Effect of convective transport mechanisms on heat transfer characteristics of nanofluids

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Nanofluids; Convective heat transfer; Two phase approach; Laminar flow.

Abstract. In this study, effect of different slip mechanisms that can produce a slip velocity between nanoparticles and base fluid in a nanofluid flow field has been investigated numerically. A two-phase Euler-Lagrange approach was applied to simulate heat transfer characteristics of different nanoparticles in a straight tube under laminar flow condition. Effect of different mechanisms such as thermophoresis, Brownian diffusion, and Saffman lift force on convective heat transfer was investigated and discussed. It is noticed that only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. In different nanofluids, effect of Brownian diffusion and thermophoresis on convective heat transfer is different. While effect of Brownian diffusion is more important in CuO-water nanofluid, thermophoresis is the main slip mechanism in Al₂O₃-water nanofluid.

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1. Introduction

Two decades ago, Masuda et al. [1] observed that thermal conductivity enhances by dispersing submicron particles in base fluid. In comparison with coolant fluids, most solids, especially metals, show much higher thermal conductivity. Therefore, fluids with suspended solid particles are expected to have higher heat transfer properties compared to conventional heat transfer fluids. Experimental applications of these fluids with suspended particles of millimeter or micrometer size show some difficulties, such as instability, erosion and flow channel clogging, and extra penalty of pressure drop in the flow channel [2–5]. The term nanofluid was first proposed by Choi [6] to indicate engineered colloids composed of nanoparticles dispersed uniformly and stably in a base fluid. Several authors have investigated several nanoparticles with different volume fractions and sizes in various base fluids [7–9]. Their results have uncovered higher thermal conductivity of nanofluids compared to that of base fluids. Eastman et al. [10] applied copper particles with 10 nm diameter and observed that thermal conductivity enhancement is about 40% at 3% volume fraction. However, some disagreements exist in the literature in terms of mechanisms of heat transfer enhancement under various conditions. Different parameters such as particle type, volume fraction, and particle diameter can affect the amount of enhancement in convective heat transfer. Akbaridost et al. [11] studied the numerically and experimentally laminar steady-state flow in helically coiled tubes at a constant wall temperature. Their results revealed that utilizing nanofluids enhances heat transfer, and more enhanced heat transfer is observed for tubes with greater curvature ratio. It has been demonstrated that the effect of particle volume fraction is more significant for turbulent flow regime than laminar flow regime [12]. It is also observed that the rate of heat transfer enhancement is different in different duct shapes [13]. Bianco et al. [14] investigated turbulent forced convection flow of water-Al₂O₃ nanofluid in a circular tube subjected to a constant and
uniform heat flux at the wall. Their results showed that heat transfer enhancement increases with the particle volume concentration and Reynolds number. Two general approaches are applied for modeling the nanofluid flow field and heat transfer characteristics: single-phase and two-phase approaches. In single-phase approach, the base fluid and nanoparticles are considered to be in thermal equilibrium, and the effect of nanoparticles are just considered in effective properties. But, according to Brownian motion of ultrafine particles, different mechanisms can produce the slip velocity between nanoparticles and base fluid. Therefore, the nanoparticles can be considered as a discrete phase, and the Lagrangian approach is applied for modeling the nanofluid flow field and heat transfer characteristics. Peng et al. [15] made comparisons between the single-phase model, Euler-Euler two-phase model, and Euler-Lagrangian two-phase model. They noticed that the Euler-Lagrangian model produces more accurate results compared to the other two approaches.

In this paper, Effect of different mechanisms of particle diffusions such as thermophoresis, Brownian, and lift on convective heat transfer of different nanofluid in a circular tube is investigated and discussed. Two-phase Euler-Lagrangian approach is applied to study the laminar forced convection flow of different nanofluids in a uniformly heated tube. The convective heat transfer of two-phase Euler-Lagrangian model and homogenous single-phase model is compared with the experimental data.

2. Mathematical model
In the single-phase approach, both the fluid and the nanoparticles are considered as a homogenous single-phase fluid. The conservative equations of continuity, momentum, and energy for pure fluid are extended to nanofluids using effective properties of the nanofluids. These effective properties are obtained from experimental or theoretical data. On the other hand, in the two-phase approach, the fluid phase is considered continuous and nanoparticles are dispersed through it. Consequently, effects of nanoparticles and base fluid interactions are entered as a source term in momentum and energy equations.

