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Development of a joint economic lot size model with stochastic demand within non-equal shipments

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Abstract. The majority of the studies on the integrated vendor-buyer inventory problem assume that the shipments are equal. In this paper, shipments are considered to be non-equal. Both demand and delivery times are also assumed to be stochastic. Moreover, unsatisfied demand can be backordered and lost, and a service level constraint is considered. The objective is to minimize both buyer and vendor costs at the same time. The problem is solved by an exact heuristic algorithm. To validate performance of the algorithm, the results are compared with those of the LINGO solver. Finally, a set of numerical problems are applied to compare the results in the integrated and independent forms.

1. Introduction

In traditional supply chain, one of the factors that cause increase in the costs is non-cooperation between vendor and buyer. In other words, according to Sajadieh et al. [1], in traditional supply chain, the inventory condition and order policies, as well as the production policies, are discussed separately. Therefore, the optimum solution in each level is quite different. To solve this problem, Goyal [2] presented the integrated vendor-buyer problem, namely JELS. The purpose of JELS is to increase the cooperation between different levels of supply chain in order to minimize costs in each level. One of the hypotheses in JELS is that after determining the amount of order, it can be divided into n unique production batches and each batch should be sent to the buyer, separately. Banerjee [3] developed the model. He assumed that order of the buyer and product of the vendor were the same. Hill [4] added an important assumption to JELS, in which the shipments were non-equal. Then, he proved that the cost, in comparison to the former model, was minimized. Other research can be categorized in four different types.

The first type includes the research that has considered the quality of products. Huang [5] presented the model in which equal shipments and failure shipments were considered. He also presumed that vendor should pay penalty to buyer for every failure in products. Wu et al. [6] studied a single vendor which was delivering one product to a single buyer with equal shipments. They considered that demand of the buyer was stochastic and each shipment contained a certain fraction of defective items. They also assumed that the buyer used sample inspection to find defective items. The second type of research has focused on minimizing both preparation and order costs. Affisco et al. [7] presented the problem with a vendor and several buyers and their objective was to minimize both preparation and order costs. They proved that by investing more, the objective became much more achievable. In the third group, however, authors have

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tried to minimize the delivery time. In this group, we can mention Chang et al. [8]. They illustrated that minimizing costs would cause decrease in the delivery time.

Finally, the last research type has added uncertainty to the model. In some papers, demands and in others, delivery times have been considered to be stochastic. Delivery time with exponential distribution function was considered by Bahri and Tarokh [9] and Sajadieh et al. [1]. Moreover, Sajadieh and Joker [10] considered delivery time to be stochastic with uniform distribution function. Hoque [11] assumed normal distribution function for delivery times. He also assumed equal and non-equal shipments. To solve the problem, he presented a new algorithm. Ouyang et al. [12] presented a model, in which demands were considered to be stochastic parameters with normal distribution function, as well as a model with empirical demand. Furthermore, backorder deficit was considered. Ben-Daya and Hariga [13] supposed that delivery time was a stochastic parameter with normal distribution function and there was a linear relation between delivery time and safety stock. The shipments were presumed to be equal and a heuristic algorithm was proposed to solve the problem. Seliaman and Ahmad [14] considered a three-level supply chain, including suppliers, manufactures, and retailers, in which demands of the retailers were assumed to be stochastic. Taleizadeh et al. [15] solved the integrated inventory model with several products. Moreover, demands were considered to be normal and delivery time depended on safety stock. Furthermore, the service-level constraint was assumed. Eventually, a meta-heuristic algorithm was applied to the problem. Glock [16] presumed demand as a stochastic parameter with normal distribution function. He proved that by decreasing setup time, shipment, safety stock, and delivery time would be reduced. Kim and Glock [17] supposed a three-level supply chain with Equal and non-equal shipments. They also assigned a penalty for long delivery time. Shahpouri et al. [18] considered JELS with normal demand and service level constraint. They entered the policy into their model to reduce both order cost and delivery time. Abdelsalam and Elassal [19] also assumed a three-level supply chain with a supplier, a manufacturer, and several retailers. Demands were considered to be stochastic. In addition, they relaxed the assumptions of constant order and holdings costs.

