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A differential Stackelberg game for pricing on a freight transportation network with one dominant shipper and multiple oligopolistic carriers

M. Mozafari, B. Karimi* and M. Mahootchi

Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, Tehran, P.O. Box 15916-34311, Iran.

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Abstract. This paper studies dynamic pricing and freight network equilibrium problem on a system consisting of one dominant producer called the shipper and multiple oligopolistic carriers who serve the shipper's origin-destination orders. The shipper sells a homogeneous commodity to spatially separated demand markets. The demand received by the shipper is price-sensitive, while the prices set for each market are influenced by the pricing strategies of the oligopolistic carriers. We formulate the problem as a differential Stackelberg-Nash game to find the equilibrium production, price, and routing decisions over a planning horizon. A finite dimensional discretization method and a penalty function algorithm are proposed to solve the model. The existence and uniqueness of properties are also explored. Finally, some numerical examples are presented and a comprehensive sensitivity analysis on the critical parameters is conducted to show the efficiency of the proposed model and solution method.

1. Introduction

With the deregulation of transport industry and the emergence of private carriers, including transportation and Third Party Logistics companies (3PLs), relationships among players of freight transportation service chain have become extremely complex. In today's competitive market place, the key stockholders involved in freight transportation networks, including shippers, carriers, and infrastructure companies, typically express a profit maximization behavior. Hence, the negotiation and pricing mechanisms applied by them noticeably impact on freight flows over the network.

Urban freight transportation markets can be characterized by oligopolistic behavior of a few large carriers or third party logistics providers who compete to win transportation contracts [1,2]. Chanut and Pache [3] noted that the French freight market is oligopolistic as the top five 3PL hold a third of the market. Arvidsson et al. [4] carried out a similar empirical study on Sweden cargo market and showed that the two dominant players in the market typically have controlled over 80% of the freight flows.

In contrast to passenger individual route choice, freight route choices reflect the collective effect of the decisions made by multiple agents. The freight flow prediction models are able to understand and analyze the impact of freight route choices on the transportation network planning. An early classification of the models predicting freight flows presented by Harker and Friesz [5] is distinguished in three categories:

1. The econometric model which uses time series

*. *Corresponding author. Tel.: +98 21 64545374;*
Fax: +98 21 66954569
E-mail addresses: m.mozafari@aut.ac.ir (M. Mozafari); b.karimi@aut.ac.ir (B. Karimi)

to predict the relationship between transportation supply and demand;

2. The spatial price equilibrium model in which the act of distributing goods by shippers between spatially separated markets to find minimum costs leads to an equilibrium of prices and transportation flows on a simplified network;
3. The freight network equilibrium model which considers the interaction of shippers and carriers on a real network to predict the equilibrium freight flows.

Recently, Tavasszy et al. [6] have reviewed freight transportation demand models by integrating the classical 4-step modelling approach into spatial computable general equilibrium models, supply chain choice models, and hyper-network models. In this paper, we focus on trip distribution on the network to estimate freight flows related to a certain commodity traded in a market where a dominant producer tries to deliver goods to his customers through oligopolistic carriers. Our model can be categorized as a freight network equilibrium model integrated with supply chain decisions, including production, inventory, price, and rout choices.

The first significant predictive freight network model was introduced by Kresge and Roberts [7] studying a multi-modal multi-commodity network, in which shippers decide on mode choice and general routing is calculated by the shortest path. Friesz et al. [8] determined freight network equilibrium flows taking into account the interaction between shippers and carriers. This model was then extended by Friesz and Harker [9] and Harker and Freisz [10]. Friesz et al. [11] proposed a sequential shipper-carrier model in which the shippers are cost-minimization agents who decide on the purchase location, the products, and the carrier to transport the products to their destinations, after which the carriers make routing decisions in response to the shippers. Hurley and Petersen [12] presented an equilibrium solution approach for the freight network problem in a system consisting of multiple profit maximizing shippers and carriers, where the carriers decide on tariffs and the shippers determine their production levels to minimize their costs. Altman and Wynter [13] gave an overview of network equilibrium models and discussed pricing issues in transportation and telecommunication networks. King and Topaloglu [14] modeled the freight transportation pricing problem which presents a linear sensitive demand function. This study was then extended by Topaloghlu and Powell [15] for the situation that the transportation demand is uncertain. Topal and Bingöl [16] studied the inventory replenishment problem for a retailer who needs the transportation services in the availability of a truckload (TL) carrier and a less-than-ruckload (LTL) carrier. Ulku and Bookbinder [17] investigated a logistic market with price- and time-sensitive demand, considering that the delivery time is guaranteed by a Third Party Logistics (3PL) provider. Lin and Lee [18] proposed a model to simultaneously determine the zone prices and operational plan for a freight carrier with limited capacity. The carrier aims to maximize his profit while meeting the expected service level and operational requirements.

Recently, many researchers have utilized game theory to analyze the interactions between freight network stockholders and studied the equilibrium flow. A dynamic freight network assignment model was developed by Agrawal and Ziliaskopoulos [19] in which the shippers choose the carrier with the lowest cost and the market reaches the equilibrium when no shipper can reduce his costs by changing the carriers for a shipment. They applied a variational inequality method to obtain the Nash equilibrium point. Xiao and Yang [20] addressed the equilibrium flow in a system consisting of shippers, carriers, and infrastructure companies as profit maximizing agents. They considered the shipper as a non-cooperative agent, while the carriers and infrastructure companies make their decisions cooperatively. Friesz and Holguin-Veras [21], and Friesz et al. [22] studied a dynamic pricing and freight network equilibrium problem on an urban transportation market consisting of multiple shippers and carriers. They formulated the problem as a differential game and applied a nonlinear complementarity problem to find the equilibrium of the non-cooperative game between the players. Mozafari and Karimi [23] investigated pricing decisions for truckload carriers in a duopoly market, considering two scenarios of non-cooperative and cooperative games. Price and frequency competitions between freight carriers were formulated by Shah and Bruckner [24] using game theoretic models. Naimi Sadigh et al. [25] examined channel pricing problem comparing two scenarios of centralized and decentralized structures using game theory. Mutlu and Cetinkaya [26] analyzed two different structures of a carrier-retailer supply chain with common-carriage option. In the former, the carrier and the retailer simultaneously compete and choose strategies to optimize their own benefit; in the latter, both the carrier and retailer try to maximize the total channel payoff. Lee et al. [27] modeled the oligopolistic behavior of the carriers in maritime fright transportation networks using a multi-level hierarchical game theoretical approach. Mozafari et al. [28] proposed a generalized Nash equilibrium problem to analyze dynamic pricing and fleet planning decisions of oligopolistic freight carriers who compete to win transportation contracts on a network. Nagurney et al. [29] developed both static and dynamic supply chain network models with multiple manufacturers and freight service providers competing on price and quality. They analyzed the Nash equilibrium of the market using variational inequality problem.

However, all the researches above studied the simultaneous decisions of the freight network stockholders, where all the agents have almost the same decision power, and there are comparatively fewer studies which concern the sequential behavior of the shipper-carrier relationship when one of the agents possesses the dominant power in the transportation service chain. Brotcorne et al. [30] focused on freight tariff-setting problem where the leader is one among a group of competing carriers and the follower is the shipper while ignoring the allocation of the freight flows among the rival carriers. Zhang et al. [31] established an equilibrium problem with equilibrium constraint for a container transportation super-network in which the upper level ports are involved in non-cooperative competition, while the lower level shippers compete for the path of minimum cost. Holguín-Veras et al. [32] discussed the theoretical and empirical evidence on the freight mode choice problem, considering the interactions between shippers and carriers. They compared the experiment of perfect cooperation with the setting in that the shipper selects the shipment size as the leader through game theory framework. Friesz et al. [33] studied a dynamic shipper carrier problem in which the carrier acts as the leader and several shippers compete on the sale of products as the followers under the Cournot-Nash behavior. A bi-level modeling approach that captures hierarchical relationships between shippers and carriers in maritime freight transportation networks was proposed by Lee et al. [34] where the carriers are the leaders and shippers are the followers.

This paper addresses the dynamic pricing behavior of the main stockholders in a freight transportation service chain, where a dominant shipper acts as a profit maximizing leader and multiple oligopolistic carriers act as Cournot-Nash followers who try to capture transportation service demand from the transactions between the shipper and spatially separated demand markets. The shipper decides on the production level and the sale's price of a homogeneous product, while the carriers compete on prices and make routing decisions simultaneously to optimize their own profits. Also, all the players' strategies can be changed continuously regarding the time. In other words, there exists continuous-time dynamic equilibrium points for the freight network game while the time value of money is considered. We propose a differential Stackelberg-Nash game using a bi-level programming approach to model the problem in such an environment. Then, the bi-level model is transformed into a single level optimization model by including the equilibrium conditions of the carriers' Nash game at the lower level as a set of constraints for the shipper's model at the upper level. A finite dimensional time discretization method is proposed to approximate the model as a mathematical program with linear constraints. The existence and uniqueness of properties are investigated. Finally, a penalty function algorithm is presented to solve the single level mathematical model.

