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A multi-objective fuzzy goal programming *P*-hub location and protection model with back-up hubs considering hubs establishment fixed costs

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Abstract. The critical and undeniable role of hubs in telecommunication and networking brings about some precautions to be taken to protect networks against any disruption. In this paper, a multi-objective model in a group of hub location problems, referred to as protection models for survivable network design, is developed. In fact, this model is a hub location problem that aims to maximize the potential flow between the origin and destination node pairs in the network, which has the minimum potential flow among all O-D pairs, including multiple assignment and back-up routes. In addition, it concerns the fixed cost of installation of the hub facilities. A fuzzy goal programming method is applied to solve the model. Different scenarios are implemented using Turkish data set and also numerical experiments are presented to illustrate the advantages of the proposed model in some aspects, compared to previous models. The results can give useful insights into telecommunication network design.

1. Introduction

Hub Location Problems (HLPs) are one of the important topics in location problems; in fact, they are on type of classic optimization problems that arise in telecommunication and airline networks. As is usual for this kind of problems, the commodities (i.e. data transmission, passengers, parcels, mail, etc.) will be sent to and received from the locations (i.e. cities, airports, post centers, etc.) and the flow between locations (nodes) will pass on through special facilitator points called hubs [1]. In other words, hubs are special facilities that function as orientation and distribution centers through which the flow goes from origins to the destinations [2]. The hub location problems include locating hub facilities and allocating non-hub nodes (spokes) to hubs in order to route the traffic between origin and destination node pairs [3,4]. Since the origin and destination node pairs in a network cannot communicate with each other without hubs, any disruption in hubs can result in whole network failure. Hence, the design of survivable networks has been a significant issue in telecommunications [5]. In this paper, the survivability of the networks is concerned. The rest of the paper is organized as follows. In the beginning comes a review of previous studies (Section 2) including some of HLPs from the time they were first introduced up to the present time. Then, the study concentrates more on hub protection models in Section 3. In Section 4, a new multi-objective mathematical model is developed, which aims at maximizing the potential flow between the origin and destination node (O-D) pairs which have the minimum potential flow among all O-D pairs in

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the network. Moreover, minimizing the total fixed cost of hub facilities installations, conducted in a fuzzy goal programming method, is performed in Section 5 to obtain a satisfactory and consistent solution for both of the objective functions. In Section 6, some computational studies based on different scenarios are carried out using Turkish data set. At the end of Section 7, a brief comparative analysis between the developed model and a previous one is done. The results show that the developed model has privilege in some aspects.

2. Literature review

Hub location problems were first introduced by O'Kelly [6]. His basic structure includes origins, destinations, and hubs; in fact, it was concerned with transferring customers from origins to the destinations using hub nodes in the network. Since then, different aspects of hub location problems have been focused on in many other studies. O'Kelly [7] offered a famous quadratic integer mathematical formulation, which is referred to as the single allocation P-hub median problem. Afterwards, Campbell [8] produced the first linear integer program formulation for the multiple-allocation P-hub median problems. He played a major role in completing hub modeling; minimum value flow on any spoke/hub connection problems, hub location problems with capacity limitation, P-hub center location problems, hub covering location problems, and hub maximal covering location problems are some of his outstanding formulations. Karimi and Bashiri [9] considered several coverage requirements in a hub covering location model and also proposed two heuristic methods to solve it.

Zanjirani et al. [10] presented a comprehensive review in which different HLP models and solution techniques had been analyzed. In a recent study, Fazel Zarandi et al. [11] proposed a Q-coverage multiple-allocation hub set-covering problem and required the distances between hubs to be greater than a lower bound. Ghodratnama et al. [12] presented a fuzzy bi-objective model for a hub covering location problem in which the first objective function aimed at minimizing network costs and the second objective function was concerned with minimizing the shipping time of products, considering different transportation vehicles and time periods. Korani and Sahraeian [13] developed a single-allocation hierarchical hub covering model in which all origin destination pairs were required to have a travel time lower than a determined value. Davari et al. [14] developed an incomplete hub-covering model with fuzzy variables for demand locations and proposed a variable neighborhood search method to solve it. Xavier et al. [15], inspired by a previous study by Bagirov et al. [16], proposed a hyperbolic smoothing hub-and-spoke algorithm to solve min-sum-min formulation for continuous multiple-allocation P-hub median problem. It should be noted that although their hyperbolic smoothing method has the capability of solving large hub-and-spoke problems, its solutions are not necessarily globally optimum. Fazel Zarandi et al. [17] in a recent study adopted two different meta-heuristic algorithms to solve hierarchical single-allocation hub median problem and then compared the performances of two methods. Correia et al. [18] developed a capacitated single-allocation hub location problem in which multiple products were considered to be transported in the network. O'Kelly et al. [19] surveyed a new aspect of hub location problem in such a way that the amount of demand depended on the price and quality of transportation service. Hult et al. [20] developed a stochastic uncapacitated single-allocation P-hub center problem in which the travel time between nodes followed normal distribution function. Meier and Clausen [21] developed a comprehensive hub location problem, considering hubs with different capacities and links with different vehicles for different week days. In addition, they proposed an innovative cost function.