The majority of studies in JELS have considered equal shipments; meanwhile, in this paper, these assumptions are preliminaries:

1. Non-equal shipments in a two-level supply chain with one vendor and one buyer are considered;
2. Demands are assumed to be stochastic parameters with normal distribution function;
3. Delivery times are assumed to be stochastic with empirical distribution function;
4. Both backorder and lost-sale are permitted;
5. The service level is also applied to buyers;
6. An exact algorithm is presented to solve the problem.

The rest of the paper is organized as follows. Section 2 presents definition of the problem. In Section 3, the model is described in detail. The proposed heuristic algorithm is developed in Section 4. Section 5 includes a problem solved by the presented heuristic algorithm. In addition, the sensitivity analysis regarding the problem parameters is performed. Finally, the paper is concluded in Section 6.

2. Problem definition

Imagine a supply chain with a buyer and a vendor. The demand is considered stochastic with normal distribution function. Also, delivery time is assumed as a non-deterministic parameter with empirical distribution function. The relation between members of the chain is as follows.

Once the inventory level reaches the reorder point, the buyer orders the products in size Q. Consequently, the buyer is in charge of costs and the vendor starts to produce the products at the rate of p. The buyer follows the non-delayed non-equal shipment policy to send the shipment. According to this policy, the vendor is able to send the product to the buyer during the production phase. As a result, the amount of shipment Q is divided into n separate shipments. This policy is called non-equal sized, because the amounts of the shipments increase by a constant rate. Hill [4] proved that the optimal proportion was between 1 and the ratio of production to demand. Moreover, the holding cost per each product is considered for both buyer and vendor. Regarding stochastic demands, the buyer may face deficit. Thereby, the buyer should consider the deficit cost. Hence, in this model, both types of deficit, namely backorder and lost-sale, are presumed.

In order to reduce the risk of deficiency, the vendor should consider the safety stock, in which the amount of the safety stock is considered as one of the decision variables. Finally, the target of the problem is to determine the reorder point, safety stock, the number of shipments, the incremental index of the size of shipments, and the amount of the first shipment sent to the buyer, regarding both demand and delivery time, are stochastic. Furthermore, both types of deficits as well as service level limitation are permitted.

3. Modeling

In this section, we define parameters, decision variables, costs to the seller and the buyer, the problem constraint, and the supply chain cost.

3.1. Buyer's and vendor's parameters

The parameters of the buyer and vendor are as follows:

- D : The annual mean demand
- μ_D : The demand mean
- σ_D : The demand variance
- A : The order cost for each shipment sent to the buyer
- A^t : The fixed transportation cost for each shipment sent to the buyer
- A^p : Preparation cost to the vendor
- P : The vendor's production rate
- h_v : The holding cost for unit of product at vendor's site
- h_b : The holding cost for unit of product at buyer's site
- I : The buyer's maximum inventory level
- $\bar{b}(r)$: The buyer's mean product deficit
- β : The percentage of the buyer's backordered demand
- B : The buyer's backordered demand mean
- S : The buyer's lost demand mean
- L : The delivery time for each shipment from vendor to buyer
- π : The demand unit for backordered cost
- $\hat{\pi}$: The demand unit for lost cost
- sl : The buyer's service level
- C_{A_b} : Annual mean ordering cost to the buyer
- C_{B_b} : Annual mean backordered cost the buyer
- C_{S_b} : Annual mean lost demand cost to the buyer
- C_{T_b} : The annual transportation cost
- C_{H_b} : Annual mean holding cost to the buyer
- C_{A_v} : Annual mean preparation cost to the vender
- C_{H_v} : Annual mean holding cost to the vender

3.2. Buyer's and vender's decision variables

The decision variables of the buyer and the vendor are defined as follows:

- r : The buyer's reorder point
- q : The size of the first transfer shipment from the vendor to the buyer in a batch
- SS : The buyer's safety stock
- n : The number of sent shipments from vendor to buyer
- λ : The increased percentage in the size of sent shipments

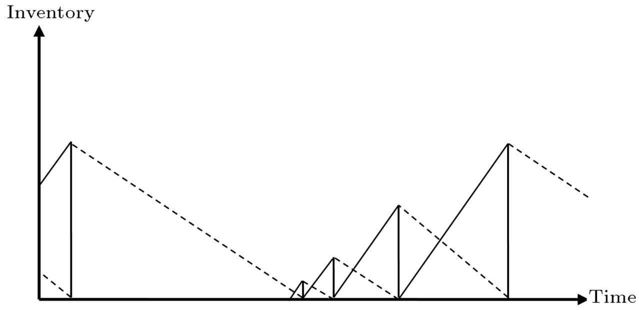

Figure 1. Sizes of the four non-equal shipments.