The rest of the paper is organized as follows: Section 2 describes the problem in detail while declaring the assumptions, notations, variables, and parameters. In Section 3, the differential Stackelberg-Nash pricing game between the shipper and carriers is formulated as a bi-level optimal control problem. In Section 4, we explore the equilibrium conditions at the lower level using a finite dimensional discretization method and the equivalent single level model is formulated. Existence and uniqueness of properties are examined in this section. Section 5 is devoted to numerical study and sensitivity analysis. Finally, concluding remarks are discussed in Section 6.

2. Problem definition

In this section, a detailed description of the problem including the structure of the network, the assumptions, and notations is presented.

2.1. Network structure

We study a freight transportation market structure in which $N+1$ decision makers interact with each other. The set of players includes a dominant shipper who is responsible for producing and shipping a homogeneous commodity to the demand markets within the network and N competing carriers who are providing freight transportation services to the shipper. The transportation network consists of, L, nodes representing the locations of the demand markets, or where the shipper's and carriers' facilities have been placed, and, A, arcs which connect the origin and destination nodes. All the players are profit maximizing agents and try to optimize their own objectives by choosing their decision variables dynamically over a planning horizon.

In the network of Figure 1, it is assumed that the shipper has two production sites at nodes 1 and 6. Each node has a market with a predetermined potential demand for the commodity produced by the shipper.

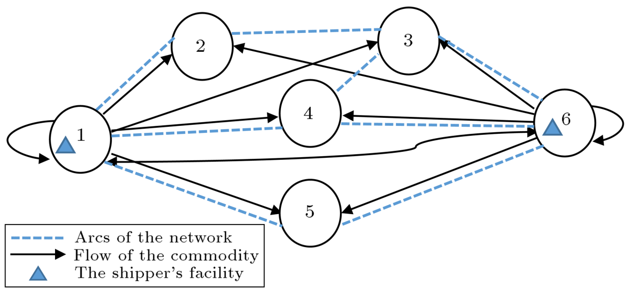

Figure 1. A schematic view of an instance of the network.

Furthermore, there are several carriers which compete to get transportation service demand of the shipper at the shipper's locations by pricing decisions. Each selected carrier makes routing decisions to optimize his benefit. For example, to serve a transportation order from node 1 to node 3, one carrier may choose the route 1-2-3 or 1-4-3.

2.2. Assumptions

The model presented in this paper is based upon the following assumptions:

1. Customers are price sensitive and the quantity demanded at each market will decrease significantly as price increases;
2. The shipper possesses A_s different locations, each producing the same commodity, but with different production capacity in the network;
3. Regardless of the site of shipping the commodity, each market has its own price which could be different from other markets;
4. The shipper as the leader, taking into account the followers' reactions, declares his strategies to optimize his own benefit function over a time-continuous planning horizon;
5. The effect of time value of money is investigated;
6. There is a competition among the followers (i.e. carriers) to achieve more demand from the transactions between the shipper at different production sites and customers at different demand markets;
7. The demand captured by each carrier between origins and destinations is influenced by his own price as well as his competitors' prices;
8. The routing decisions made by the carriers collectively determine the freight flows in the market over the planning horizon;
9. Each carrier is able to hire extra fleets whenever the demand is more than his own fleet, thus the transportation fleet capacity of each carrier is assumed to be unlimited.

2.3. Notations

The notations used to formulize the game theoretical model are listed in the following:

Sets:

L	The set of nodes in the network;
A	The set of arcs connecting nodes in the network;
L_s	The set of locations for the shipper's production facilities;
R_{ij}	The set of paths between origin i and destination j;
$\mathcal{C}$	The set of competing carriers;
T	The planning horizon $T = [t_0, t_f]$.

Shipper's variables:

$p_j(t)$	The price charged by the shipper to demand market placed on node $j \in L$ at time t;
$d_j(t)$	The amount of demand received from market $j \in L$ at time t;
$q_i(t)$	The production quantity at the shipper's facility placed on node $i \in L_S$ at time t;
$I_i(t)$	The inventory held by the shipper at the facility placed on node $i \in L_S$ at time t.

Carriers' variables:

$\varphi^c_{ij}(t)$	The price charged by carrier c to the shipper for providing transportation services between origin $i \in L_S$ and destination $j \in L$ at time t;
$v^c_{ij}(t)$	The flow of shipments received by carrier c for transportation services between origin $i \in L_S$ and destination $j \in L$ at time t;
$h^c_{r,ij}(t)$	The flow of shipments from carrier c on path r between origin $i \in L_S$ and destination $j \in L$ at time t;
$f^c_a(t)$	The flow of shipments from carrier c on arc $a \in A$ in the network at time t.

Parameters:

ρ	The constant nominal discount rate of future cash flows representing the time value of money;
$\psi_i(t)$	The inventory cost of the shipper at location $i \in L_S$ and time t;
$\theta_i(t)$	The production cost of the shipper at location $i \in L_S$ and time t;
Cap_i	The maximum production capacity of the shipper at his facility placed on node $i \in L_S$;
$\bar{D}_j(t)$	The potential market demand at node $j \in L$ and time t;
α_j	The price elasticity of the market demand at node $j \in L$;
$\bar{I}_i$	The initial inventory at node $i \in L_S$;
k^c_a	The travel cost on arc a at time t;
$\delta_{a,r}$	This parameter is equal to 1 if arc a exists in the path r, and 0 otherwise;
γ_c	The elasticity of demand for carrier c to his own price;
$\beta_{c,-c}$	The elasticity of demand for carrier c to his rivals' prices.

3. The differential Stackelberg-Nash game model

In this section, we apply a bi-level modeling approach to formulize the Stackelberg-Nash game of the shipper-carrier problem defined in the previous section. Since the decisions made by the players are changing continuously by time, the corresponding game is a differential Stackelberg-Nash game.

3.1. The shipper's objective functional, dynamics, and constraints

The shipper in the upper level acts as the leader and tries to maximize his net profit over the finite planning horizon, $T = [t_0, t_f]$, considering the followers' reactions. The shipper dynamically determines the price of the commodity charged to each demand market, the quantity produced at each production site, and the quantity shipped from each production site to each market. The quantity shipped to each market is affected by the price of the commodity set for that demand market, as well as the price charged by the carriers for the transportation. The shipper's model is defined as follows:

$$\max_{p,d,q} \int_{t_0}^{t_f} e^{-\rho t}.\Bigg\{\sum_{j\in L} p_j(t).d_j(t) - \sum_{i\in L_S} \psi_i(I_i(t),t) - \sum_{i\in L_S} \theta_i(q_i(t),t) - \sum_{i\in L_S}\sum_{j\in L}\sum_{c\in C} \varphi^c_{ij}(t).v^c_{ij}(t)\Bigg\}dt. \tag{1}$$

Subject to:

$$\frac{dI_i(t)}{dt} = q_i(t) - \sum_{j\in L}\sum_{c\in C} v^c_{ij}(t) \qquad \forall i \in L_s, \tag{2}$$

$$I_i(t_0) = \overline{I_i} \qquad \forall i \in L_s, \tag{3}$$

$$I_i(t) \geq 0 \qquad \forall i \in L_s, \tag{4}$$

$$d_j(t) = \bar{D}_j(t) - \alpha_j.p_j(t) \qquad \forall j \in L, \tag{5}$$

$$0 \leq q_i(t) \leq \mathrm{Cap}_i \qquad \forall i \in L_s, \tag{6}$$

$$p_j(t) \geq 0 \qquad \forall j \in L. \tag{7}$$

Eq. (1) represents the objective functional of the shipper as its revenue minus the inventory, production, and transportation costs, respectively, where the transportation term is determined by the followers' reactions at the lower level. Constraint (2) shows the state dynamics as the changes in the inventory level of each production site $i \in L_S$. Constraint (3) determines the initial level of the inventory at the beginning of the planning horizon $t = t_0$. Constraint (4) ensures positive inventory and prohibits shortage or backlog. The demand is defined as a linear price-sensitive function by Constraint (5). Constraint (6) guarantees the capacity of each production site. Finally, boundaries on decision variables are defined by Constraint (7).