In the classical facility location problems, it is usually assumed that all facilities are perfectly available and optimal facility locations are derived under this ideal assumption. However, in a real-world situation, disruptions, failures, damages, and unexpected events may lead to unavailability of some facilities [22]. Hence, design of survivable and robust systems subjected to disruptions has been one of the important research topics in recent years. Survivability refers to the ability of a network to provide services uninterruptedly, despite damage to network components such as nodes and links [23]. The research on survivable network problems was addressed by Baybars and Edahl [24], in which they applied a heuristic method for facility location in telecommunication networks. In terms of reliability, Shishebori et al. [25] developed a mixed integer programming model for facility location, considering networks reliability and financial budget restriction. In another study, Li and Savachkin [26] presented a reliable P-median problem to design a lean distribution network which could resist unforeseeable failures.

In fact, survivable network problems try to protect and support the flow from origin to destination nodes, without any loss, using alternative routes in a given network [27]. Kim and O'Kelly [23] introduced a group of models which are called *P-hub protection models* (PHPRO) and focus on survivable network design for telecommunications. This type of models is known as protection models, because they take precautions to make the networks manageable in the presence of unpredicted disruptions. This could be considered as an advantage in telecommunication systems [28].

3. *P*-hub protection models

P-hub protection models have first been introduced by Kim and O'Kelly [23]. Indeed, the most important feature that makes protection models different from the other models is the consideration of some back-up hubs in the network in order to support the flow going through primary hubs. A brief explanation of elements, introduced by Medhi [29], is presented here to address the probability of traffic delivery rate in a given route.

RP_{ij}^{km} represents the reliability of the primary route between origin i and destination j, which passes through hubs k and m ($i \rightarrow k \rightarrow m \rightarrow j$). Every route includes three links with serial connections and consequently the total reliability of the route could be computed by sequentially multiplying the reliabilities of each link, i.e. r_{ik}, r_{km}, and r_{mj}.

$$RP_{ij}^{km} = r_{ik} \times r_{km} \times r_{mj}. \tag{1}$$

Kim [5] presented a protection model by assuming some back-up hubs which could be applied for transferring traffic in situations where primary hubs were not available. In this paper, let RB_{ij}^{n} show the reliability of the back-up route between origin i and destination j, which passes through back-up hub n ($i \rightarrow n \rightarrow j$), where primary route passes through primary hubs k and m that are not accessible. The reliability of the back-up route is computed as follows:

$$RB_{ij}^{n} = r_{in} \times r_{nj}. \tag{2}$$

Finally, total probability of the potential traffic delivery rate between the origin and destination node pairs (i, j) by primary route and back-up route can be calculated as presented below [5]:

$$R_{ij}^{km} = RP_{ij}^{km} + \overline{RP_{ij}^{km}} \times RB_{ij}^{n} = RP_{ij}^{km} + (1 - RP_{ij}^{km}) \times RB_{ij}^{n}, \tag{3}$$

where R_{ij}^{km} is total probability of the potential traffic delivery rate between the origin and destination node pairs (i, j) by primary route and back-up route; and $\overline{RP_{ij}^{km}}$ represents complement probability of the primary route ($\overline{RP_{ij}^{km}} = 1 - RP_{ij}^{km} = 1 - (r_{ik} \times r_{km} \times r_{mj})$).

Eq. (3) shows that the total reliability of the potential traffic delivery rate between the origin and destination node pairs (i, j) by primary and back-up routes (R_{ij}^{km}) is increased while the reliability of back-up route (RB_{ij}^{n}) is included. In addition, the traffic flow between the origin and destination node pairs (i, j) will be delivered unless both primary and back-up routes disrupt simultaneously.

The reliability factors, α and γ ($0 \leq \alpha, \gamma \leq 1$), which account, respectively, for the inter-hub and intra-hub capabilities in transferring the flow are another components in reliable P-hub location problems. The inter-hub reliability factor, α, is imposed on the inter-hub link term in the form of $r_{km}^{(1-\alpha)}$. Thus, the reliability is calculated as $RP_{ij}^{km} = r_{ik} \times r_{km}^{(1-\alpha)} \times r_{mj}$ in the route between origin i and destination j, which passes through hubs k and m ($i \rightarrow k \rightarrow m \rightarrow j$); and intra-hub reliability factor γ is applied to the hub term r_{nn}, which is the element of computation in such routing schemes as $i \rightarrow n \rightarrow j$. By applying these factors, Eq. (3) is expanded to Eq. (4), as shown below, to compute the potential amount of traffic flow between origin and destination node pairs (i, j) [23]:

$$R_{ij}^{km} = RP_{ij}^{km} + (1 - RP_{ij}^{km}) \times RB_{ij}^{n} = \left(r_{ik} \times r_{km}^{(1-\alpha)} \times r_{mj}\right) + \left[1 - (r_{ik} \times r_{km}^{(1-\alpha)} \times r_{mj})\right] \times (r_{in} \times \gamma \times r_{nj}). \tag{4}$$

