Large-amplitude free vibration of magneto-electro-elastic curved panels

A. Shooshtari* and S. Razavi

Department of Mechanical Engineering, Bu-Ali Sina University, Hamedan, P.O. Box 65175-461, Iran.

Received 13 December 2014; received in revised form 20 October 2015; accepted 28 November 2015

KEYWORDS
Large-amplitude free vibration; Magneto-electro-elastic material; Curved panel; Donnell shell theory; Multiple timescales method.

Abstract. In this study, the large-amplitude free vibration of magneto-electro-elastic curved panels was investigated. The panel was considered to be simply supported on all edges and the magneto-electro-elastic body was subjected to the electric and magnetic fields along z direction. To obtain the governing equations of motion, the Donnell shell theory and the Gauss’s laws for electrostatics and magnetostatics were used. The first mode of vibration of these smart panels was studied in this paper. To this end, the nonlinear partial differential equations of motion were reduced to a nonlinear ordinary differential equation by introducing trial functions for displacements and rotations and then by applying the single-mode Galerkin method on the obtained equation. The resulting equation was solved by multiple timescales perturbation method. Some numerical examples were presented to validate the study and to investigate the effects of several parameters such as geometry of the panel and the magneto-electric boundary conditions on the vibration behavior of these smart panels.

* Corresponding author. Tel.: +98 8138272410
E-mail address: shooshta@bsu.ac.ir (A. Shooshtari)

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1. Introduction

Magneto-electro-elastic smart materials exhibit a coupling between mechanical, electric, and magnetic fields and are able to convert energy among these energy forms, which makes them suitable for energy harvesting, vibration control, etc.


Tsai and Wu [11,12] presented free vibration analysis of doubly curved functionally graded magneto-electro-elastic shells with open-circuit and closed-circuit surface conditions, respectively. Chen et al. [13] presented an analytical solution for the static deformation of a magneto-electro-elastic hollow sphere. Xin and Hu [14] presented a semi-analytical model based on the three-dimensional elasticity theory to study the free vibration of simply supported magneto-electroelastic plates. However, there are few studies dealing with the nonlinear vibration response of magneto-electro-elastic structures. Xue et al. [15] studied the large deflection of a rectangular magneto-electro-elastic thin plate for the first time based on the classical plate theory. Shadek et al. [16] used a meshless local Petrov-Galerkin method to study the large deflection of magneto-electro-elastic thick plates. Milazzo [17] derived a shear deformable model for the large deflection analysis of MEE laminated plates. Alaimo et al. [18] presented an original shear deformable finite element model for the analysis of large deflections.
of magneto-electro-elastic laminated plates. Rao et al. [19] proposed a finite-element model for large deflection static analysis of layered magneto-electro-elastic structures. Razavi and Shoshtari [20] studied nonlinear vibration of symmetrically laminated magneto-electro-elastic rectangular plates. They [21, 22] also investigated the effects of electric and magnetic potentials on the nonlinear vibration response of laminated magneto-electro-elastic plates and doubly-curved shells, respectively, with movable simply supported boundary condition. Nonlinear forced vibration of magneto-electro-elastic nanobeams has also been investigated [23, 24]. Kattimani and Ray [25, 26] studied the active control of large-amplitude vibrations of magneto-electro-elastic plates and doubly curved shell, respectively.

According to the published literature, there are not any studies about the large-amplitude free vibration of multiphase magneto-electro-elastic curved panels. Thus, this study deals with this topic to fill the gap. In this paper the large-amplitude free vibration of multiphase smart curved panel with immovable simply-supported boundary condition is investigated. The panel is considered to be made of transversely isotropic magneto-electro-elastic material. The magnetic and electric fields are applied along z direction. The Donnell shell theory without in-plane and rotary inertias along with Gauss’s laws for electrostatics and magnetostatics is used to model the panel. The first mode of vibration is studied here. To achieve this goal, after transforming the equations of motion to a nonlinear ordinary differential equation by single-mode Galerkin method, it is solved analytically and then a closed-form relation for nonlinear frequency is obtained. This model can be used to study the nonlinear and linear free vibrations of simply-supported single-layered curved panels with magneto-electro-elastic, piezoelectric, piezomagnetic, orthotropic, or isotropic material properties. Several numerical studies are presented to validate the study and to investigate the effects of several parameters on the behavior of these smart panels.

2. Theoretical formulations

For a curved panel with $R_x$ and $R_y$ being the radii of curvature (Figure 1), the strain-displacement relations are given as [27]:

\[
\varepsilon_x = \left( w_{0,x} + w_0/R_x + \frac{1}{2} w_{0,x}^2 \right) + z\theta_{x,x},
\]

\[
\varepsilon_y = \left( w_{0,y} + w_0/R_y + \frac{1}{2} w_{0,y}^2 \right) + z\theta_{y,y},
\]

\[
\gamma_{xy} = (w_{0,y} + w_{0,x} + w_{0,x} w_{0,y}) + z(\theta_{x,y} + \theta_{y,x}).
\]

Figure 1. Schematic of the studied curved panel.