3.2. The carriers' objective functional, dynamics, and constraints

The carriers in the lower level are in oligopolistic Cournot-Nash competition. They simultaneously set the price of transportation services and make the routing decisions for different origins and destinations on the network. This is with respect to the decisions made by the shipper with the goal of maximizing their own net profits over a finite planning horizon, $t \in [t_0, t_f]$. The routing decisions made by the carriers collectively determine the overall freight flow on the network. Each carrier tries to achieve more demand from the transactions between the shipper and his customers at different demand markets. We define the transportation demand for each carrier as a linear function in which the potential demand is influenced by the carrier's own price as well as the competitors' price given as:

$$v^c_{ij}(t) = \bar{D}_j(t) - \gamma_c\varphi^c_{ij}(t) + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi^g_{ij}(t). \tag{8}$$

No shortage or backlog is allowed, thus the whole demand of each market must be shipped by the set of carriers. The sub-model of each carrier is defined as follows:

$$\max_{\varphi,v,h} J^c = \int_{t_0}^{t_f} e^{-\rho t}.\Bigg\{\sum_{i\in L_s}\sum_{j\in L}\varphi^c_{ij}(t).v^c_{ij}(t) - \sum_{a\in A} k^c_a.f^c_a(t)\Bigg\}dt. \tag{9}$$

Subject to:

$$v^c_{ij}(t) = \bar{D}_j(t) - \gamma_c\varphi^c_{ij}(t) + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi^g_{ij}(t) \qquad \forall i \in L_s, \quad \forall j \in L, \tag{10}$$

$$f^c_a(t) = \sum_{i\in L_s}\sum_{j\in L}\sum_{r\in R_{ij}} \delta_{a,r}.h^c_{r,ij}(t) \qquad \forall a \in A, \tag{11}$$

$$\sum_{r\in R_{ij}} h^c_{r,ij}(t) = v^c_{ij}(t) \qquad \forall i \in L_s, \quad \forall j \in L, \tag{12}$$

$$\sum_{c\in C}\sum_{i\in L_s} v^c_{ij}(t) = d_j(t) \qquad \forall i \in N_s, \quad \forall j \in L, \tag{13}$$

$$\varphi^c_{ij}(t) \geq 0, \quad v^c_{ij}(t) \geq 0 \qquad \forall i \in L_s, \quad \forall j \in L, \tag{14}$$

$$f^c_a(t) \geq 0 \qquad \forall a \in A, \tag{15}$$

$$h^c_{r,ij}(t) \geq 0 \qquad \forall i \in L_s, \quad \forall j \in L, \quad \forall r \in R_{ij}. \qquad (16)$$

Eq. (9) represents the objective functional of each carrier as its revenue minus the transportation costs. Constraint (10) shows the transportation demand function. Constraint (11) defines the flow on different arcs of the network. Constraint (12) states that the sum of flows on the paths connecting origin i to destination j must be equal to the total transportation demand between i and j. Constraint (13) ensures that the demand of each market is collectively shipped by the carriers. Finally, Constraints (14) to (16) define the boundaries on the decision variables.

4. The Stackelberg-Nash equilibrium

In this section, we investigate the equilibrium conditions for the proposed differential Stackelberg game between the shipper and the carriers; we explore the existence and uniqueness of properties of the equilibrium.

4.1. The Cournot-Nash price equilibrium condition at the carriers' level

The game among competing carriers at the lower level is a Generalized Nash Equilibrium Problem (GNEP) with joint constraint, in which both the objective function and the constraint set for each player depend on the strategies taken by rival players [35,36]. To find the best responses, we first take the time discretization approach for the optimal control problem of each carrier. The optimal control models (9)-(16) become time discretized by assigning a discrete instant of time $t_z = t_0 + z\Delta t$, where Δt is the length of each time step. The number of time discretization can be calculated by $M = \frac{t_f - t_0}{\Delta t}$ and $t_M = t_f$. In this way, we convert the optimal control problem to a finite dimensional mathematical program for which the optimal solution can be achieved through Karush-Kuhn-Tucker (KKT) optimality conditions. When the optimal control model is a convex model, the discretized solution will be a good approximation of the optimal controls.

Proposition 1. (Convexity) For every carrier ($c \in \mathcal{C}$), the objective function is concave and the feasible set is convex.

Proof. See Appendix A.

A necessary and sufficient condition for a given solution to be an optimal one for each carrier's sub problem is that a suitable constraint qualification holds and there exists Lagrangian multiplier vectors $(\mu^t_j, \lambda^{c,t}_{ij})$ and $(\zeta^{c,t_z}_{ij}, \varpi^{c,t_z}_{r,ij}) \geq \vec{0}$ which satisfy the following system of equations:

$$\begin{aligned}\mathcal{L}^c = \sum_{z=0}^{M} e^{-\rho t_z} \Bigg\{ & \sum_{i\in L_s}\sum_{j\in L} \varphi^{c,t_z}_{ij} \\ & .\Bigg(\bar{D}^{t_z}_j - \gamma_c \varphi^{c,t_z}_{ij} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi^{g,t_z}_{ij}\Bigg) \\ & - \sum_{a\in A} k^c_a. \sum_{i\in L_s}\sum_{j\in L}\sum_{r\in R_{ij}} \delta_{a,r}.h^{c,t_z}_{r,ij} + \sum_{i\in L_s}\sum_{j\in L}\lambda^{c,t_z}_{ij} \\ & .\Bigg(\bar{D}^{t_z}_j - \gamma_c \varphi^{c,t_z}_{ij} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi^{g,t_z}_{ij} - \sum_{r\in R_{ij}} h^{c,t_z}_{r,ij}\Bigg) \\ & + \sum_{j\in L}\mu^{t_z}_j.\Bigg(\bar{D}^{t_z}_j - \alpha_j.p^{t_z}_j - \sum_{i\in L_s}\sum_{c\in C}\Bigg(\bar{D}^{t_z}_j - \gamma_c\varphi^{c,t_z}_{ij} \\ & + \sum_{g\in C-\{c\}}\beta_{c,g}.\varphi^{g,t_z}_{ij}\Bigg)\Bigg) + \sum_{i\in L_s}\sum_{j\in L}\zeta^{c,t_z}_{ij}.\varphi^{c,t_z}_{ij} \\ & + \sum_{i\in L_s}\sum_{j\in L}\sum_{r\in R_{ij}}\varpi^{c,t_z}_{r,ij}.h^{c,t_z}_{r,ij}\Bigg\}. \end{aligned} \qquad (17)$$

$$\begin{aligned}\frac{\partial \mathcal{L}^c}{\partial \varphi^{c,t_z}_{ij}} = e^{-\rho t_z}.\Bigg\{ & \bar{D}^{t_z}_j - 2\gamma_c\varphi^{c,t_z}_{ij} + \sum_{g\in C-\{c\}}\beta_{c,g}.\varphi^{g,t_z}_{ij} \\ & - \gamma_c.\lambda^{c,t_z}_{ij} + \mu^{t_z}_j.\Bigg(\gamma_c - \sum_{g\in C-\{c\}}\beta_{g,c}\Bigg) + \zeta^{c,t_z}_{ij}\Bigg\} = 0, \end{aligned} \qquad (18)$$

$$\frac{\partial \mathcal{L}^c}{\partial h^{c,t_z}_{r,ij}} = e^{-\rho t_z}.\Bigg\{-\sum_{a\in A}\delta_{a,r}.k^c_a + \varpi^{c,t_z}_{r,ij} - \lambda^{c,t_z}_{ij}\Bigg\} = 0, \qquad (19)$$

$$\bar{D}^{t_z}_j - \gamma_c\varphi^{c,t_z}_{ij} + \sum_{g\in C-\{c\}}\beta_{c,g}.\varphi^{g,t_z}_{ij} - \sum_{r\in R_{ij}} h^{c,t_z}_{r,ij} = 0, \qquad (20)$$

$$\begin{aligned}\bar{D}^{t_z}_j - \alpha_j.p^{t_z}_j - \sum_{i\in L_s}\sum_{c\in C}\Bigg(& \bar{D}^{t_z}_j - \gamma_c\varphi^{c,t_z}_{ij} \\ & + \sum_{g\in C-\{c\}}\beta_{c,g}.\varphi^{g,t_z}_{ij}\Bigg) = 0, \end{aligned} \qquad (21)$$

$$\zeta^{c,t_z}_{ij}.\varphi^{c,t_z}_{ij} = 0, \qquad (22)$$

$$\varpi^{c,t_z}_{r,ij}.h^{c,t_z}_{r,ij} = 0, \qquad (23)$$

where Eq. (17) defines the Lagrange function, Eqs. (18) and (19) show stationary conditions, and Eqs. (20) to (23) are complementary slackness equations.

Proposition 2. The Slater constraint qualification holds for every carrier's sub problem.

Proof. See Appendix B.

When the GNEP satisfies the convexity assumption (proposition 1), the vector $(\bar{\varphi}_{ij}^{c,t_z}, \bar{h}_{r,ij}^{c,t_z}, \bar{\mu}_j^t, \bar{\lambda}_{ij}^{c,t}, \bar{\zeta}_{ij}^{c,t_z}, \bar{\varpi}_{r,ij}^{c,t_z})$ which solves the system of equations obtained by concatenating the KKT optimality conditions of all carriers is an equilibrium point of the GNEP [35,37]. According to Proposition 1, $X(-c)$ is defined as the feasible set of each carrier depending on price strategies of the rival carriers, which is convex for every carrier. Therefore, if $X(-c)$ is closed, then there exists at least one Nash equilibrium point for the Cournot GNEP among carriers.

Proposition 3. (Existence property) The GNEP of the carriers has an equilibrium point.

Proof. See Appendix C.