Not surprisingly, potential amount of traffic flow between the origin and destination node pairs (i, j) increases with higher values of inter- and intra-hub reliability factors (α and γ). However, the behavior of the model in locating hubs and in finding the optimal route and the distribution of spatial interactions between the origin and destination node pairs changes due to various values of both factors, α and γ. In general, a higher α encourages the use of two hub stop routes, while higher γ produces a tendency to use one hub stop or direct routes [23].

4. Mathematical formulation

4.1. Proposed formulation

Kim [5] developed a multiple-assignment P-hub protection model with back-up hubs (PROBA). This model takes reliability-relevant components in a classical hub location model with multiple assignments into account, and furthermore enhances the total amount of potential network flow by employing back-up hubs and routes. PROBA assigns some primary routes for every origin and destination node pair (i, j) through different primary hubs and moreover assigns it some back-up routes through back-up hubs, which prepare the most potential flow for the whole network. It should be considered that the back-up routes should be disjoint from the corresponding primary routes and the flow of back-up routes should be routed via disjoint back-up hubs. This model maximizes the total potential network flow via primary as well as back-up routes for a set of node pairs (i, j).

In this paper, a multi-objective model of a multiple-assignment P-hub protection model with back-up hubs (PROBA) is developed, which consists of two objective functions; the first one concerns maximization of the potential flow between the origin and destination node pairs, which has the minimum potential flow among all O-D pairs in the network; in other words, it is a Max-Min objective function. The second objective function aims at minimizing the total fixed cost of installation of hub facilities. In fact, another version of PROBA is developed. Additional assumptions and constraints intended to make the model more realistic and practical. The objective functions and their related constraints are as follows:

$$\text{Obj.Function1: Max Min}_{i,j}T = \sum_k \sum_m W_{ij} \times \left(RP_{ij}^{km} \times X_{ij}^{km} + \overline{RP_{ij}^{km}} \times X_{ij}^{km} \times \sum_n RB_{ij}^n \times Y_{ij}^n \right), \tag{5}$$

$$\text{Obj. Function2: Min}C = \sum_k C_k \left(ZP(k) + ZB(k) \right), \tag{6}$$

Subject to:

$$\sum_k ZP(k) = P \quad (P \geq 2), \tag{7}$$

$$\sum_k ZB(k) = Q \quad (Q \geq 2), \tag{8}$$

$$ZP(k) + ZB(k) \leq 1 \ \forall k, \tag{9}$$

$$\sum_k \sum_m X_{ij}^{km} = 1 \quad \forall i,j, \ X_{ij}^{km} \in S, \tag{10}$$

$$\sum_n Y_{ij}^n = 1 \quad \forall i,j, \ Y_{ij}^n \in S', \tag{11}$$

$$\sum_k X_{ij}^{km} - ZP(m) \leq 0 \quad \forall i,j,m, X_{ij}^{km} \in S, \tag{12}$$

$$\sum_m X_{ij}^{km} - ZP(k) \leq 0 \quad \forall i,j,k, X_{ij}^{km} \in S, \tag{13}$$

$$Y_{ij}^n - ZB(n) \leq 0 \quad \forall i,j,n, Y_{ij}^n \in S', \tag{14}$$

$$ZP(k), \ ZB(k) \in \{0,1\} \ \forall k, \tag{15}$$

$$X_{ij}^{km} \in \{0,1\} \quad X_{ij}^{km} \in S, \tag{16}$$

$$Y_{ij}^n \in \{0,1\} \quad Y_{ij}^n \in S'. \tag{17}$$

where:

- P Number of primary hubs that should be located;
- Q Number of back-up hubs that should be located;
- C_k Fixed cost of installing hub facilities in candidate node k;
- S Set of practical routes X_{ij}^{km};
- S' Set of disjoint routes Y_{ij}^n.

$$ZP(k) = \begin{cases} 1, & \text{If node } k \text{ is a primary hub} \\ 0, & \text{Otherwise} \end{cases}$$

$$ZB(k) = \begin{cases} 1, & \text{If node } k \text{ is a back-up hub} \\ 0, & \text{Otherwise} \end{cases}$$

$$X_{ij}^{km} = \begin{cases} 1, & \text{If there is a route from origin } i \text{ to destination } j \text{ via primary hubs } k \text{ and } m \ (i \to k \to m \to j) \\ 0, & \text{Otherwise} \end{cases}$$

$$Y_{ij}^{n} = \begin{cases} 1, & \text{If there is a route from origin } i \text{ to destination } j \text{ via back-up hub } n \ (i \to n \to j) \\ 0, & \text{Otherwise} \end{cases}$$