\[
\gamma_{yz} = w_{0,y} + \theta_y, \quad \gamma_{xz} = w_{0,x} + \theta_x,
\]

\[
\begin{align*}
\sigma_x & = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{55} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \varepsilon_x \\
\sigma_y & = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \varepsilon_y \\
\sigma_xz & = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \phi_{x,z} \\
\sigma_yz & = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \phi_{y,z} \\
\sigma_{xy} & = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \psi_{x,y}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} & = \begin{bmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \phi_{x,z} \\ \phi_{y,z} \\ \psi_{x,y} \end{bmatrix}
\end{align*}
\]
\[
\begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & q_{15} & 0 & 0 & \varepsilon_x \\
0 & 0 & q_{24} & 0 & 0 & \gamma_y \\
q_{31} & q_{32} & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\gamma_y \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix}
- \begin{bmatrix}
d_{11} & 0 & 0 & 0 \\
0 & d_{22} & 0 & 0 \\
0 & 0 & d_{33} & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
\phi_z
\end{bmatrix},
\]

where \( \sigma_x, \sigma_y, \sigma_{xz}, \sigma_{xy} \) is stress vector; \( \{D_x, D_y, D_z\}^T \) and \( \{B_x, B_y, B_z\}^T \) are the electric displacement and magnetic flux density vectors, respectively; \( [C_{ij}], [\eta_{ij}], \) and \( [\mu_{ij}] \) are the elastic, dielectric, and magnetic permeability coefficient matrices, respectively; \( [E_{ij}], [\eta_{ij}], \) and \( [H_{ij}] \) are the piezoelectric, piezomagnetic, and magneto-electric coefficient matrices, respectively; and \( \phi \) and \( \psi \) are electric and magnetic potentials.

By neglecting in-plane and rotary inertia effects, the equations of motion of a curved panel can be expressed in the following form [27]:

\[
N_{x,x} + N_{x,y,y} = 0, 
\]

\[
N_{x,y,x} + N_{y,y} = 0, 
\]

\[
Q_{x,x} + Q_{y,y} + (N_x w_{0,x} + N_y w_{0,y}) \psi_x + (N_x w_{0,x} + N_y w_{0,y}) \psi_y = 0,
\]

\[
M_{x,x} + M_{x,y,y} = 0,
\]

\[
M_{x,y,x} + M_{y,y} = 0,
\]

where \( I_0 = \int_{-h/2}^{h/2} \rho dz \) is the mass moment of inertia, in which \( \rho \) is the density of the material of the panel. The in-plane force resultants (i.e., \( N_x, N_y, N_{xy} \)), the transverse force resultants (i.e., \( Q_x, Q_y \)) and the moment resultants (i.e., \( M_x, M_y, M_{xy} \)) are obtained by the following equations:

\[
\begin{bmatrix}
N_x & N_y & N_{xy}
\end{bmatrix}^T = \int_{-h/2}^{h/2} \begin{bmatrix}
\sigma_x & \sigma_y & \sigma_{xy}
\end{bmatrix}^T dz,
\]

\[
\begin{bmatrix}
M_x & M_y & M_{xy}
\end{bmatrix}^T = \int_{-h/2}^{h/2} \begin{bmatrix}
\sigma_x & \sigma_y & \sigma_{xy}
\end{bmatrix}^T \times dz,
\]

\[
\begin{bmatrix}
Q_x & Q_y
\end{bmatrix}^T = K \int_{-h/2}^{h/2} \begin{bmatrix}
\varepsilon_x & \varepsilon_y
\end{bmatrix}^T dz,
\]

where \( K \) is shear correction factor.

To obtain the resultants of Eq. (10), the gradients of the electric and magnetic potentials in Eq. (2) must be obtained in terms of \( z \). To this end, Gauss’s laws for electrostatics and magnetostatics, i.e.:

\[
D_{x,x} + D_{y,y} + D_{z,z} = 0,
\]

\[
B_{x,x} + B_{y,y} + B_{z,z} = 0,
\]

along with Eqs. (3) and (4) are considered from which one obtains:

\[
\phi = M_1 z^2 + \phi_0 z + \phi_1, \quad \psi = M_2 z^2 + \psi_0 z + \psi_1.
\]

In the above equation, \( M_1 \) and \( M_2 \) are obtained by:

\[
M_1 = \frac{1}{2} (a_1 w_{0,x} + a_2 w_{0,y} + a_3 \theta_{x,x} + a_4 \theta_{y,y}),
\]

\[
M_2 = \frac{1}{2} (a_5 w_{0,x} + a_6 w_{0,y} + a_7 \theta_{x,x} + a_8 \theta_{y,y}),
\]

where:

\[
\delta_1 = d_{x3} / (d_{x3}^2 - \mu_{x3} \eta_{x3}), \quad \delta_2 = d_{y3} / (d_{y3}^2 - \mu_{y3} \eta_{y3}), \quad \delta_3 = \eta_{x3} / (d_{x3}^2 - \mu_{x3} \eta_{x3}),
\]

\[
a_1 = \delta_1 q_{15} - \delta_2 e_{15}, \quad a_2 = \delta_1 q_{24} - \delta_2 e_{24}, \quad a_3 = \delta_1 (q_{15} + q_{31}) - \delta_2 (e_{15} + e_{31}), \quad a_4 = \delta_1 (q_{24} + q_{31}) - \delta_2 (e_{24} + e_{31}), \quad a_5 = \delta_1 e_{15} - \delta_3 q_{15}, \quad a_6 = \delta_1 e_{24} - \delta_3 q_{24}, \quad a_7 = \delta_1 (e_{15} + e_{31}) - \delta_3 (q_{15} + q_{31}), \quad a_8 = \delta_1 (e_{24} + e_{31}) - \delta_3 (q_{24} + q_{31}).
\]

