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Research Note

A two-warehouse inventory model with preservation technology investment and partial backlogging

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Two-warehouse;
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Inflation.

Abstract. In today's global world, preservation technology becomes very important due to rapid changes in environment. To study this concept, in this article, we have developed the two-warehouse inventory model with the consideration of preservation technology investment. Demand rate is treated as linear function of instantaneous stock level. The limited storage area (OW) is used for the storage of the stock. The extra amount of stock beyond the capacity of the Owned Warehouse (OW) is stored in the Rented Warehouse (RW). The holding cost of rented warehouse is greater than the holding cost of Owned Warehouse (OW); thus, the stock of Rented Warehouse (RW) is used first and then, the stock of Owned Warehouse (OW) is consumed. The shortages are allowed and partially backlogged under the effect of inflation. The main aim of this study is to find the optimal value of the total cost function. Numerical illustrations and sensitivity analysis are given at the end of this article.

1. Introduction

Every organization stores items to maintain the long-term relationship with its customers. It is a very common phenomenon that the amount of stock exceeds the capacity of the storage space of the owned warehouse, which is limited to a certain level. In such situation, the organization hires a rented warehouse at a high holding cost. In literature, the concept of two-warehouse has first been introduced by Hartley [1]. Sarma [2] is the first who has used the concept of limited storage capacity of owned warehouse in inventory control modeling. After that, many authors have studied the two-warehouse inventory problem. Kumari et al. [3], Agrawal and Banerjee [4], Kumar et al. [5], Singh et al. [6], Singh and Rathore [7], and Singh and Rathore [8] have studied the two-warehouse problem with partially backlogged shortages. Sarkar et al. [9], Sarkar and Moon [10], Khanra et al. [11], and Sarkar et al. [12] have studied the inventory control model with partially backlogged shortages.

Zhou and Yang [13] have studied the two-inventory problem with stock-dependent demand rate. Further, the stock-dependent demand rate with the concept of two-warehouse has been studied by Yadav et al. [14], Singh et al. [15], Gayen and Pal [16] etc. Dye and Ouyang [17], Ghiami et al. [18], and Sarkar and Sarkar [19] concentrated on the concept of stock-dependent demand with partially backlogged shortages.

Due to rapid changes in environment, preservation technology becomes very important as well as essential to preserve the items. In the abovementioned papers, like Sarkar et al. [20] and Sarkar and Sarkar [21], the deterioration rate is treated as an uncontrollable variable. Through some procedural

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changes and preservation techniques, the deterioration rate can be controlled to a certain level. Recently, this concept has been a main point of concentration in the field of inventory control modeling. In literature, Hsu et al. [22] developed a deteriorating inventory model in which they considered that the retailer could invest in preservation technology to reduce deterioration. For further review, we can go through the work of Lee and Dye [23], Hsieh and Dye [24], Dye [25], Xue et al. [26], Zhang et al. [27], Tayal et al. [28], Zhang et al. [29], etc.

The concept of inflation has first been studied by Buzacott [30]. After that, many authors have extended the work of Buzacott. Wee et al. [31] and Yang [32] focused on two-warehouse problem with partially backlogged shortages under the effect of inflation. Singh and Rathore [33-35] have studied the effect of preservation technology in an inflationary environment. Many other authors have considered the effect of inflation in their inventory control modeling, like Singh et al. [36], Singh et al. [37], Patra and Ratha [38], Sarkar et al. [39-40], Sarkar et al. [41], Singh and Rathore [42-43], etc.

In the present article, we have developed a two-warehouse inventory model with controllable deterioration rate. The demand rate is a linear function of instantaneous stock level. The shortages are permitted and partially backlogged. For the storage of items, two-warehouse system is considered with limited storage capacity at Owned Warehouse (OW) and unlimited storage capacity at Rented Warehouse (RW). In the second section, assumptions and notations are given for mathematical model formulation. Solution of the mathematical model is described in the third section. At the end of this paper, numerical illustration and sensitivity analysis are provided. The convexity of the model is shown in Figures 1 and 2.

