An extended fuzzy VIKOR for group decision-making based on fuzzy distance to supplier selection

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Abstract. Supplier Selection Problem (SSP) has become a critical objective of purchasing departments because of its significant effect on successful logistic and Supply Chain Management (SCM). In real-life situations, SSP parameters are often imprecise, vague, uncertain, or incomplete. In this respect, fuzzy sets theory is the best developed approach to formulate these uncertainties. In this paper, we have extended fuzzy VIKOR using an efficient fuzzy distance measure to solve applicable SSP under group decision-making process. In our study, an efficient fuzzy VIKOR for solving SSP under group decision-making process is presented in which decision makers have different weights in decision-making process and their opinions are collected in the form of linguistic variables. In our methodology, preference ratio method is applied to rank the alternatives. Ultimately, several numerical illustrations and sensitivity analyses are performed to demonstrate the applicability of the proposed method.

1. Introduction

Supplier selection plays a key role in achieving strategic goals of the companies. Nowadays, changing the level of customer satisfaction, general purchasing rules, and configuration of the organizations have caused us to face complicated decisions in purchasing processes [1]. Much research has been carried out on Supplier Selection Problem (SSP). Jiang et al. [2] carried out an empirical study to examine whether different supplier selection criteria and integration mechanisms could improve customer satisfaction and business performance. In the literature, SSP has been investigated in terms of Multi-Criteria Decision-Making (MCDM) and various exact and efficient heuristic methodologies are proposed for solving SSPs [3].

De Boer et al. [1] proposed a well-known framework for analyzing a supplier selection problem consisting of (I) problem definition, (II) formulation of criteria, (III) qualification, and (IV) choice.

In SSPs, criteria formulation is one of the most important steps to select the suppliers [4,5]. Dickson [6] presented a seminal study and introduced quality, delivery, and performance history as three most important criteria. Weber et al. [7] analyzed 74 papers on supplier selection criteria and identified price as the most cited criterion, followed by delivery and quality. Kannan and Tan and Kannan [8] investigated more than 30 selection criteria and found that on-time delivery and quality were ranked as the most important. However, in another research by FrÖdell [9], cost is the most important criterion in 12 zones of industries. Ku et al. [10] reviewed literature of SSP and summarized supplier selection criteria as cost or price, quality, service, supplier's profile, risk, buyer-supplier partnership, cultural and communication barriers, and trade restrictions. Kahraman et al. [11] presented four parts

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of SSP criteria, consisting of supplier characteristics, product efficiency, service quality, and cost.

One of the features of SSP is uncertainty in making decisions because of inherent ambiguity in evaluating qualitative criteria. Fuzzy sets theory is the most important and applicable approach to facing uncertainty in supplier selection process [12]. Fuzzy sets theory, introduced by Zadeh [13], has been presented to deal with this incomplete information and model it. In SSP, many experts utilize linguistic variables as fuzzy numbers in order to determine important features and performance of alternatives [14].

Much research concentrates on SSP in fuzzy environment. Wang et al. [15] proposed fuzzy hierarchical TOPSIS for solving a SSP. Carrera and Mayorga [16] proposed a modular fuzzy inference system approach in supplier selection for new product development. Keskin et al. [17] presented fuzzy adaptive resonance theory to evaluate and select the suppliers. Hsu et al. [18] used fuzzy preference relations with using fuzzy quality data to rank suppliers and select the best one. BüyükÖzkan and Cifci [19] developed a novel approach based on fuzzy analytic network process within multi-person decision-making schema under incomplete preference relations in sustainable supply chain. Shemshadi et al. [20] proposed fuzzy VIKOR for SSP based on entropy measure for objective weighting. Amindoust et al. [21] proposed a ranking method based on fuzzy interference system for sustainable supplier selection. BuyükÖzkan and Cifci [22] hybridized three MCDM methods including fuzzy DEMATEL, fuzzy ANP, and fuzzy TOPSIS to evaluate green suppliers. Ozaki et al. [23] described irregular ANP as minor ANP, which utilized the calculation with only the alternative matrix consisting of the missing values or the non-square matrix. Pang and Bai [24] developed a hybrid supplier evaluation approach based on the ANP and fuzzy synthetic evaluation using linguistic variables. Dursun and Karsak [25] developed a fuzzy multi-criteria group decision-making approach that made use of the Quality Function Deployment (QFD) concept for supplier selection process. Omurca [26] proposed a hybrid method of fuzzy c-means and rough set theory techniques for supplier selection, evaluation, and development problem. Boran et al. [27] proposed a TOPSIS method combined with intuitionistic fuzzy set to select appropriate supplier in group decision-making environment. Sharma and Balan [28] proposed an integrative approach considering Taguchi's loss function, TOPSIS, and Multi-criteria goal programming. Ayag and Samanlioglu [29] proposed an intelligent approach to SSP based on fuzzy ANP using uncertain human preferences as input information in the decision-making process. Carpinetti and Junior [30] combined fuzzy rule based system and TOPSIS technique to solve SSP. Cao et al. [31] proposed a TOPSIS method based on

intuitionistic fuzzy judgment matrix for green supplier selection.

2. Related works

Compromise ranking method of VIKOR, proposed by Opricovic and Tzeng [32,33], has been developed to solve Multi-Attribute Decision-Making (MADM) problem in complex systems with non-commensurable and incompatible criteria. Recently, VIKOR method has been widely used in various applications, especially in uncertain environments, such as supplier selection [20,34-37], material selection [38-40], personnel selection [41], performance evaluation [42,43], water resources planning [44], robot selection [45], marketing [46], and waste management [47,48]. Wang and Chang [49] introduced fuzzy VIKOR for solving multicriteria group decision-making problem.