4.2. The equivalent single level optimization problem

A solution for the lower level Nash game can be expressed implicitly as a function of the upper level problem's controls. If the uniqueness of solution for the lower level problem is guaranteed, we can transform the bi-level differential problem into an equivalent single level dynamic model by adding the optimality conditions of the lower level Nash game to the shipper's model constraints set.

Proposition 4. (Uniqueness of property) The equilibrium point of GNEP among rival carriers is unique.

Proof. See Appendix D.

As it is shown in Propositions 3 and 4, there exists a unique equilibrium point for the followers' GNEP at the lower level. Therefore, we can convert the bi-level dynamic Stackelberg-Nash game between shipper and carriers to the following time discretized single level dynamic optimization problem:

$$\max_{p,q,\varphi,h} \hat{J}_s \sum_{z=0}^{M} e^{-\rho t_z}.\Bigg\{\sum_{j\in L} p_j^{t_z}.\left(\bar{D}_j^{t_z} - \alpha_j.p_j^{t_z}\right) - \sum_{i\in L_S} \psi_i^{t_z}.I_i^{t_z} - \sum_{i\in L_S} \theta_i^{t_z}.q_i^{t_z} - \sum_{i\in L_S}\sum_{j\in L}\sum_{c\in\mathcal{C}} \varphi_{ij}^{c,t_z} .\left(\bar{D}_j^{t_z} - \gamma_c\varphi_{ij}^{c,t_z} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi_{ij}^{g,t_z}\right)\Bigg\}. \tag{24}$$

Subject to:

$$I_i^{t_z} = I_i^{t_z-1} + q_i^{t_z} - \sum_{j\in L}\sum_{c\in C}(\bar{D}_j^{t_z} - \gamma_c\varphi_{ij}^{c,t_z} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi_{ij}^{g,t_z}) \quad \forall i\in L_s, \quad \forall z\in\{0,1,\cdots,M\}, \tag{25}$$

$$I_i^{t_0} = \overline{I_i} \qquad \forall i\in L_s, \quad \forall z\in\{0,1,\cdots,M\}, \tag{26}$$

$$\bar{D}_j^{t_z} - \alpha_j.p_j^{t_z} \geq 0 \quad \forall j\in L, \quad \forall z\in\{0,1,\cdots,M\}, \tag{27}$$

$$0 \leq q_i^{t_z} \leq \mathrm{Cap}_i \quad \forall i\in L_s, \quad \forall z\in\{0,1,\cdots,M\}, \tag{28}$$

$$p_j^{t_z} \geq 0 \qquad \forall j\in L, \quad \forall z\in\{0,1,\cdots,M\}, \tag{29}$$

$$I_i^{t_z} \geq 0 \qquad \forall i\in L_s, \quad \forall z\in\{0,1,\cdots,M\}, \tag{30}$$

$$\bar{D}_j^{t_z} - 2\gamma_c\varphi_{ij}^{c,t_z} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi_{ij}^{g,t_z} - \gamma_c.\lambda_{ij}^{c,t_z} + \mu_j^{t_z}.\left(\gamma_c - \sum_{g\in C-\{c\}} \beta_{g,c}\right) + \zeta_{ij}^{c,t_z} = 0 \quad \forall i\in L_s, \quad \forall j\in L, \quad \forall c\in\mathcal{C}, \quad \forall z\in\{0,1,\cdots,M\}, \tag{31}$$

$$\varpi_{r,ij}^{c,t_z} - \sum_{a\in A} \delta_{a,r}.k_a^c - \lambda_{ij}^{c,t_z} = 0 \quad \forall i\in L_s, \quad \forall j\in L, \quad \forall r\in R_{ij}, \quad \forall c\in\mathcal{C}, \quad \forall z\in\{0,1,\cdots,M\}, \tag{32}$$

$$\bar{D}_j^{t_z} - \gamma_c\varphi_{ij}^{c,t_z} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi_{ij}^{g,t_z} - \sum_{r\in R_{ij}} h_{r,ij}^{c,t_z} = 0 \quad \forall i\in L_s, \quad \forall j\in L, \quad \forall c\in\mathcal{C}, \quad \forall z\in\{0,1,\cdots,M\}, \tag{33}$$

$$\bar{D}_j^{t_z} - \alpha_j.p_j^{t_z} - \sum_{i\in L_s}\sum_{c\in C}\left(\bar{D}_j^{t_z} - \gamma_c\varphi_{ij}^{c,t_z} + \sum_{g\in C-\{c\}} \beta_{c,g}.\varphi_{ij}^{g,t_z}\right) = 0 \quad \forall j\in L, \quad \forall c\in\mathcal{C}, \quad \forall z\in\{0,1,\cdots,M\}, \tag{34}$$

$$\zeta_{ij}^{c,t_z}.\varphi_{ij}^{c,t_z} = 0 \qquad \forall i\in L_s, \quad \forall j\in L, \quad \forall c\in\mathcal{C}, \quad \forall z\in\{0,1,\cdots,M\}, \tag{35}$$

$$\varpi_{r,ij}^{c,t_z}.h_{r,ij}^{c,t_z} = 0 \qquad \forall i\in L_s, \quad \forall j\in L, \quad \forall c\in\mathcal{C}, \quad \forall r\in R_{ij}, \quad \forall z\in\{0,1,\cdots,M\}, \tag{36}$$

$$\zeta_{ij}^{c,t_z} \geq 0, \quad \varphi_{ij}^{c,t_z} \geq 0, \quad \varpi_{r,ij}^{c,t_z} \geq 0, \quad h_{r,ij}^{c,t_z} \geq 0. \tag{37}$$

Note that the state variables $I_i^{t_z}$ in the discretized model are intermediate variables and can be easily replaced by the control variables $q_i^{t_z}$ and φ_{ij}^{c,t_z} as follows:

$$I_i^{t_z} = I_i^{t_0} + \sum_{n=1}^{z} \left\{ q_i^{t_n} - \sum_{j \in L} \sum_{c \in C} \left(\bar{D}_j^{t_n} - \gamma_c \varphi_{ij}^{c,t_n} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_n} \right) \right\}. \tag{38}$$

Since setting the transportation price equal to zero makes the objective function of the carrier negative, we can be sure that at the equilibrium point of the Nash game, all the carriers choose non-zero transportation prices and $\varphi_{ij}^{c,t_z} \geq 0$. Hence, we can fix the Lagrange multiplier, $\zeta_{ij}^{c,t_z} = 0$, and remove Constraint (34). According to Propositions 3 and 4, the GNEP of the carriers at the lower level has a unique equilibrium solution for any given $(\bar{p}_j^{t_z}, \bar{q}_j^{t_z})$ from the shipper, and also the shipper's objective function is concave and the feasible set is convex regarding the shipper's decision variables (see Appendix E for the proof). Then, it is concluded that the Stackelberg-Nash problem has a unique equilibrium point. This equilibrium can be obtained by solving the model (24)-(37).

4.3. Penalty function method

To handle nonlinear constraints, we employ a penalty function method which appends the nonlinear constraints to the objective function. It assigns a penalty when a given nonlinear constraint is violated and approximates the solution of the original problem [38]. So in this case, a penalty vector:

$$\Theta = (\eta_{r,ij}^{c_1,t_z}, \eta_{r,ij}^{c_2,t_z}, \cdots, \eta_{r,ij}^{c_N}, t_z),$$

is defined, where $\eta_{r,ij}^{c,t_z} \in \mathbb{R}_+$ is a large number. When $\eta_{r,ij}^{c,t_z}$ approaches infinity, the approximation becomes increasingly accurate. Then, we can reformulate the model (24) to (36) as a nonlinear programming model with linear constraints as follows:

$$\begin{aligned} \max_{p,q,\varphi,h} \hat{J}_s \sum_{z=0}^{M} e^{-\rho t_z} . \Bigg\{ & \sum_{j \in L} p_j^{t_z} . \left(\bar{D}_j^{t_z} - \alpha_j . p_j^{t_z} \right) \\ & - \sum_{i \in L_S} \psi_i^{t_z} . \left(I_i^{t_0} + \sum_{n=1}^{z} \left\{ q_i^{t_n} - \sum_{j \in L} \sum_{c \in C} \left(\bar{D}_j^{t_n} - \gamma_c \varphi_{ij}^{c,t_n} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_n} \right) \right\} \right) - \sum_{i \in L_S} \theta_i^{t_z} . q_i^{t_z} \\ & - \sum_{i \in L_S} \sum_{j \in L} \sum_{c \in \mathcal{C}} \varphi_{ij}^{c,t_z} . \left(\bar{D}_j^{t_z} - \gamma_c \varphi_{ij}^{c,t_z} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} \right) \\ & - \sum_{c \in \mathcal{C}} \sum_{i \in L_S} \sum_{j \in L} \sum_{r \in R_{ij}} \eta_{r,ij}^{c,t_z} . (\varpi_{r,ij}^{c,t_z} . h_{r,ij}^{c,t_z}) \Bigg\}. \end{aligned} \tag{39}$$