2. Assumptions and notations

The assumptions and notations which are used in the mathematical model formulation are as follows:

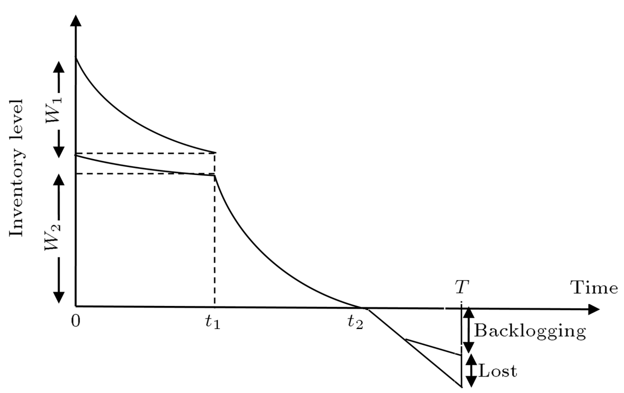

Figure 1. Inventory functioning in two-warehouse system.

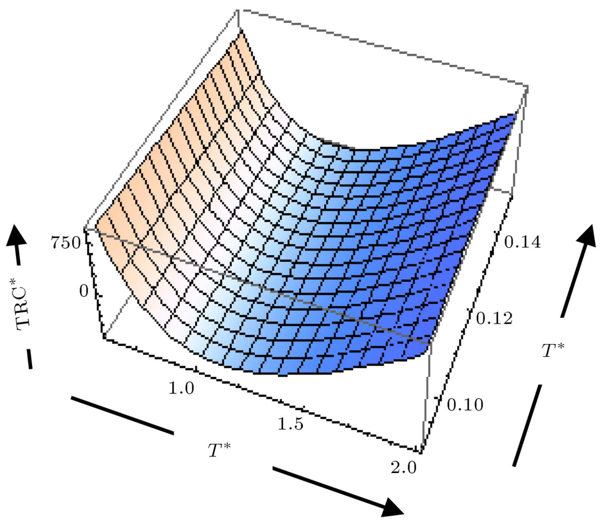

Figure 2. Convexity of TRC* with respect to T^* and t_1^*.

2.1. Notation

- $D(I(t))$: Instantaneous stock-level-dependent demand rate;
- Q: The order quantity;
- θ: Constant deterioration rate;
- $m(\xi) : (= \theta(1 - e^{-a\xi}))$ reduced deterioration rate, a $\xi > 0$;
- ξ: The Preservation Technology (PT) cost, $\xi > 0$;
- τ_θ: Resultant deterioration rate, $\tau_\theta = (\theta - m(\xi))$;
- r: Difference of inflation and time discounting;
- A: Ordering cost;
- h_1: The holding cost (per unit per time unit) of Rented Warehouse (RW);
- h_2: The holding cost (per unit per time unit) of Owned Warehouse (OW);
- C: The unit purchasing cost;
- C_s: The shortage cost;
- C_L: The lost sale cost;
- W_2: The limited space area of OW;
- W_1: Maximum inventory level in RW;
- B: Maximum backorder level;
- t_1: The time period at which inventory level in RW reaches zero;
- t_2: The time period at which inventory level in OW reaches zero;
- T: The cycle length;
- $I_1(t)$: The inventory level in RW during time period $[0,\ t_1]$;
- $I_2(t)$: The inventory level in OW during time period $[0,\ t_1]$;

- $I_3(t)$: The inventory level in OW during time period $[t_1, t_2]$;
- $I_4(t)$: The inventory level in OW during time period $[t_2, T]$;
- TRC: The present worth of the total relevant cost;
- PC: The purchase cost;
- HC: The present worth of holding cost;
- SC: The present worth of shortage cost;
- LC: The present worth of lost sale cost.

2.2. Assumptions

- Demand rate is stock-dependent and taken as the following form:

$$D\left(I(t)\right)=\begin{cases}\alpha+\beta I(t); I(t)>0;\\ \alpha; I(t)\leq 0;\end{cases}$$

- Shortages are allowed and partially backlogged where backlogging rate is:

$$B(t)=\frac{1}{(1+t\delta)},$$

  where t is the waiting time and $0<\delta<1$ is the backlogging parameter;
- Time horizon is infinite;
- Preservation technology is used to reduce the deterioration rate;
- Replenishment rate is infinite and lead time is zero;
- The Owned Warehouse (OW) has the limited space of W_2 units, whereas the rented warehouse has unlimited space area;
- The holding cost (h_1) of RW is greater than the holding cost (h_2) of OW; therefore, consumption of inventory starts only when inventory level of RW reaches zero;
- The charges for transportation as well as time between RW and OW are negligible.