Some researchers developed fuzzy VIKOR and used it in various applications [44,50-54]. Sayadi et al. [55] proposed a VIKOR method with using interval number. Sanayei et al. [35] and Shemshadi et al. [20] used linguistic values to assess the ratings for a supplier selection problem. Kuo and Liang [42] proposed interval-value based fuzzy VIKOR. Devi [45] extended VIKOR method in intuitionistic fuzzy environment (IF-VIKOR) and then Park et al. [56] developed VIKOR method for Dynamic Intuitionistic Fuzzy Multiple-Attribute Decision-Making (DIF-MADM). Wan et al. [41] extended VIKOR method with triangular intuitionistic fuzzy numbers. Chang and Wang [49] extended fuzzy VIKOR with using 2-tuple linguistic decision matrix. Liu et al. [39] proposed an interval 2-tuple linguistic VIKOR method for a material selection problem. Ju and Wang [57] considered linguistic information for both the criteria values and criteria weights. Next, thay transformed the individual linguistic decision matrix given by each decision maker into 2-tuple linguistic decision matrix, and then aggregated into collective 2-tuple linguistic decision matrix by 2-tuple linguistic arithmetic mean operation. Yücenur and Demirel [58] extended fuzzy VIKOR with uncertain Decision Maker (DM)'s degree of optimism. Minatour et al. [59] used VIKOR technique for dam site selection problem. Some papers proposed hybrid approaches of VIKOR and other techniques such as DEMATEL, ANP, and AHP for MADM problems [43,46,52,60-62].

3. Preliminary concepts

3.1. VIKOR method
Suppose that the set of alternatives is denoted by $a_1, a_2, \cdots a_i, \cdots, a_m (i = 1, \cdots, m)$ and the set of criteria is denoted by $C_1, C_2, \cdots, C_j, \cdots, C_n (j =$

$1, 2, \ldots, n$). For an alternative a_i, the rating of the jth criterion is asked from decision maker as f_{ij}.

The compromise ranking algorithm of VIKOR is briefly reviewed as follows:

1. Determine the best f_j^* and the worst f_j^- values of all criteria, $j = 1, 2, \cdots, n$. If the jth criterion represents a benefit, then $f_j^* = \max_i f_{ij}$ or setting f_j^* is the aspired/desired level and $f_j^- = \min_i f_{ij}$ or setting f_j^- is the worst level;

2. Compute the values S_k and R_k, $k = 1, 2, \cdots, n$, by the following relations:

$$S_i = \sum_{j=1}^{n} w_j \left| f_j^* - f_{ij} \right| / \left| f_j^* - f_j^- \right|, \tag{1}$$

$$R_i = \max_j \left\{ \left| f_j^* - f_{ij} \right| / \left| f_j^* - f_j^- \right|; j = 1, 2, \cdots, n \right\}, \tag{2}$$

where w_j is the weight of the jth criterion expressing its relative importance;

3. Compute the value Q_i, $i = 1, 2, \cdots, m$ as:

$$Q_i = \nu(S_i - S^*) / (S^- - S^*)$$
$$+ (1 - \nu)(R_i - R^*) / (R^- - R^*), \tag{3}$$

where:

$$S^* = \min_i S_i \quad \text{or let} \quad S^* = 0,$$

$$S^- = \max_i S_i \quad \text{or let} \quad S^- = 1,$$

$$R^* = \min_i R_i \quad \text{or let} \quad R^* = 0,$$

$$R^- = \max_i R_i \quad \text{or let} \quad R^- = 1.$$

Therefore, we can rewrite $Q_i = \nu S_i + (1 - \nu) R_i$, when $S^* = 0$, $S^- = 1$, $R^* = 0$ and $R^- = 1$. ν is introduced as the weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here $v = 0.5$;

4. Rank the alternatives, sorting by the values S, R, and Q in ascending order. The results are three ranking lists;

5. Propose a compromise solution as the alternative (a'), which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

  (I) Acceptable advantage:

  $$Q(a'') - Q(a') \geq DQ,$$

  where a'' is the alternative with the second position in the ranking list by Q; $DQ = 1/J - 1$; and J is the number of alternatives;

  (II) Acceptable stability in decision-making: Alternative a' must also be the best ranked by S or/and R.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed which consists of:

- Alternatives a' and a'' if only condition (II) is not satisfied, or;

- Alternatives $a', a'', \cdots, a^{(m)}$ if condition (I) is not satisfied; and $a^{(m)}$ is determined by the relation $Q(a^{(m)} - Q(a')) < DQ$ for maximum m (the positions of these alternatives are "in closeness").

The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives and the compromise solution with the "advantage rate".

3.2. Preliminaries of the fuzzy sets theory

Fuzzy sets theory has been used for modeling decision-making processes based on imprecise and vague information such as judgment of decision makers [14].

Let X be the universe of discourse, $X = \{x_1, x_2, \cdots, x_n\}$; a fuzzy set $\tilde{A}$ of X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which is associated with each element x in X, a real number in the interval $[0, 1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$. Some basic definitions of fuzzy sets and numbers are reviewed [63].

Definition 1. A Triangular Fuzzy Number (TFN) can be denoted as a triplet $\tilde{A} = (a_1, a_3, a_3)$; the membership function of the fuzzy number $\tilde{A}$ is taken as:

$$f_{\tilde{A}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1); & a_1 \leq x \leq a_2 \\ 1; & x = a_2 \\ (a_3 - x)/(a_3 - a_2); & a_2 \leq x \leq a_3 \\ 0; & \text{otherwise} \end{cases} \tag{4}$$

Definition 2. $\tilde{m} = (a, b, c)$ is called a positive TFN if $a \geq 0$. a, b, and c are not identical.

Definition 3. Let $\tilde{A}$ and $\tilde{B}$ be two positive fuzzy numbers parameterized by the triplets (a_1, a_2, a_3) and (b_1, b_2, b_3) and r be a positive scalar; the operations (addition, subtraction, multiplication, and division) of TFN can be performed as:

$$\tilde{A}(+)\tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3),$$

$$\tilde{A}(-)\tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1),$$

$$\tilde{A}(\times)\tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3),$$

$$\tilde{A}(\div)\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1),$$

$$\tilde{A}(\times)r = (ra_1, ra_2, ra_3). \tag{5}$$

3.3. Fuzzy distance

Many proposed methods for measuring distance between fuzzy numbers obtain a crisp value for distance between fuzzy numbers [64]. Moreover, a logical and acceptable consequence of defining a fuzzy distance measure for generalized fuzzy numbers is that the distance between two uncertain numbers should also be an uncertain and fuzzy number [65].

Voxman [66] for the first time introduced a fuzzy distance measure between two fuzzy numbers based on $\alpha - cut$ and also stated how it was possible for the distance between two fuzzy numbers to be a crisp number. Chakraborty and Chakraborty [67] proposed another fuzzy distance in which the general fuzzy number was calculated by LR-Type fuzzy. Its main logical disadvantage is that this distance will be negative in some cases when calculated from the left point. To solve this problem, Guha and Chakraborty [68] introduced a new distance for general fuzzy numbers through using the $\alpha - cut$ concept. Allahviranloo et al. [69] proposed a method based on $\alpha - cut$ for calculating fuzzy distance between two trapezoidal fuzzy numbers.