3.3. Costs formulation

To obtain the total cost of the supply chain, we calculate costs to the buyer and the vendor, separately. Then, the summation is considered.

3.3.1. The cost to the buyer

The cost to the buyer is equal to the sum of the holding cost, transportation cost, deficit cost, and ordering cost. In Figure 1, an example of 4 non-equal shipments is shown. As soon as the inventory level of the buyer reaches the reorder point, the vendor produces a shipment at the rate of p. Once each shipment is ready, it will be sent to the buyer. Moreover, the transportation cost for the buyer is considered. Q subject to non-equal shipments is defined through Eq. (1):

$$Q = \sum_{i=1}^{n} \lambda^{i-1} q = \frac{q(\lambda^n - 1)}{(\lambda - 1)}. \tag{1}$$

We can compute the ordering, holding, and transportation costs via Eqs. (2)-(4):

$$C_{A_b} = \frac{AD(\lambda - 1)}{q(\lambda^n - 1)}, \tag{2}$$

$$C_{T_b} = \frac{nA^t D(\lambda - 1)}{q(\lambda^n - 1)}, \tag{3}$$

$$C_{H_b} = h_b \left(\frac{q(\lambda^n + 1)}{2(\lambda + 1)} + SS \right). \tag{4}$$

The safety stock, considering both backorder and lost-sale, is defined through Eq. (5):

$$SS = \beta SS^{\text{Backorder}} + (1 - \beta) SS^{\text{Lost-sale}}. \tag{5}$$

According to Barzegar and Seifbarghy [20], $SS^{\text{Backorder}}$ and $SS^{\text{Lost-sale}}$ can be obtained via Eqs. (6) and (7):

$$SS^{\text{Backorder}} = \sum_{\forall L_i}\left(-\frac{\sqrt{L_i}\sigma_D}{\sqrt{2\pi}}e^{-\left(\frac{1}{2}\right)\left(\frac{\sqrt{L_i}\mu_D}{\sigma_D}\right)^2} - L_i\left(\frac{\mu_D}{2}\right) + \frac{r}{2}\right).P(L=L_i), \tag{6}$$

$$SS^{\text{Lost-Sale}} = \sum_{\forall L_i}\left(\int_0^{\infty}(r-D)f_L(D)d_D + \int_r^{+\infty}(D-r)f_L(D)d_D\right).P(L=L_i). \tag{7}$$

Therefore, the safety stock can be obtained by Eq. (8):

$$SS = \sum_{\forall L_i}\left[\left(-\frac{\sqrt{L_i}\sigma_D}{\sqrt{2\pi}}e^{-\left(\frac{1}{2}\right)\left(\frac{\sqrt{L_i}\mu_D}{\sigma_D}\right)^2} - L_i\left(\frac{\mu_D}{2}\right)\right).P(L=L_i)\right] + (1-\beta) . \sum_{\forall L_i}\left[\left(\sqrt{L_i}\sigma_D.G_u(z_{sl})\right).P(L=L_i)\right] + \frac{r}{2}. \tag{8}$$

The stock cost consists of backordered and lost demand costs. The mentioned costs can be calculated through Eqs. (9) and (10):

$$C_{B_b} = \frac{\pi\beta D\bar{b}(r)(\lambda-1)}{q(\lambda^n-1)}, \tag{9}$$

$$C_{S_b} = \frac{\hat{\pi}(1-\beta)D\bar{b}(r)(\lambda-1)}{q(\lambda^n-1)}. \tag{10}$$

When both demand and delivery times are stochastic, $\bar{b}(r)$ can be obtained via Eq. (11):

$$\bar{b}(r) = \sum_{\forall L_i}\left(\int_r^{+\infty}(D-r)\frac{1}{\sigma_{DL}\sqrt{2\pi}}e^{-\frac{(D-\mu_{DL})^2}{2\sigma_{DL}^2}}d_D\right).P(L=L_i) = \sum_{\forall L_i}(\sigma_{DL}.G_u(z_{sl})).P(L=L_i) = \sum_{\forall L_i}\left(\left(\sqrt{L_i}\sigma_D.G_u(z_{sl})\right).P(L=L_i)\right). \tag{11}$$