On the other hand, the integration constants, i.e. \( \phi_0, \phi_1, \psi_0, \) and \( \psi_1, \) are obtained by applying the magneto-electric boundary condition on top and bottom surfaces of the plate.

Closed-circuit and open-circuit magneto-electric boundary conditions are considered in this study. These boundary conditions are expressed in the following form:

\[
\phi = \psi = 0, \quad (z = \pm h/2) \quad \text{(closed-circuit)},
\]

\[
B_z = D_z = 0, \quad (z = \pm h/2) \quad \text{(open-circuit)}.
\]

Eqs. (5)-(10) and (13) give the Partial Differential Equations (PDEs) of motion in terms of displacements.
Table 1. Dimensionless fundamental frequencies of curved isotropic panels \((a = b, a = 10b, \text{ and } \nu = 0.3)\).

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\infty)</td>
<td>(\infty)</td>
<td>0.0597</td>
<td>0.0577</td>
<td>0.0588</td>
<td>0.0597</td>
<td>0.0581</td>
<td>0.05812</td>
</tr>
<tr>
<td>2</td>
<td>(\infty)</td>
<td>0.0648</td>
<td>0.0629</td>
<td>0.0622</td>
<td>0.0648</td>
<td>0.0632</td>
<td>0.06327</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.0779</td>
<td>0.0762</td>
<td>0.0751</td>
<td>—</td>
<td>0.0767</td>
<td>0.07667</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>0.0597</td>
<td>0.0580</td>
<td>0.0563</td>
<td>—</td>
<td>0.0592</td>
<td>0.05812</td>
</tr>
</tbody>
</table>

and rotations. The immovable simply supported boundary condition is expressed in the following form:

\[
u_0 = v_0 = w_0 = \theta_y = w_{0,xx} = 0 \text{ at } x = 0, a,
\]

\[
u_0 = v_0 = w_0 = \theta_x = w_{0,yy} = 0 \text{ at } y = 0, a,
\]

for which the displacements and rotations can be obtained by:

\[
\begin{bmatrix}
u_0 \\
v_0 \\
w_0 \\
\theta_x \\
\theta_y
\end{bmatrix} = \begin{bmatrix}
\frac{hU \sin(2\pi x/a) \sin(\pi y/b)}{hV \sin(\pi y/a) \sin(2\pi y/b)} \\
\frac{hV \sin(\pi y/a) \sin(2\pi y/b)}{hW \sin(\pi x/a) \sin(\pi y/b)} \\
\frac{hW \sin(\pi x/a) \sin(\pi y/b)}{Y \sin(\pi x/a) \sin(\pi y/b)} \\
\end{bmatrix}.
\]

and by applying the Galerkin method to the PDEs of motion, one obtains:

\[L_1 W^2 + L_2 W + L_3 U + L_4 V = 0, \quad (21)\]

\[L_5 W^2 + L_6 W + L_7 U + L_8 V = 0, \quad (22)\]

\[L_9 W^3 + L_{10} W^2 + L_{11} U W + L_{12} V W + L_{13} V
\]

\[+ L_{14} U + L_{15} X + L_{16} Y + L_{17} W + L_{18} W = 0, \quad (23)\]

\[L_{19} W + L_{20} X + L_{21} Y = 0, \quad (24)\]

\[L_{22} W + L_{23} X + L_{24} Y = 0, \quad (25)\]

where \(L_i\) \((i = 1, 2, \ldots, 24)\) are constant coefficients and are given in Appendix A for two magneto-electric boundary conditions.

Obtaining \(U\) and \(V\) from Eqs. (21) and (22), and \(X\) and \(Y\) from Eqs. (24) and (25), and then substituting the obtained values into Eq. (23) give:

\[\tilde{W} + \omega_0^2 W + \alpha W^2 + \beta W^3 = 0, \quad (26)\]

where the coefficients are given in Appendix B.

Eq. (26) can be analytically solved by using multiple timescales method. To do this, a small dimensionless parameter (\(\varepsilon\)) is inserted to Eq. (26) to scale the nonlinear terms [28]:

\[\tilde{W} + \omega_0^2 W = -\varepsilon \alpha W^2 - \varepsilon^2 \beta W^3. \quad (27)\]

Solving Eq. (27), one can simply obtain the nonlinear frequency ratio [28]:

\[\omega_{NL}/\omega_L = \left[ 1 + \frac{9/\beta \omega_0^2 - 10 \alpha \omega_0^2}{12 \omega_0^2} \right]^{1/2}, \quad (28)\]

where \(\rho_0 = w_{max}/h\) is the dimensionless initial displacement.