3. Mathematical model formulation

After satisfying the backlogged shortages of the previous period, the inventory level at time $t=0$ is S, out of which W units are stored in OW and the remaining SW units are stored in RW.

Inventory level of RW decreases due to demand and deterioration during the time interval $[0,t_1]$ and at $t=t_1$, it reaches the zero level. After time t_1, demand of items is fulfilled by using inventory of OW during time interval $[t_1,t_2]$. In the time period $[t_2,T]$, shortages occur and are partially backlogged. The description model is presented in Figure 3.

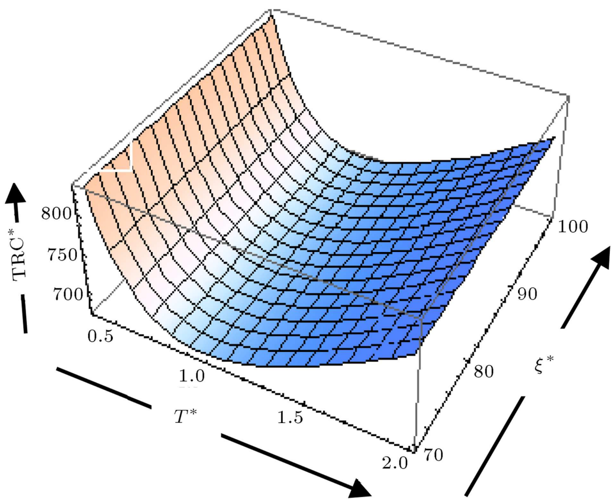

Figure 3. Convexity of TRC* with respect to T^* and ξ^*.

The inventory depletion in RW is represented by differential equation, given as follows:

$$\frac{d\left(I_1(t)\right)}{dt}+\tau_\theta I_1(t)=-\left(\alpha+\beta I_1(t)\right);\quad 0\leq t\leq t_1. \tag{1}$$

And in OW, functioning of inventory is as follows:

$$\frac{d\left(I_2(t)\right)}{dt}+\tau_\theta I_2(t)=0;\qquad 0\leq t\leq t_1, \tag{2}$$

$$\frac{d\left(I_3(t)\right)}{dt}+\tau_\theta I_3(t)=-\left(\alpha+\beta I_3(t)\right);\quad t_1\leq t\leq t_2, \tag{3}$$

$$\frac{d\left(I_4(t)\right)}{dt}=-\left(\frac{\alpha}{(1+\delta(T-t))}\right);\quad t_2\leq t\leq T. \tag{4}$$

Under the boundary conditions, $I_1(t=0)=W_1$, $I_1(t=t_1)=0$, $I_2(t=0)=W_2$, $I_2(t=t_1)=I_3(t=t_1)$, $I_3(t=t_d)=I_4(t=t_d)$, $I_4(t=t_2)=0$, $I_5(t=t_2)=0$. Now, by solving the above equations, we obtain:

$$I_1(t)=\frac{\alpha}{(\beta+\tau_\theta)}\left(e^{(\beta+\tau_\theta)(t_1-t)}-1\right), \tag{5}$$

$$I_2(t)=e^{-t\tau_\theta}W_2, \tag{6}$$

$$I_3(t)=\frac{\alpha}{(\beta+\tau_\theta)}\left(e^{(\beta+\tau_\theta)(t_2-t)}-1\right), \tag{7}$$

$$I_4(t)=-\alpha\left[(t-t_2)\right], \tag{8}$$

$$W_1=\frac{\alpha}{(\beta+\tau_\theta)}\left(e^{(\beta+\tau_\theta)t_1}-1\right). \tag{9}$$

From $I_2(t=t_1)=I_3(t=t_1)$:

$$t_2=t_1+\frac{1}{(\tau_\theta+\beta)}\log\left(W_2e^{t_1\tau_\theta}+\frac{\alpha}{(\tau_\theta+\beta)}\right).$$