Recently, Sadi-Nezhad et al. [70] proposed a method to compute fuzzy distance between two TFNs. Their method, in addition to covering disadvantages of past methods, had a simple mechanism. They only used the left, middle, and right points and developed their method for K-dimensional space. The result of this approach is a positive TFN which indicates the fuzzy distance between any two TFNs; also, when two fuzzy numbers are overlapped, this fuzzy distance includes zero value. Because of the mentioned reasons, we use fuzzy distance method, proposed by Sadi-Nezhad et al. [70], in our extended fuzzy VIKOR.

Let $\tilde{X} = (x_1, x_2, x_3)$ and $\tilde{Y} = (y_1, y_2, y_3)$ be two TFNs. In Sadi-Nezhad et al. [70], $D_{XY} = (d_1, d_2, d_3)$ is defined as a fuzzy distance between $\tilde{X}$ and $\tilde{Y}$ in the following equation:

$$d_1 = \begin{cases} \max\{x_1 - y_3, 0\}; & x_2 \geq y_2 \\ \max\{y_1 - x_3, 0\}; & x_2 \leq y_2 \end{cases}$$

$$d_2 = |x_2 - y_2|,$$

$$d_3 = \{\max(y_3 - x_1, x_3 - y_1)\}. \tag{6}$$

Figure 1 graphically shows the mentioned equation for fuzzy distance. More details and various examples can be found in their paper [70].

3.4. Ranking fuzzy numbers

In many cases, especially in MADM problem, ranking of fuzzy numbers is an important component in the decision-making process. Many methods have been
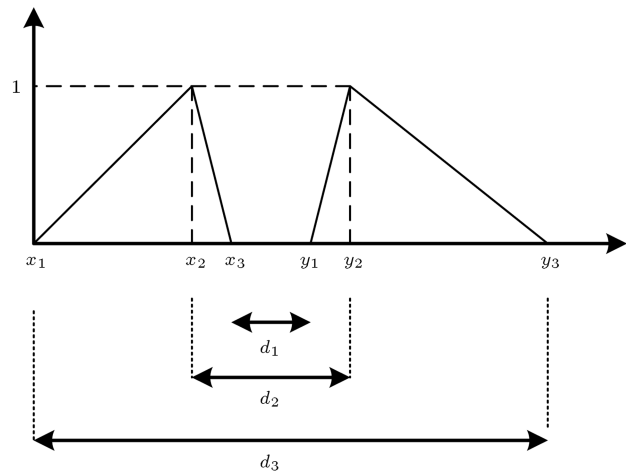

Figure 1. Fuzzy distance measure.

proposed for ranking fuzzy numbers that can be found in a brief review in paper [71]. In this paper, the method of ranking fuzzy numbers proposed by Modarres and Sadi-Nezhad [72], namely preference ratio (Pr), is used. In this method, fuzzy numbers are evaluated point by point and ranked at each point. Then, the overall preference over all points is calculated. Therefore, in this method, the numbers are compared relatively and not absolutely. More details of computation procedure can be found in their paper [73]. For two fuzzy numbers, $\tilde{N}_1$ and $\tilde{N}_2$, Modarres and Sadi-Nezhad [74] developed an algorithm to find scalar K such as:

$$\tilde{N}_1 \overset{\text{Pr}}{\cong} K \times \tilde{N}_2.$$

Sadi-Nezhad and Ghaleh-Assadihave [75] proposed an algorithm based on preference ratio to rank n TFNs as follows:

Let $\tilde{N}_1, \tilde{N}_2, \cdots, \tilde{N}_n$ be as n TFNs.

Step 1 :　Calculate $\tilde{N}_{\text{sum}} = \sum\limits_{i=1}^{n} \tilde{N}_i$;

Step 2 :　Find scalar K_i such as
　　　　　$\tilde{N}_i \overset{\text{Pr}}{\cong} K_i \times \tilde{N}_{\text{sum}}$ for $i = 1, 2, \cdots, n$; $\qquad(7)$

Step 3 :　Sort K_i and, respectively, $\tilde{N}_i$.

4. Research gaps

Supplier evaluation and selection process has a critical role and significant impact on purchase management in supply chain. It is also a complicated MCDM problem with several conflicting criteria. Many real-world decision-making problems inherently involve uncertainty, vagueness, and impreciseness, particularly when human judgments are considered by fuzzy modeling. Fuzzy sets theory has been widely used to provide

a consistent and reliable mechanism for evaluating the alternatives in MCDM problems with uncertain or vague variables. The VIKOR method as an applicable MCDM technique has recently been applied for solving decision-making problems. Some research, such as Sanayei et al. [35] and Shemshadi et al. [20], extended fuzzy VIKOR and used it for SSP. It aggregated fuzzy rating of DMs and defuzzified the decision matrix in initialization of its methods. In other words, it converted different DM's opinions into one decision-making matrix; then, it estimated fuzzy matrix by a precise one. It is logical that estimating the fuzzy numbers by a crisp value would be faced by lack of accuracy. In this study, we keep all data in form of fuzzy numbers during the solving process. Therefore, obtaining distance between fuzzy numbers is required. To do so, unlike some papers (such as [64]), which calculated the distance between fuzzy numbers as a crisp value, we have applied an efficient fuzzy distance measure presented by Sadi-Nezhad et al. [70] to increase accuracy in decision-making process. As another novelty of our study, we solve the problem for each individual DM without aggregating the rating of DMs; then, the group decision-making matrix with using DMs decisions is constructed at the end of the proposed method. Finally, aggregating the results for final decision is carried out with considering different weights for DMs. As mentioned, in our method, data remains in the form of fuzzy numbers during the process; therefore, preference ratio method was applied for ranking fuzzy numbers.

5. The proposed fuzzy VIKOR

After explaining some preliminary concepts in Section 3, in this section, a fuzzy VIKOR based on fuzzy distance and preference ratio is extended. The main contribution of the proposed method is utilizing novel fuzzy distance for measuring distance between fuzzy numbers and well-known preference ratio for ranking fuzzy numbers. Utilization a fuzzy distance [70] instead of a crisp value between fuzzy numbers helps us to increase accuracy and efficiency in analyzing real-world problem.

