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# Hydro-magnetic mixed convection in a lid-driven cavity with partially thermally active walls

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**KEYWORDS** Mixed convection; Magnetic field; Lid-driven cavity; Partially active walls. **Abstract.** A numerical analysis has been made on mixed convection in a lid-driven square cavity when both vertical sidewalls are partially heated and cooled in the presence of uniform magnetic field. Three parallel locations and two dissimilar locations for heating and cooling on left and right sidewalls are considered. The top and bottom walls with the remaining locations free from heating and cooling on both vertical sidewalls are considered to be adiabatic. The governing equations are solved by the finite volume method. The resulting effects on combined convection flow and heat transfer for different heating and cooling locations are exhibited graphically for the variations of the Richardson and Hartmann numbers. The average heat transfer rate enhances in the similar locations than dissimilar locations

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## 1. Introduction

Mixed convection is a result of interaction between the forced convection, due to the moving wall of the cavity, and free convection, due to the temperature gradients within the cavity. Practical applications of mixed convection in lid-driven cavities are found in chemical and industrial engineering, lubrication technologies, cooling of electronic devices, food processing, producing plane glasses. and nuclear reactors. Guo and Sharif [1] described the mixed convection in a rectangular enclosure with discrete heat flux from bottom wall and moving isothermal sidewalls. It is found that the heat-affected region becomes larger when the length of heat source is increased for symmetric and asymmetric locations of the heat source. Sharif [2] discussed the combined convection in a shallow two-dimensional rectangular cavity for various Richardson numbers. He concluded that increase of the average Nusselt number

with cavity inclination is low for Ri = 0.1 and much more rapid for Ri = 10. The effects of wide range of Reynolds, Prandtl, and Grashof numbers on mixed convection in a lid-driven cavity with uniform and non-uniform heating were numerically investigated by Shokouhmand et al. [3]. Bhuvaneswari et al. [4] made a numerical approach to study the mixed convection flow, heat, and mass transfer with Soret effect in a two-sided lid-driven square cavity by considering three different directions of the movement of walls. It is found that heat and mass transfer rates decrease as the Richardson number increases for all the cases.

The study on convective heat transfer in cavities with partially thermally active walls is significant in applications, like solar energy collection and cooling of electronic components, because the active walls may be subject to abrupt temperature non-uniformities due to shading or other effects. Kuhn and Oosthuizen [5] performed an analysis on a two-dimensional unsteady free convective flow in a partially heated rectangular cavity. They have observed that temporal changes in the heat transfer rate are slightly affected by the

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position of the heated element unless the heater is very close to a horizontal wall. Frederick and Quiroz [6] explored numerically the steady natural convection in a cubical enclosure with a cold vertical wall and a hot square sector on the opposite wall. They revealed that the value of the Nusselt number reduces for partially heated wall than that of fully heated wall. Erbay et al. [7] investigated the entropy generation due to the transient heat transfer and the buoyancy-driven flow in a partially heated square enclosure. They obtained that the most effective portion to enhance heat transfer is at the upper corner of the heated part of the sidewall.

Oztop [8] numerically analyzed the combined convection in a partially heated porous lid-driven enclosure. On changing the locations of a heater of finite length, it is shown that the highest heat transfer is obtained when the heater is located on the left wall and the heat transfer decreases on increasing the Richardson number. Oztop and Abu-Nada [9] numerically studied free convection in a partially heated rectangular enclosure filled with nanofluids. They found that heater location influences flow pattern and temperature distributions. The heat transfer rate enhances upon the increase of the heater height. The effect of the heater location on heat transfer in an enclosure was numerically examined by Delavar et al. [10]. They found that the maximum heat transfer is found when the heater is located near the cold wall.

