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On the solution of a contact problem for a rhombus weakened with a full-strength hole

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Abstract. This paper addresses the problem of plane elasticity theory for a doubly connected body whose external boundary is a rhombus with its diagonals lying at the coordinate axes OX and OY. The internal boundary is the required full-strength hole and the symmetric axes are the rhombus diagonals. Smooth stamps with rectilinear bases are applied to the linear parts of the boundary and the middle points of these stamps are under the action of concentrated forces; thus, there are no friction forces between the stamps and the elastic body. The hole boundary is free from external load and the tangential stresses are zero along the entire boundary of the rhombus. Using the methods of complex analysis, the analytical image of Kolosov-Muskhelishvili's complex potentials (characterizing an elastic equilibrium of the body) and the equation of an unknown part of the boundary are determined under the condition that the tangential normal stress arising at it takes a constant value. Such holes are called full-strength holes. Numerical analyses are performed and the corresponding graphs are constructed.

1. Introduction

Mixed and contact problems belong to the category of problems of mechanics of deformed rigid bodies, which are highly important for application and are the most difficult ones from the mathematical stand-point. It is through the mixed contact that loads are transferred to deformed bodies and it is in the contact zone that stress concentrations occur, which cause body failure, and emergence and spread of cracks. These phenomena cannot be prevented in the conditions of modern technological processes of production of mechanisms and machine parts.

Boundary-value problems of the plane theory of elasticity and plate bending for infinite plates weakened by unknown full-strength holes with normal stresses acting on their boundaries and forces applied at infinity were analyzed in [1-3].

Boundary-value problems for a finite doubly connected domain, with a part of its boundary being unknown full-strength and the other part being a polygonal line, are solved in [4].

The axis-symmetric and cycle-symmetric problems of the plane theory of elasticity and plate bending with partially unknown boundaries are studied in [5-11]. The most effective methods for studying these problems are the methods of the theory of analytical

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functions of a complex variable.

In this article, the axially symmetric problem of plane elasticity theory for a rhombus weakened with a full-strength hole is considered. The formulae of Kolosov-Muskhelishvili are used for investigating this problem. The solution is written in quadratures and the unknown full-strength hole of the plate is constructed

2. Problem formulation and solution technique

Let an isotropic elastic body on the plane $z = x + iy$ occupy a doubly connected domain S, whose external boundary is a rhombus with its diagonals lying at the coordinate axes OX and OY. The internal boundary is the required full-strength hole and the symmetric axes are the rhombus diagonals (Figure 1).

Let every link of the broken line (outer boundary of the given body) be applied to absolutely smooth, rigid stamps with rectilinear bases, which displace the normal under the action of concentrated, normally compressive forces, P, applied to the stamp midpoints of polygon's sides. There is no friction between the given elastic body and stamps and the unknown full-strength contour is free from outer actions.

Under the above assumptions, the tangential stresses are zero $\tau_{ns} = 0$, along the entire boundary of the domain, S, and the normal displacements of every link of the external boundary, $\nu_n = \nu$.

Consider the following problem: Find the shape of the unknown hole and the stress state of the given body such that the tangential normal stress, σ_s, arising at it would take the constant value, $\sigma_s = K = \text{const}$.

Since the problem is axially symmetric, to investigate it, it is sufficient to consider the curvilinear quadrangle, $A_1A_2A_3A_4$, denoted by D. The normal displacements and the tangential stresses are equal to zero, $\nu_n = \tau_{ns} = 0$, at each segment $[A_1, A_2]$, $[A_3, A_4]$.

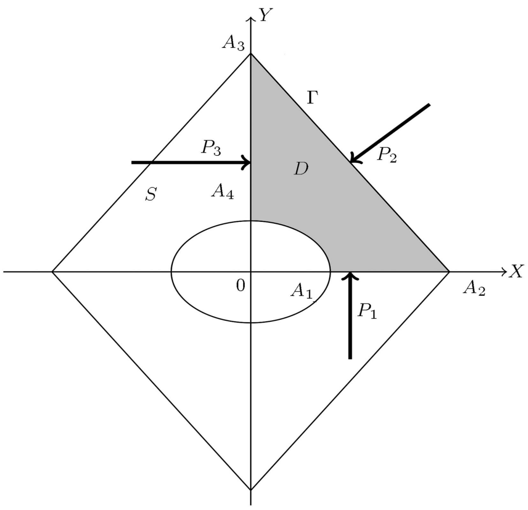

Figure 1. Graph of the posed problem.