2.1. Single-phase approach
In single-phase method, the conservation equations are written in the same form of the base fluid, but thermo physical properties will be replaced with appropriate properties of the nanofluids like below:

- Continuity equation:
  \[ \nabla \cdot (\rho_n \mathbf{v}) = 0. \]  

- Momentum equation:
  \[ \nabla \cdot (\rho_n \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot (\mu_n \nabla \mathbf{v}). \]  

- Energy equation:
  \[ \nabla \cdot (\rho_n c_p n T) = \nabla \cdot (k_n \nabla T). \]  

2.1.1. Thermo physical properties of nanofluids
In this study, the following correlations have been applied to compute properties of the Al₂O₃-water and CuO-water nanofluids:

\[ \rho_{nf} = \phi \rho_p + (1 - \phi) \rho_f, \]  

\[ c_{p,nf} = \phi c_{p,p} + (1 - \phi) c_{p,f}. \]  

Correlation of Yu and Choi [16] has been used to calculate thermal conductivity of the Al₂O₃-water and CuO-water nanofluids. This correlation only considers the effect of nanoparticle volume fraction on thermal conductivity:

\[ k_{nf} = k_f \left[ \frac{k_p + 2k_f + 2(k_p - k_f)(1 + \beta)^{3\phi}}{k_p + 2k_f - (k_p - k_f)(1 + \beta)^{3\phi}} \right], \]  

where \( \beta \) is the nanolayer thickness to radius of nanoparticle; \( \beta = 0.1 \) has been considered in order to compute effective thermal conductivity of the Al₂O₃-water and CuO-water nanofluids [17].

In order to consider the effect of temperature variation on heat transfer conductivity of nanofluid, Chon correlation [18] was used for calculating the effective thermal conductivity:

\[ \frac{k_{nf}}{k_f} = \frac{1}{1 + 64.7 \times \phi^{0.749} \left( \frac{d_f}{d_p} \right)^{0.3000} \left( \frac{k_s}{k_f} \right)^{0.7476} \times Pr^{1.0555} \times Re^{1.2921}}, \]  

where Pr and Re in Eq. (7) are defined as:

\[ Pr = \frac{\mu}{\rho_f c_f}, \]

\[ Re = \frac{\rho_f B_t \bar{a}}{3\pi \mu l_{BF}}. \]

\( l_{BF} \) is the mean free path of water and the calculated value of 0.17 nm is used for water in modeling, \( B_t \) is Boltzmann constant; and \( \mu \) is calculated by the following equation:

\[ \mu = 4 \times 10^{-5} \pi n, \quad C = 140, \quad B = 247, \quad A = 2.414e - 5. \]

Effective viscosity of CuO-water nanofluid can be calculated by Birkman’s model [19] as follows:

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}. \]  

Maiga [20] equation is used to calculate the viscosity of Al₂O₃-water nanofluid:

\[ \mu_{nf} = (123\phi^2 + 7.3\phi + 1) \mu_f. \]
2.2. Two-phase approach

The Euler-Lagrange two-phase approach is applied to obtain the effect of nanoparticles slip velocity on base fluid flow and heat transfer characteristics. The influence of nanoparticles in base fluid is considered as a source term in momentum and energy equations. Therefore, the conservative equations are presented as below:

- **Continuity equation:**
  \[ \nabla \cdot \mathbf{v}_f = 0. \]  
  \[ (12) \]

- **Momentum equation:**
  \[ \nabla \cdot (\mathbf{v}_f \mathbf{v}_f) = -\frac{1}{\rho_f} \nabla p + \nabla (\mu_f \nabla \mathbf{v}_f) + \frac{S_m}{\rho_f}, \]  
  \[ (13) \]

where \( S_m \) in Eq. (13) is the momentum transfer between base fluid and nanoparticles and can be obtained from the instant variation of nanoparticles momentum when they pass from the control volume:

\[ S_m = \sum m_p \mathbf{F} \Delta t, \]  
\[ (14) \]

where \( \mathbf{F} \) is the force per unit particle mass and \( m_p \) is particle mass. This force is equal to the rate of momentum within the control volume:

\[ \mathbf{F} = \frac{d\mathbf{v}_p}{dt}, \]  
\[ (15) \]

where \( \mathbf{v}_p \) is the nanoparticle velocity. Different forces that cause slip velocity between nanoparticles and base fluid are considered as below:

\[ \mathbf{F} = \mathbf{F}_G + \mathbf{F}_L + \mathbf{F}_V + \mathbf{F}_D + \mathbf{F}_T + \mathbf{F}_B, \]  
\[ (16) \]

where \( \mathbf{F}_G \) is the gravity force and can be calculated by:

\[ \mathbf{F}_G = \frac{(\rho_p - \rho_f)g}{\rho_p} \mathbf{g}, \]  
\[ (17) \]

and \( \mathbf{F}_L \) is the Saffman lift force and can be obtained by [21]:

\[ \mathbf{F}_L = \frac{2K}{\rho_p d_i d_{ij}} (\mathbf{v}_f - \mathbf{v}_p), \]  
\[ (18) \]

where \( K = 2.594 \) and \( d_{ij} \) is deformation tensor.