Let us consider an MADM problem with m alternatives and n criteria. Assume that k DMs are asked about the performance ratings and the importance weight of the criteria. Ratings of DMs are collected in the form of linguistic variables and then transformed into TFNs. Consider $\tilde{x}_{ij}^k$ is as performance rating of the ith alternative with regards to the jth criterion for the kth DM. The weights of criteria have been asked about in the form linguistic variables and then converted into fuzzy numbers. Therefore, $\tilde{W}^k$ matrix is the fuzzy importance of the criteria for the kth DM. In addition, the relative fuzzy importance of the kth

DM in decision-making procedure is defined as $\tilde{W}'_k$. By these definitions, the problem can be represented more formally as follows:

$$\tilde{D}^k = \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1j}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2j}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{i1}^k & \tilde{x}_{i2}^k & \cdots & \tilde{x}_{ij}^k \end{bmatrix},$$

$$\tilde{W}^k = \begin{bmatrix} \tilde{w}_1^k & \tilde{w}_2^k & \cdots & \tilde{w}_j^k \end{bmatrix},$$

$$\tilde{W}'_k = (w'_{Lk}, w'_{Mk}, w'_{Rk}), \tag{8}$$

where $\tilde{D}^k$ is the fuzzy decision matrix of the kth DM. It has i rows and j columns which are related to alternatives and criteria, respectively. Also, $\tilde{w}_j^k$ is weight of the jth criterion for the kth DM in form of TFN.

Executive procedure of the proposed method can be designed in four stages including:

1. Preliminary and parameters definition;
2. Applying extended fuzzy VIKOR for each DM;
3. Applying extended fuzzy VIKOR for the group;
4. Final fuzzy ranking of alternatives.

Graphical procedure of the proposed method can be considered as shown in Figure 2.

Stage (1): Preliminary and parameters definition:

- **Step 1.1.** Selection of a committee of decision makers;
- **Step 1.2.** Determining the set of criteria for decision-making;
- **Step 1.3.** Determining the final candidate of alternatives after initial screening;
- **Step 1.4.** Selecting appropriate linguistic variables and their related fuzzy numbers;
- **Step 1.5.** Determining weights of decision makers in form of fuzzy numbers;
- **Step 1.6.** Taking DM's opinion about weights of criteria in form of linguistic variables;
- **Step 1.7.** Taking DM's opinion about scores of the alternative with respect to criteria;
- **Step 1.8.** Transferring linguistic variables into appropriate fuzzy numbers.

Stage (2): Applying extended fuzzy VIKOR for each DM:

At this stage, the proposed extended fuzzy VIKOR is utilized for each DM and the parameters $\tilde{R}$, $\tilde{S}$, and $\tilde{Q}$ are calculated for each of them. By these assumptions, the steps of the proposed algorithm of extended VIKOR in this stage are as follow:
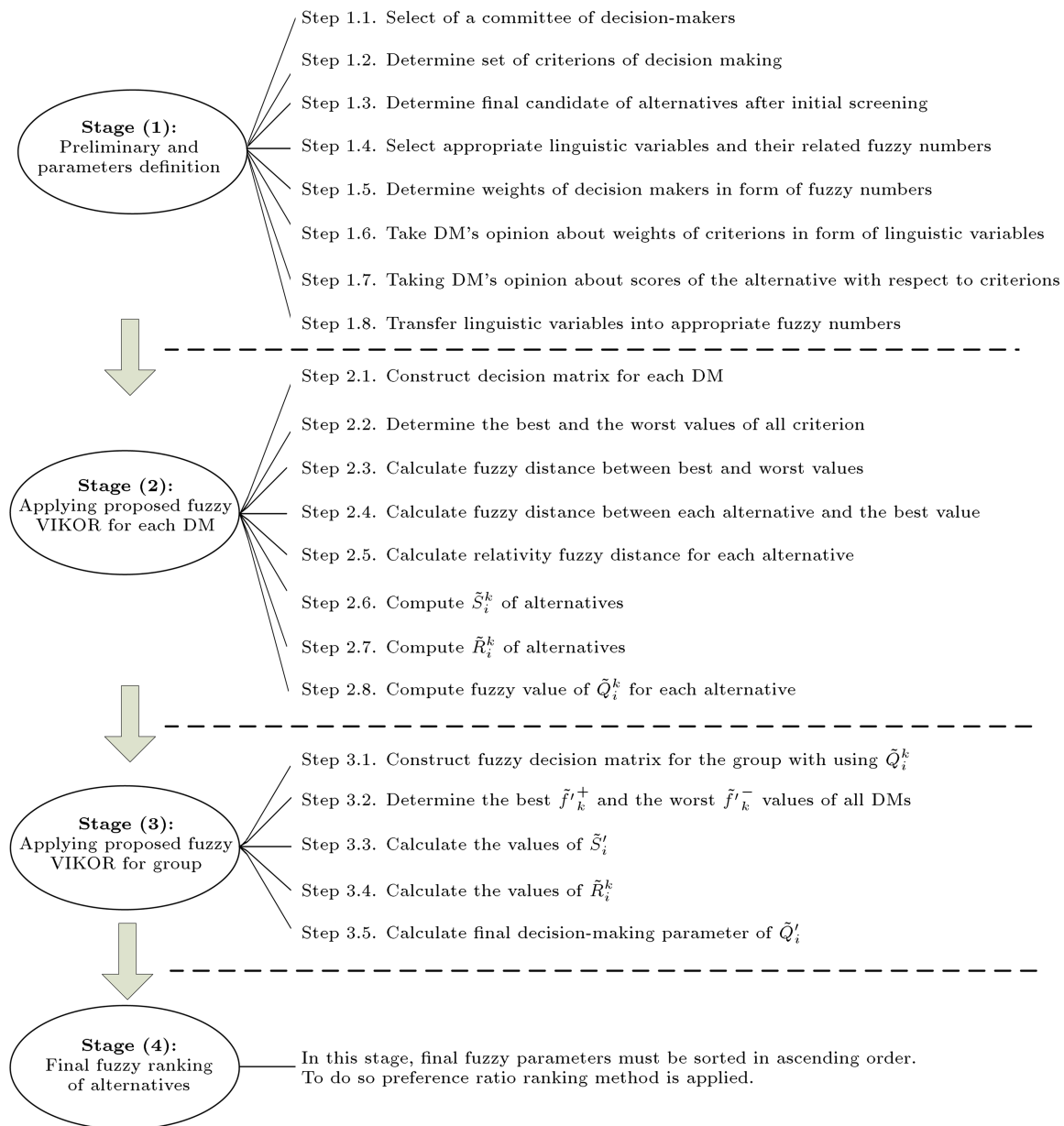
Figure 2. Conceptual framework of the proposed approach.