Sivakumar et al. [11] carried out a numerical study on mixed convection in a lid-driven cavity with different lengths of the heating portion and different locations of it. They have found that the heat transfer rate enhances when the location of heating portion is in the middle or the top on the left wall of the cavity. Sivasankaran et al. [12] studied the effect of discrete heating on free convection in a rectangular porous enclosure containing a heat-generating substance. They concluded that the heat transfer rates are high at both heaters for smaller heater length ratio, and the effect of heater length ratio is not significant for higher aspect ratio. Kahveci and Ogut [13] observed from the study on mixed convection of nanofluids in a liddriven enclosure that heat transfer increases upon the decrease of the Richardson number and the heater length. Sivakumar et al. [14] numerically examined the effect of partial heating/cooling on mixed convection in a lid-driven cavity with internal heat generation or absorption. They found that heat transfer is enhanced at Top-Top heating and cooling regions in the forced convection-dominated regime.

The study involving the dynamics of electrically conducting fluids in the presence of electromagnetic field is referred to as Magneto-hydrodynamics (MHD). Magneto-convection occurs in many situations and has received a great attention in geophysics, astrophysics, and aerodynamics. In particular, magneto-convection in cavities is involved in the applications of gas cooled reactors, solar technologies, and material manufacturing technology. The study of convection flow connected with MHD becomes very useful in industries due to its wide variety of applications in engineering, such as electromagnetic casting, liquid-metal cooling of nuclear reactors, and plasma confinement. Piazza and Ciofalo [15] studied the buoyancy-driven magnethydrodynamic flow in a liquid-metal filled cubic enclosure. They concluded that increase in Hartmann number results in the suppression of convective motions. Chamkha [16] examined the mixed convection in a square cavity in the presence of the magnetic field and an internal heat generation or absorption. The average Nusselt number decreases when the strength of a magnetic field is increased. Jalil and Tae'y [17] numerically investigated the natural convection of molten sodium in a square enclosure. Their result showed that the Nusselt number decreases upon the increase of the Hartmann number. Kang and Hyun [18] carried out a numerical analysis on buoyant convection of air in an enclosure under constant gravity and time-periodic magnetizing forces. Suppression in convective activities and periodic variations in temperature were observed in the system since the magnetizing force exists.

Sivasankaran and Ho [19] performed a numerical approach to analyze the effects of temperaturedependent properties of water near its density maximum in the existence of uniform magnetic field on convective flow and heat transfer. They concluded that the heat transfer rate increases according to the increase in Ravleigh number, and it decreases when Hartmann number is increased. Sivasankaran et al. [20] numerically examined the effects of sinusoidal boundary condition on mixed convection in a square cavity in the presence of a magnetic field. Thev revealed that increasing the Hartmann number results in the decrease of the total heat transfer rate in natural convection regime, whereas it does not affect the heat transfer in forced convection regime. Sivasankaran and Bhuvaneswari [21] investigated the effect of thermally active zones and direction of the external magnetic field on hydro-magnetic convection in an enclosure. Malleswaran et al. [22] investigated the effects of various lengths and locations of heater on magnetoconvection in a square lid-driven cavity. They observed that the heat transfer enhances at the center location of the heater along the left wall. Kefayati [23] explored from the study on magneto-convection in an inclined square cavity that the heat transfer rate decreases with increase of the Hartmann number for different Rayleigh numbers and inclination of the cavity. Sheikholeslami and Ganji [24] numerically studied the hydrodynamic flow in a permeable channel in the presence of magnetic field.

Mixed convection has been discussed broadly in the literature with a greater importance since it has the most powerful practical applications. The problems dealing with the existence or non-existence of magnetic field have attracted significant attention in many available works due to its widespread applications in science and engineering. The works involving the partially heated sidewalls extend many of its applications in the areas such as oil extraction, cooling of electronic devices, and heat transfer improvement in heat exchanger devices. All kinds of the above works cited previously have their own significance in the literature due to their applications. Hence, any numerical approach involving mixed convection, magnetic field, and partially thermal active walls would be a special kind of interest to study. In this paper, such a numerical investigation is carried out to obtain the effects on the mixed convection in a lid-driven cavity of partially thermally active walls in the presence of magnetic field.