Maximum backlogged amount IB $=-I_4(t=T)$:

$$\text{IB}=\alpha\left[(T-t_2)\right].$$

Order quantity $Q=B+W_1+W_2$:

$$Q = \frac{\alpha}{(\beta+\tau_\theta)}\left(e^{(\beta+\tau_\theta)t_1}-1\right)+W_2+\alpha(T-t_2). \tag{10}$$

The total relevant cost includes the following cost parameters:

1. The ordering cost = A.
2. The purchase cost = CQ.
3. The present worth of Holding Cost (HC) is:

$$\text{HC} = h_1\left[\int_0^{t_1} I_1(t)e^{-rt}dt\right]+h_2\left[\int_0^{t_1} I_2(t)e^{-rt}dt + \int_{t_1}^{t_2} I_3(t)e^{-rt}dt\right],$$

$$\begin{aligned}\text{HC} =& h_1\left[\frac{\alpha}{\beta+\tau_\theta}\left(e^{(\beta+\tau_\theta)t_1}\left(\frac{1-e^{-(\beta+\tau_\theta+r)t_1}}{(\beta+\tau_\theta+r)}\right)+\left(\frac{e^{-rt_1}-1}{r}\right)\right)\right]\\ &+h_2\left[W_2\left(\frac{1-e^{-(r+\tau_\theta)^{t_1}}}{(r+\tau_\theta)}\right)+\frac{\alpha}{(\beta+\tau_\theta)}\left(\frac{e^{-rt_2}-e^{-rt_1}}{r}\right)\right]\\ &h_2\left[\frac{\alpha}{(\beta+\tau_\theta)}\left(e^{(\beta+\tau_\theta)t_2}\left(\frac{e^{-(\beta+\tau_\theta+r)t_1}-e^{-(\beta+\tau_\theta+r)}t_2}{(\beta+\tau_\theta+r)}\right)\right)\right].\end{aligned} \tag{11}$$

4. The present worth of Shortage Cost (SC) is:

$$\text{SC} = C_s\left(-\int_{t_1}^{T} I_4(t)e^{-rt}dt\right),$$

$$\text{SC} = \alpha C_s\left(t_2\left(r\frac{T^2-t_2^2}{2}-(T-t_2)\right)+\left(\frac{T^2-t_2^2}{2}-r\frac{T^3-t_2^3}{2}\right)\right). \tag{12}$$

5. The present worth of Lost sale Cost (LC) is:

$$\text{LC} = C_L\left(\alpha\int_{t_2}^{T}\left(1-\frac{1}{(1+\delta(T-t))}\right)e^{-rt}dt\right),$$

$$\begin{aligned}\text{LC} =& \alpha C_L\left((T-t_2)-r\frac{T^2-t_2^2}{2}-\frac{1}{\delta}\log\left(1+\delta(T-t_2)\right)\right)\\ &+\alpha C_L\left(\frac{r}{\delta}\left(-(\delta T+1)\log\left(1+\delta(T-t_2)\right)-(T-t_2)\right)\right).\end{aligned} \tag{13}$$

Therefore, the total relevant cost is as follows:

$$\text{TRC}(t_1,\xi,T) = (1/T)[A+PC+HC+SC+LC]. \tag{14}$$

To minimize total relevant cost, we differentiate total relevant cost, $K = TC(t_1,\xi,T)$ with respect to to t_1, ξ and T, for optimal value necessary conditions are:

$$\frac{\partial TC(t_1,\xi,T)}{\partial t_1}=0, \qquad \frac{\partial TC(t_1,\xi,T)}{\partial \xi}=0,$$

$$\frac{\partial TC(t_1,\xi,T)}{\partial T}=0,$$

provided the determinant of principal minor of hessian matrix are positive definite, i.e. det(H1)>0, det(H2)>0, det(H3)>0 where H1, H2, H3 is the principal minor of the Hessian-matrix.