- **Step 2.1.** Construct decision matrix for each DM as follows:

$$\tilde{D}^k = \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1j}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2j}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{i1}^k & \tilde{x}_{i2}^k & \cdots & \tilde{x}_{ij}^k \end{bmatrix},$$

$$\tilde{x}_{ij}^k = \left(x_{ijL}^k, x_{ijM}^k, x_{ijR}^k \right). \tag{9}$$

- **Step 2.2.** Determine the best $\tilde{f}_j^*$ and the worst $\tilde{f}_j^-$ values for each criterion, $j = 1, 2, \cdots, n$:

$$\begin{cases} \tilde{f}_j^{*k} = (\max_i(x_{ijL}^k), \max_i(x_{ijM}^k), \\ \quad \max_i(x_{ijR}^k)); \quad j \in B \\ \tilde{f}_j^{*k} = (\min_i(x_{ijL}^k), \min_i(x_{ijM}^k), \\ \quad \min_i(x_{ijR}^k)); \quad j \in C \end{cases} \quad k = 1, 2, \cdots, K$$

$$\begin{cases} \tilde{f}_j^{-k} = (\min_i(x_{ijL}^k), \min_i(x_{ijM}^k), \\ \quad \min_i(x_{ijR}^k)); \quad j \in B \\ \tilde{f}_j^{-k} = (\max_i(x_{ijL}^k), \max_i(x_{ijM}^k), \\ \quad \max_i(x_{ijR}^k)); \quad j \in C \end{cases} \quad k = 1, 2, \cdots, K \tag{10}$$

where, B and C are benefit and cost criteria index

sets, respectively;

- **Step 2.3.** Fuzzy distance by using Eq. (8) is calculated between best and worst values $|\tilde{f}_j^{*k} - \tilde{f}_j^{-k}|$ for each criterion;

- **Step 2.4.** Calculate fuzzy distance between each pair of alternatives and best value for DMs as follows:

$$\left| \tilde{f}_j^{*k} - \tilde{x}_{ij}^k \right|; \quad (i = 1, 2, \cdots, m; \quad j = 1, 2, \cdots, n),$$

$$\text{for} \quad k = 1, 2, \cdots, K. \tag{11}$$

- **Step 2.5.** Calculate relativity fuzzy distance for each alternative for DMs as follows:

$$\left| \tilde{f}_j^{*k} - \tilde{x}_{ij}^k \right| \Big/ \left| \tilde{f}_j^{*k} - \tilde{f}_j^{-k} \right|,$$

$$(i = 1, 2, \cdots, m; \quad j = 1, 2, \cdots, n),$$

$$\text{for} \quad k = 1, 2, \cdots, K. \tag{12}$$

- **Step 2.6.** Compute $\tilde{S}_i^k$ of alternatives for DMs as follows:

$$\tilde{S}_i^k = \sum_{j=1}^m \tilde{w}_j^k \left| \tilde{f}_j^{*k} - \tilde{x}_{ij}^k \right| \Big/ \left| \tilde{f}_j^{*k} - \tilde{f}_j^{-k} \right|,$$

$$i = 1, 2, \cdots, m, \qquad k = 1, 2, \cdots, K. \tag{13}$$

- **Step 2.7.** Compute $\tilde{R}_i^k$ of alternatives for DMs as follows:

$$\tilde{R}_i^k = \max_j \Big\{ \left| \tilde{f}_j^{*k} - \tilde{x}_{ij}^k \right| \Big/ \left| \tilde{f}_j^{*k} - \tilde{f}_j^{-k} \right|,$$

$$j = 1, 2, \cdots, m \Big\},$$

$$i = 1, 2, \cdots, m, \qquad k = 1, 2, \cdots, K. \tag{14}$$

In order to implementat Eq. (14), preference ratio ranking method is applied and the biggest relativity fuzzy distance of $|\tilde{f}_j^{*k} - \tilde{x}_{ij}^k| / |\tilde{f}_j^{*k} - \tilde{f}_j^{-k}|$ for alternatives between criteria is obtained as $\tilde{R}_i^k$;

- **Step 2.8.** Compute fuzzy value of $\tilde{Q}_i^k$ for each alternative and DM as follows:

$$\tilde{Q}_i^k = \nu \tilde{S}_i^k + (1 - \nu) \tilde{R}_i^k, \tag{15}$$

where ν is a scalar as decision-making, which is considered $\nu = 0.5$, in this paper.

Stage (3): Applying the proposed fuzzy VIKOR for group

The obtained $\tilde{Q}_i^k$ in the previous stage are still TFNs. Applying the proposed fuzzy VIKOR for group decision-making is as the following steps:

- **Step 3.1.** with using $\tilde{Q}_i^k$ ($i = 1, 2, \cdots, m$ and $k = 1, 2, \cdots, K$), fuzzy decision matrix for the group is constructed as follows:

$$\tilde{D}' = [\tilde{Q}_i^k]_{m \times K},$$

$$\tilde{D}' = \begin{bmatrix} \tilde{Q}_1^1 & \tilde{Q}_1^2 & \cdots & \tilde{Q}_1^K \\ \tilde{Q}_2^1 & \tilde{Q}_2^2 & \cdots & \tilde{Q}_2^K \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{Q}_m^1 & \tilde{Q}_m^2 & \cdots & \tilde{Q}_m^K \end{bmatrix},$$

$$\tilde{Q}_i^k = \left(Q_{iL}^k, Q_{iM}^k, Q_{iR}^k \right).$$

- **Step 3.2.** Determine the best $\tilde{f}_k'^*$ and the worst $\tilde{f}_k'^-$ values of all DMs ($k = 1, 2, \cdots, K$).

$$\begin{cases} \tilde{f}_k'^* = \left(\min_i (Q_{iL}^k), \min_i (Q_{iM}^k), \min_i (Q_{iR}^k) \right) \\[2mm] \tilde{f}_k'^- = \left(\max_i (Q_{iL}^k), \max_i (Q_{iM}^k), \max_i (Q_{iR}^k) \right) \end{cases} \tag{16}$$