Virtual mass force arises due to acceleration from relative velocity [22] and can be obtained by:

\[ \mathbf{F}_V = \frac{\rho_f}{2 \rho_p} (\dot{\mathbf{v}}_f - \dot{\mathbf{v}}_p). \]  
\[ (19) \]

For sub-micron particles due to the small relative Reynolds number, the drag force is the computed form of Stokes’ drag law [23]:

\[ \mathbf{F}_D = \frac{18 \mu_f}{d_p^2 \rho_p C_c} (\mathbf{v}_f - \mathbf{v}_p), \]  
\[ (20) \]

where \( C_c \) is Cunningham correction factor to Stokes' drag law and can be calculated from:

\[ C_c = 1 + \frac{2 \lambda}{d_p} \left( 1.257 + 0.4e^{-(1.1d_p/2\lambda)} \right), \]  
\[ (21) \]

where \( \lambda \) is the molecular mean free path.

Thermophoresis is the motion of particles in a fluid, which is subject to a steady temperature gradient. At sufficiently long times, the stronger molecular impulses in the hotter fluid region drive nanoparticles towards the colder region, where the molecular impulses are weaker. The following formula which was introduced by Epstein [24] is used for the thermophoretic force:

\[ \mathbf{F}_T = -9\pi \mu_f d_i^2 \frac{k_f}{2k_f + k_p} \frac{\nabla T}{\rho_f m_p T}, \]  
\[ (22) \]

Nanoparticles are the same order of fluid molecules and the collision between nanoparticles; fluid molecules can cause random motion and slip velocity. This random motion of nanoparticles are called Brownian, and the Brownian force (\( \mathbf{F}_B \)) is modeled as a Gaussian function with spectral intensity of \( S_{n,ij} [25] \):

\[ S_{n,ij} = \delta_{ij}, \]  
\[ (23) \]

\[ S_0 = \frac{216\nu k_B T}{\pi^2 \rho_p d_p} \left( \frac{\rho_f}{\rho_p} \right)^2 C_c, \]  
\[ (24) \]

where \( \delta_{ij} \) is the Kronecker delta function, \( T \) is the fluid temperature, \( \nu \) is the kinematic viscosity, and \( k_B \) is the Boltzmann constant. Energy equation is obtained as follows:

\[ \nabla \cdot (\mathbf{v}_f T_f) = \nabla \cdot (\kappa_f \nabla T_f) + \frac{S_c}{\rho_f^c \rho_{p,f}}, \]  
\[ (25) \]

where \( S_c \) in Eq. (26) is the source term related to energy transferred between nanoparticle and base fluid:

\[ S_c = \sum \dot{\mathcal{Q}} \Delta t, \]  
\[ (26) \]

where \( \dot{\mathcal{Q}} \) is the total convective heat transfer from the particle and is obtained by:

\[ \dot{\mathcal{Q}} = \text{Nu} \pi d_p k_f (T_p - T_f), \]  
\[ (27) \]

where \( \text{Nu} \) is computed using the Ranz and Marshall correlation [26]:

\[ \text{Nu} = 2.0 + 0.6 \text{Re}^{1/2} \text{Pr}^{1/3}. \]  
\[ (28) \]
3. Geometry and boundary conditions

The geometry of considered problem is shown in Figure 1. It consists of 8 mm diameter tube with 1 m length. For water (continous phase), a uniform distribution of velocity ($V_0$) and temperature ($T_0$) is assumed at tube inlet. At the tube outlet, the fully developed condition is assumed, which states that all axial derivatives are zero. For CuO and Al2O3 nanoparticles (dispersed phase), the constant amount of velocity and temperature is prevailed at tube inlet. It is also assumed that all particles have the same diameter ($d_p$). A constant heat flux condition ($q''$) is assumed at the tube wall. A no-slip condition is employed at wall for the continuous phase and the reflect condition for the dispersed phase. Table 1 shows the considered boundary conditions.