Subject to:

$$\bar{D}_j^{t_z} - \alpha_j . p_j^{t_z} \geq 0 \quad \forall j \in L, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{40}$$

$$0 \leq q_i^{t_z} \leq \mathrm{Cap}_i \quad \forall i \in L_s, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{41}$$

$$p_j^{t_z} \geq 0 \quad \forall j \in L, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{42}$$

$$I_i^{t_0} + \sum_{n=1}^{z} \left\{ q_i^{t_n} - \sum_{j \in L} \sum_{c \in C} \left(\bar{D}_j^{t_n} - \gamma_c \varphi_{ij}^{c,t_n} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_n} \right) \right\} \geq 0 \quad \forall i \in L_s, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{43}$$

$$\bar{D}_j^{t_z} - 2\gamma_c \varphi_{ij}^{c,t_z} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} - \gamma_c . \lambda_{ij}^{c,t_z} + \mu_j^{t_z} . \left(\gamma_c - \sum_{g \in C - \{c\}} \beta_{g,c} \right) + \zeta_{ij}^{c,t_z} = 0$$
$$\forall i \in L_s, \quad \forall j \in L, \quad \forall c \in \mathcal{C}, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{44}$$

$$\varpi_{r,ij}^{c,t_z} - \sum_{a \in A} \delta_{a,r} . k_a^c - \lambda_{ij}^{c,t_z} = 0 \qquad \forall i \in L_s, \quad \forall j \in L,$$
$$\forall r \in R_{ij}, \quad \forall c \in \mathcal{C}, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{45}$$

$$\bar{D}_j^{t_z} - \gamma_c \varphi_{ij}^{c,t_z} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} - \sum_{r \in R_{ij}} h_{r,ij}^{c,t_z} = 0$$
$$\forall i \in L_s, \quad \forall j \in L, \quad \forall c \in \mathcal{C}, \quad \forall z \in \{0, 1, \cdots, M\}, \tag{46}$$

$$\bar{D}_j^{t_z} - \alpha_j . p_j^{t_z} - \sum_{i \in L_s} \sum_{c \in C} \left(\bar{D}_j^{t_z} - \gamma_c \varphi_{ij}^{c,t_z} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} \right) = 0$$
$$\forall j \in L, \quad \forall c \in \mathcal{C}, \ \forall z \in \{0, 1, \cdots, M\}, \tag{47}$$

$$\varphi_{ij}^{c,t_z} \geq 0, \quad \varpi_{r,ij}^{c,t_z} \geq 0, \quad h_{r,ij}^{c,t_z} \geq 0. \tag{48}$$

We employ an initial penalty parameter vector and iteratively increase the penalty parameters until the algorithm converges. In each iteration, we apply an optimization technique to solve the model (39) to (48) by starting from the optimum solution of the previous iteration. The steps of the penalty function method are listed below:

- **Step 1:** Set the iteration counter $K = 0$, choose an initial solution $S^0 = (p_j^{t_z^0}, q_i^{t_z^0}, \varphi_{ij}^{c,t_z^0}, h_{r,ij}^{c,t_z^0}, \varpi_{r,ij}^{c,t_z^0})$ which is the optimal solution of the model (24) to (37), ignore Constraint (36). Also, set the initial values for penalty parameters vector, Θ^0;
- **Step 2:** Solve the model (39) to (48) and set $S^{K+1} = (p_j^{t_z*}, q_i^{t_z*}, \varphi_{ij}^{c,t_z*}, h_{r,ij}^{c,t_z*}, \varpi_{r,ij}^{c,t_z*})$;
- **Step 3.** If the penalty function is equal to or less than a predefined $\varepsilon > 0$, i.e.:

$$\sum_{c\in C}\sum_{i\in L_S}\sum_{j\in L}\sum_{r\in R_{ij}} \eta_{r,ij}^{c,t_z^K}.(\varpi_{r,ij}^{c,t_z K+1}.h_{r,ij}^{c,t_z K+1}) \leq \varepsilon,$$

then stop, the optimal solution is found, otherwise set:

$$\eta_{r,ij}^{c,t_z K+1} = \sigma.\eta_{r,ij}^{c,t_z^K}, \quad \forall i \in L_s, \quad j \in L, \quad r \in R_{ij},$$

$$c \in C, \quad z \in \{0, 1, \cdots, M\},$$

where $\sigma > 1$. Also, set $K = K + 1$ and go to Step 2.

To solve the mathematical model in Step 2, we employ a multi-start global optimization algorithm in GAMS software using the MINOS solver. Bringing together all we discussed, the penalty function algorithm will converge to the equilibrium point of the Stackelberg-Nash game among a dominant shipper and multiple competing carriers.

5. Numerical results and sensitivity analysis

In this section, we present several examples to demonstrate the applicability of the proposed differential Stackelberg-Nash game model and examine the effectiveness of the proposed solution approach. Furthermore, we conduct a comprehensive sensitivity analysis on the main parameters, which illustrates some important features of the model and highlights several managerial aspects.

In order to solve each example, we transform the bi-level differential Stackelberg-Nash game into a single level optimization model by including the equilibrium conditions of the lower level Nash game in the shipper's problem as a set of constraints. A finite dimensional time discretization and a penalty function method are then applied to approximate the model as a mathematical program with linear constraints. Finally, the penalty function algorithm is coded in GAMS software solved using the MINOS solver as a multi start global optimization tool.

Example 1. Consider a network consisting of 8 nodes and 13 bidirectional arcs. There is one dominant shipper who possesses two production sites with limited capacities over the nodes 3 and 8. Also, there are two freight carriers who serve transportation demands from the shipper's nodes to customers' nodes on the network. The carriers compete with each other by setting their pricing decisions simultaneously in an oligopolistic market. Every node on the network is assumed as a particular demand market for the commodity produced by the shipper. Figure 2 illustrates the schematic network of the example.

The planning time interval is considered to be [0,10]. As can be seen in Figure 2, the origin and destination of a commodity flow may be connected through different routes. The relationships between arcs and routes for O-D pairs are summarized in Table 1. The transportation costs for carrier c on different network arcs are generated randomly from the uniform distribution $k_a^c \sim U(1, 10)$. Table 2 shows the values of the other parameters used in this example.

In the Stackelberg-Nash equilibrium point, the Net Present Value (NPV) of benefit for the shipper is $J_s^* = 67655161.32$, while the carriers c_1 and c_2 gain net present values of benefit equal to $J_{c_1}^* = 14065980.17$ and $J_{c_2}^* = 7956465.29$, respectively. Figure 3 shows the inventory and production trajectories for the shipper at different production sites. As it can be seen, the trajectories start from the initial inventory levels $\overline{I_i}$ at time t_0, then the shipper gradually reduces his

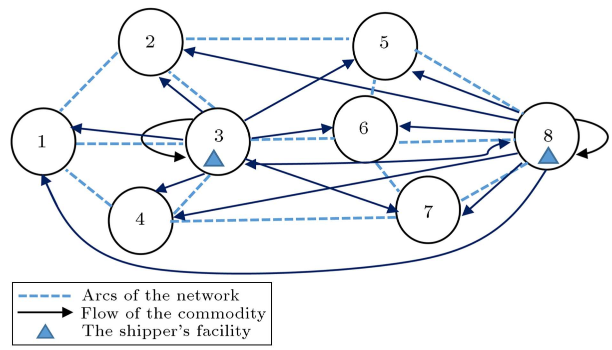

Figure 2. Transportation network structure of Example 1.

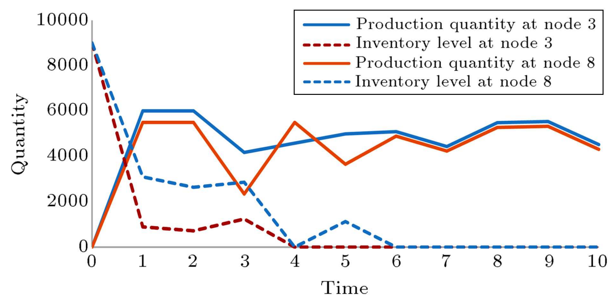

Figure 3. Inventory and production trajectories ($I_i^*(t)$ and $q_i^*(t)$) of the shipper at production nodes.

Table 1. Route-arc relationship for O-D pairs.

O-D pairs	R_{ij}s' arc sequences	O-D pairs	R_{ij}s' arc sequences
(3,1)	{3.1}	(8,1)	{8.6,6.3,3.1},{8.5,5.2,2.1},{8.7,7.4,4.1}
(3,2)	{3.2}	(8,2)	{8.5,5.2}
(3,3)	{3.3}	(8,3)	{8.6,6.3}
(3,4)	{3.4}	(8,4)	{8.7,7.4}
(3,5)	{3.2,2.5},{3.6,6.5}	(8,5)	{8.5}
(3.6)	{3.6}	(8.6)	{8.6}
(3.7)	{3.4,4.7},{3.6,6.7}	(8.7)	{8.7}

inventory levels to keep the inventory holding cost low and meet the terminal inventory condition $I_i^{t_f} = 0$. In contrast, the production quantity levels at different production sites start from zero at time t_0 to allow the shipper to sell off his existing inventories, and then increase to meet the quantity of commodity demanded by different markets up to predefined production capacities Cap_i. The capacity constraints force the shipper to produce in advance for future demand, when the quantity demanded in future will be more than the available production capacity.