3. Results and discussion

In the numerical examples, the shear correction factor \((K)\) is taken to be 5/6. Dimensionless fundamental frequencies, \(\omega = \omega_0 h \rho_0 / E\), of curved isotropic panels for different radii of curvature are obtained and compared with the previously published results (Table 1). The results of the presented model are compared with the results of Alijani et al. [29] based on the Donnell’s nonlinear shell theory, Chorfi and Honmat [30] based on the first-order shear deformation theory, Matsunaga [31] based on the two-dimensional higher-order theory, and Bich et al. [32,33] based on the classical shell theory and first-order shear deformation theory, respectively. It is seen that although the panels are relatively thick, the results are in good agreement with the higher-order results reported by Matsunaga [31].

Table 2 shows the dimensionless frequencies of piezoelectric BaTiO\(_3\) and piezomagnetic CoFe\(_2\)O\(_4\) square thick plates, which are compared with the results obtained by higher-order shear deformation theory [34] and 3D approach [35]. For BaTiO\(_3\), the material properties are: \(C_{11} = C_{22} = 166\ GPa\), \(C_{12} = 77\ GPa\), \(C_{44} = C_{55} = 43\ GPa\), \(C_{66} = 44.5\ GPa\), \(\varepsilon_{31} = \varepsilon_{32} = -4.4\ \text{Cm}^{-2}\), \(\varepsilon_{15} = \varepsilon_{24} = 11.6\ \text{Cm}^{-2}\), \(\eta_{11} = 12.6 \times 10^{-9}\ \text{Cm}^3\text{V}^{-1}\), \(\mu_{33} = 10 \times 10^{-6}\ \text{Ns}^2\text{C}^{-2}\), and \(\rho_0 = 5800\ \text{kgm}^{-3}\); and the material properties of CoFe\(_2\)O\(_4\) are: \(C_{11} = C_{22} = 286\ GPa\), \(C_{12} = 173\ GPa\), \(C_{44} = C_{55} = 45.3\ GPa\), \(C_{66} = \)

<table>
<thead>
<tr>
<th>Method</th>
<th>BaTiO(_3)</th>
<th>CoFe(_2)O(_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSDT [34]</td>
<td>1.2629</td>
<td>1.1358</td>
</tr>
<tr>
<td>3D [35]</td>
<td>1.2660</td>
<td>1.0212</td>
</tr>
<tr>
<td>Present study</td>
<td>1.3268</td>
<td>1.1735</td>
</tr>
</tbody>
</table>
56.5 GPa, \( q_{31} = q_{32} = 580.3 \, \text{N/(Am)}^{-1} \), \( q_{15} = q_{24} = 550 \, \text{N/(Am)}^{-1} \), \( \eta_{33} = 0.033 \times 10^{-9} \, \text{C/(Vm)}^{-1} \), \( \mu_{33} = 157 \times 10^{-6} \, \text{Ns}^2\text{C}^{-2} \), and \( \rho_0 = 5300 \, \text{kgm}^{-3} \). The geometric properties of the plates are \( a = b = 1 \, \text{m} \) and \( h = 0.3 \, \text{m} \), and the dimensionless frequencies are obtained by
\[
\omega = \omega_0 \sqrt{\frac{\rho_0}{C_{\text{max}}}}
\]
where \( C_{\text{max}} \) denotes the maximum of \( C_{ij} \) of the material of the plate. There is a discrepancy between the results of the present study with highly accurate HSĐT and 3D results. This is because the plates are thick and the rotary inertia is not included in the formulation. Thus, the proposed approach can predict the vibration of thin and relatively thick panels with an acceptable precision.

As the last comparison, the nonlinear frequency ratios (\( \omega_{NL}/\omega_L \)) of an isotropic square plate with immovable simply-supported boundary condition are obtained and compared with the results obtained by a shear deformable finite-element model [36] (Table 3). Good agreement is observed between the results of the present approach and the ones obtained by Singh and Daripa [36].

In Tables 4 and 5, the nonlinear frequency ratios of magneto-electro-elastic curved panels with the following material properties are presented [37]:
\[
C_{11} = 226 \, \text{GPa}, \quad C_{12} = 124 \, \text{GPa}, \quad C_{22} = 216 \, \text{GPa}, \quad C_{44} = C_{55} = 44 \, \text{GPa}, \quad C_{66} = 51 \, \text{GPa}, \quad e_{31} = e_{32} = -2.2 \, \text{Cm}^{-2}, \quad q_{31} = q_{32} = 290.2 \, \text{N/(Am)}^{-1}, \quad \eta_{33} = 6.35 \times 10^{-9} \, \text{C/(Vm)}^{-1}, \quad d_{33} = 273.75 \times 10^{-12} \, \text{Ns}(\text{CV})^{-1}, \quad \mu_{33} = 83.5 \times 10^{-6} \, \text{Ns}^2\text{C}^{-2}, \quad \rho_0 = 5500 \, \text{kgm}^{-3}.
\]
Three curved panels are considered in these tables, which are spherical, cylindrical, and hyperbolic paraboloidal panels. The hyperbolic paraboloidal panel has the highest and spherical panel has the lowest nonlinear frequency ratios among the three panels. Moreover, it is seen that the nonlinear frequency ratio is slightly higher for the open-circuit magneto-electric boundary condition, which means that in the open-circuit boundary condition, the nonlinearity of the panel is more than that of the closed-circuit case. However, the spherical panel has the same nonlinear behavior in the open-circuit and closed-circuit cases for vibration amplitudes equal to or smaller than the shell thickness. By contrast, magneto-electric boundary condition has the greatest effect on the response of hyperbolic paraboloidal panel.