<table>
<thead>
<tr>
<th>Boundary index</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$L$</td>
<td>1 m</td>
</tr>
<tr>
<td>$D$</td>
<td>8 mm</td>
</tr>
<tr>
<td>$V_0$</td>
<td>0.17 m/s</td>
</tr>
<tr>
<td>$T_0$</td>
<td>300 K</td>
</tr>
<tr>
<td>$d_p$</td>
<td>40 nm</td>
</tr>
<tr>
<td>$q''$</td>
<td>9000 W/m²</td>
</tr>
</tbody>
</table>

4. Numerical method and validation

The dimensioned governing equations, i.e., Eqs. (1) to (3), (12), (13), and (25), are discretized using a control volume method. The second order upwind and power low schemes are, respectively, used to approximate convection and diffusion terms in the differential equations, while SIMPLE procedure has been applied for the velocity-pressure coupling. The solution domain is discretized with a non-uniform spacing mesh in the radial direction with grids clustering toward the walls, which allows the boundary layers to be resolved without an excessive number of grids. The coupling be-

5. Results and discussion

Results have been obtained by employing the single-phase and two-phase models for different Reynolds number and volume fractions. In all cases, the size of the spherical particles is considered equal to 40 nm. The initial simulations are performed on single-phase model for both constant and temperature-dependent properties, and two-phase Euler-Lagrange method to check the reliability and accuracy of each numerical method. The results are compared with the experimental data of Asirvatham et al. [27] at $\phi = 0.3\%$ and $Re = 1350$ in Figure 3. It can be observed that the single-
phase model based on constant properties (Yu and Choi model [16]) does not predict convective heat transfer of the nanofluid through the tube length in contrast with the two-phase Euler-Lagrange approach. While the average discrepancy of the two-phase model with the experimental data is less than 2%, this discrepancy for single-phase [16] is 8%. However, the single-phase model that considers the effect of random motion of nanoparticles [18] shows the better results and its average discrepancy is 3.6%.

Figures 4 and 5 exhibit the effect of nanoparticle volume fraction on convective heat transfer of CuO-water and Al₂O₃-water nanofluids at Re = 100. It is observed that the amount of enhancement in heat transfer coefficient increases with concentration of the nanoparticles. While the amount of enhancement increases along the tube for CuO-water nanofluid, this amount is almost constant for Al₂O₃-water nanofluid. The main reason is concerned with the higher thermal conductivity of CuO nanoparticles than Al₂O₃ nanoparticles. Through flow direction, fluid temperature increases and CuO nanoparticles accelerate the heat transfer between fluid layers.

The effect of different particle transport mechanisms on convective heat transfer of CuO-water nanofluid is studied in Figures 6-8. The evaluation is done at Re = 100 and volume concentration of 1%. They illustrate the effect of Saffman lift force, thermophoretic force, and Brownian force on convective heat transfer coefficient, respectively.

It is observed that the effect of Saffman lift force on convective heat transfer against thermophoretic and Brownian forces is negligible. For CuO-water nanofluid, Brownian effects on convective heat transfer are more important than thermophoresis effects. While elimination of the thermophoretic force from simulation has affected the amount of this enhancement and caused some 0.25% relative decrease in the convective heat transfer, eliminating Brownian force from model incurred 0.35% relative decrease in the convective heat transfer.
Figure 7. Effect of thermophoretic force on convective heat transfer of CuO-water nanofluid at Re = 100 and \( \phi = 1\% \).

Figure 8. Effect of Brownian force on convective heat transfer of CuO-water nanofluid at Re = 100 and \( \phi = 1\% \).

Figure 9. Effect of Saffman lift force on convective heat transfer of Al2O3-water nanofluid at Re = 100 and \( \phi = 1\% \).

Figure 10. Effect of thermophoretic force on convective heat transfer of Al2O3-water nanofluid at Re = 100 and \( \phi = 1\% \).

Figure 11. Effect of Brownian force on convective heat transfer of Al2O3-water nanofluid at Re = 100 and \( \phi = 1\% \).

transfer. Similar to the thermophoretic force, the role of Brownian force becomes more significant along the tube length. It is observed that the effect of thermophoretic and Brownian force on the rate of heat transfer enhancement is more prominent at greater volume fractions.

Figures 9-11 show the effect of particle transport mechanisms on convective heat transfer of Al2O3-water nanofluid. The evaluation is performed at Re = 100 and volume concentration of 1%. The effect of Saffman lift force, thermophoretic force, and Brownian force on convective heat transfer coefficient of Al2O3 is investigated.