Figures 4 illustrates the price trajectories for the commodity sold to different demand nodes and the realized demand received by the shipper from each market. As expected, the demand market at node 8 received higher prices from the shipper, because its demand is less price sensitive regarding the parameter $\alpha_8 = 1$, and there is no competitor for the shipper at the market; however, the quantity demanded from node 8 is about the demand from other nodes.

Table 2. Values of the parameters.

Parameter	Value	Parameter	Value
α_j	$\sim U(1,5)$	ρ	0.01
$\bar{D}_j^t$	$\sim U(1000,5000)$	γ_{c_1}	15
θ_i^t	$\sim U(8,30)$	γ_{c_2}	17
ψ_i^t	$0.2 * \theta_i^t$	β_{c_1,c_2}	0.4
$\text{Cap}_{i(i=3,8)}$	{6000,5500}	β_{c_2,c_1}	0.6
$\bar{I}_{i(i=3,8)}$	{9000,9000}	Δ	1

The transportation price trajectories for different Origin-Destinations (ODs) and the allocations of demand to freight carriers in the equilibrium point of the GNEP are illustrated in response to the shipper's actions in Figures 5 and 6. To enhance clarity of these figures, the transportation demands originated from node 8 are ignored.

As can be observed in Figures 5 and 6, carrier c_1 offers higher prices and captures more demand from transactions between the shipper and the demand markets. The reason is the lower elasticity to own price as well as the rival's price that carrier c_1 has. The lower price elasticity factor typically denotes the higher reputation for the carrier in the market, which enables him to set higher prices, get more transportation demand, and gain more benefit at the end of the planning horizon. In the following, we present several examples to analyze the sensitivity to the main parameters involved in the proposed model.

In order to investigate the sensitivity of the Stackelberg-Nash equilibrium to the price elasticity parameter of the demand nodes, we change parameter α_j at the interval $[0.6\alpha_j, 1.5\alpha_j]$ for Example 1, while all the other parameters are fixed.

As it can be seen in Figure 7, when α_j increases, the quantity demanded by the shipper decreases; the

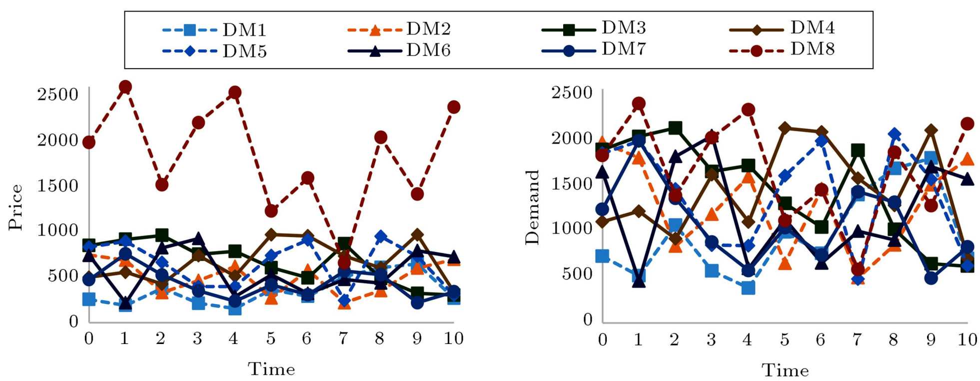

Figure 4. Price and demand trajectories ($p_j^*(t)$ and $d_j^*(t)$) of the shipper's commodity for different markets.

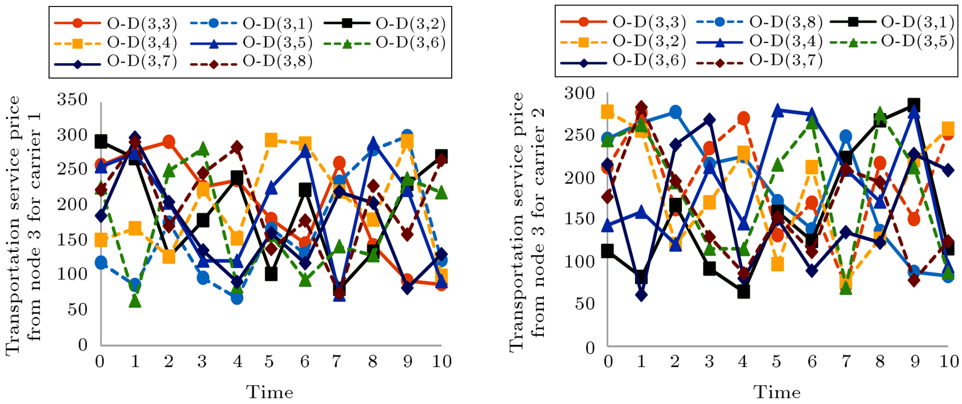

Figure 5. Transportation price trajectories, $\varphi_{ij}^{c*}(t)$, between different O-D pairs for the two carriers.

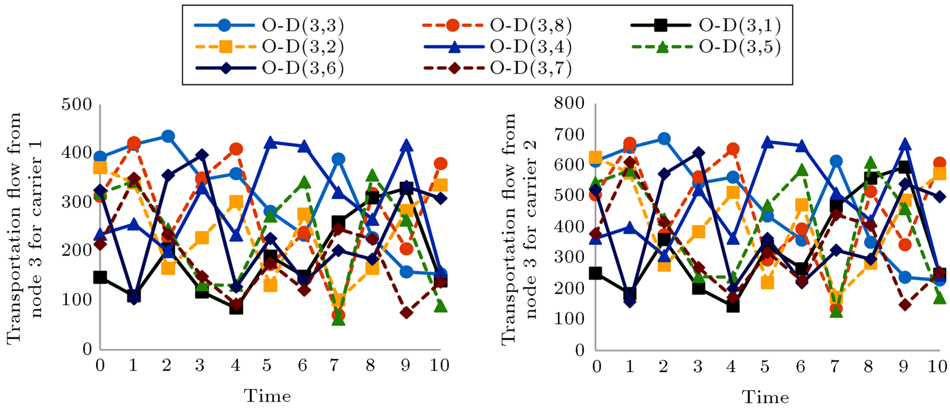

Figure 6. Transportation flow trajectories, $v_{ij}^{c*}(t)$, on different O-D pairs for the two carriers.

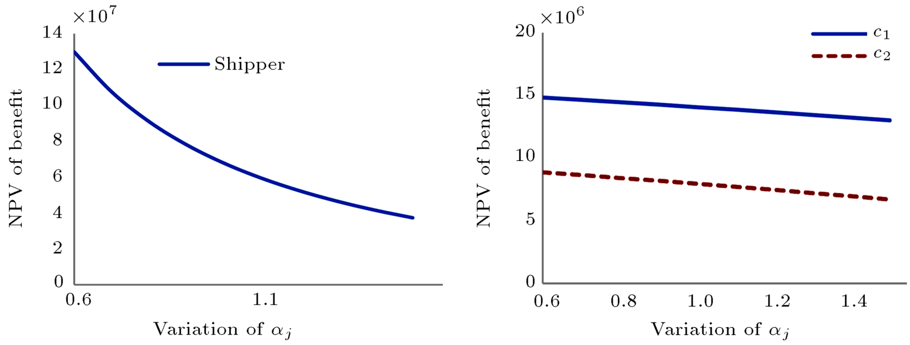

Figure 7. The impact of α_j on the players' benefit in the equilibrium point.

shipper chooses lower rates to keep his demand level, and his benefit will decline. On the other side, both carriers gain degraded demand according to the smaller market of the shipper. We have enhanced α_j up to 50% and observed that carriers c_1 and c_2 lose up to 8% and 16% of their benefits, respectively. The reason is the lower pricing power which carrier c_2 possesses in the freight transportation market regarding parameter γ_{c_2}.

To investigate the sensitivity of the Stackelberg-Nash equilibrium to the potential demand factor, we change parameter $\bar{D}_j^t$ at the interval $[0.6\bar{D}_j^t, 1.5\bar{D}_j^t]$ in Example 1, while all the other parameters are fixed.

As it is observed in Figure 8, the shipper chooses higher prices when he faces with a larger potential market demand and he gains more benefit at the end of the planning horizon. Since the carriers' market is defined by the shipper's transactions, the realized demand for both carriers' increases for a larger $\bar{D}_j^t$.