According to Relations (1)-(11), total cost of the buyer can be calculated through Eq. (12):

$$TC_{\text{Buyer}} = \frac{AD(\lambda-1)}{q(\lambda^n-1)} + h_b\left(\frac{q(\lambda^n+1)}{2(\lambda+1)} + SS\right) + \frac{(\pi\beta+\hat{\pi}(1-\beta))D\bar{b}(r)(\lambda-1)}{q(\lambda^n-1)} + \frac{nA^tD(\lambda-1)}{q(\lambda^n-1)}. \tag{12}$$

3.3.2. The costs to the vendor

The cost to the vender includes holding cost, transportation cost, and deficit and ordering cost. The total preparation cost is defined via Eq. (13):

$$C_{A_v} = \frac{A^pD(\lambda-1))}{q(\lambda^n-1)}. \tag{13}$$

In addition, holding cost to the vendor, which is proved by Hill [4], can be obtained through Eq. (14):

$$C_{H_v} = h_v\left[\frac{Dq}{P} + \frac{(P-D)q(\lambda^n-1)}{2P(\lambda-1)} - \frac{q(\lambda^n+1)}{2(\lambda+1)}\right]. \tag{14}$$

Therefore, total cost to the vendor is obtained by Eq. (15):

$$TC_{\text{Vendor}} = C_{A_v} + C_{H_v} = \frac{A^pD(\lambda-1))}{q(\lambda^n-1)} + h_v\Big[\frac{Dq}{P} + \frac{(P-D)q(\lambda^n-1)}{2P(\lambda-1)} - \frac{q(\lambda^n+1)}{2(\lambda+1)}\Big]. \tag{15}$$

3.3.3. The service-level constraint

The service level is equal to the possibility of not facing any deficit during delivery time. It is obtained by Eqs. (16) and (17):

$$p\ (\text{the demands during delivery time} \leq r) \geq \text{the service level}, \tag{16}$$

$$p(\mu_{DL} \leq r) \geq sl \Rightarrow p\left(z \leq \frac{r-\mu_D\mu_L}{\sigma_{DL}}\right) \geq sl \Rightarrow r \geq (\sigma_{DL}z_{sl}) + \mu_D\mu_L. \tag{17}$$

Since both demand and delivery time are supposed to be stochastic, the mean and the variance can be defined through Eqs. (18) and (19), respectively;

$$\mu_{DL} = \sum_{\forall L_i} L_i.\mu_D, \tag{18}$$

$$\sigma_{DL} = \sum_{\forall L_i}\sqrt{L_i}\sigma_D. \tag{19}$$

Through computing mean and variance, the service level can be obtained by Eq. (20):

$$r \geq \sum_{\forall L_i}\left(\sqrt{L_i}\sigma_D.z_{sl} + L_i.\mu_D\right).p(L=L_i). \tag{20}$$

Note that safety stock cannot be negative ($SS \geq 0$). Therefore, another constraint with respect to r is defined through Eq. (21):

$$r \geq -2\sum_{\forall L_i}\left[\left(-\frac{\sqrt{L_i}\sigma_D}{\sqrt{2\pi}}e^{-\left(\frac{1}{2}\right)\left(\frac{\sqrt{L_i}\mu_D}{\sigma_D}\right)^2} - L_i\left(\frac{\mu_D}{2}\right)\right).P(L=L_i)\right] - 2(1-\beta) .\sum_{\forall L_i}\left[\left(\sqrt{L_i}\sigma_D.G_u(z_{sl})\right).P(L=L_i)\right]. \quad (21)$$

3.3.4. The mathematical model

According to Relations (1)-(21), the mathematical model is given as follow:

$$\min TC = \frac{AD(\lambda-1)}{q(\lambda^n-1)} + h_b\left(\frac{q(\lambda^n+1)}{2(\lambda+1)} + SS\right) + \frac{(\pi\beta+\hat{\pi}(1-\beta))D\bar{b}(r)(\lambda-1)}{q(\lambda^n-1)} + \frac{nA^tD(\lambda-1)}{q(\lambda^n-1)} + \frac{A^pD(\lambda-1)}{q(\lambda^n-1)} + h_v\left[\frac{Dq}{P} + \frac{(P-D)q(\lambda^n-1)}{2P(\lambda-1)} - \frac{q(\lambda^n+1)}{2(\lambda+1)}\right]. \quad (22)$$