The effect of panel thickness on the response has also been investigated and the results are shown in Tables 6 and 7. For each panel \( a/h, \ a/h, \ R_x/a, \) and \( R_y/b, \) ratios are constant whereas \( h \) can be changed. It is seen that thinner spherical and cylindrical panels have higher nonlinear frequency ratios. However, no change is observed in the nonlinear frequency ratio of the hyperbolic paraboloidal panel.

In Figure 2, backbone curves of piezoelectric \( \text{BaTiO}_3, \) piezomagnetic \( \text{CoFe}_2\text{O}_4, \) and magneto-electro-elastic panels are presented. The magneto-electric boundary condition is considered to be closed-circuit. It is observed that for the spherical panel, the backbone curves of magneto-electro-elastic and

<table>
<thead>
<tr>
<th>Method</th>
<th>( w_{\text{max}}/h )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>Singh and Daripa [36]</td>
<td>1.01967</td>
</tr>
<tr>
<td>Present study</td>
<td>1.02085</td>
</tr>
</tbody>
</table>

Table 4. Nonlinear frequency ratios of curved panels with open-circuit magneto-electric boundary condition \((a = b = 100h)\).

<table>
<thead>
<tr>
<th>( R_x/a )</th>
<th>( R_y/b )</th>
<th>( w_{\text{max}}/h )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.00033</td>
</tr>
<tr>
<td>2</td>
<td>\infty</td>
<td>1.00036</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>1.00038</td>
</tr>
</tbody>
</table>

Table 5. Nonlinear frequency ratios of curved panels with closed-circuit magneto-electric boundary condition \((a = b = 100h)\).
Table 6. Nonlinear frequency ratios of open-circuit magneto-electric curved panels with different thicknesses ($a = b = 100h$).

<table>
<thead>
<tr>
<th>$R_m/a$</th>
<th>$R_y/b$</th>
<th>$w_{max}/h$</th>
<th>$w_{max}/h$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h$</td>
<td>$h$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.00114</td>
<td>1.00113</td>
</tr>
<tr>
<td>2</td>
<td>$\infty$</td>
<td>1.00331</td>
<td>1.00326</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>1.00339</td>
<td>1.00339</td>
</tr>
</tbody>
</table>

Table 7. Nonlinear frequency ratios of closed-circuit magneto-electric curved panels with different thicknesses ($a = b = 100h$).

<table>
<thead>
<tr>
<th>$R_m/a$</th>
<th>$R_y/b$</th>
<th>$w_{max}/h$</th>
<th>$w_{max}/h$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h$</td>
<td>$h$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.00114</td>
<td>1.00113</td>
</tr>
<tr>
<td>2</td>
<td>$\infty$</td>
<td>1.00330</td>
<td>1.00325</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>1.00336</td>
<td>1.00336</td>
</tr>
</tbody>
</table>

CoFe$_2$O$_4$ panels are almost identical. Moreover, in cylindrical and hyperbolic paraboloidal panels, CoFe$_2$O$_4$ and BaTiO$_3$ have the most and the least nonlinearity, respectively.

4. Conclusions

Large-amplitude free vibration of magneto-electro-elastic curved panels is investigated in this paper. The Donnell shell theory, Gauss’s laws for electrostatics and magnetostatics, and Galerkin and multiple timescales methods are used to model and solve the problem. The effects of geometry of the panel and the magneto-electric boundary conditions on the vibration behavior of these smart panels are studied by using some numerical examples and it is found that:

(a) The hyperbolic paraboloidal panel has the most and spherical panel has the least nonlinear frequency ratio among the spherical, cylindrical, and hyperbolic paraboloidal panels;

(b) The nonlinear frequency ratio is higher for the open-circuit magneto-electric boundary condition, meaning that in the open-circuit case, the nonlinearity of the panel is more than that of the closed-circuit case;

(c) Thinner spherical and cylindrical panels have higher nonlinear frequency ratios;

(d) Backbone curves of magneto-electro-elastic and CoFe$_2$O$_4$ spherical panels are almost identical.

Figure 2. Backbone curves of (a) spherical, (b) cylindrical, and (c) hyperbolic paraboloidal panels with closed-circuit magneto-electric boundary conditions ($a = b = 100h$).