This model has two objective functions; the first one, Eq. (5), maximizes the potential flow between the origin and destination node pairs, which has the minimum potential flow among all O-D pairs in the network. The other one, Eq. (6), concerns minimizing the total fixed cost of installation of hub facilities in candidate nodes. Constraints (7) and (8) require the numbers of primary and back-up hubs to be respectively equal to P and Q. Constraint (9) ensures the primary and back-up hubs not to be located in the same nodes. Constraint (10) represents that between any O-D pair (i,j), there is just one primary route through primary hubs k and m. In the same way, Constraint (11) shows that between any O-D pair (i,j), there is just one back-up route through back-up hub n. Constraints (12) and (13) ensure that non-hub nodes are not connected directly in the network and prohibit the flow between origin i to destination j from being routed through non-hub nodes. Constraint (14) is like constraints (12) and (13), with exception that this constraint concerns back-up hubs and back-up routes instead of primary ones. Constraints (15) to (17) show the values which decision variables could adopt. In fact, they restrict $ZP(k)$, $ZB(k)$, X_{ij}^{km}, and Y_{ij}^n variables to accept only zero-one values. In this study, X_{ij}^{km} is required to be in set $S = \{(i,k,m,j)|(i \neq j) \cap [(k = i)U(k = m = j)U(k \neq i \cap k \neq j \cap m \neq j)]\}$; otherwise, there may occur some impractical solutions (e.g. $(i \rightarrow i \rightarrow i \rightarrow i)$ or $(i \rightarrow j \rightarrow i \rightarrow j)$) in the solution space. In the same way, Y_{ij}^n is required to be in set $S' = \{(i,n,j)|(i \neq j)\}$ to avoid producing impractical solutions in back-up routes. In other words, all of the impractical primary and back-up routes could be eliminated by defining S and S' sets. Furthermore, these requirements reduce the number of decision variables, which lead to a considerable decrease in the computation time of the problem.

In fact, this model is an extended version of Kim's model, but the existence of some differences in the assumption and problem structure have changed its nature and application. The most important difference between these two models is in the ways they treat the potential flow in a given network. Kim's model, which could be classified as a median location problem, aims at maximization of total potential flow (the summation of potential flow between all origin-destination pairs in the network); while the proposed model in this study is a Max-Min model which seeks the origin-destination node pairs that have the poorest potential flow in the network and then tries to maximize the flow between them by using other routing alternatives. In addition, the former model (Kim's model) is not concerned with fixed cost of installation of new hub facilities, which is a critical point when there are budget restrictions in the real-world problems. Thus, it would be beneficial to optimize both objective functions simultaneously.

4.2. Model linearization

As mentioned above, the first objective function is a Max-Min, which can be presented in a constraint form as follows:

Max T

S.t.:

$$T \leq \sum_k \sum_m W_{ij} \times \left(RP_{ij}^{km} \times X_{ij}^{km} + \overline{RP_{ij}^{km}} \times X_{ij}^{km} \times \sum_n RB_{ij}^n \times Y_{ij}^n \right) \forall i,j. \tag{18}$$

But, as it is obvious, this constraint is nonlinear. In order to linearize the model, Constraint (18) must be expressed in a different way. Let M present a large positive value. Constraint (19) is the linear restatement of Constraint (18), as demonstrated below:

Max T

S.t.:

$$T \leq W_{ij} \times \left(RP_{ij}^{km} + \overline{RP_{ij}^{km}} \times RB_{ij}^n \right) + (2 - X_{ij}^{km} - Y_{ij}^n) \times M \ \forall i,j,k,m,n. \tag{19}$$

According to Constraint (19), the right-hand side of the inequality will be a large positive value, which will subsequently convert this constraint to a redundant constraint, unless both binary variables (X_{ij}^{km}, Y_{ij}^n) receive non zero values simultaneously.

The linear model can be solved using linear programming methods.

5. Fuzzy goal programming

5.1. Introduction to fuzzy goal programming

The Goal Programming (GP) method is a Multi-Criteria Decision Making Method which considers several objectives (goals) simultaneously. In other words, it aims to minimize the absolute deviation between the achievement values and the desired values of some conflicting objective functions. In fact, GP is like a standard linear programming model with the exception that it has more than one objective function.

Goal programming model was first developed by Charnes and Cooper [30] and due to its ease of use and functionality, it attracted the attention of many researchers who dealt with multi-objective programming. The standard formulation of GP is presented as follows:

$$\text{Min } Z = \sum_{i=1}^{n} d_i^+ + d_i^-, \tag{20}$$

S.t.:

$$F_i(x) + d_i^+ - d_i^- = g_i \quad \text{for} \quad i = 1, ..., n, \tag{21}$$

$$x \in X, \tag{22}$$

$$d_i^+, d_i^- \geq 0 \quad \text{for} \quad i = 1, ..., n, \tag{23}$$

where variables d_i^+ and d_i^- present, respectively, the positive and the negative deviations from the desired values g_i, and X denotes the feasible solutions set.