#### 2. Mathematical formulation

Figure 1(a) represents the schematic diagram of a twodimensional square cavity of height L. The origin of the Cartesian coordinate system is taken at the lower left corner of the cavity. The flow is assumed to be twodimensional, unsteady, laminar, and incompressible. The velocity components are u along x-direction and v along y-direction, respectively. The lid of the cavity moves in its own plane with constant speed  $U_0$ . The heating surface on left sidewall and the cold surface on right sidewall are of equal area kept at two different temperatures, respectively,  $\theta_h$  and  $\theta_c$ , such that  $\theta_h >$  $\theta_c$ . The (constant) length of the heating/cooling region is taken as L/2. The heating region on left sidewall and the cooling region on right sidewall, both kept at top of the cavity, are referred as a Top-Top (parallel) location (Figure 1(b)). When both of the heating and cooling regions are located in middle of their respective walls, the parallel location is known as Middle-Middle.



Figure 1(a). Schematic diagram of physical configuration and coordinate system.



Figure 1(b). Different thermally active locations.

Similarly, Bottom-Bottom is a parallel location when both the regions are at bottom of the cavity. Furthermore, two dissimilar locations under investigation are Bottom-Top and Top-Bottom. Bottom-Top occurs by placing the heating and cooling regions at bottom and top on their respective sidewalls of the cavity, respectively; the locations are reversed for the Top-Bottom location. All remaining surfaces including top and bottom walls are adiabatic. The gravity acts in the downward direction. The cavity is filled with an electrically conducting fluid of low Prandtl number like liquid metal. A uniform magnetic field is applied in the horizontal direction with a constant magnitude  $B_0$ . The electric current J and the electromagnetic force Fare defined by  $J = \sigma_e(V \times B)$  and  $F = \sigma_e(V \times B) \times B$ , respectively. The induced magnetic field due to the motion of the electrically conducting fluid is very small compared to the applied magnetic field. Further, the viscous dissipation and Joule heating are assumed to be negligible. The governing equations of motion for incompressible electrically conducting fluid can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),$$
(2)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) 
+ g\beta \left( \theta - \theta_c \right) - \frac{\sigma_e B_0^2 v}{\rho_0},$$
(3)

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{k}{\rho_0 c_p} \left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right). \tag{4}$$

The appropriate initial and boundary conditions for the three parallel locations and the two dissimilar locations are written as follows: For t = 0:

$$u = v = 0, \qquad \theta = 0, \qquad 0 \le (x, y) \le L.$$

For t > 0:

$$u = v = 0, \qquad \frac{\partial \theta}{\partial y} = 0, \qquad y = 0,$$
  

$$u = U_0; v = 0, \qquad \frac{\partial \theta}{\partial y} = 0, \qquad y = L,$$
  

$$u = v = 0, \qquad \theta = \theta_c \quad \text{cooling region},$$
  

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{elsewhere} \quad x = L,$$
  

$$u = v = 0 \qquad \theta = \theta_h \quad \text{heating region},$$
  

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{elsewhere} \qquad x = 0.$$
(5)

To write Eqs. (1) to (5) in the non-dimensional form, the following dimensionless variables are used:

$$X = \frac{x}{L}, \qquad Y = \frac{y}{L}, \qquad U = \frac{u}{U_0},$$
$$V = \frac{v}{U_0}, \qquad T = \frac{\theta - \theta_c}{\theta_h - \theta_c}, \qquad \tau = \frac{tU_0}{L},$$
$$P = \frac{p}{\rho U_o^2}, \qquad \zeta = \omega L/U_0, \qquad \Psi = \psi/LU_0.$$

The dimensionless form of the governing equations in vorticity-stream function formulation is as follows:

$$\frac{\partial \zeta}{\partial \tau} + U \frac{\partial \zeta}{\partial X} + V \frac{\partial \zeta}{\partial Y} = \frac{1}{\text{Re}} \left( \frac{\partial^2 \zeta}{\partial X^2} + \frac{\partial^2 \zeta}{\partial Y^2} \right) + \text{Ri} \frac{\partial T}{\partial X} - \frac{\text{Ha}^2}{\text{Re}} \frac{\partial V}{\partial X}, \tag{6}$$

$$\nabla^2 \Psi = -\zeta,\tag{7}$$

$$\frac{\partial T}{\partial \tau} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{\Pr \operatorname{Re}} \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right), \quad (8)$$