Similar to CuO-water nanofluid, the effect of Saffman lift force on convective heat transfer against thermophoretic and Brownian forces is negligible. For Al2O3-water nanofluid, thermophoresis effects on convective heat transfer are of the same order or more
important than Brownian effects. While elimination of the thermophoretic force from simulation has affected the amount of this enhancement and caused 0.45% relative decrease in average in the convective heat transfer, eliminating Brownian force from model incurred 0.35% relative decrease in the convective heat transfer. According to higher thermal conductivity of CuO nanoparticles which makes lower thermal diffusion, thermophoresis effects in Al2O3-water nanofluid are more important than CuO-water nanofluid. Moreover, according to higher density of CuO nanoparticles, collisions between particles and fluid molecules become more important, and effect of Brownian motion on convective heat transfer is more considerable.

6. Conclusions
A two-phase Euler-Lagrange approach has been applied to investigate the effect of transform mechanisms on heat transfer characteristics of nanoparticles in a straight tube. A comparison between two-phase Euler-Lagrange and homogenous single-phase model has been implemented to predict heat transfer characteristics of nanofluids. It was observed that the two-phase Euler-Lagrange model is more precise than the homogenous model. Slip mechanisms between particles and base fluid were not predicted by single phase model. Different mechanisms have been considered that can produce slip velocity between nanoparticles and base fluid. It has been observed that only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. In different nanofluids, effect of each one on convective heat transfer is different. While in CuO-water nanofluid effect of Brownian diffusion is more important, thermophoresis is the main slip mechanism in Al2O3-water nanofluid.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>ε &lt;sub&gt;p&lt;/sub&gt;</td>
<td>Specific heat</td>
</tr>
<tr>
<td>C&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Cunningham correction factor to Stokes’ drag law</td>
</tr>
<tr>
<td>D</td>
<td>Tube diameter</td>
</tr>
<tr>
<td>d&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Particle diameter</td>
</tr>
<tr>
<td>d&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Deformation tensor</td>
</tr>
<tr>
<td>F</td>
<td>Total force applied to particle</td>
</tr>
<tr>
<td>F&lt;sub&gt;B&lt;/sub&gt;</td>
<td>Brownian force</td>
</tr>
<tr>
<td>F&lt;sub&gt;D&lt;/sub&gt;</td>
<td>Drag force</td>
</tr>
<tr>
<td>F&lt;sub&gt;G&lt;/sub&gt;</td>
<td>Gravity force</td>
</tr>
<tr>
<td>F&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Lift force</td>
</tr>
<tr>
<td>F&lt;sub&gt;T&lt;/sub&gt;</td>
<td>Thermophoretic force</td>
</tr>
<tr>
<td>F&lt;sub&gt;V&lt;/sub&gt;</td>
<td>Virtual force</td>
</tr>
<tr>
<td>g</td>
<td>Gravity acceleration</td>
</tr>
<tr>
<td>h</td>
<td>Convective heat transfer coefficient</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>k&lt;sub&gt;B&lt;/sub&gt;</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>Kn</td>
<td>Knudsen number</td>
</tr>
<tr>
<td>k&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Coefficient</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
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<tr>
<td>Nu</td>
<td>Nusselt number</td>
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<tr>
<td>p</td>
<td>Pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>Q</td>
<td>Total heat flux</td>
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<tr>
<td>S&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Source term</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>X</td>
<td>Axial distance</td>
</tr>
<tr>
<td>ν</td>
<td>Velocity</td>
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Greek Symbols

<table>
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<tbody>
<tr>
<td>β</td>
<td>Ratio of the nanolayer thickness to the original particle radius</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
</tr>
<tr>
<td>λ</td>
<td>Mean free path of the fluid</td>
</tr>
<tr>
<td>φ</td>
<td>Particle volume fraction</td>
</tr>
<tr>
<td>δ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>Kronecker delta function</td>
</tr>
<tr>
<td>ζ</td>
<td>Zero-mean, unit-variance-independent Gaussian random number</td>
</tr>
<tr>
<td>μ</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>ν</td>
<td>Kinematic viscosity</td>
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Subscripts

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<th>Description</th>
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<tr>
<td>n&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Nanofluid</td>
</tr>
<tr>
<td>f</td>
<td>Fluid</td>
</tr>
<tr>
<td>p</td>
<td>Particle</td>
</tr>
<tr>
<td>m</td>
<td>Mean</td>
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References


Biographies

Ebrahim Mohammadpour obtained his BS in Mechanical Engineering (fluid mechanics) from Iran University of Science and Technology, in 2008. He received his MSc degree in Mechanical Engineering from K.N. Toosi University of Technology, in 2010. He is the author of more than 7 papers on various subjects in nanofluids, power plants, turbo machinery, and numerical modeling.

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