S.T.:

$$r \geq \sum_{\forall L_i}\left[\left(\sqrt{L_i}\sigma_D.z_{sl} + L_i.\mu_D\right).P(L=L_i)\right], \quad (23)$$

$$r \geq -2\sum_{\forall L_i}\left[\left(-\frac{\sqrt{L_i}\sigma_D}{\sqrt{2\pi}}e^{-\left(\frac{1}{2}\right)\left(\frac{\sqrt{L_i}\mu_D}{\sigma_D}\right)^2} - L_i\left(\frac{\mu_D}{2}\right)\right).P(L=L_i)\right] - 2(1-\beta) .\sum_{\forall L_i}\left[\left(\sqrt{L_i}\sigma_D.G_u(z_{sl})\right).P(L=L_i)\right], \quad (24)$$

$$SS = \sum_{\forall L_i}\left[\left(-\frac{\sqrt{L_i}\sigma_D}{\sqrt{2\pi}}e^{-\left(\frac{1}{2}\right)\left(\frac{\sqrt{L_i}\mu_D}{\sigma_D}\right)^2} - L_i\left(\frac{\mu_D}{2}\right)\right).P(L=L_i)\right] + (1-\beta).\sum_{\forall L_i}\left[\left(\sqrt{L_i}\sigma_D.G_u(z_{sl})\right).P(L=L_i)\right] + \frac{r}{2}, \quad (25)$$

$$\bar{b}(r) = \sum_{\forall L_i}\left(\left(\sqrt{L_i}\sigma_D.G_u(z_{sl})\right).P(L=L_i)\right), \quad (26)$$

$$q, r, SS, \lambda \geq 0, \quad (27)$$

$$n \geq 0, \text{Integer}. \quad (28)$$

4. The proposed method

For solving most of the JELS problems, researchers have applied different heuristic approaches; however, in this paper, after proving that the objective function and the constraints are convex, an exact heuristic algorithm is presented. To show that the objective function and the constraints are convex, we have to use the concept of Hessian, which is explained in the Appendix.

To compute the optimal q, we differentiate from the objective function with respect to q. to calculate q^*, we consider the model equal to zero and solve it. The relative relation is demonstrated in Eq. (29):

$$\frac{\partial(TC)}{\partial q} = 0,$$

$$\frac{-AD(\lambda-1)}{(\lambda^n-1)q^2} + \frac{h_b(\lambda^n+1)}{2(\lambda+1)} - \frac{(\pi\beta+\hat{\pi}(1-\beta))D\bar{b}(r)(\lambda-1)}{(\lambda^n-1)q^2} - \frac{nA^tD(\lambda-1)}{(\lambda^n-1)q^2} - \frac{A^pD(\lambda-1)}{(\lambda^n-1)q^2} + \frac{h_vD}{P} + \frac{h_v((P-D)(\lambda^n-1))}{2P(\lambda-1)} - \frac{h_v(\lambda^n+1)}{2(\lambda+1)} = 0. \quad (29)$$

For simplification, the value of q^* is defined through Eq. (30), as shown in Box I.

4.1. The solution method

Here, the overall structure of the proposed algorithm is presented:

1. Compute $\bar{b}(r)$ through Eq. (26);
2. Obtain the values of r_1 and r_2 through Eqs. (23) and (24), respectively;
3. The value of SS is given by Eq. (25);
4. Put $Z = 0$ and imagine that TC^{OPT} and Z^{OPT} are great numbers;
5. For $\lambda = [1, P/D]$, do steps 6 to 21;
6. Put $n = 1$;

$$q^* = \sqrt{\frac{D(\lambda-1)\left(A+nA^t+\left(\left(\pi\beta+\hat{\pi}(1-\beta)\right)\bar{b}(r)\right)+A^p\right)}{(\lambda^n-1)\left(h_b\frac{(\lambda^n+1)}{2(\lambda+1)}+h_v\left(\frac{D}{P}+\frac{(P-D)(\lambda^n-1)}{2P(\lambda-1)}-\frac{(\lambda^n+1)}{2(\lambda+1)}\right)\right)}}. \tag{30}$$