Here, we examine whether any changes in each carrier's own price elasticity can influence the equilibrium point of the game or not. We fix γ_{c_1} at its value in Example 1 and alter γ_{c_2} at the interval $[0.6\gamma_{c_2}, 1.5\gamma_{c_2}]$. A growth in parameter γ_{c_2} implies a reduction in the pricing power of carrier c_2. Hence, the lower pricing

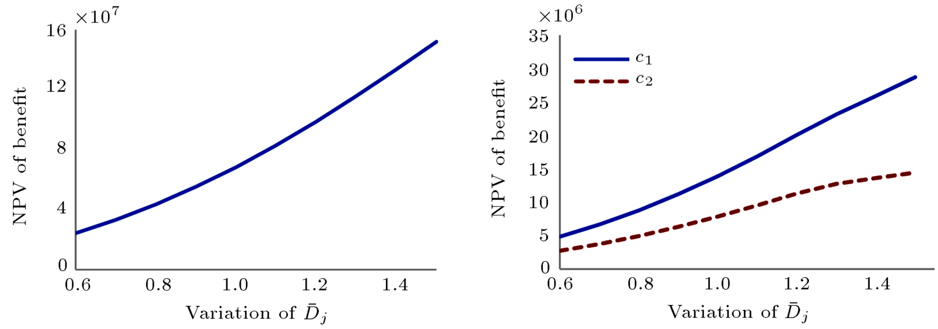

Figure 8. The impact of $\bar{D}_j^t$ on the players' benefit in the equilibrium point.

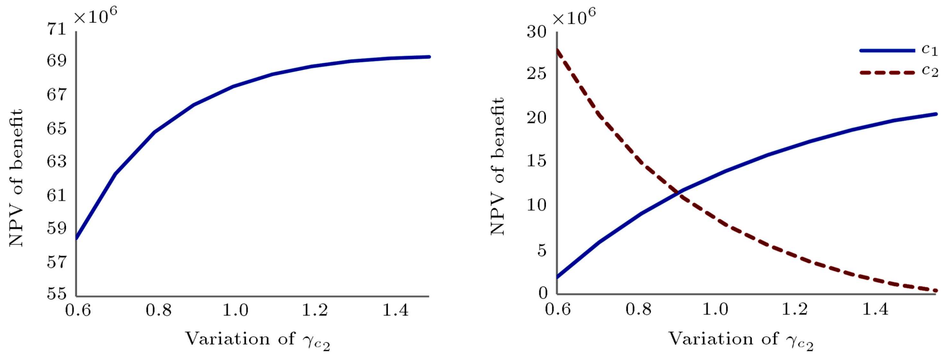

Figure 9. The impact of γ_c on the players' benefit in the equilibrium point.

power carrier c_2 has, the lower prices he may choose and the less benefit he will gain. On the other side, by decreasing the pricing power of c_2, c_1 becomes much powerful in the market, captures more demand from the shipper's transactions, and gets more benefit. When γ_{c_2} approaches γ_{c_1}, carriers achieve almost the same demand shares (Figure 9).

We examine whether any changes in the rivals' price elasticity can influence the equilibrium point of the game. We fix β_{c_2,c_1} at its value in Example 1 and change β_{c_1,c_2} at the interval $[1/5\beta_{c_1,c_2}, 5\beta_{c_1,c_2}]$. Regarding the demand function defined in Eq. (8), a larger β_{c_1,c_2} implies that a small increment in the prices set by carrier c_2 leads to a significant enhancement in the transportation service demand, and consequently to a higher benefit for carrier c_1. Since the carriers compete for a constant demand defined by the shipper; the demand for carrier c_1 will decrease as the demand for carrier c_2 increases. Therefore, carrier c_2 is encouraged to reduce his prices for keeping his demand share, therefore, he will gain lower benefit (Figure 10).

Example 2. In order to investigate the impact of the competitive environment on the equilibrium point of the game, here we add a new rival carrier servicing on the network of Example 1. Since the new carrier has lower reputation in the market, we assume higher price elasticity for him:

$$\gamma_{c_3} = 19 \quad \text{and} \quad \beta_{c,g} = \begin{pmatrix} 0 & 0.4 & 0.4 \\ 0.6 & 0 & 0.6 \\ 0.7 & 0.7 & 0 \end{pmatrix}.$$

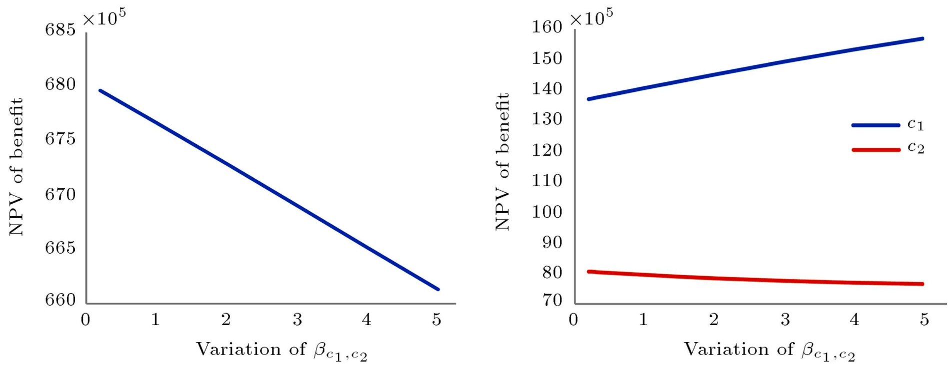

Figure 10. The impact of β_{c_1,c_2} on the players' benefit in the equilibrium point.

In the Stackelberg-Nash equilibrium point of Example 2, the shipper achieves $J_s^* = 66865521.04$, while the carriers c_1, c_2, and c_3 gain $J_{c_1}^* = 13718644.67$, $J_{c_2}^* = 7486998.65$, and $J_{c_3}^* = 1523108.71$, respectively. As it is expected, when a new carrier is added to the market, a certain demand share will be allocated to him regarding his prices. Thus, the previous carriers may lose some demand and their benefit will decrease consequently.

6. Concluding remarks and future work

This paper addressed a shipper-carrier problem in which a dominant shipper aims at selling a homogeneous commodity to several spatially separated markets with price sensitive demand and pursues the goal of maximizing his benefit over a finite planning horizon. The shipper requires transportation services to deliver goods to his customers, and multiple oligopolistic carriers compete to capture these service demands on a geographic network. The freight carriers dynamically set their prices for origin-destination services and make their routing decisions to gain more transportation demand from the transactions between the shipper and his customers. All the players' strategies can be changed dynamically regarding the time; there exists continuous-time dynamic equilibrium for the freight network game while the time value of money is considered.

The problem has been formulated as a differential Stackelberg-Nash game to find the equilibrium price trajectories and routing decisions over the planning horizon. A finite dimensional discretization approach has been applied to append the equilibrium conditions of the carriers to the shipper's model and to transform the bi-level model to an equivalent single level mathematical program; a penalty function algorithm has been proposed to solve the resulting model. Some numerical studies have been done to show how the mathematical model and the proposed solution approach can approximate the equilibrium trajectories of the differential Stackelberg-Nash game of the shipper-carrier problem. Finally, a comprehensive sensitivity analysis has been conducted on the critical parameters and some managerial highlights have been discussed. For future research, one can consider nonlinear demand functions and the uncertainty of the demand function parameters. Furthermore, investigating other decisions of the carriers such as mode choice or fleet assignment are worthwhile.

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Appendix A

Time discretizing of each carrier's optimal control problem and substituting Eqs. (10) and (11) into Eq. (9) give that:

$$\hat{J}_c = \sum_{z=0}^{M} e^{-\rho t_z} . \Bigg\{ \sum_{i \in L_s} \sum_{j \in L} \varphi_{ij}^{c,t_z} . \Bigg(\bar{D}_j^{t_z} - \gamma_c \varphi_{ij}^{c,t_z} + \sum_{g \in C - \{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} \Bigg) - \sum_{a \in A} k_a^c . \sum_{i \in L_s} \sum_{j \in L} \sum_{r \in R_{ij}} \delta_{a,r} . h_{r,ij}^{c,t_z} \Bigg\}. \quad (A.1)$$

If a function f is convex, then $-f$ is concave [39]. The Gradient of $-\hat{J}_c(\varphi_{ij}^{c,t_z}, h_{r,ij}^{c,t_z})$ can be calculated as follows:

$$\nabla\left(-\hat{J}_c\left(\varphi_{ij}^{c,t_z}, h_{r,ij}^{c,t_z}\right)\right) = \begin{bmatrix} -\frac{\partial \hat{J}_c(\varphi,h)}{\partial \varphi} \\ \\ -\frac{\partial \hat{J}_c(\varphi,h)}{\partial h} \end{bmatrix}$$

$$= \begin{bmatrix} -\left(\bar{D}_j^{t_z} - 2\gamma_c \varphi_{ij}^{c,t_z} + \sum\limits_{g\in C-\{c\}} \beta_{c,g}.\varphi_{ij}^{g,t_z}\right) \\ \\ \sum\limits_{a\in A} \delta_{a,r}.k_a^c \end{bmatrix}. \tag{A.2}$$

The Hessian of $-\hat{J}_c(\varphi_{ij}^{c,t_z}, h_{r,ij}^{c,t_z})$ can be given as:

$$H\left(-\hat{J}_c\left(\varphi_{ij}^{c,t_z}, h_{r,ij}^{c,t_z}\right)\right) = \begin{bmatrix} -\frac{\partial^2 \hat{J}_c(\varphi,h)}{\partial \varphi^2} & -\frac{\partial^2 \hat{J}_c(\varphi,h)}{\partial \varphi \partial h} \\ \\ \frac{-\partial^2 \hat{J}_c(\varphi,h)}{\partial h \partial \varphi} & \frac{-\partial^2 \hat{J}_c(\varphi,h)}{\partial h^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2\gamma_c & 0 \\ 0 & 0 \end{bmatrix}. \tag{A.3}$$

Then, we have:

$$(\varphi, h).\left[H\left(-\hat{J}_c(\varphi,h)\right)\right].(\varphi,h)^T$$

$$= (\varphi, h).\begin{bmatrix} 2\gamma_c & 0 \\ 0 & 0 \end{bmatrix}.(\varphi,h)^T = 2\gamma_c\varphi^2 \geq 0. \tag{A.4}$$

Thus, the Hessian matrix, $H(-\hat{J}_c(\varphi_{ij}^{c,t_z}, h_{r,ij}^{c,t_z}))$, is positive-semi-definite, $-\hat{J}_c$ is convex, and consequently $\hat{J}_c$ is concave. Since all the constraints are linear, the feasible set of each carrier is convex and the proof is completed.□

Appendix B

If the feasible set is convex and there exists a feasible point such that every inequality constraint is satisfied strictly, then the Slater's constraint qualification holds, [39].