Let us introduce the following notations:

$$\Gamma_1 = A_1A_2, \quad \Gamma_2 = A_2A_3, \quad \Gamma_3 = A_3A_4,$$

$$\gamma = A_4A_1, \quad \Gamma = \cup_{j=1}^{3}\Gamma_j, \quad P_1 = \int_{\Gamma_1} \sigma_n ds,$$

$$P_2 = \int_{\Gamma_2} \sigma_n ds, \quad P_3 = \int_{\Gamma_3} \sigma_n ds,$$

σ_n is the normal stress:

$$P_2 = \int_{\Gamma_2} \sigma_n ds = -P.$$

Since D is in the equilibrium state, we have:

$$P_1 = P_2 \cos\beta = -P\cos\beta,$$

$$P_3 = P_2 \sin\beta = -P\sin\beta,$$

where $\beta = \angle A_1A_2A_3$.

Following the statement of the problem, the boundary conditions have the form:

$$\nu_n = \begin{cases} 0, & t \in \Gamma_3 \cup \Gamma_1 \\ \nu, & t \in \Gamma_2 \end{cases} \tag{1}$$

$$\tau_{ns} = 0, \qquad t \in \Gamma \cup \gamma, \tag{2}$$

$$\sigma_n = 0, \qquad \sigma_s = K, \qquad t \in \gamma, \tag{3}$$

$$P_2 = -P, \qquad P_1 = -P\cos\beta, \qquad P_3 = -P\sin\beta. \tag{4}$$

Let the points A_1, A_2, A_3, and A_4 be counted in a positive direction and its affixes be denoted by the same symbols. Also, assume that A_1 is the origin of the broken line Γ.

On the basis of the well-known Kolosov-Muskhelishvili's formulae [12], the problem is reduced to finding the functions ψ and φ, which are holomorphic in the domain D with the following conditions:

$$\text{Re}\, e^{-i\alpha(t)}\Big(\chi\varphi(t) - t\overline{\varphi'(t)} - \overline{\psi(t)}\Big) = 2\mu\nu_n(t), \quad t \in \Gamma, \tag{5}$$

$$\text{Re}\, e^{-i\alpha(t)}\Big(\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)}\Big) = C(t), \quad t \in \Gamma, \tag{6}$$

$$\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)} = 0, \quad t \in \gamma, \tag{7}$$

$$\text{Re}\varphi'(t) = \frac{\sigma_n + \sigma_s}{4} = \frac{K}{4}, \quad t \in \gamma, \tag{8}$$

where χ and μ are elasticity constants, $C(t)$ is a piecewise-constant function, $\alpha(t)$ is the angle formed between the external normal n to contour, and the abscissa axis Ox.

$$\alpha(t)=\alpha_k, \qquad t\in\Gamma_k, \qquad k=1,2,3,$$

$$\alpha_1=-\frac{\pi}{2}, \qquad \alpha_2=\frac{\pi}{2}-\beta, \qquad \alpha_3=\pi, \tag{9}$$

$$C(t)=\mathrm{Re}\left(e^{-i\alpha(t)}i\left(\int_{A_1}^{t}\sigma_n(s_0)e^{i\alpha(s_0)}ds_0\right)\right). \tag{10}$$

Taking Eqs. (4) and (9) into account, Eq. (10) has the following form:

$$C(t)=\begin{cases}0, & t\in\Gamma_1\\ -P\cos\beta\sin\beta, & t\in\Gamma_2\\ 0, & t\in\Gamma_3\end{cases} \tag{11}$$

Let $t\in A_kA_{k+1}$, $k=1,2,3$; then, $t-A_k=i|t-A_k|e^{i\alpha_k}$. Hence, we obtain:

$$\mathrm{Re}\ te^{-i\alpha(t)}=\mathrm{Re}\ e^{-i\alpha(t)}A(t), \tag{12}$$

where $A(t)$ is a piecewise-constant function, $A(t)=A_k$, $t\in A_kA_{k+1}$, and $k=1,2,3$. Taking Eq. (9) into account, one obtains:

$$\mathrm{Re}\ e^{-i\alpha(t)}A(t)=\begin{cases}d_1\cos\alpha_2, & t\in\Gamma_2\\ 0, & t\in\Gamma_1\cup\Gamma_3\end{cases} \tag{13}$$

where $d_1=|0A_2|$.