References


Appendix A

\begin{equation}
\lambda_1 = \frac{d_{42}q_{11} - \mu_{33}q_{31}}{(d_{43}^2 - \mu_{33}q_{33})},
\end{equation}

\begin{equation}
\lambda_2 = \frac{d_{42}q_{31} - \mu_{33}q_{11}}{(d_{43}^2 - \mu_{33}q_{33})}. \tag{A.1}
\end{equation}

For the open-circuit boundary condition we have the following equations:

\begin{align*}
L_1 &= \frac{\pi^2 h^3}{6a} (C_{12} - C_{06} + e_{32} \lambda_1 + q_{32} \lambda_2) - \frac{\pi^2 a h^3}{3b^2} (C_{11} + e_{31} \lambda_1 + q_{31} \lambda_2), \\
L_2 &= \frac{2b h^2}{3R_x} (C_{11} + e_{31} \lambda_1 + q_{31} \lambda_2) + \frac{2b h^2}{3R_y} (C_{12} + e_{32} \lambda_1 + q_{32} \lambda_2), \\
L_3 &= \frac{\pi^2 b h^3}{a} (C_{11} + e_{31} \lambda_1 + q_{31} \lambda_2) - \frac{\pi^2 a h^2}{4b} C_{06}, \\
L_4 &= -\frac{16 b h^2}{9} (C_{12} + C_{06} + e_{32} \lambda_1 + q_{32} \lambda_2). \tag{A.2}
\end{align*}

\begin{align*}
L_5 &= \frac{\pi^2 h^3}{6a} (C_{12} - C_{06} + e_{32} \lambda_1 + q_{32} \lambda_2) - \frac{\pi^2 a h^3}{3b^2} (C_{22} + e_{32} \lambda_1 + q_{32} \lambda_2), \\
L_6 &= \frac{2a h^2}{3R_x} (C_{12} + e_{32} \lambda_1 + q_{32} \lambda_2) + \frac{2a h^2}{3R_y} (C_{22} + e_{32} \lambda_1 + q_{32} \lambda_2). \\
L_7 &= -\frac{16 b h^2}{9} (C_{12} + C_{06} + e_{32} \lambda_1 + q_{32} \lambda_2), \\
L_8 &= -\frac{\pi^2 a h^2}{b} (C_{22} + e_{32} \lambda_1 + q_{32} \lambda_2) - \frac{\pi^2 b h^2}{4a} C_{06}. \tag{A.3}
\end{align*}

\begin{align*}
L_9 &= \frac{4 b h^3}{9 a b R_x R_y} \left[ 1 - C_{12} + 2 C_{06} \right] + \frac{9 b^2}{a^2} (C_{11} + e_{31} \lambda_1 + q_{31} \lambda_2) + \frac{9 b^2}{b^2} (C_{22} + e_{32} \lambda_1 + q_{32} \lambda_2) + \lambda_1 (e_{31} + e_{32}) \nonumber \\
&\quad + \lambda_2 (q_{31} + q_{32}), \\
L_{10} &= -\frac{4 b h^3}{9 a b R_x R_y} \left[ 3 b^2 R_y (C_{11} + e_{31} \lambda_1 + q_{31} \lambda_2) + 3 a^2 R_x (C_{22} + e_{32} \lambda_1 + q_{32} \lambda_2) + 3 C_{12} (a^2 R_y + b^2 R_x) + a^2 R_y (e_{31} + 2 e_{32}) \nonumber \\
&\quad + b^2 R_x (e_{31} + e_{32}) + 2 b R_y (q_{31} + q_{32}) \right], \\
L_{11} &= \frac{\pi^2 h^3}{3b} (C_{12} - C_{06} + e_{32} \lambda_1 + q_{32} \lambda_2) - \frac{2 \pi^2 a h^3}{3b^2} (C_{11} + e_{31} \lambda_1 + q_{31} \lambda_2), \\
L_{12} &= \frac{\pi^2 h^3}{3a} (C_{12} - C_{06} + e_{31} \lambda_1 + q_{31} \lambda_2) - \frac{2 \pi^2 a h^3}{3b^2} (C_{22} + e_{32} \lambda_1 + q_{32} \lambda_2), \\
L_{13} &= \frac{2 a h^2}{3 b R_x} (C_{12} + e_{31} \lambda_1 + q_{31} \lambda_2). \tag{A.4}
\end{align*}
\[L_{14} = \frac{2b h^2}{3R_y} (C_{11} + \epsilon_{31} \lambda_1 + q_{31} \lambda_2),\]
\[L_{15} = -\frac{1}{4} K \pi h C_{55},\]
\[L_{16} = -\frac{1}{4} K \pi a h C_{44},\]
\[L_{17} = -\frac{ab h^2}{4R_y} (C_{11} + \epsilon_{31} \lambda_1 + q_{31} \lambda_2)\]
\[-\frac{ab h^2}{4R_y} (C_{22} + \epsilon_{32} \lambda_1 + q_{32} \lambda_2)\]
\[-\frac{a \pi^2 h^2}{4b} K C_{44} - \frac{b \pi^2 h^2}{4a} K C_{55}\]
\[-\frac{a b h^2}{4R_y R_y} [2C_{12} + \lambda_1 (\epsilon_{31} + \epsilon_{32}) + \lambda_2 (q_{31} + q_{32})],\]
\[L_{18} = -\frac{1}{4} I_o a b h,\] (A.4)
\[L_{19} = \frac{\pi^3 h^4}{48b} [e_{24} \epsilon_{31} \delta_2 - (e_{24} q_{31} + \epsilon_{31} q_{24}) \delta_1]\]
\[+ q_{24} q_{31} \delta_3 - \frac{b h^2}{4b} K C_{55} + \frac{\pi^3 h^4}{48a^2} (e_{15} e_{31} \delta_2)\]
\[- (e_{15} q_{31} + \epsilon_{31} q_{15}) \delta_1 + q_{15} q_{31} \delta_3],\]
\[L_{20} = \frac{\pi^2 b h^3}{48a} [(e_{31} + e_{15} + e_{31} \epsilon_{2}) - C_{11} + (q_{21} + q_{15} \epsilon_{2}) \delta_3]\]
\[- (e_{15} q_{31} + (q_{15} + 2q_{31}) \epsilon_{31}) \delta_1 - \frac{\pi^2 a h^3}{4b} C_{60}\]
\[-\frac{a b h}{4} K C_{55},\]
\[L_{21} = \frac{1}{48} \pi^2 h^3 [(e_{31} + e_{24} \epsilon_{31}) \delta_2 - C_{12} - C_{60}\]
\[+ (q_{31} + q_{24}) \delta_3 - (e_{24} q_{31} + q_{24} \epsilon_{31}) \delta_1],\] (A.5)
\[L_{22} = \frac{\pi^3 h^4}{48a} [e_{15} e_{32} \delta_2 - (e_{15} q_{32} + e_{32} q_{15}) \delta_1]\]
\[+ q_{15} q_{32} \delta_3 - \frac{b h^2}{4b} K C_{44} + \frac{\pi^2 a h^4}{48b^2} [e_{24} e_{32} \delta_2\]
\[- (e_{24} q_{32} + e_{32} q_{24}) \delta_1 + q_{24} q_{32} \delta_3],\]
\[L_{23} = \frac{1}{48} \pi^2 h^3 [(e_{15} + e_{31}) \epsilon_{32} \delta_2 - C_{12} - C_{60}\]
\[+ (q_{15} + q_{31}) \epsilon_{32} \delta_3 - (q_{31} e_{15} + e_{31}) \delta_1]\]
\[+ (q_{15} + q_{31}) \epsilon_{32} \delta_1],\]
\[L_{24} = \frac{\pi^2 a h^3}{48b} [(e_{24} + e_{32}) \epsilon_{32} \delta_2 - C_{22}\]
\[-(e_{32} q_{24} + q_{31}) + (e_{24} + e_{31}) q q_{32} \delta_1\]
\[+ (q_{24} + q_{31}) \epsilon_{32} \delta_3] - \frac{\pi^2 b h^3}{48a} C_{60} \frac{a b h}{4} K C_{44},\] (A.6)