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X} \quad \text{and} \quad \zeta = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}.$$
 (9)

The dimensionless parameters in the Eqs. (6) and (8) are defined as follows:  $\Pr = \nu/\alpha$ , the Prandtl number;  $\operatorname{Gr} = g\beta\Delta TL^3/\nu^2$ , the Grashof number;  $\operatorname{Ha} = B_0L\sqrt{\sigma_e/\mu}$ , the Hartmann number;  $\operatorname{Re} = U_0L/\nu$ , the Reynolds number; and  $\operatorname{Ri} = \operatorname{Gr/Re}^2$ , the Richardson number which describes the relative strengths of the buoyancy convection and forced convection.

The initial and boundary conditions in the nondimensional form for the considered problem can be written as follows:

For 
$$\tau = 0$$
:  
 $U = V = 0, \quad T = 0, \quad 0 \le (X, Y) \le 1,$   
For  $\tau > 0$ :  
 $U = V = 0, \quad \frac{\partial T}{\partial Y} = 0, \quad Y = 0,$   
 $U = 1; V = 0, \quad \frac{\partial T}{\partial Y} = 0, \quad Y = 1,$   
 $U = V = 0, \quad T = 0, \quad X = 1,$   
 $U = V = 0 \quad T = 0 \quad \text{cooling region},$   
 $\frac{\partial T}{\partial X} = 0 \quad \text{elsewhere} \quad X = L,$   
 $U = V = 0, \quad T = 1, \quad \text{heating region},$   
 $\frac{\partial T}{\partial X} = 0 \quad \text{elsewhere} \quad X = 0.$  (10)

The Nusselt number is a ratio of convective heat transfer and conductive heat transfer across the cavity, and it provides the relative strengths between them. The local Nusselt number for the considered problem along the heater is obtained from  $\operatorname{Nu} = -(\frac{\partial T}{\partial X})_{X=0}$  and the average Nusselt number for overall heat transfer is calculated as  $\overline{\operatorname{Nu}} = \frac{1}{L_h} \int_0^{L_h} \operatorname{Nu} dY$ , where  $L_H$  is the length of the heating location.

## 3. Numerical method and code validation

The discretization of the dimensionless equations (Eqs. (6)-(9)) is performed by the finite volume method. The approximations of the convection and the diffusion terms have been obtained by applying the upwind scheme and the central difference scheme, respectively. The implicit scheme is used for time steps. By expressing the stream function using Taylor series expansion near the walls, the second order expression for  $\zeta_w$  is obtained and can be written in the form of:

$$\zeta_w = \frac{\psi_{w+2} - 8\psi_{w+1}}{2(\Delta\eta^2)} + O(\Delta\eta^2),$$

where w denotes the boundary node and  $\Delta \eta$  is the spatial interval in the direction normal to the boundary. The velocity components at every grid point are evaluated using the central difference approximation. For the solution of the resulting sets of algebraic equations, the Gauss-Seidel point by point iteration technique is employed. The trapezoidal rule is employed for the calculations of average Nusselt number. The above process is repeated until the following convergence criterion (for local and global) for temperature, vorticity, and stream function is satisfied:

$$\left|\frac{\Phi_{ij}^{n+1} - \Phi_{ij}^{n}}{\Phi_{ij}^{n+1}}\right| < 10^{-6},$$

where n is any time level and  $\Phi$  represents the field variables  $(T, \zeta, \text{ or } \Psi)$ .

The numerical solutions presented in this paper are obtained by choosing a uniform grid system for computations. The grid sensitivity tests are executed to find the field variables grid-independency solutions. The test is carried out by experimenting with various uniform grid sizes ranging from  $41 \times 41$  to  $161 \times 161$ for Ri = 1, Pr = 0.054, and Ha = 0. The grid independence tests confirmed that a uniform grid with size  $81 \times 81$  yields the desired accuracy of results, and further increase in grid size does not affect the solution. Accuracy of the present results is established when the present code is validated with the available works in the literature on convective flow in lid-driven cavities [2,25]. A comparison table is presented to show the good agreement of the obtained results with the existing solution, see Table 1.