Let functions $\varphi'(z)$ and $\psi(z)$ be continuously extendable everywhere on the boundary of domain D, except, perhaps, the vertices of broken line Γ; in the neighborhood of vertices A_k, the following condition holds:

$$|\varphi'(z)|<M|z-A_k|^{-\delta_k},$$

$$|\psi(z)|<M\,|z-A_k|^{-\delta_k}, \tag{14}$$

where $0\le\delta_k<1$, $k=2,3$; $0\le\delta_k<1/2$, $k=1,4$.

Combining Eqs. (5) and (6), then, differentiating with respect to the arc abscissa s, and noting that $\alpha(t)$, $C(t)$, and $\nu_n(t)$ are piecewise constant functions, one obtains:

$$\mathrm{Im}\varphi'(t)=0, \qquad t\in\Gamma. \tag{15}$$

Eqs. (8) and (15) are the Keldysh-Sedov problem for domain D:

$$\mathrm{Re}\left(\varphi'(t)-\frac{K}{4}\right)=0, \qquad t\in A_4A_1,$$

$$\mathrm{Im}\left(\varphi'(t)-\frac{K}{4}\right)=0, \qquad t\in\Gamma. \tag{16}$$

Problem (16) has a unique solution [13] (see more details in [14]):

$$\varphi'(z)=\frac{K}{4}. \tag{17}$$

Hence, one obtains:

$$\varphi(z)=\frac{K}{4}z, \tag{18}$$

where the constant is neglected.

Substituting the values of $\varphi(t)$ and $C(t)$, defined by Eqs. (18) and (11), into the boundary conditions (6)-(7) and taking Eq. (13) into account, one gets the following problem:

$$\mathrm{Re}\left[e^{-i\alpha(t)}\left(\frac{K}{2}t+\overline{\psi(t)}\right)\right]=C(t)$$

$$=\begin{cases}0, & t\in\Gamma_1\\ -P\cos\beta\sin\beta, & t\in\Gamma_2\ ,\\ 0, & t\in\Gamma_3\end{cases} \tag{19}$$

$$\mathrm{Re}\ te^{-i\alpha(t)}=\mathrm{Re}\ e^{-i\alpha(t)}A(t), \qquad t\in\Gamma, \tag{20}$$

$$\frac{K}{2}t+\overline{\psi(t)}=0, \qquad t\in\gamma. \tag{21}$$

Let the function $z=\omega(\zeta)$ and $\zeta=\xi+i\eta$ map the semi-circle $|\zeta|<1$ and $\mathrm{Im}\zeta>0$ conformally onto the domain D. It is assumed that the vertices A_k of rhombus line correspond to the points a_k of the semicircle $|\zeta|=1$, $\mathrm{Im}\zeta>0$, $a_k=\omega^{-1}(A_k)$, $k=1,2,3,4$. It is assumed that $a_1=1$, $a_4=-1$, and $a_3=i$.

Here, we can fix three points and the remaining ones are to be defined. Then, the diameter $-1\le\xi\le1$ is mapped onto arc A_4A_1 and the semi-circumference $\gamma_0:|\gamma_0|=1$, $\mathrm{Im}\zeta>0$ is mapped onto the broken line Γ. The point $a_2=e^{i\theta_2}$, $0<\theta_2<\pi/2$, is to be defined. Hence, by virtue of Eqs. (19), (20), and (21) for functions $\psi_0(\zeta)=\psi(\omega(\zeta))$ and $\omega(\zeta)$, one obtains:

$$\mathrm{Re}\ e^{-i\alpha(\sigma)}\overline{\psi_0(\sigma)}=-\frac{K}{2}\mathrm{Re}\ e^{-i\alpha(\sigma)}A(\sigma)+C(\sigma),$$

$$\sigma\in\gamma_0, \tag{22}$$

$$\mathrm{Re}\ e^{-i\alpha(\sigma)}\omega(\sigma)=\mathrm{Re}\ e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma\in\gamma_0, \tag{23}$$

$$\frac{K}{2}\omega(\sigma)+\overline{\psi_0(\sigma)}=0, \qquad \sigma\in(-1,1), \tag{24}$$

$$\psi_0(\zeta)=\psi(\omega(\zeta)). \tag{25}$$

For simplicity, the piecewise constant functions $\alpha(\omega(\sigma))$, $A(\omega(\sigma))$, and $C(\omega(\sigma))$ will again be denoted by $\alpha(\sigma)$, $A(\sigma)$, and $C(\sigma)$.