In the classic GP formulations, it is assumed that the desired values for goals are deterministic and precise; but, due to ambiguity, complexity, and uncertainty of real-world problems, these values cannot be stated with precision. In order to overcome this difficulty, the concept of Fuzzy Goal Programming (FGP) was developed in the 1980s [31].

The fuzzy sets theory was initially introduced by Zadeh [32]. Then, Zimmermann [33,34] proposed the membership function based on the fuzzy sets theory. The general formulation of the membership function is as follows [35,36]:

$$\mu_i(x) = \begin{cases} 1 & \text{if } f_i(x) \leq b_i \\ 1 - \frac{f_i(x) - b_i}{\Delta_i} & \text{if } b_i < f_i(x) \leq b_i + \Delta_i \\ 0 & \text{if } f_i(x) > b_i + \Delta_i \end{cases} \tag{24}$$

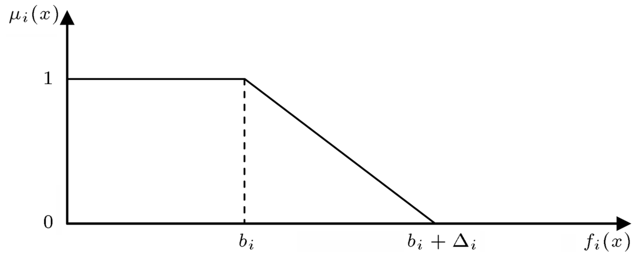

Figure 1. Fuzzy sets membership function diagram.

where $\mu_i(x)$ is the membership function, and b_i is the aspired level for the ith fuzzy goal and finally, Δ_i is the maximum tolerable level associated with the objective function, $f_i(x)$. Figure 1 shows this membership function.

Fuzzy multi-objective programming method using the membership functions was initially developed by Zimmerman [35] as follows:

$$\text{Max } Z = \lambda, \tag{25}$$

S.t.:

$$\lambda \leq 1 - (f(x)_i - b_i)/\Delta_i \quad \text{for } i = 1, ..., n, \tag{26}$$

$$x \in X, \tag{27}$$

$$\lambda \geq 0. \tag{28}$$

Fuzzy goal programming method is widely applied in various domains; however, to the best knowledge of the authors of this paper, only little research has been done in the field of Hub Location Problems (HLPs) using fuzzy goal programming methods. Wang and Jiang [37] developed a multi-objective model for extended hub-and-spoke regional port transportation network that consisted of two objective functions; the first one aimed at minimizing total costs and the second one was concerned with the total transit times. Finally, a fuzzy goal programming method was used to minimize both objective functions simultaneously. In another study, Zarandi et al. [38] applied an interactive fuzzy goal programming method to solve a single-allocation hub-and-spoke model.

The method employed to solve the proposed model in this study is based on Zimmermann's fuzzy multi-objective programming method.

5.2. Problem formulation using fuzzy goal programming method

As mentioned before, the multi-objective formulation developed in this study consists of two conflicting objective functions. Also, in order to make a trade-off between these two conflicting objectives, a fuzzy goal programming method is developed.

The first goal, which concerns maximizing the potential flow between the O-D pairs that have the minimum potential flow among all O-D pairs in the network, can be written in fuzzy systems as follows:

$$\text{Goal 1}: \sum_k \sum_m W_{ij} \times \Big(RP_{ij}^{km} \times X_{ij}^{km} + \overline{RP_{ij}^{km}} \times X_{ij}^{km} \times \sum_n RB_{ij}^{n} \times Y_{ij}^{n} \Big) \tilde{\geq} T \; \forall i,j. \tag{29}$$

And the second goal, which tries to minimize the costs related to opening of the hubs in the determined nodes, can be presented in fuzzy systems as follows:

$$\text{Goal 2}: \sum_K C_k \left(ZP(k) + ZB(k) \right) \tilde{\leq} C_{\max}. \tag{30}$$

According to Zimmermann's fuzzy multi-objective programming method and the linearization technique, which was employed in Section 4.2, the FGP model of the attempted problem can be summarized and presented as follows:

$$\text{Max } Z = \lambda, \tag{31}$$

S.t.:

$$\lambda \leq 1 - \frac{b_1 - W_{ij} \times (RP_{ij}^{km} + \overline{RP_{ij}^{km}} \times RB_{ij}^{n})}{\Delta_1} + (2 - X_{ij}^{km} - Y_{ij}^{n}) \times M \; \forall i,j,k,m,n, \tag{32}$$

$$\lambda \leq 1 - \frac{\sum_k C_k \left(ZP(k) + ZB(k) \right) - b_2}{\Delta_2}. \tag{33}$$

Constraints (7) – (17),

$$\lambda \geq 0, \tag{34}$$

where, b_i is the aspiration level of the ith objective function.