Hessian matrix of the total cost function is as follows:

$$\begin{bmatrix}\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\partial\xi} & \frac{\partial^2 TC}{\partial t_1\partial T}\\ \frac{\partial^2 TC}{\partial\xi\partial t_1} & \frac{\partial^2 TC}{\partial\xi^2} & \frac{\partial^2 TC}{\partial\xi\partial T}\\ \frac{\partial^2 TC}{\partial T\partial t_1} & \frac{\partial^2 TC}{\partial T\partial\xi} & \frac{\partial^2 TC}{\partial T^2}\end{bmatrix}$$

4. Numerical illustration

For illustration of the proposed model, we consider the following inventory system in which values of different parameters in proper units are:

$A=500;\ \alpha=270;\ \beta=30;\ C=2;\ C_s=1.6;$

$C_L=1.5;\ \theta=0.05;\ \delta=0.1;\ r=0.02;$

$h_1=0.3;\ h_2=0.2;\ W=50;\ a=3.$

Using mathematical software Mathematica 6, we get the optimal values of $t_1^*=0.11729$, $t_2^*=1.07937$, $T^*=1.34624$, $\xi^*=88.1963$, $Q^*=252.544$, and $\text{TRC}^*=691.346$.

5. Sensitivity analysis

To test the sensitivity of this model, we performed a sensitivity analysis by varying values of some important parameters like demand parameters 'α' and 'β', deterioration rate θ, etc. The effect of change in parameters is shown in Table 1. Keen observation of all the three tables above reveals the following facts:

Table 1. Sensitivity analysis of optimal values different parameters with respect to other parameters.

	t_1^*	t_2^*	T^*	ξ^*	Q^*	TRC*
Change in α						
265	0.11779	1.07436	1.84416	87.4587	254.35	688.325
275	0.116804	1.0844	1.81278	88.9354	250.649	694.168
285	0.115871	1.09449	1.78314	90.4211	255.057	704.129
Change in β						
25	0.127449	1.55469	1.82933	83.8983	124.483	531.698
35	0.108774	0.753198	1.82753	94.6152	340.425	777.641
45	0.0965022	0.336423	1.82665	108.938	452.725	860.515
Change in θ						
0.04	0.11729	1.07818	1.82824	73.5525	252.865	690.997
0.055	0.11729	1.07997	1.82824	80.2088	252.382	691.17
0.06	0.11729	1.08056	1.82824	110.162	252.222	691.695
Change in δ						
0.15	0.112751	1.07491	1.80149	90.0809	246.5	678.325
0.17	0.111669	1.07384	1.79537	90.5794	245.13	675.398
0.2	0.110454	1.07265	1.78844	91.1642	243.574	672.164
Change in r						
0.015	0.11382	1.07596	1.78898	89.5996	242.844	679.354
0.025	0.120645	1.08266	1.8723	87.0001	263.571	704.562
0.03	0.123892	1.08584	1.92161	85.9734	276.045	719.121
Change in W						
40	0.115078	0.74409	1.79946	98.4872	325.267	765.808
60	0.119408	1.41462	1.85669	81.4136	179.733	599.777
80	0.123845	1.51462	1.91253	72.3996	149.733	367.155

1. Increase in α results in decrement in t_1^* and T^*, but increment in t_2^*, TRC*, Q^*, and ξ^*;
2. Increase in β results in decrement in t_1^*, t_2^*, and T^*, but increment in TRC*, Q^*, and ξ^*;
3. Increase in θ results in decrement in Q^*, but increment in t_2^*, TRC*, and ξ^*;
4. Increase in r results in decrement in T^* and TRC*, but increment in t_1^*, t_2^*, ξ^*, and Q^*;
5. Increase in W results in decrement in ξ^*, Q^*, and TRC*, but increment in t_1^*, t_2^*, and T^*;
6. Increase in δ results in increment in ξ^*, but decrement in t_1^*, t_2^*, T^*, TRC*, and Q^*.

6. Conclusion

In this paper, we have developed a two-warehouse inventory model for deteriorating items with non-instantaneous deterioration rates. The storage capacity of the owned warehouse is limited, whereas there is infinite storage space in the rented warehouse. Demand rate is a linear function of stock level with partially backlogged shortages and the backlogging rate is inverse function of time under the effect of inflation. The optimal values of total cost, total cycle length, and order quantity are calculated and sensitivity analysis is performed. Our model can be used in the cases of fruits, vegetables, cosmetic products, etc. This model can be extended by incorporating other parameters of inventory system.

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