Consider the new unknown function $W(\zeta)$ defined by:

$$W(\zeta)=\begin{cases}\frac{K}{2}\omega(\zeta), & |\zeta|<1, \quad \mathrm{Im}\zeta>0\\ -\overline{\psi_0(\bar{\zeta})}, & |\zeta|<1, \quad \mathrm{Im}\zeta<0\end{cases} \tag{26}$$

By virtue of Eq. (24), it is easy to verify that $W(\zeta)$ is a holomorphic function in the circle $|\zeta| < 1$.

From Eq. (26), one obtains:

$$W^{+}(\xi)=\frac{K}{2}\omega(\xi),$$

$$W^{-}(\xi)=-\overline{\psi_0(\xi)}, \quad -1<\xi<1, \tag{27}$$

$$W^{+}(\sigma)=\frac{K}{2}\omega(\sigma), \qquad \sigma\in\gamma_0,$$

$$W^{-}(\sigma)=-\overline{\psi_0(\bar{\sigma})}, \qquad \sigma\in\gamma_0^{*}. \tag{28}$$

The signs $(+)$ and $(-)$ refer to the upper and lower edges, respectively. γ_0^* is the reflection of γ_0 with respect to X-axis. By virtue of Eqs. (18) and (20), one obtains:

$$W^{+}(\xi)-W^{-}(\xi)=0, \qquad -1<\xi<1,$$

that means that $W(\zeta)$ is a holomorphic function in the circle $|\zeta|<1$.

By virtue of Eqs. (22) and (23), the function $W(\zeta)$ defined by Eq. (26) satisfies the boundary conditions:

$$\mathrm{Re}\ e^{-i\alpha(\sigma)}W(\sigma)=\frac{K}{2}\mathrm{Re}\ e^{-i\alpha(\sigma)}A(\sigma),$$

$$\sigma\in\gamma_0, \tag{29}$$

$$\mathrm{Re}\ e^{-i\alpha(\sigma)}W(\sigma)=-C(\sigma)+\frac{K}{2}\mathrm{Re}\ e^{-i\alpha(\sigma)}A(\sigma),$$

$$\sigma\in\gamma_0^{*}. \tag{30}$$

Conditions (29) and (30) can be rewritten in the following form:

$$\mathrm{Re}\ e^{-i\alpha(\sigma)}W(\sigma)=f(\sigma),$$

$$\sigma\in\gamma', \tag{31}$$

where:

$$\gamma'=\gamma_0\cup\gamma_0^{*}, \quad \alpha(\sigma)=\alpha(\bar{\sigma}),$$

$$\sigma\in\gamma_0^{*}, \quad \alpha_1=-\pi/2,$$

$$\alpha_2=\pi/2-\beta, \quad \alpha_3=\pi,$$

$$\beta=\angle A_1A_2A_3.$$

If $\sigma\in\gamma_0$, then:

$$f(\sigma)=\frac{K}{2}\mathrm{Re}\ e^{-i\alpha(\sigma)}A(\sigma)$$

$$=\begin{cases}\frac{K}{2}d_1\cos\alpha_2, & \sigma\in(a_2,a_3)\\ 0, & \sigma\in(a_1,a_2)\cup(a_3,a_4)\end{cases},$$

$$d_1=|0A_2|. \tag{32}$$

If $\sigma\in\gamma_0^{*}$, then:

$$f(\sigma)=\frac{K}{2}\mathrm{Re}\ e^{-i\alpha(\sigma)}A(\sigma)-C(\sigma)$$

$$=\begin{cases}0, & \sigma\in(\bar{a}_2,a_1^{*})\\ P\cos\beta\sin\beta+\frac{K}{2}d_1\cos\alpha_2, & \sigma\in(\bar{a}_3,\bar{a}_2)\\ 0, & \sigma\in(a_4,\bar{a}_3)\end{cases}$$

$$a_1^{*}=e^{2\pi i}. \tag{33}$$

Thus, the problem in question has been reduced to the Riemann-Hilbert problem with piecewise-constant coefficients. The solution to this problem was obtained in [15] (by reducing it to a linear-conjugation problem). Here, the problem is reduced to the Dirichlet problem for a circle and its solution is presented by Schwarz formula, which is computationally convenient.