## 4. Results and discussion

A two-dimensional numerical investigation is performed in this paper to examine the effects on combined convection in a lid-driven square cavity while the two vertical bounding surfaces of the cavity are partly heated and cooled simultaneously in the presence of magnetic field. By varying heating regions on left sidewall and cooling regions on right sidewall, five different cases of thermally active locations, namely Top-Top, Middle-Middle, Bottom-Bottom, Bottom-Top, and Top-Bottom, are considered. All results are obtained for an electrically conducting fluid of low Prandtl number (Pr = 0.054). The fluid flow and heat transfer characteristics are determined for variations of thermally active locations, Richardson number ( $\operatorname{Ri} = \operatorname{Gr}/\operatorname{Re}^2$ ), and Hartmann number (Ha). The dominance of the buoyancy or the magnetic force on the flow field inside the cavity is described according to the magnitudes of Richardson and Hartmann

numbers. The Hartmann number (Ha) values are taken to be 0, 25, and 100. The computations have been performed extensively for different values of the Richardson number in the range of  $0.01 \leq \text{Ri} \leq 100$  and for a constant value of Grashof number,  $\text{Gr} = 10^4$ , by varying the Reynolds number.

#### 4.1. Effect of thermally active locations

Figure 2(a)-(e) depict typical temperature contours for different partially thermally active locations which are



Figure 2. Isotherms for different locations with Re = 100 and Ha = 25.

Table 1. Comparison of average Nusselt number with available results in the literature for lid-driven cavity.

		Re = 4	400		${ m Re}=1000$		
$\mathbf{Gr}$	$\mathbf{Present}$	Sharif	Iwatsu et al.	Present	Sharif	Iwatsu et al.	
	work	[2]	[25]	work	[2]	[25]	
$10^{2}$	4.08	4.05	3.84	6.48	6.55	6.33	
$10^4$	3.84	3.82	3.62	6.47	6.50	6.29	
$10^{6}$	1.10	1.17	1.22	1.66	1.81	1.77	

partly heated and cooled simultaneously on their respective walls for the variations of Richardson number such as Ri = 0.01, 1, and 100. Setting the Hartmann number constantly at Ha = 25, the effects of the thermally active locations have been discussed. The isotherms for the parallel location Top-Top is displayed in Figure 2(a). Since the thermally active regions are in the upper half of the cavity, the temperature distribution is strong in the upper half of the cavity for all Ri = 0.01, 1, and 100. Though the parallel location of heat source is changed to Middle-Middle, the effect on temperature distribution is identical to that of Top-Top. This is exposed in Figure 2(b). In both of these cases, it is obvious that the isotherms are displaced closer to the thermally active regions. Figure 2(c) shows the isotherms for the parallel location Bottom-Bottom. In this case, temperature distribution becomes weak in the upper half of the cavity and an analogue result has been observed for all the values Ri = 0.01, 1, and 100. The temperature distribution for the dissimilar locations Bottom-Top and Top-Bottom is illustrated in Figure 2(d)-(e), respectively. The isotherms are spread out in the entire cavity and an enhancement in heat transfer is observed for all values of the Richardson number. Since the heated particles raise parallel to the vertical heating region and move towards the cooling region due to the forces generated by buoyancy and shear, heat energy is transported throughout the cavity.

The fluid flow for different locations of heating and cooling and for various Richardson numbers is exemplified by Figure 3(a)-(e). For all the cases of parallel and dissimilar locations, the flow field presented in the form of streamlines consists of a one-cell pattern with clockwise rotation. When forced convection dominates, the center of the cell is viewed to be near the cold wall. The clustered streamlines near the top wall indicate steep velocity gradients near the top wall of the cavity. The rates of the fluid circulation are very close to all locations when shear force dominates. The formation of flow pattern in the forced convection regime occurs only due to the shear force induced by the velocity of the lid. There is no considerable change in flow pattern when changing the thermal (heating & cooling) locations in forced convection regime. The cell is stretched out further for all cases of thermally active locations in the mixed convection regime. However, the core region of the vortex is retained near the top wall and slightly moves to the left side. But, a significant change in the flow behavior can be noticed when buoyancy force dominates. The buoyancy force which raises the fluid particles heated near the heating regions acts parallel to the hot wall, moves horizontally, and falls down towards the cold location. Hence, for all parallel and dissimilar locations, the rotating eddy appears almost in the entire cavity at Ri = 100, and