6. Model application and computational studies

In order to evaluate the performance of the model introduced in this study, some computational results are presented in the following. With the purpose of knowing the optimum value (desired value) of the objective functions (b_i), the model was solved considering each objective function solely in the absence of the other one and finally, in order to obtain a satisfactory solution to the proposed problem, both objective functions were considered together. All of the numerical experiments were executed using CPLEX 12 solver on an Intel Core i5 with 4 GB of RAM.

The experiments are based on a data set frequently used by many of the hub location researchers in

Table 1. Results of the model performance regardless of cost constraint in the different scenarios for the case study problem.

α, γ	P, Q	Primary hubs	Back-up hubs	Objective function	Solution time (s)	Hubs fixed costs
0.9	2,2	2,5	3,4	10035.244	193.54	1396.25
	3,2	2,3,7	4,5	10098.363	235.19	1781.28
0.7	2,2	2,5	7,8	9437.66	359.61	1508.90
	3,2	2,5,7	3,8	9562.94	215.91	1836.05
0.5	2,2	2,5	4,8	8735.03	282.27	1636.56
	3,2	2,5,8	3,4	8958.52	181.68	1963.03
0.3	2,2	5,7	4,8	7828.19	297.73	1711.19
	3,2	2,5,7	4,6	8464.30	634.32	1684.54
0.1	2,2	5,7	2,4	7164.354	243.94	1454.80
	3,2	2,5,7	1,3	7411.831	222.60	1748.2

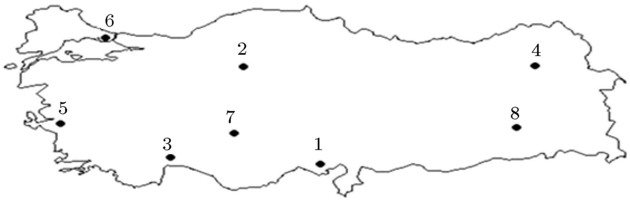

Figure 2. Potential 8 cities of the locating hubs for the case study problem.

the literature, i.e. Turkish data set, which contains the data on 81 cities on Turkish network. W-matrix and R-matrix are two input matrices to the presented model. W-matrix contains the amounts of flow between 8 most important Turkish cities, which have been provided from the mentioned data set; on the other hand, for R-matrix, which represents the reliability of links between nodes, there is no information in the stated data set; thus, R-matrix has been generated randomly. Figure 2 illustrates the location potentials for 8 cities.

6.1. Model performance regardless of cost constraint

In this section, it is assumed that there is no concern about budget and the model has been relaxed from cost constraint. In other words, the model tries to find the optimum places for hub facilities in order to maximize the potential flow between the O-D pairs that have the minimum potential flow, among all O-D pairs in the network, where the total fixed cost of hubs is not an issue of significant importance. In this way, the model can be summarized as follows:

Max T

S.t.:

Constraint (19),

Constraints (7) – (17).

Different scenarios with different levels of α and γ and different numbers of primary (P) and back-up (Q) hub nodes are considered in a numerical experiment and the results consisting of objective functions and allocation of primary and back-up hubs are reported and summarized in Table 1. Several findings can be deduced from the results, which are worth being noted. One of the most distinguished perceivable relationships in the table is the relationship between the number of hubs and the amount of objective function (which could be interpreted as network performance). The more the number of the considered hubs in the model, the greater the network performance becomes. This is because the model offers more choices for the routing of each (i, j) pair with an increasing number of hubs [39]. In addition, different combinations of primary hubs (P) and back-up hubs (Q) result in different objective functions. Another important point in findings is the effects of inter-hub reliability factor ($0 \leq \alpha \leq 1$) and intra-hub reliability factor ($0 \leq \gamma \leq 1$) on the network performance, which reveal the ability of inter-hub links and hubs in transferring and delivering the traffic without delay or disruption. Greater α and γ result in greater objective function, which could be predicted easily. Installing more reliable facilities in hubs and connecting them with more reliable links will enhance the network performance.

Figure 3 illustrates the mentioned points. It can be seen that increase in the number of hubs intensifies the objective function gradually with a gentle slope and in addition, changes in inter- and intra-hub reliability factors, α and γ, result in changes in the objective function, too.

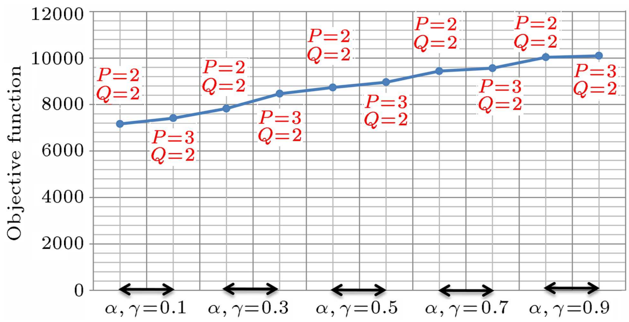

Figure 3. Incremental trend of objective function (network performance) according to different values of α and γ and P, Q, regardless of cost constraint.