- **Step 3.3.** With using fuzzy algebraic operations, the values of $\tilde{S}_i'$ are calculated as:

$$\tilde{S}_i' = \sum_{k=1}^K \tilde{w}_k' \left| \tilde{f}_k'^* - \tilde{Q}_i^k \right| \Big/ \left| \tilde{f}_k'^* - \tilde{f}_k'^- \right|;$$

$$i = 1, 2, \cdots, m, \tag{17}$$

where $\tilde{W}_k' = (w_{Lk}', w_{Mk}', w_{Rk}')$ is fuzzy weight of the kth DM in decision-making process;

- **Step 3.4.** Obtain $\tilde{R}_i'$ as follows:

$$\tilde{R}_i' = \max_k \left\{ \left| \tilde{f}_k'^* - \tilde{Q}_i^k \right| \Big/ \left| \tilde{f}_k'^* - \tilde{f}_k'^- \right| \right\};$$

$$i = 1, 2, \cdots, m. \tag{18}$$

This step is utilized same as Step 2.6 in Stage (2);

- **Step 3.5.** Calculate the final decision-making parameter of $\tilde{Q}_i'$ for each alternative as:

$$\tilde{Q}_i' = \nu \tilde{S}_i' + (1 - \nu) \tilde{R}_i', \tag{19}$$

where ν is a scalar as decision-making, which is considered $\nu = 0.5$ in this paper.

Stage (4): Final fuzzy ranking of alternatives:

Final decision-making parameters in VIKOR method are R, S, and Q, which have been sorted in ascending order. The proposed fuzzy VIKOR decision-making parameters are $\tilde{R}_i'$, $\tilde{S}_i'$, and $\tilde{Q}'$, which are TFNs. Therefore, in this stage, the fuzzy parameters are sorted in ascending order. To do so, preference ratio ranking method of fuzzy numbers described in Section 3.4 is applied as algorithm (7).

6. Numerical illustration

In this section, we evaluate performance of our proposed method for SSP. To do so, a numerical example is first applied; then, sensitivity analysis on some parameters is performed to demonstrate validity of the proposed method.

6.1. A numerical example

Suppose that a company wants to select a suitable supplier for the main material which strongly affects the production process. In Stage (1) of decision-making process, a committee of four DMs is established. After the initial screening, five candidate suppliers (A_1, A_2, A_3, A_4, and A_5) remain for further evaluation. All the four DMs were asked to provide a list of criteria that could be used to evaluate suppliers; ultimately, five criteria consisting of C_1: quality, C_2: price, C_3: lead time delivery, C_4: technical level, and C_5: flexibility were selected. DM's opinions are considered as linguistic variables. We consider five linguistic variables for the performance rating of the alternative to determine the importance weight of the attributes. These linguistic variables and their TFNs are shown in Table 1.

In order to determine weights of criteria, DMs are asked to rate criteria. Ratings of the importance weight of the criteria provided by each individual DM are presented in Table 2. Also, DMs are asked about rating alternatives with respect to the criteria using linguistic variables mentioned in Table 1. The performance scores of the alternative suppliers with respect to the five criteria provided by each individual DM are presented in Table 3. These ratings are done by linguistic variables and then linguistic variables are transformed into associated TFNs for criteria and alternatives. At the end of Stage (1), we can construct fuzzy decision matrix for each individual MD. Table 4 shows fuzzy decision matrix for DM1.

After completion of Stage (1), decision-making process is continued by implementing the proposed fuzzy VIKOR for individual DMs in Stage (2). To

Table 2. The importance weight of the criteria provided by the four DMs.

Attributes	DM1	DM2	DM3	DM4
C_1	MH	H	H	VH
C_2	M	M	MH	MH
C_3	H	H	H	VH
C_4	H	H	MH	VH
C_5	L	ML	M	M

do so, the best and the worst values of each criterion are determined by using Eq. (10) and then fuzzy distances ($|\tilde{f}_j^{*k} - \tilde{f}_j^{-k}|$) between best and worst values for each criterion are calculated by using Eq. (6). Next, fuzzy distances $|\tilde{f}_j^{*k} - \tilde{x}_{ij}^k|$ for each pair of alternatives and with respect to each criterion are calculated by Eq. (11). Then, in order to obtain $\tilde{S}_i^k$ and $\tilde{R}_i^k$ parameters, values of $|\tilde{f}_j^{*k} - \tilde{x}_{ij}^k|/|\tilde{f}_j^{*k} - \tilde{f}_j^{-k}|$ are calculated by Eq. (12). Ultimately, fuzzy values of $\tilde{S}_i^k$, $\tilde{R}_i^k$, and $\tilde{Q}_i^k$ are calculated by using Eqs. (13), (14), and (15). The same steps are performed and the final parameters are achieved for all DMs. The final result of Stage (2) for DM1 is shown in Table 5. All the performed calculations in Stage (2) must be repeated for each individual DM.

Stage (3) of decision-making process is started after obtaining $\tilde{Q}_i^k$ values for all DMs. In the first step of this stage, group decision-making matrix is established as shown in Table 6 by using $\tilde{Q}_i^k$ values. With using group decision-making matrix, the best and the worst values for each DM are determined by Eq. (16) and then fuzzy distances of $|\tilde{f}_k'^* - \tilde{f}_k'^-|$ are calculated. Fuzzy distance values of $|\tilde{f}_k'^* - \tilde{Q}_i^k|$ and also relative fuzzy distance values of $|\tilde{f}_k'^* - \tilde{Q}_i^k|/|\tilde{f}_k'^* - \tilde{f}_k'^-|$ for alternatives with respect to DMs are calculated. Using the fuzzy weight vector of the DMs determined as $\tilde{W}_1' = (0.6, 0.7, 0.8)$, $\tilde{W}_2' = (0.4, 0.5, 0.6)$, $\tilde{W}_3' = (0.7, 0.8, 0.9)$, and $\tilde{W}_4' = (0.7, 0.8, 0.9)$, the values of $\tilde{S}_i'$ are calculated by Eq. (17). Also, with using Eqs. (18) and (19), the values of final parameters of decision-making, $\tilde{R}_i'$ and $\tilde{Q}_i'$, are calculated as shown in Table 7.

Table 1. Linguistic variables and their fuzzy numbers.

