reynolds number
Ri$_T$ Thermal Richardson number
Ri$_c$ Solutal Richardson number
S Heat generation parameter
Sc Schmidt number
Sh Sherwood number
Sr Soret number
T Temperature
u, v Velocity components
x, y Cartesian coordinates

Greek letters
\( \beta \) Coefficient of thermal expansion
\( \beta^* \) Coefficient of solutal expansion
\( \Gamma_0 \) Chemical reaction rate
\( \eta \) Similarity variable
\( \theta \) Dimensionless temperature
\( \mu \) Dynamic viscosity
\( \nu \) Kinematic viscosity
\( \rho \) Density
\( \sigma^* \) Stefan-Boltzmann constant
\( \sigma_e \) Electrical conductivity
\( \phi \) Dimensionless concentration
\( \psi \) Stream function

Subscripts
W At wall
\( \infty \) At free stream

References


Biographies

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