Function $e^{2i\alpha(\sigma)}$ is given by:

$$e^{2i\alpha(\sigma)}=\frac{X(\sigma)}{\overline{X(\sigma)}},$$

$$|\sigma|=1, \tag{34}$$

where $X(\zeta)$ is presented by Eq. (35) as shown in Box I.

By virtue of Condition (34), Condition (31) will have the form:

$$\frac{W(\sigma)}{X(\sigma)}+\frac{\overline{W(\sigma)}}{\overline{X(\sigma)}}=\frac{2fe^{i\alpha(\sigma)}}{X(\sigma)}. \tag{36}$$

Condition (36) presents the boundary condition of Dirichlet problem, whose solution is presented by Schwartz formula:

$$\frac{W(\zeta)}{X(\zeta)}=\frac{1}{2\pi i}\int_{\gamma'}\frac{f(\sigma)e^{i\alpha(\sigma)}(\sigma+\zeta)d\sigma}{X(\sigma)\sigma(\sigma-\zeta)}. \tag{37}$$

$$X(\zeta)=\frac{(\zeta-a_2)^{\frac{\alpha_1-\alpha_2}{\pi}+1}(\zeta-a_3)^{\frac{\alpha_2-\alpha_3}{\pi}+1}(\zeta-\bar{a}_3)^{\frac{\alpha_3-\alpha_2}{\pi}}(\zeta-\bar{a}_2)^{\frac{\alpha_2-\alpha_1}{\pi}}}{\zeta}\sqrt{\bar{a}_2\bar{a}_3}. \tag{35}$$

Box I

Since function $X(\zeta)$ has a simple pole at the point $\zeta = 0$, function $W(\zeta)/X(\zeta)$ has the first-order zero at point $\zeta = 0$ and from Eq. (37), one gets:

$$\int_{\gamma'} \frac{f(\sigma)e^{i\alpha(\sigma)}d\sigma}{X(\sigma)\sigma} = 0. \tag{38}$$

i.e.:

$$\frac{K}{2}d_1 \cos\alpha_2 e^{\alpha_2 i}\int_{a_2}^{a_3} \frac{d\sigma}{X(\sigma)\sigma} + \left(P\cos\beta\sin\beta + \frac{K}{2}d_1\cos\alpha_2\right)e^{\alpha_2 i}\int_{\bar{a}_3}^{\bar{a}_2}\frac{d\sigma}{X(\sigma)\sigma} = 0. \tag{39}$$

Thus, we obtain an equation with respect to two unknown parameters, a_2 and K.

We could choose some value for K and, then, determine the parameter a_2. In this case, the problem becomes more complicated. For the purpose of doing the computations, it is more convenient to calculate K for each fixed point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \pi/2$, and for the given P.

Taking Eq. (38) into account, Eq. (37) will have the form:

$$W(\zeta) = \frac{\zeta X(\zeta)}{\pi i}\int_{\gamma'}\frac{f(\sigma)e^{i\alpha(\sigma)}d\sigma}{X(\sigma)\sigma(\sigma-\zeta)}, \tag{40}$$

i.e.:

$$W(\zeta) = \frac{\zeta X(\zeta)}{\pi i}\left(\frac{K}{2}d_1\cos\alpha_2 e^{\alpha_2 i}\int_{a_2}^{a_3}\frac{d\sigma}{X(\sigma)\sigma(\sigma-\zeta)} + \left(P\cos\beta\sin\beta + \frac{K}{2}d_1\cos\alpha_2\right)e^{\alpha_2 i}\int_{\bar{a}_3}^{\bar{a}_2}\frac{d\sigma}{X(\sigma)\sigma(\sigma-\zeta)}\right). \tag{41}$$

By virtue of Eq. (26), equation of the contour $z = \omega(\xi)$ is presented by:

$$\omega(\xi) = \frac{2W(\xi)}{K}, \qquad -1 < \xi < 1. \tag{42}$$

From Eq. (26), the functions ω and ψ are defined. Thus, equation of the contour and stress state of body is defined.