As mentioned before, binary variables, X_{ij}^{km} and Y_{ij}^{n}, account for primary and back-up hubs, respectively. Indeed, they show if there is any primary or back-up routes between origin-destination pair (i, j). According to Table 1, where α, $\gamma = 0.9$ and P, $Q = 2$, 2, nodes 2 and 5 are determined as primary hubs and nodes 3 and 4 are determined as back-up hubs; in other words, for a given i, j:

$$X_{ij}^{km} = \begin{cases} 1, & \text{If } k, m = 2 \text{ or } 5 \\ 0, & \text{Otherwise} \end{cases}$$

And in a similar way:

$$Y_{ij}^{n} = \begin{cases} 1, & \text{If } n = 3 \text{ or } 4 \\ 0, & \text{Otherwise} \end{cases}$$

About the effect of Y_{ij}^{n}, it should be noted that this variable addresses the back-up hubs and neglecting this variable leads to minor network potential flows (the second part in the right side of Eq. (5) will be omitted). Table 2 shows how the objective function decreases without considering back-up hubs in the network.

6.2. Hub equipment fixed costs

In this section, the second objective function is considered solely in the absence of the first one. In fact, the purpose of this section is to calculate the minimum possible fixed cost of opening new hubs for different combinations of P and Q, where the network potential flow is not important. It can be presented as follows:

$$\text{Min } C = \sum_{k} C_k \left(ZP(k) + ZB(k) \right)$$

S.t.

Constraints (7) – (17).

Two scenarios with different values of P and Q are implemented in the model and the computational results are reported and summarized in Table 3.

A brief comparison of Tables 1 and 3 shows that for the same numbers of primary and back-up hubs (P, Q), the fixed costs of hub equipment in Table 1 are considerably greater than those in Table 2 and demonstrate that there is an opportunity for minimizing the fixed costs.

Table 2. Results of the model performance without consideration of the back-up hubs for the case study problem.

α, γ	P, Q	Primary hubs	Objective function	Solution time (s)
0.9	2,2	2,4	7449.11	1.57
	3,2	2,5,8	9070.93	1.43
0.1	2,2	5,7	6184.02	1.51
	3,2	4,5,8	6964.26	1.38

Table 3. Minimum fixed cost of opening new hubs.

P, Q	Primary hubs	Back-up hubs	Objective function	Solution time (s)
2,2	2,6	3,5	1113.962	0.05
3,2	3,5,7	2,6	1498.997	0.010

6.3. Computational results for the proposed multi-objective model using fuzzy goal programming method

In the previous sections, the best possible and desired values of each objective function (b_i) were calculated solely in the absence of the other one. In the following, both objective functions are considered simultaneously in a fuzzy linear goal programming model, aiming to minimize the absolute deviations of objective functions from their desired values. The first objective function should be maximized not lower than the tolerable value $(b_1 - \Delta_1)$, and the second objective function must be minimized not greater than the related tolerable value $(b_2 + \Delta_2)$, where δ_1 and Δ_2 are determined by decision maker.

In order to evaluate the performance of the model and the FGP method conducted in the model, some computational experiments were implemented and the results are reported in Table 4. The results show that where α and $\gamma = 0.9$, the desired values of the first and second goals are achieved; but where α and $\gamma = 0.1$, although the desired values of the second goal are achieved in different scenarios, the obtained values of the first goal have some deviations from their desired levels; in other words, the minimum fixed cost of hub equipment has been achieved, but it results in a lower potential flow in the network.

Figure 4 illustrates the locations of primary and back-up hubs on 8 Turkish cities. When α and $\gamma = 0.9$ and $(P, Q) = (3, 2)$, cities 2, 5, and 6 (Ankara, Izmir, and Istanbul) are determined as primary hubs

Table 4. Computational results of the model using FGP method.

α, γ	P, Q	Δ_1, Δ_2	λ	Goal 1	Goal 2	Primary hubs	Back-up hubs	Solution time (s)
0.9	2,2	100, 100	1	10035.244	1113.962	2,5	3,6	12.06
	3,2	100, 100	1	10098. 363	1498.997	2,5,6	3,7	104.38
0.1	2,2	100, 100	0.98	7162.354	1113.962	3,5	2,6	6.22
	3,2	100, 100	0.93	7403.831	1498.997	3,5,7	2,6	6.43

Table 5. Results for the median model.