Box I

7. If $\lambda > 1$, go to the next step, otherwise go to step 5;
8. Find q via Eq. (30). Then, for q, n, and λ, compute cost by Eq. (22) and put Z equal to the obtained value;
9. If $Z < Z^{\text{OPT}}$, go to the next step, otherwise go to Step 15;
10. If $n > 1$, go to the next step, otherwise go to Step 12;
11. If $Z < Z^{\text{OPT}}$, then $Z = Z^{\text{OPT}}$ and go to the next step;
12. Find q via Eq. (30). For q, n, and λ, cost is defined through Eq. (22); put Z equal to the obtained value and then put $Z = TC$;
13. If $TC < TC^{\text{OPT}}$, then put $TC^{\text{OPT}} = TC$, $\lambda^{\text{OPT}} = \lambda$, $q^{\text{OPT}} = q$ and $n^{\text{OPT}} = n$;
14. Put $n = n + 1$ and go to Step 9;
15. Put $n = 1$;
16. For $\lambda = [1, P/D]$, do steps 17 to 22;
17. If $\lambda > 1$, go to the next step, otherwise go to step 16;
18. Find q via Eq. (30). Then, for q, n, and λ, compute cost by Eq. (22) and put Z equal to the obtained value;
19. If $Z < Z^{\text{OPT}}$, then $Z = Z^{\text{OPT}}$, $\lambda^{\text{OPT}} = \lambda$, $n^{\text{OPT}} = n$ and $q^{\text{OPT}} = q$;
20. Put $n = n + 1$;
21. If $n \leq n^{\text{OPT}}$, go to step 16, otherwise go to step 5;
22. The obtained value is the optimal solution.

5. Numerical examples

In Tables 1 and 2, the data for a two-level supply chain are considered. The data for the numerical examples are taken from Ben-Daya and Hariga [13].

To understand whether the proposed method is better than former methods regarding the chain members or not, we compare the outcome with the policies using equal shipments and non-cooperation. As a result, we face three different problems (a), (b), and (c) that are categorized as follows:

(a) The independent chain performance;

(b) The integrated chain performance regarding equal shipments;

(c) The integrated chain performance regarding non-equal shipments.

The results are demonstrated in Table 3. To validate performance of the algorithm, the model is also solved with LINGO solver. The result shows that mode (c) is 43.1 and 4.7 percent better than modes (a) and (b), respectively. Consequently, the chain members become more attracted to use the cooperative policy as well as non-equal shipment policy, because it can reduce cost more than equal shipment policy.

Furthermore, the number of shipments in mode (b) is two, in which the amount of each shipment is equal to 360.67. Nevertheless, mode (c) has four shipments with $\lambda = 1.69$. Thus, the shipment sizes are 66.7, 112.73, 190.51, and 321.97, respectively. As mentioned in Table 3, the outcomes of both LINGO and the proposed algorithm are exactly similar. Hence, it is proved that the proposed algorithm is an exact method.

In order to analyze the effectiveness of parameters in the total cost, a sensitivity analysis on P, sl, and β is applied. In this sensitivity analysis, the parameters will increase and decrease up to 20%, 30%, and 50%, respectively. Moreover, the effectiveness of L with increasing delivery time up to 20%, 30%, 50%, 70%, 90%, and 100% is discussed. It should be noted that the increased value of L is an integer value which rounds up to $[L + s\% * L]$. The summery of the result is demonstrated in Figures 2 and 3, and Table 4. According to Figures 2 and 3, by increasing the values of P and L, the total cost will increase. However, for β, it is vice versa. Increasing β can reduce the total cost. According to Table 4, by increasing delivery time, variables including reorder point, safety

Table 1. Computational data of the example.

D	μ_D	σ_D	sl	β	π	$\hat{\pi}$	A	A_t	h_b	A_p	h_v	P
1000	40	5	0.5	0.5	100	110	50	25	5	400	4	6000

Table 2. Delivery time data.

	L_i					
	1	**2**	**3**	**4**	**5**	**6**
$P(L_i)$	0.1	0.25	0.35	0.15	0.1	0.05

Table 3. Computational results for variables value in various mode.

	Buyer				Vendor				Chain cost
	q	SS	r	TC_{Buyer}	λ	n	Q_v	TC_{Vendor}	
Independent	415.81	306.7	122	3612.55	—	—	415.81	1793.60	5406.15
Integrated with equal shipment	308.62	306.7	122	3045.84	—	2	617.25	1265.29	4311.13
Integrated with non-equal shipment	66.70	306.7	122	2834.29	1.69	4	691.92	1321.62	4155.91
Lingo solving	66.70	306.7	122	2834.29	1.69	4	691.92	1321.62	4155.91

Table 4. The results of the sensitivity analysis.