Remark 1. *Let us introduce the following notations:*

$$A = \int_{a_2}^{a_3}\frac{d\sigma}{X(\sigma)\sigma},$$

$$B = \int_{\bar{a}_3}^{\bar{a}_2}\frac{d\sigma}{X(\sigma)\sigma}.$$

From Eq. (39), one obtains:

$$K = -\frac{2PB\,ctg\beta}{d_1(A+B)}. \tag{43}$$

The solution of problem exists if:

$$A + B \neq 0. \tag{44}$$

3. Construction of the rhombus full-strength hole

Let us consider some concrete cases. Let the length $|0A_2| = 1.5$ and the value of angle $\beta = \angle A_1A_2A_3$ be changed for $0 < \beta < \pi/2$. The full-strength contours are defined for different rhombuses, which are obtained by changing the parameter β, $0 < \beta < \pi/2$.

To construct the required full-strength hole of the rhombus, at first, the arc $\gamma = A_4A_1$ of the required full-strength contour is constructed. Having defined K by Eq. (43) for every fixed point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \pi/2$, $a_1 = 1$, $a_4 = -1$, $a_3 = i$ and given P:

$$K = -\frac{2PB\mathrm{ctg}\beta}{d_1(A+B)}.$$

The equation of the contour $z = \omega(\xi)$ is presented by Eq. (42):

$$\omega(\xi) = \frac{2W(\xi)}{K}, \qquad -1 < \xi < 1,$$

where $W(\xi)$ is defined as Eq. (41):

$$W(\xi) = \frac{\xi X(\xi)}{\pi i}\left(\frac{K}{2}d_1\cos\alpha_2 e^{\alpha_2 i}\int_{a_2}^{a_3}\frac{d\sigma}{X(\sigma)\sigma(\sigma-\xi)} + \left(P\cos\beta\sin\beta + \frac{K}{2}d_1\cos\alpha_2\right)e^{\alpha_2 i}\int_{\bar{a}_3}^{\bar{a}_2}\frac{d\sigma}{X(\sigma)\sigma(\sigma-\xi)}\right), \qquad -1 < \xi < 1,$$

for each pair of parameters (θ_2, K), $(a_2 = e^{i\theta_2})$ and the function $X(\zeta)$ are presented by Eq. (35) as shown in Box I, where $\alpha_1 = -\pi/2$, $\alpha_2 = \pi/2 - \beta$, and $\alpha_3 = \pi$.

Thus, the part A_4A_1 of the required full-strength contour is constructed by $z = \omega(\xi)$. Since the problem is axially symmetric, the other parts of this graphic are obtained by symmetric mapping of its axis with respect to coordinate axes Ox and Oy.

Here, as an illustration, some graphics of full-strength contours are presented for the following parameters (Figure 2).