α, γ	P, Q	Median model			Proposed model		
		Primary hubs	Back-up hubs	Min potential flow	Primary hubs	Back-up hubs	Min potential flow
0.9	2,2	4,7	2,6	7991.41	2,5	3,4	10035.244
	3,2	1,3,4	6,7	8332.90	2,3,7	4,5	10098.363
0.7	2,2	4,7	2,6	7661.36	2,5	7,8	9437.66
	3,2	1,4,5	6,7	7823.36	2,5,7	3,8	9562.94
0.5	2,2	4,7	2,6	6894.01	2,5	4,8	8735.03
	3,2	1,4,5	6,7	7101.36	2,5,8	3,4	8958.52
0.3	2,2	1,4	6,7	5598.39	5,7	4,8	7828.19
	3,2	1,4,5	6,7	5691.23	2,5,7	4,6	8464.30
0.1	2,2	1,4	6,7	5245.23	5,7	2,4	7164.354
	3,2	1,4,5	6,7	5512.01	2,5,7	1,3	7411.831

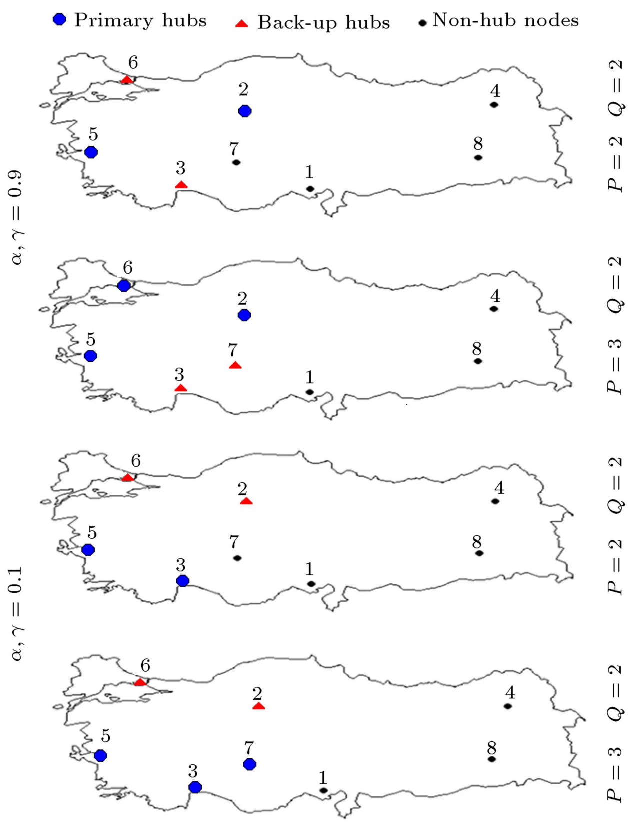

Figure 4. Locations of primary and back-up hubs for different values of factors α and γ.

and cities 3 and 7 (Antalya and Konya) are back-up hubs. However, when α and $\gamma = 0.1$ and $(P, Q) = (3, 2)$, cities 3, 5, and 7 (Antalya, Izmir, and Konya) are primary hubs and cities 2 and 6 (Ankara and Istanbul) are back-up hubs.

7. Analyzing the results

It is worth making a comparison between the proposed model and the one which Kim [5] presented. As stated before, the most important difference between these two models is in the ways they treat network potential flow. In fact, the former model, which could be classified as a median location problem, is not concerned with the reliability of the potential flow between origin-destination pairs in the network. In other words, in Kim's approach, there is the possibility of flows between origin-destination pairs through the risky routes with low reliabilities, while the presented model in this study assures that under all restrictions, the routes in the network are as reliable as possible.

Table 5 provides a comparison between the minimum potential flows of the two approaches. The results show that there is a sensible difference between the values for minimum potential flow in the models and the reliability of the network performance is increased in the proposed approach in comparison with the former one.

8. Conclusions

With daily increasing application in telecommunications and transportation networks, survivable network design has attracted the attention of many researchers in recent years. Even small disruptions in traffic delivery could result in disastrous consequences in different networks. Despite such importance, few comprehensive research has examined the issue from various perspectives (e.g., reliability and cost). In this study, a nonlinear multi-objective model of a multiple-assignment P-hub protection model with back-up hubs (PROBA) was developed, which consisted of two objective functions. The first one aimed to maximize the potential flow between the origin and destination node pairs, which had the minimum potential flow among all O-D pairs in the network. The second objective function concerned minimization of total fixed cost of opening new hub in candidate nodes. A technique was used to linearize the proposed model and the FGP method was employed to make a balance between two conflicting objective functions.

The optimum and desired value of each goal was achieved by taking each objective function into account solely for the model in the absence of the other one. Finally, in order to obtain a satisfactory solution to the attempted problem, both goals were considered simultaneously in the proposed FGP. Computational results demonstrated that the ideal and desired values of both objective functions can be obtained in scenarios with high values of α and γ; but in scenarios with low values of α and γ, there exist some deviations between objective functions and their desired values. This study can prepare a useful attitude for managers and decision makers who deal with finding optimum places for hubs where there are budget restrictions.

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