Figure 3. Streamlines for different locations with Re = 100 and Ha = 25.

the core region of the circular eddy tends to move in accordance with the change in locations of thermally active walls. The deformation of the circulating single eddy in the upper side of the cavity is notable at Ri = 100 for Bottom-Bottom location. Thus, locations of thermal active walls play an important role for the changes in the flow behavior in the free convection mode.

Figure 4(a)-(c) depict the local Nusselt number variations for different thermal active locations and for the Richardson numbers, Ri = 0.01, 1, and 100, when Ha = 25. From Figure 4(a), it can be witnessed that Top-Top case yields a higher heat transfer rate in the leading edge of the heater when forced convection dominates. For all Richardson numbers, the local Nusselt number graph of the Middle-Middle location is seen as a U-shaped curve. It is observed that the local Nusselt number for the parallel location Middle-Middle shows higher heat transfer rate on both ends of



Figure 4. Local Nusselt number for different locations and Richardson numbers with Ha = 25.

the heater. Furthermore, heat transfer rate increases at the leading edge of the heating region for Top-Top and Top-Bottom locations, whereas it increases at the trailing edge of the heating portion for Bottom-Top and Bottom-Bottom cases. This is because a high heat transfer rate occurs at the trailing edge of the heating portion in bottom location. Further scrutinizing the curves, the curves for Bottom-Top and Bottom-Bottom cases are overlapped in the region  $0.4 \leq Y \leq 0.5$ . Also, the curves for Top-Top and Top-Bottom locations are overlapped in the region  $0.5 \leq Y \leq 0.6$ .

The overall heat transfer rate for all cases of



Figure 5. Average Nusselt number versus Richardson numbers for different locations and Hartmann number.

thermally active locations against Richardson numbers is exposed in Figure 5(a)-(c). When the magnetic field is not present, the average Nusselt number is boosted up well for the Middle-Middle location compared to the other parallel locations. Even if the Hartmann number is increased, the Middle-Middle location provides enhancement on heat transfer rate for all Richardson numbers except at Ri = 0.01. The heat transfer rate is slightly increased for the Top-Top location at Ri = 0.01 when the magnitude of the magnetic field is increased to 25 and 100. In general, heat transfer rate is high for the parallel locations Top-Top and Middle-Middle than the dissimilar locations. Furthermore, heat transfer rate is too low for Bottom-Bottom location when compared with all other locations up to Ri < 1. But, heat transfer rate of Bottom-Bottom location is higher than that of the dissimilar locations and still less than the other parallel locations when Ri  $\geq 1$ .

## 4.2. Effect of the Richardson number

The influence of the Richardson numbers, Ri = 0.01, 1, and 100, on temperature distributions and fluidflow are demonstrated in Figures 2(a)-(e) and 3(a)-(e), respectively, for a constant magnetic field parameter, Ha = 25. For Ri = 0.01, Figures 2(a)-2(e) show that the isotherms are concentrated near the thermal active zones and indicate that the convective mode of heat transfer prevails all over the cavity. When the forced convection dominates, heat transfer is weakened at the top and the bottom of the cavity for Bottom-Bottom location and Top-Top location, respectively. But, the heat energy is transported through moderate convection only. Meanwhile, the isotherms of different locations for the increase of Richardson numbers, such as Ri = 1 and 100, are almost vertical in the entire cavity. In particular, the mode of heat transfer in these regimes is almost through conduction. Energy transportation for the mixed convection regime and for the buoyancy-driven convection mode seems to be very similar. For the increase in Richardson number, temperature distribution is enhanced much for the parallel location Middle-Middle. This can be seen in Figure 2(b). On increasing Richardson numbers, it is observed that the dominant heat transfer mechanism is conduction for all the thermally active locations in the presence of magnetic field.