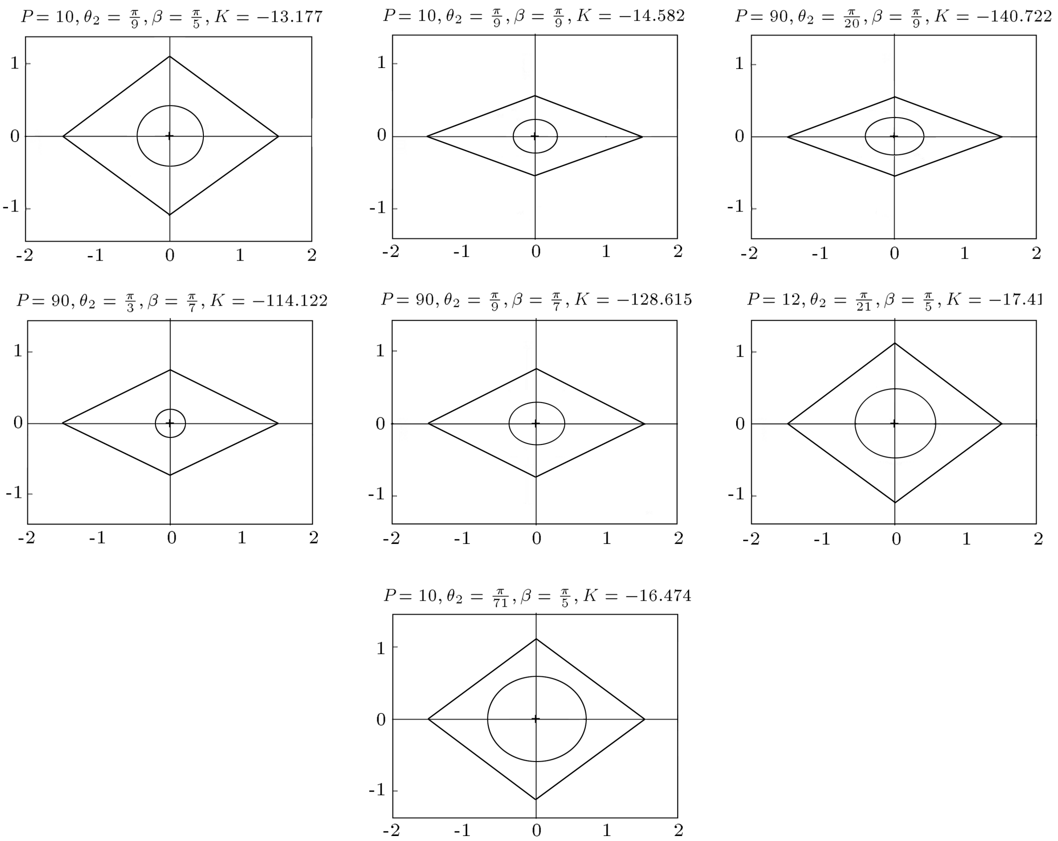

Figure 2. Graphics of full-strength contours for different parameters.

For illustration, we present some numerical calculations and plot for the concrete case in Figure 3, where:

$$x=-1.5,-1.4,\cdots,1.5,$$

$$g_0(x)=\begin{cases} f_2(x) & \text{if } \quad -1.5<x\leq 0 \\ f_1(x) & \text{if } \quad 0\leq x<1.5 \\ 0 & \text{otherwise} \end{cases}$$

$$g_1(x)=\begin{cases} f_4(x) & \text{if } \quad -1.5<x\leq 0 \\ f_5(x) & \text{if } \quad 0\leq x<1.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_1(\xi)=\frac{2W_1(\xi)}{K},$$

$$\xi_4=-0.9,-0.8,\cdots,0.9,$$

$$W(\xi_4)=\omega_1(\xi_4),$$

$$h(\xi_4)=\text{Re}(W(\xi_4))-v.\text{Im}(W(\xi_4)),$$

$$u(\xi_4)=-\omega_1(\xi_4),$$

$$g(\xi_4)=-\text{Re}(W(\xi_4))+v.\text{Im}(W(\xi_4)).$$

Numerical results can be found in Table 1.

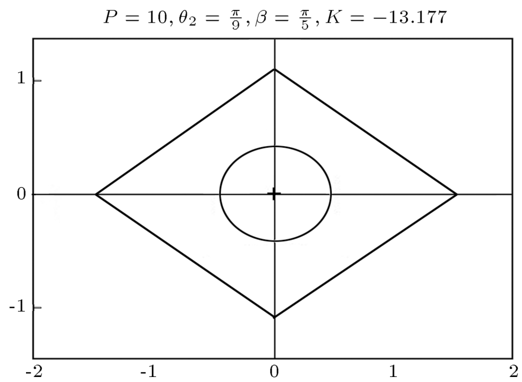

Figure 3. An example for numerical calculations.

4. Conclusion

The shape of the contour of the required hole and the stress state of the given body are determined, provided that the tangential normal stress, σ_s, arising at contour of the required hole, takes a constant value. Full-strength contours are found by means of complex analyses. The considered problem with partially unknown boundaries is reduced to the known boundary value problem of the theory of analytic functions by means of the developed method. The solutions are presented in quadratures and full-strength contours are constructed.

The plates weakened by a hole with full-strength

Table 1. An example for numerical calculations.

$u(\xi_4)$	$\omega_1(\xi_4)$	$h(\xi_4)$	$g(\xi_4)$
$-0.011-0.416i$	$0.011+0.416i$	$0.011-0.416i$	$-0.011+0.416i$
$-0.024-0.416i$	$0.024+0.416i$	$0.024-0.416i$	$-0.024+0.416i$
$-0.038-0.415i$	$0.038+0.415i$	$0.038-0.415i$	$-0.038+0.415i$
$-0.054-0.413i$	$0.054+0.413i$	$0.054-0.413i$	$-0.054+0.413i$
$-0.071-0.41i$	$0.071+0.41i$	$0.071-0.41i$	$-0.071+0.41i$
$-0.09-0.407i$	$0.09+0.407i$	$0.09-0.407i$	$-0.09+0.407i$
$-0.111-0.402i$	$0.111+0.402i$	$0.111-0.402i$	$-0.111+0.402i$
$-0.134-0.395i$	$0.134+0.395i$	$0.134-0.395i$	$-0.134+0.395i$
$-0.158-0.387i$	$0.158+0.387i$	$0.158-0.387i$	$-0.158+0.387i$
$-0.185-0.376i$	$0.185+0.376i$	$0.185-0.376i$	$-0.185+0.376i$
$-0.214-0.362i$	$0.214+0.362i$	$0.214-0.362i$	$-0.214+0.362i$
$-0.244-0.345i$	$0.244+0.345i$	$0.244-0.345i$	$-0.244+0.345i$
$-0.276-0.324i$	$0.276+0.324i$	$0.276-0.324i$	$-0.276+0.324i$
$-0.309-0.298i$	$0.309+0.298i$	$0.309-0.298i$	$-0.309+0.298i$
$-0.343-0.266i$	$0.343+0.266i$	$0.343-0.266i$	$-0.343+0.266i$
$-0.377-0.228i$	$0.377+0.228i$	$0.377-0.228i$	$-0.377+0.228i$