		Integrated with non-equal shipment						Integrated with equal shipment		Lingo solving	
		q	r	SS	n	λ	Chain cost	Chain cost	$PS1$	Chain cost	$PS2$
P	+20%	74.85	122	306.70	4	1.59	4156.28	4311.13	3.73%	4186.28	0.007%
	+30%	78.50	122	306.70	4	1.55	4197.58	4311.13	2.7%	4197.58	0%
	+50%	85.51	122	306.70	4	1.48	4215.20	4311.13	2.3%	4215.20	0%
	-20%	45.83	122	306.70	5	1.6	4106.78	4311.13	4.98%	4106.78	0%
	-30%	41.85	122	306.70	5	1.66	4068.57	4311.13	5.96%	4068.57	0%
	-50%	32.95	122	306.70	5	1.83	3938.95	4780.31	21.36%	3938.95	0%
sl	+20%	62.89	124.13	312.62	4	1.69	4035.46	4172.23	3.4%	4035.46	0%
	+30%	61.16	125.24	315.74	4	1.69	3982.98	4111.16	3.2%	3982.98	0%
	+50%	57.94	127.71	322.78	4	1.69	3891.94	4003.79	2.9%	3891.96	0%
	-20%	57.31	119.87	300.89	5	1.5	4297.09	4474.75	4.1%	4297.09	0%
	-30%	59.25	118.76	297.91	5	1.5	4376.69	4569.91	4.4%	4376.69	0%
	-50%	63.84	116.28	291.35	5	1.5	4567.80	4790.20	4.9%	4567.80	0%
β	+20%	66.58	122	306.36	4	1.69	4149.29	4303.91	3.73%	4149.29	0%
	+30%	66.51	122	306.19	4	1.69	4145.97	4300.30	3.72%	4145.97	0%
	+50%	66.39	122	306.85	4	1.69	4139.34	4293.06	3.71%	4139.34	0%
	-20%	66.83	122	306.04	4	1.69	4162.52	4318.34	3.74%	4162.53	0%
	-30%	66.89	122	307.21	4	1.69	4165.83	4321.94	3.75%	4165.83	0%
	-50%	67.02	122	307.55	4	1.69	4172.43	4329.13	3.76%	4172.43	0%
L	+20%	69.01	168	422.01	4	1.69	4824.66	4990.93	3.45%	4824.66	0%
	+30%	55.90	174	437.04	5	1.5	4908.95	5076.54	3.4%	4908.95	0%
	+50%	57.13	212	532.26	5	1.5	5445.16	5621.54	3.24%	5445.16	0%
	+70%	57.69	232	582.35	5	1.5	5723.20	5903.57	3.15%	5723.20	0%
	+90%	57.98	244	612.41	5	1.5	5887.46	6069.85	3.1%	5887.45	0%
	+100%	59.14	284	712.61	5	1.5	6444.83	6635.28	2.95%	6444.82	0%

stock, and number of shipments will be increased. In addition, column PS1 shows that if we use the non-equal shipment policy instead of equal shipment policy, the costs will be reduced. However, the column PS2 displays the cost difference between LINGO and the proposed algorithm.

6. Conclusions and future research

In this paper, we discussed an integrated vendor-buyer problem with respect to non-equal shipment. We also assumed that there was only one type of product. Unlike previous research, which considered only demand or delivery time as a stochastic parameter, in this paper, we considered both factors to be stochastic. Moreover, the service level was considered. Both backorder and lost-sale were also presumed. We formulated the problem as a mixed non-linear model. To find the optimal amount of order, reorder point, number of shipments, and safety stock, an exact heuristic algorithm was applied. In order to validate performance of the algorithm, a problem was

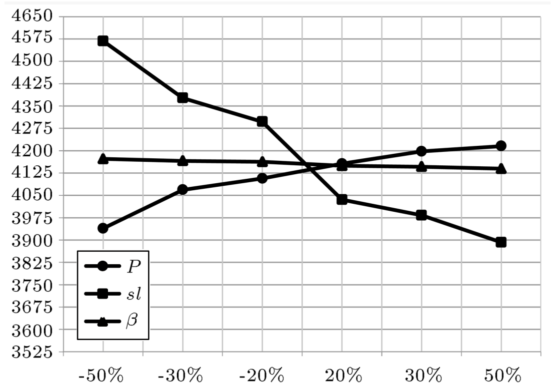

Figure 2. Effectiveness of P, sl, and β in total cost.