Figure 3(a)-(e) illustrate the fluid flow in the form of streamlines for different Richardson numbers and for different thermally active locations. For Ri = 0.01, the flow field consists of a primary clockwise circulating eddy that derives from the rising of the hot fluid adjacent to the heating region and its descent along the opposite cooled wall. Due to the domination of the inertia force generated by the moving lid, no considerable change in flow pattern can be noticed on changing the thermally active locations at Ri = 0.01. Forced convection and applied transverse magnetic field compress the flow pattern towards the horizontal top boundary and the flow in the lower part of the cavity is almost stagnant. When mixed convection occurs, the circulating eddy is further elongated. The core region of the eddy still exists near the top wall. When the buoyancy-driven convection dominates, the core region moves towards the centre of the cavity and the circulating eddy appears in the entire cavity, Ri = 100.

The effects on the local Nusselt number for different Richardson numbers and for the locations of thermally active regions are presented in Figure 4(a)-(c) for Ha = 25. In all the five parallel and dissimilar locations, heat transfer rate is decreased upon the increase of the Richardson numbers. When the Richardson number is increased such as Ri = 1 and Ri = 100, the variation on local Nusselt number is less for all locations. The average heat transfer rate for the variations of Richardson numbers, thermal active locations, and different Hartmann numbers is shown in Figure 5(a)-(c). In the absence of the magnetic field, the average heat transfer decreases even though the Richardson number is increased. However, the overall heat transfer is high for the parallel location Middle-Middle than any other locations under consideration regardless of the variations in Richardson number.

#### 4.3. Effect of the Hartmann number

The impacts for the active location Middle-Middle in the form of isotherms and streamlines for different Hartmann numbers are shown in Figure 6(a)-(c) when Ri = 0.01, 1, and 100. Figure 6(a) confirms that the temperature distribution is improved much through convection mode due to the shear force stimulated by the motion of top wall when Ri = 0.01 in the absence of magnetic field. On increasing the values of Hartmann number 25 and 100, the isotherms are more straightened out. In other words, the mechanism of heat transfer is changed to almost conduction mode. The thermal boundary layers formed when Ha = 0vanish due to the decrease in temperature gradients as a result of the increased values of Hartmann number. When the forced convection dominates, the flow is described by a clockwise rotating cell, which appears in the whole cavity if the magnetic field is not present. At Ha = 25, the core region moves towards the cold wall when mixed convection occurs. For a strong magnetic field, Ha = 100, the magnetic field resists the flow and the convection is totally suppressed inside the cavity. Since the shear force dominates due to the motion of the lid and a resistive force occurs due to the application of the magnetic field in the horizontal direction, the motion of the fluid flow is suppressed. As a result, the core region of the circulating eddy inside the cavity is near to the top of the cavity. Hence, it is observed that the convective motion is totally inhibited gradually with the increase of Hartmann numbers.

When the mixed and buoyancy-driven convection dominates, the effect on heat distribution is very similar to Ha = 0, 25, and 100. The isotherms present in Figure 6(b) and (c) do not show any considerable variation in heat transfer in the presence or the absence of the applied magnetic field. In the streamline pattern, the presence of a primary circular eddy rotating in the clockwise direction is seen and its core region appears



**Figure 6.** Isotherms and streamlines of Middle-Middle location for different Hartmann numbers and Richardson numbers.

in the centre of the cavity, see Figure 6(c). The intensities in the flow decrease owing to the increase in the magnetic field. This is expected since the presence of magnetic field usually retards the flow. When Ha = 100, the streamlines are elongated, and the core region of the eddy is very close to the top wall of the cavity. The flow becomes broadly stagnated. There exists an irregularity in the flow speed and the flow pattern. It is important to notice that the fluid flow does not depend on the thermally active location due to the significant effect of magnetic field on either

of the cases: the forced convection and the natural convection. These are illustrated in Figure 6(c).