contours have the highest strength and the least weight (in comparison with the other holes). It is proved in [2] that the weight of a plate weakened by hole with full-strength contour is less than 40% of that of a plate weakened by circular hole with the same strength.

Hence, finding an optimal shape is of great practical importance and the investigation of problems of plane elasticity with a partially unknown boundary is topical and of great practical and theoretical values.

References

1. Banichuk, N.V. "Optimality conditions in the problem of seeking the hole shapes in elastic bodies", *Journal of Applied Mathematics and Mechanics*, **41**(5), pp. 946-951 (1977).
2. Cherepanov, G.P. "Inverse problems of the plane theory of elasticity", *J. Appl. Math. Mech.*, **38**(6), pp. 915-931 (1975).
3. Vigdergauz, S.B. "On a case of the inverse problem of the two-dimensional theory of elasticity", *J. Appl. Math. Mech.*, **41**(5), pp. 927-933 (1979).
4. Bantsuri, R.D. "Some inverse problems of plane elasticity and of bending of thin plates", In *Continuum Mechanics and Related Problems of Analysis*, pp. 100-107, Metsniereba, Tbilisi (1991).
5. Bantsuri, R. "On one mixed problem of the plane theory of elasticity with a partially unknown boundary", *Proc. A. Razmadze Math. Inst.*, **140**, pp. 9-16 (2006).
6. Bantsuri, R. "Solution of the mixed problem of plate bending for a multi-connected domain with partially unknown boundary in the presence of cyclic symmetry", *Proc. A. Razmadze Math. Inst.*, **145**, pp. 9-22 (2007).
7. Bantsuri, R. and Mzhavanadze, Sh. "The mixed problem of the theory of elasticity for a rectangle weakened by unknown equiv-string holes", *Proc. A. Razmadze Math. Inst.*, **145**, pp. 23-33 (2007).
8. Odishelidze, N. "Solution of the mixed problem of the plane theory of elasticity for a multiply connected domain with partially unknown boundary in the presence of axial symmetry", *Proceedings of A. Razmadze Mathematical Institute*, **146**, pp. 97-112 (2008).
9. Odishelidze, N. and Criado-Aldeanueva, F. "A mixed problem of plane elasticity for a domain with partially unknown boundary", *Acta Mech.*, **199**, pp. 227-240 (2008).
10. Odishelidze, N. and Criado-Aldeanueva, F. "A mixed problem of plate bending for a doubly connected domains with partially unknown boundary in the presence of cycle symmetry", *Science China Physics, Mechanics and Astronomy*, **53**(10), pp. 1884-1894 (2010).
11. Odishelidze, N.T. "Solution of the mixed problem of the plane theory of elasticity for a multiply connected domain with partially unknown boundary in the presence of axial symmetry", *Proc. A. Razmadze Math. Inst.*, **146**, pp. 97-112 (2008).
12. Muskhelishvili, N.I., *Some Basic Problems of the Mathematical Theory of Elasticity*, Noordhoff International Publishing, Leiden, reprint of the second English edition (1977).
13. Keldysh, M.V. and Sedov, L.D. "The effective solution of some boundary problems for harmonic-functions", *Dokl. Akad. Nauk SSSR*, **16**(1), pp. 7-10 (1937) (Russian).
14. Gakhov, F.D., *Boundary Value Problems*, Dover Pub-

lications, Inc., New York (1990). Translated from the Russian; Reprint of the 1966 translation.

15. Muskhelishvili, N.I., *Singular Integral Equations*, Dover Publications, Inc., New York (1992).

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