As it is shown in Proposition 1, the feasible set for every carrier is convex. Moreover, we can easily define a price vector $\bar{\varphi}_{ij}^{t_z} = (\bar{\varphi}_{ij}^{c_1,t_z}, \bar{\varphi}_{ij}^{c_2,t_z}, \cdots, \bar{\varphi}_{ij}^{c_N,t_z}) = (\varepsilon, \varepsilon, \cdots, \varepsilon)$, where ε is a small positive number, then the transportation demand will be divided between rival carriers and $v_{ij}^c(t) > 0$, $\forall c \in \mathcal{C}$. In addition, each carrier is able to decompose his own origin-destination transportation demand into the possible paths, then we have $h_{r,ij}^c(t) > 0$, $\forall r \in R_{ij}$ and consequently $f_a^c(t) > 0$, $\forall a \in A$. Therefore, the interior feasible set of every carrier is nonempty with regard to the rival carriers' action sets, and the Slater's constraint qualification holds. So, the proof is completed.□

Appendix C

We can easily define conceptual upper bounds for the decision variables. According to Eq. (8), if $\varphi_{ij}^{c,t}$ goes to infinity, the transportation demand for carrier c will be negative which is not allowed; thus, the variable is bounded from above and we have $0 \leq \varphi_{ij}^{c,t} \leq \bar{\varphi}$. In addition, according to Eqs. (12) and (13), it is concluded that $h_{r,ij}^{c,t}$ and $v_{ij}^{c,t}$ are bounded from above and we have $0 \leq h_{r,ij}^{c,t} \leq v_{ij}^{c,t}$ and $0 \leq v_{ij}^{c,t} \leq d_j$. Since:

$$f_a^{c,t} = \sum_{i\in L_s}\sum_{j\in L}\sum_{r\in R_{ij}} \delta_{a,r}.h_{r,ij}^{c,t},$$

then $f_a^{c,t}$ is also a bounded variable. Thereby, the feasible set of every carrier is closed and the proof is completed.□

Appendix D

Defining a functional vector $F(x)$ on X, such that:

$$F(x) := \left(\nabla_{x^c}\hat{J}_c(x)\right)_{c=1}^{c=N},$$

and X is the joint feasible set of all players, if $F(x)$ is monotone, then the GNEP has a unique equilibrium point which can be obtained by concatenating KKT system of equations for all the players [35].

According to Facchinei and Kanzow [35], for a maximization problem, $F(x)$ will be monotone if for any two $x, y \in X$, it holds that:

$$\langle (F(x) - F(y)), \qquad (x - y)\rangle \leq 0. \tag{D.1}$$

For the GNEP of the carriers, $F(x)$ can be formed as:

$$F(x) = \begin{pmatrix} \bar{D}_j^{t_z} - 2\gamma_{c_1}\varphi_{ij}^{c_1}, t_z + \sum\limits_{g\in C-\{c_1\}}^{\beta_{c_1},g} .\varphi_{ij}^{g,t_z} \\ \vdots \\ \bar{D}_j^{t_z} - 2\gamma_{c_N}\varphi_{ij}^{c_N,t_z} + \sum\limits_{g\in C-\{c_N\}}^{\beta_{c_N},g} .\varphi_{ij}^{g,t_z} \\ \\ -\sum\limits_{a\in A}\delta_{a,r}.k_a^{c_1} \\ \vdots \\ -\sum\limits_{a\in A}\delta_{a,r}.k_a^{c_N} \end{pmatrix}. \tag{D.2}$$

Let $S' = (\varphi_{ij}^{c,t'_z}, h_{r,ij}^{c,t'_z})$ and $S'' = (\varphi_{ij}^{c,t''_z}, h_{r,ij}^{c,t''_z})$ be two solutions in X, then for Inequality (D.1), we have:

$$\langle (F(S') - F(S'')), (S' - S'') \rangle$$
$$= \sum_{z=0}^{M} e^{-\rho t_z} . \sum_{c\in\mathcal{C}} \sum_{i\in L_S} \sum_{j\in L} \left(\left(\bar{D}_j^{t_z} - 2\gamma_c \varphi_{ij}^{c,t'_z} + \sum_{g\in C-\{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} \right) - \left(\bar{D}_j^{t_z} - 2\gamma_c \varphi_{ij}^{c,t''_z} + \sum_{g\in C-\{c\}} \beta_{c,g} . \varphi_{ij}^{g,t_z} \right) \right) . (\varphi_{ij}^{c,t'_z} - \varphi_{ij}^{c,t''_z})$$
$$- \sum_{i\in L_S} \sum_{j\in L} \sum_{r\in R_{ij}} \left(\sum_{a\in A} \delta_{a,r} . k_a^c - \sum_{a\in A} \delta_{a,r} . k_a^c \right) . \left(h_{r,ij}^{c,t'_z} - h_{r,ij}^{c,t''_z} \right) = \sum_{z=0}^{M} e^{-\rho t_z} . \sum_{c\in\mathcal{C}} \sum_{i\in L_S} \sum_{j\in L} (-2\gamma_c) . \left(\varphi_{ij}^{c,t'_z} - \varphi_{ij}^{c,t''_z} \right)^2 \leq 0. \tag{D.3}$$

Since $\gamma_c > 0$, we conclude that Eq. (D.3) is negative. Therefore, the GNEP has a unique equilibrium point, and the proof is completed.□

Appendix E

Proposition E.1. The shipper's objective function is concave and his feasible set is convex regarding his own decision variables.

Considering the objective function of the shipper's model in Eq. (1), we can immediately remove the last term from the objective function, because the decision variables φ_{ij}^{c,t_z} and v_{ij}^{c,t_z} are obtained uniquely from the lower level's KKT system of Eqs. (43) to (47). The second and the third terms are linear; consequently, they are concave regarding $q_i^{t_z}$. For the first term, the second order optimality condition regarding $p_j^{t_z}$ is defined by:

$$-\frac{\partial^2 \hat{J}_s(p,q,\varphi,h)}{\partial p_j^2} = -2\alpha_j < 0. \tag{E.1}$$

Since $\alpha_j > 0$, Eq. (E.1) is negative and the first term is concave. Regarding the fact that the sum of concave functions is a concave function, we can conclude that the shipper's objective is concave with respect to his decision variables. In addition, since all the constraints of the model are linear, the feasible set is convex regarding the shipper's variables. Then, the proof is completed.□

Biographies

Marzieh Mozafari is a PhD candidate in Industrial Engineering at Amirkabir University of Technology. She received her MSc degree in Industrial Engineering from Amirkabir University of Technology and BSc degree in Industrial Engineering from K. N. Toosi University of Technology. Her research interests are dynamic pricing and revenue management, game theory, freight transportation, and supply chain optimization.

Behrooz Karimi is a Professor of Faculty of Industrial Engineering and Management Systems at Amirkabir University of Technology in Iran. He received his PhD degree in Industrial Engineering from Amirkabir University of Technology and his MSc degree in Industrial Engineering from Iran University of Science and Technology. He has also received his BSc in Industrial Engineering from Amirkabir University of Technology. His research interest areas include supply chain and logistics management, inventory control & production planning, and meta-heuristics algorithms. He has published books, several papers, and book chapters in reputable journals. He serves as the editor/editorial board member for a number of journals.

Masoud Mahootchi is an Assistant Professor of faculty of Industrial Engineering and Management Systems at Amirkabir University of Technology in Iran. He received his PhD degree in Systems Design Engineering from University of Waterloo in Canada. He received his MSc and BSc degrees in Industrial Engineering both from Amirkabir University of Technology, in Iran. His research interest areas include stochastic programming, storage management, reinforcement learning, and portfolio optimization. He has published several papers and book chapters in reputable journals.