Linguistic variables for rating of alternatives	Abbreviation	TFN	Linguistic variables for importance weights of the criteria/DMs	Abbreviation	TFN
Very Poor	VP	(1,1,2)	Very Low	VL	(0.1,0.1,0.2)
Poor	P	(2,3,4)	Low	L	(0.2,0.3,0.4)
Medium Poor	MP	(3,4,5)	Medium Low	ML	(0.3,0.4,0.5)
Fair	F	(4,5,6)	Medium	M	(0.4,0.5,0.6)
Medium Good	MG	(6,7,,8)	Medium High	MH	(0.6,0.7,0.8)
Good	G	(7,8,9)	High	H	(0.7,0.8,0.9)
Very Good	VG	(9,10,10)	Very High	VH	(0.9,1,1)

Table 3. The performance scores of the alternatives with respect to the criteria provided by DMs.

	DM1						DM2				
	C_1	C_2	C_3	C_4	C_5		C_1	C_2	C_3	C_4	C_5
A_1	G	G	VG	G	VG	A_1	MG	MG	F	P	P
A_2	MG	MG	G	F	G	A_2	VG	VG	MG	MG	G
A_3	G	G	F	MG	G	A_3	G	VG	VG	VG	VG
A_4	F	VG	MG	VG	VG	A_4	MP	G	MG	G	VG
A_5	VG	P	VG	VG	MP	A_5	G	MG	F	F	F
	DM3						DM4				
	C_1	C_2	C_3	C_4	C_5		C_1	C_2	C_3	C_4	C_5
A_1	G	VG	MG	VG	F	A_1	MG	MG	G	MP	VG
A_2	F	G	MP	MG	VG	A_2	VG	P	VG	F	G
A_3	VG	G	MP	MG	MG	A_3	G	MG	G	G	VG
A_4	MG	G	G	G	MP	A_4	VG	G	G	MG	G
A_5	MG	MG	VG	G	G	A_5	G	VG	P	VG	MG

Table 4. The fuzzy decision matrix for the DM1.

Weights of attributes	(0.6, 0.7, 0.8)	(0.4, 0.5, 0.6)	(0.7, 0.8, 0.9)	(0.7, 0.8, 0.9)	(0.2, 0.3, 0.4)
	C_1	C_2	C_3	C_4	C_5
A_1	(7, 8, 9)	(7, 8, 9)	(9, 10, 10)	(7, 8, 9)	(9, 10, 10)
A_2	(6, 7, 8)	(6, 7, 8)	(7, 8, 9)	(4, 5, 6)	(7, 8, 9)
A_3	(7, 8, 9)	(7, 8, 9)	(4, 5, 6)	(6, 7, 8)	(7, 8, 9)
A_4	(4, 5, 6)	(9, 10, 10)	(6, 7, 8)	(9, 10, 10)	(9, 10, 10)
A_5	(9, 10, 10)	(2, 3, 4)	(9, 10, 10)	(9, 10, 10)	(3, 4, 5)

Table 5. The final parameters of the proposed fuzzy VIKOR for DM1.

	$\tilde{S}_i^k$	$\tilde{R}_i^k$	$\tilde{Q}_i^k$
A_1	(0.00, 0.47, 2.46)	(0.00, 0.40, 1.00)	(0.00, 0.57, 1.73)
A_2	(0.50, 1.85, 4.55)	(0.50, 1.00, 2.00)	(0.50, 1.43, 3.27)
A_3	(0.47, 1.80, 4.46)	(0.50, 1.00, 2.00)	(0.48, 1.40, 3.23)
A_4	(0.42, 1.18, 3.32)	(0.50, 1.00, 2.00)	(0.46, 1.09, 2.66)
A_5	(0.36, 0.80, 2.53)	(0.57, 1.00, 1.75)	(0.47, 0.90, 2.14)

After achieving values of $\tilde{Q}_i'$ for each alternative, sorting of alternatives in ascending order is implemented in Stage (4). As the values of $\tilde{Q}_i'$ are in the form of fuzzy numbers, preference ratio method is applied for ranking alternatives as described in Eq. (7). The final ranking of alternatives is shown in Table 8. As we can see in Table 8, the ranking of the five alternatives is $A_4 \succ A_1 \succ A_3 \succ A_2 \succ A_5$ in accordance with $\tilde{Q}_i'$ in ascending order.

6.2. Sensitivity analysis

According to the abovementioned results presented in the previous numerical example, two sensitivity analyses on the parameter ν and weights of DMs are

presented. ν is a great effective parameter in VIKOR technique to determine the value of the index rank. Normally, the value of ν is considered as 0.5. Therefore, a sensitivity analysis by changing value of ν in the interval $[0, 1]$ is performed for the obtained results. The rankings for five alternatives under different ν values are illustrated in Table 9. As can be seen, when ν is changed, there are some deviations in ranking of alternatives. A_4 is the best ranked alternative for $\nu < 0.75$; also, A_1 has the best rank for $\nu \geq 0.75$. Moreover, A_5 is the worst ranked alternative for different values of ν.

Another parameters that affect decision-making are weights of DMs. To present a sensitivity analysis on weights of DMs, we consider some compositions of DMs weights and apply the proposed method based on information of the mentioned example. To do so, we use linguistic variables and their TFNs as DMs weights exhibited in Table 1. The related results according to the different compositions of DMs weights are illustrated in Table 10. It is obvious from Table 10 that for the same given data, the final ranking orders of the alternatives may be influenced by weights of DMs. For instance, if all DMs have the same weights for the evaluation (Problem No. 6

Table 6. Group decision-making matrix.

Weights of DMs	(0.6, 0.7, 0.8)	(0.4, 0.5, 0.6)	(0.7, 0.8, 0.9)	(0.7, 0.8, 0.9)
	DM1	DM2	DM3	DM4
A_1	(0.00, 0.57, 1.73)	(0.71, 1.95, 5.67)	(0.30, 0.89, 3.13)	(0.53, 1.79, 5.80)
A_2	(0.50, 1.43, 3.27)	(0.19, 0.77, 2.19)	(0.49, 1.88, 7.13)	(0.38, 1.27, 4.39)
A_3	(0.48, 1.40, 3.23)	(0.00, 0.13, 1.43)	(0.35, 1.61, 6.60)	(0.04, 1.13, 4.30)
A_4	(0.46, 1.09, 2.66)	(0.26, 1.25, 4.11)	(0.17, 1.29, 5.48)	(0.06, 0.99, 3.94)
A_5	(0.47, 0.90, 2.14)	(0.54, 1.71, 5.28)	(0.26, 1.41, 6.08)	(0.46, 1.58, 5.71)

Table 7. Final parameters of the proposed fuzzy VIKOR for group.