The changes on overall heat transfer rate is shown in Figure 5(a)-(c) by plotting the average Nusselt numbers against Richardson numbers for different locations and Hartmann numbers. In Figure 5(a)-(c), the overall heat transfer rate is represented for the thermally active zones and for the increased values of magnetic field parameter. When the magnetic field is not present, the average Nusselt number is increased well for the Middle-Middle location only. Even if the Hartmann number is increased such as Ha = 25 and 100, average heat transfer rate remains high for the same Middle-Middle location except for Ri = 0.01. It is seen that a slight increase in heat transfer rate is observed for the Top-Top location at Ri = 0.01 when the magnitude of the magnetic field is increased to 25 and 100.

## 5. Conclusion

In this paper, a numerical investigation has been carried out to examine the effects on mixed convection in a lid-driven square cavity when two vertical bounding sidewalls are partly heated and cooled simultaneously in the presence of the magnetic field. The heating region is placed on left sidewall, whereas the cooling region is positioned on right sidewall. The five different partially heated and cooled regions under investigation are referred to as Top-Top, Middle-Middle, Bottom-Bottom, Bottom-Top, and Top-Bottom. The following are the conclusions made after numerically analyzing the problem considered.

- 1. The overall heat transfer decreases on increasing the Richardson number, and it does not depend on either the parallel or the dissimilar locations;
- 2. Among the five different thermally active locations, the total heat transfer is enhanced in the Middle-Middle location in the presence of magnetic field;
- 3. In the absence of magnetic field, overall heat transfer is high at Middle-Middle location. For the increase in Hartmann number, average heat transfer rate is higher at Middle-Middle location except for Ri = 0.01, and it is high at Top-Top location for Ri = 0.01;
- 4. Presence of the magnetic field results in the suppression of the convective flow and heat transfer;
- 5. For all Hartmann numbers, heat transfer rate is enhanced for the parallel locations, Top-Top and Middle-Middle, than the dissimilar locations. Generally, heat transfer rate is too low for Bottom-Bottom location when compared with all other locations up to Ri < 1. But, heat transfer rate of Bottom-Bottom location is higher than that of

the dissimilar locations and still less than the other parallel locations when  $Ri \ge 1$ .

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## Nomenclature

- $B_0$  Strength of the magnetic field (T)
- $c_p$  Specific heat (J/(kg.K))
- *F* Electromagnetic force
- g Gravitational acceleration (m/s<sup>2</sup>)
- Gr Grashof number,  $q\beta\Delta TH^3/\nu^2$
- Ha Hartmann number,  $B_0 L \sqrt{\sigma_e/\mu}$
- J Electric current
- k Thermal conductivity (W/(m.K))
- L Cavity size (m)
- $L_H$  Heater length (m)
- Nu Local Nusselt number
- Nu Average Nusselt number
- p Pressure (Pa)
- Pr Prandtl number,  $\nu/\alpha$
- Re Reynolds number,  $U_0 L/\nu$
- Ri Richardson number,  $Gr/Re^2$
- T Dimensionless temperature
- t Time (s)
- u, v Velocity components (m/s)
- U, V Dimensionless velocities
- $U_0$  Lid velocity (m/s)
- x, y Cartesian coordinates (m)
- X, Y Dimensionless coordinates, (x, y)/L

## Greek symbols

- $\begin{array}{ll} \alpha & \text{Thermal diffusivity } (\text{m}^2/\text{s}) \\ \beta & \text{Coefficient of thermal expansion } (\text{K}^{-1}) \end{array}$
- $\zeta$  Dimensionless vorticity
- $\theta$  Temperature (K)
- $\mu$  Dynamic viscosity (Pa s)
- $\nu$  Kinematic viscosity (m<sup>2</sup>/s)
- $\rho$  Density (kg/m<sup>3</sup>)
- $\sigma_e$  Electrical conductivity, S/m
- au Dimensionless time,  $tU_0/L$
- $\psi$  Stream function (m<sup>2</sup>/s)

- $\Psi$  Dimensionless stream function,  $\psi/LU_0$
- $\omega$  Vorticity (1/s)

## Subscripts

c Cold

h Hot/heater

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