For the closed-circuit boundary condition, we have the following equations.
\[L_1 = \frac{\pi^2 h^3}{6 a^2 b} \{a^2 (C_{12} - C_{60}) - 2 b^2 C_{11}\},\]
\[L_2 = \frac{2 h^2}{3 R_x} C_{11} + \frac{2 b h^2}{3 R_y} C_{12},\]
\[L_3 = -\frac{\pi^2 a h^2}{a} C_{11} - \frac{\pi^2 a h^2}{4 b} C_{60},\]
\[L_4 = -16 h^2 \frac{9}{9} (C_{12} + C_{60}),\] (A.7)
\[L_5 = \frac{\pi^2 h^3}{6 a} (C_{12} - C_{60}) - \frac{\pi^2 a h^3}{3 b^2} C_{22},\]
\[L_6 = \frac{2 a h^2}{3 R_x} C_{12} + \frac{2 a h^2}{3 R_y} C_{22},\]
\[L_7 = -16 h^2 \frac{9}{9} (C_{12} + C_{60}),\]
\[L_8 = -\frac{\pi^2 a h^2}{b} C_{22} - \frac{\pi^2 b h^3}{4 a} C_{60},\] (A.8)
\[L_9 = -\frac{\pi^4 h^4}{128 a b} \left\{2 (C_{12} + 2 C_{60}) + \frac{9 h^2}{a^2} C_{11} + \frac{9 h^2}{b^2} C_{22}\right\},\]
\[L_{10} = -\frac{4 h^3}{9 a b R_x R_y} [3 b^2 R_y C_{11} + 3 a^2 R_x C_{22}\]
\[+ 3 C_{12} (a^2 R_y + b^2 R_x)],\]
\[L_{11} = \frac{\pi^2 h^3}{3 b} (C_{12} - C_{60}) - \frac{2 h^3}{3 a} C_{11},\]
\[L_{12} = \frac{\pi^2 h^3}{3 a} (C_{12} - C_{60}) - \frac{2 h^3}{3 b} C_{22},\]
\[L_{13} = \frac{2 a h^2}{3 R_x} C_{12} + \frac{2 a h^2}{3 R_y} C_{22},\]
\[ L_{14} = \frac{2bh^2}{3R_z} C_{11} + \frac{2bh^2}{3R_y} C_{12}, \]
\[ L_{15} = -\frac{1}{4} K \pi bh C_{55}, \quad L_{16} = -\frac{1}{4} K \pi ah C_{44}, \]
\[ L_{17} = -\frac{ab h^2}{4R_z^2} C_{11} - \frac{ab h^2}{4R_y^2} C_{22} - \frac{\pi^2 h^2}{4a} K C_{44} \]
\[ - \frac{bh^2}{4a} K C_{55} - \frac{ab h^2}{2R_z R_y} C_{12}. \]
\[ L_{18} = -\frac{1}{4} \pi ab h. \tag{A.9} \]
\[ L_{19} = \frac{\pi^2 h^4}{48b} [e_{24} e_{31} e_2 - (e_{24} q_{31} + e_{31} q_{24}) \delta_1 \]
\[ + q_{24} q_{31} \delta_3 - \frac{\pi bh^2}{4} K C_{55} + \frac{\pi^3 bh^4}{48a} [e_{15} e_{31} e_2 \]
\[ - (e_{15} q_{31} + e_{31} q_{15}) \delta_1 + q_{15} q_{31} \delta_3], \]
\[ L_{20} = \frac{\pi^3 h^4}{48a} [e_{31} e_1 + e_{15} q_{31}] \delta_2 - C_{11} \]
\[ + (q_{31}^2 + q_{15} q_{31}) \delta_3 - (e_{15} q_{31} + (q_{15} + 2q_{31}) e_{31}) \delta_1 \]
\[ - \frac{\pi^2 a h^3}{48b} C_{66} - \frac{ab h}{4} K C_{55}. \]
\[ L_{21} = \frac{1}{48} \pi^2 h^3 [(e_{31}^2 + e_{24} e_{31}) \delta_2 - C_{12} - C_{66} \]
\[ + (q_{31}^2 + q_{24} q_{31}) \delta_3 - (e_{24} q_{31} + (q_{24} + 2q_{31}) e_{31}) \delta_1]. \tag{A.10} \]
\[ L_{22} = \frac{\pi^2 h^4}{48a} [e_{15} e_{32} \delta_2 - (e_{15} q_{32} + e_{32} q_{15}) h \delta_1 \]
\[ + q_{15} q_{32} \delta_3 - \frac{\pi a h^2}{4} K C_{44} + \frac{\pi^3 a h^4}{48b^2} [e_{24} e_{32} \delta_2 \]
\[ - (e_{24} q_{32} + e_{32} q_{24}) \delta_1 + q_{24} q_{32} \delta_3], \]
\[ L_{23} = \frac{1}{48} \pi^2 h^3 [(e_{15} + e_{31}) e_{32} \delta_2 - C_{12} - C_{66} \]
\[ + (q_{15} + q_{31}) \delta_3 - (q_{32} (e_{15} + e_{31} \]
\[ + (q_{15} + q_{31}) \delta_3)], \]
\[ L_{24} = \frac{\pi^2 a h^3}{48b} [(e_{24} + e_{31}) \delta_2 - C_{22} \]
\[ - (e_{24} q_{24} + q_{31}) + (e_{24} + q_{31}) \delta_1 \]
\[ + (q_{24} + q_{31}) \delta_3 - \frac{\pi^2 h^2}{4a} C_{66} - \frac{ab h}{4} K C_{44}. \tag{A.11} \]