	$\tilde{S}'_i$	$\tilde{R}'_i$	$\tilde{Q}'_i$
A_1	(0.00, 0.68, 12.62)	(0.00, 0.64, 5.67)	(0.00, 0.66, 9.15)
A_2	(0.00, 0.96, 14.11)	(0.00, 0.50, 6.95)	(0.00, 0.73, 10,53)
A_3	(0.00, 0.66, 13.06)	(0.00, 0.36, 6.43)	(0.00, 0.51, 9.74)
A_4	(0.00, 0.56, 12.88)	(0.00, 0.20, 5.30)	(0.00, 0.38, 9.09)
A_5	(0.00, 0.88, 15.29)	(0.00, 0.26, 5.90)	(0.00, 0.57, 10.60)

Table 8. Final ranking of alternatives.

	1	2	3	4	5
By $\tilde{S}'_i$	A_3	A_4	A_1	A_2	A_5
By $\tilde{R}'_i$	A_4	A_1	A_5	A_3	A_2
By $\tilde{Q}'_i$	A_4	A_3	A_1	A_2	A_5

Table 9. Ranking orders of alternatives under different ν values.

Values of ν	Ranking index				
	1	2	3	4	5
$\nu = 0$	A_4	A_2, A_1, A_5	A_3	–	–
$\nu = 0.25$	A_4	A_1	A_3	A_2	A_5
$\nu = 0.5$	A_4	A_1	A_3	A_2	A_5
$\nu = 0.75$	A_1	A_4	A_3	A_2	A_5
$\nu = 1$	A_1	A_4	A_3	A_2	A_5

in Table 10), A_1 is the suitable alternative for the considered application example. If DM1 and AM4 have more weights (Problem No. 1), A_4 is introduced as the best ranked alternative. Also, whenever DM2 and DM3 have more weights (Problem No. 10), A_3 is the suitable alternative. Thus, the sensitivity analysis shows that the proposed method can represent a more realistic problem by considering different weights for DMs in the decision-making process. It is necessary to mention that weights of DMs can be determined by a higher management level based on their different backgrounds and expertise in problem.

6.3. Comparison

In this subsection, we compare our proposed method with two current methods in the field of SSP. To do so, we have applied our proposed method for the considered example in Chen et al. [64] and Sanayei et al. [35] and then compared our result with their result. Comparative results are shown in Table 11.

From the results of Example 1 given in Table 11, it can be observed that Fuzzy TOPSIS [64] and the proposed method suggest the same ranking for suppliers: $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$. This demonstrates the validity of the proposed method. From Example 2, we observe that both fuzzy VIKOR [35] and our proposed method suggest suppliers 4 and 5 as the fourth and fifth choices, respectively. But, for suppliers 1, 2, and 3, our proposed method suggests a different ranking for fuzzy VIKOR [35]. This inconsistency can be understood by the fact that the fuzzy numbers are converted into crisp values and DM's opinions are aggregated into a single matrix in fuzzy VIKOR [35] method, which may produce the consequent loss of information and hence the lack of precision in the final results. The main advantages of our proposed supplier selection method are that we:

1. Keep all data in form of fuzzy numbers during the solving process;

2. Solve the problem for each individual DM without aggregating DMs rating at first;

3. Consider different weights for each DM;

4. Use a simple method to calculate fuzzy distance and to rank fuzzy numbers.

7. Conclusion

Supplier evaluation and selection have become a very important issues in today's highly competitive global business environment. In this study, we proposed an extension of fuzzy VIKOR with considering a new efficient, robust, and simple fuzzy distance measure for SSP. To do so, unlike some papers which calculated the distance between fuzzy numbers as a crisp value, we applied an efficient fuzzy distance measure presented

Table 10. Ranking orders of alternatives under different weights of DMs.

Problem no.	Weights of DMs	Ranking	Pro. no.	Weights of DMs	Ranking
1	DM1=H, DM2=L DM3=M, DM4=H	$A_4 \succ A_1 \succ A_3 \succ A_2 \succ A_5$	9	DM1=L, DM2=M DM3=M, DM4=H	$A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5$
2	DM1=L, DM2=M DM3=MH, DM4=H	$A_4 \succ A_1, A_3 \succ A_2 \succ A_5$	10	DM1=L, DM2=H DM3=H, DM4=L	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$
3	DM1=L, DM2=L DM3=H, DM4=H	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	11	DM1=L, DM2=L DM3=L, DM4=H	$A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5$
4	DM1=H, DM2=H DM3=L, DM4=L	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$	12	DM1=L, DM2=H DM3=L, DM4=L	$A_3 \succ A_2, A_4 \succ A_1 \succ A_5$
5	DM1=H, DM2=L DM3=H, DM4=L	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	13	DM1=L, DM2=L DM3=H, DM4=L	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
6	DM1=M, DM2=M DM3=M, DM4=M	$A_1 \succ A_3 \succ A_4 \succ A_2 \succ A_5$	14	DM1=H, DM2=L DM3=L, DM4=L	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
7	DM1=L, DM2=H DM3=L, DM4=H	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$	15	DM1=L, DM2=M DM3=M, DM4=H	$A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5$
8	DM1=H, DM2=L DM3=L, DM4=H	$A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5$	16	DM1=H, DM2=L DM3=H, DM4=H	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$

Table 11. Ranking comparisons for examples.

Alternatives	Example 1		Example 2	
	Fuzzy TOPSIS (Chen et al. [64])	Proposed method	Fuzzy VIKO (Sanayei et al. [35])	Proposed method
A_1	4	4	1	3
A_2	1	1	3	2
A_3	2	2	2	1
A_4	3	3	4	4
A_5	5	5	5	5

by Sadi-Nezhad et al. [70] to increase accuracy of the decision-making process. Also, we solved the problem for each individual DM without aggregating DMs rating; then, the group decision-making matrix with using DMs decisions was constructed at the end of the proposed method. Finally, aggregation of the results for the final decision was carried out with considering different weights for DMs. To increase accuracy of our method, data remained in the form of fuzzy numbers during the process; therefore, preference ratio method was applied for ranking fuzzy numbers. In the present study, the executive procedure of our proposed fuzzy VIKOR was illustrated by applying it to the problem of supplier selection. At the end of the numerical illustration, two sensitivity analyses on (1) the parameter ν and (2) weights of DMs were performed. Results showed that the final ranking orders of the alternatives might be influenced by the parameter ν and weights of DMs. Therefore, the proposed method can model a more realistic problem by considering different weights for DMs in the decision-making process.

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