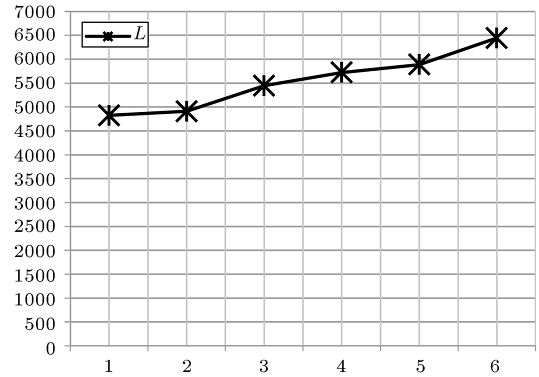

Figure 3. Effectiveness of L in total cost.

solved with LINGO and the results were compared. Since their results are the same, it is proved that the proposed algorithm is a precise method. Furthermore, by solving the example, the superiority of the proposed model, considering non-equal shipments, in comparison to equal shipments, is proved. The results obtained from the solved example illustrate that cooperation between chain members is more profitable than non-cooperative policy. Furthermore, by using the non-equal shipments policy, the cost will also be decreased.

Finally, the potential for further research can be determined with attention to stochastic prices. In addition, we can consider multiple vendors and buyers in the model. Different kinds of discount can also be added to the model.

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Appendix

The proof for convex of objective function and constraint

It is obvious that q, r, and SS are continuous variables and n is a discrete one. Hence, the problem is a mixed non-linear programming problem. To prove that the objective function for a specific ratio of n to r and q, Hessian matrix is applied via Eq. (A.1):

$$H=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}=\begin{bmatrix} \frac{\partial^2 TC}{\partial q^2} & \frac{\partial^2 TC}{\partial q \partial r} \\ \frac{\partial^2 TC}{\partial r \partial q} & \frac{\partial^2 TC}{\partial r^2} \end{bmatrix}. \quad (A.1)$$

After that Eq. (A.1) is simplified, Eq. (A.2) is obtained:

$$H=\begin{bmatrix} \frac{2AD(\lambda-1)}{(\lambda^n-1)q^3}+\frac{2(\pi\beta+\hat{\pi}(1-\beta))D\bar{b}(r)(\lambda-1)}{(\lambda^n-1)q^3} + \frac{2nA^tD(\lambda-1)}{(\lambda^n-1)q^3}+\frac{2A^pD(\lambda-1)}{(\lambda^n-1)q^3} & 0 \\ 0 & 0 \end{bmatrix}. \quad (A.2)$$

Determination of Hessian matrix is given through Eqs. (A.3) and (A.4):

$$\Delta_1=\frac{2AD(\lambda-1)}{(\lambda^n-1)q^3}+\frac{2(\pi\beta+\hat{\pi}(1-\beta))D\bar{b}(r)(\lambda-1)}{(\lambda^n-1)q^3} +\frac{2nA^tD(\lambda-1)}{(\lambda^n-1)q^3}+\frac{2A^pD(\lambda-1)}{(\lambda^n-1)q^3}. \quad (A.3)$$

Because all the parameters, including q, are positive, Δ_1 would be positive.

$$\Delta_2=\begin{vmatrix} \frac{2AD(\lambda-1)}{(\lambda^n-1)q^3}+\frac{2(\pi\beta+\hat{\pi}(1-\beta)D\bar{b}(r)(\lambda-1)}{(\lambda^n-1)q^3} + \frac{2nA^tD(\lambda-1)}{(\lambda^n-1)q^3}+\frac{2A^pD(\lambda-1)}{(\lambda^n-1)q^3} & 0 \\ 0 & 0 \end{vmatrix}=0. \quad (A.4)$$

As $\Delta_1 \geq 0$ and $\Delta_2 = 0$, the objective function is positive and convex. On the other hand, because all the constraints are linear and since it was proved that the linear constraints are convex, the model is proved to be convex.

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