Appendix B

\[ g_1 = (L_{21} L_{22} - L_{19} L_{24})/(L_{20} L_{24} - L_{21} L_{23}), \]
\[ g_2 = (L_{19} L_{23} - L_{20} L_{22})/(L_{20} L_{24} - L_{21} L_{23}), \]
\[ g_3 = (L_4 L_5 - L_2 L_3)/(L_3 L_8 - L_4 L_7), \]
\[ g_4 = (L_4 L_5 - L_1 L_8)/(L_3 L_8 - L_4 L_7), \]
\[ g_5 = (L_2 L_7 - L_3 L_8)/(L_3 L_8 - L_4 L_7), \]
\[ g_6 = (L_1 L_7 - L_3 L_5)/(L_3 L_8 - L_4 L_7), \]
\[ g_0 = \sqrt{(L_1 + L_3 g_5 + L_1 g_3 + L_5 g_1 + L_5 g_2)/L_{18}}, \]
\[ \alpha = (L_{10} + L_{11} g_4 + L_{13} g_8 + L_{13} g_6 + L_{14} g_4)/L_{18}, \]
\[ \beta = (L_9 + L_{11} g_4 + L_{12} g_8)/L_{18}. \]

Biographies

Alireza Shooshtari is Associate Professor of Mechanical Engineering at Bu-Ali Sina University. He received his BS degree in Mechanical Engineering from Tehran University, Tehran, Iran; and his MS and PhD (2006) degrees in Applied Mechanics, respectively, from Bu-Ali Sina University, Hamedan, Iran, and Tarbiat Modarres University, Tehran, Iran. His research interests are nonlinear dynamics of engineering structures, modal analysis, and random vibrations.

Soleil Razavi is candidate for the PhD degree in Applied Mechanics. He received his MSc in Applied Mechanics from Bu-Ali Sina University, Hamedan, Iran, in 2010. His MSc thesis was about single-mode analysis of hybrid laminated plates. His PhD dissertation deals with the analytical study of nonlinear dynamics of smart plates and shells.