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Joint economic lot-sizing problem for a two-stage supply chain with price-sensitive demand

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Abstract. This paper studies Joint Economic Lot-Sizing problem (JELS) for a single-vendor single-buyer system while demand is dependent on selling price. This problem is modeled for geometric shipment policy and a solution procedure is developed to find a well approximation of the global optimal solution of the problem. Since the equal-size shipment policy or geometric shipment policy may yield more joint profit compared to each other, the most important factor that affects the break-even point of geometric and equal-size policies is determined. The JELS problem for dependent demand is also modeled for geometric-then-equal size and optimal shipment policies. Solution procedures to find a well approximation of the global optimum of each problem are also developed for these models. The models and solution procedures for geometric, geometric-then-equal size, and shipment policies are novel in the literature. Numerical results of the models show considerable improvement in the joint profit of the chain compared to lot-for-lot and equal-size shipment policies for chains with price-sensitive demand and it could be very interesting for supply chain coordinators and practitioners.

1. Introduction

In today's competitive global market, companies are pushed towards not only integrating different decision processes within their operational borders, but also towards closely collaborating with their customers and suppliers [1]. Therefore, all parties need to seek Economic Order Quantity (EOQ) based on their integrated total cost function, rather than each party's individual cost functions. Such a problem is generally called Joint Economic Lot-sizing Problem [2]. A thorough review of the research dealing with coordinated vendor-buyer Supply Chain (SC) is gathered by Glock [3]. The Joint Economic Lot-Sizing problem (JELS) was developed in different aspects and focusing on shipment policies is one of the earliest attempts in this context.

The first shipment policy studied in the JELS literature was lot-for-lot shipment policy (LP). In this policy, a production batch is shipped to the buyer as a single shipment. The first study modeled total joint cost of a single-vendor single-buyer SC with infinite production rate and was published in 1977 [4]. Then, this policy was developed by assuming finite production rate [5]. Kim et al. [6] also studied the LP for a system consisting of a manufacturer and a retailer with price-dependent demand while full coordination, partial coordination, and non-coordination mechanisms were applied to the system. Sana [7] adopted LP for a chain consisting of a manufacturer and a retailer while demand was dependent on the sale initiatives provided by the retailer.

The second shipment policy considered in the JELS literature was equal-size shipment policy (EP). In this policy, a production batch is divided to equal shipments. Two variants of EP are developed in the literature: delayed EP introduced by Goyal [8] and

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non-delayed EP applied by Lu [9]. In the delayed EP, it is assumed that shipping the products from the vendor to the buyer is delayed until producing the entire production batch is finished. This assumption is relaxed in non-delayed EP and the first shipment will be sent to the buyer when it is produced at the vendor. EP was investigated by Buscher and Lindner [10] for a production system with rework considerations. The focus of the paper is on the determination of production and rework lot sizes, but the optimal number of shipments is also determined by the proposed solution procedure. Total cost of an integrated two-stage system was optimized by Shu and Zhou [11] when vendor invested on setup cost reduction, process quality improvement, and EP as the shipment policy. Non-delayed EP was also studied for a three-layer vertically integrated SC involving a supplier, a manufacturer, and multi retailers with constant demand and increasing inventory holding costs in the downstream direction [12]. Abdelsalam and Elassal [13] extended the proposed model of [12] by applying stochastic demand while each retailer had its own holding and ordering costs that were not necessarily equal to those of other retailers. More examples of considering EP in the JELS problem optimization can be found in [14-16].

The third shipment policy was introduced in 1995 [17] that is known as geometric shipment policy (GP). In this policy, a growth factor that is equal to the ratio of production rate to demand rate, $\frac{p}{D}$, is applied to the size of shipments. Therefore, if size of the first shipment is named q_1, the nth shipment size will be equal to $q_1\left(\frac{p}{D}\right)^{n-1}$. Later, Hill [18] assumed the growth factor as a decision variable and developed a solution procedure to find a locally optimal solution of the resultant problem. GP with a constant growth factor has recently been considered, in which the production of defective items has been taken into account and the manufacturer should pay a warranty cost for each identified defective product [19].

The fourth shipment policy in the JELS literature is known as Geometric-then-Equal size Policy (GEP). First, Goyal and Nebebe [20] considered the ratio of the first shipment to the remaining equal shipments equal to $\frac{p}{D}$. So they considered:

$$q_1, q_1\frac{p}{D}, q_1\frac{p}{D}, q_1\frac{p}{D}, \dots$$

as the size of shipments.

Later, Ben-Daya et al. [1] generalized the proposed policy of [20], considering that the first m shipments followed the geometric policy and the size of equal shipments was similar to that of the last geometric shipment.

Hill [21] developed a new shipment policy as the result of relaxing the assumption of predetermined shipment policy using Lagrangian multiplier. He proved that the developed policy, named Optimal Policy (OP), is similar to GEP, but the size of $n-m$ equal shipments is not necessarily equal to the size of the last geometric shipment. He also developed a solution procedure to solve the joint cost model of the chain.

Integrating pricing decision with ordering, inventory, and shipment decisions is another stream of research in JELS literature. Whitin [22] was the first one who modeled EOQ for price-sensitive demand for an SC in 1995. This stream was continued by Jokar and Sajadieh [23]. They optimized total joint profit of an SC by integrating ordering, shipment, and pricing policies considering LP as shipment policy and price-dependent demand. Another effort to optimize joint profit of a chain with price-sensitive demand while EP is applied can be found in [24]. Joint-pricing and lot-sizing policies for price-dependent demand was investigated by Kim et al. [25] for a system while LP was adopted for shipment of the products. In another study, Huang et al. [26] tried to coordinate pricing and inventory decisions for a three-stage SC consisting of multiple suppliers, single manufacturer, and multiple retailers as a non-corporative game. More examples of recent research in this area can be found in [27-29]. The JELS problem was also modeled for price and environmentally dependent demand when EP was adopted [30]. Joint-pricing and lot-sizing problem for a two-stage system was studied by Ghasemy Yaghin et al. [31] when supplier followed EP and demand was price-sensitive through a logit function with a price discount for the orders arrived prior to the sale period. A two-stage system was also investigated for a multi-product chain where demand for each product at each retailer was dependent on its price, price of other competitors, and the price of substitutable products [32]. As another effort to integrate lot-sizing and pricing policies, Taleizadeh and Noori-daryan [33] modeled a three-stage SC to find the minimum total cost of the chain using Stackelberg-Nash equilibrium to optimize the cost incurred by the members. Wang et al. [34] also modeled the JELS problem for price-dependent demand with infinite production rate. They considered both linear and non-linear dependency functions and solved the corresponding models for centralized and decentralized conditions.

Besides the extensions focusing on the various shipment policies and efforts to integrate joint-pricing and lot-sizing policies, the JELS problem was also extended by other considerations. Integrated vendor-buyer model with stock-dependent demand was studied for coordinated and non-coordinated SCs [35]. Three-stage SC with imperfect quality products was considered by Sanaz [36] while assuming production rate as a decision variable and different probability distribution functions for defective items. Lee and Fu [37] investigated a make-to-order two-echelon SC while considering transportation cost between the vendor and the buyer.

To the best of our knowledge, joint lot-sizing and pricing is only studied for LP and EP and there is no research considering it for neither GP, nor GEP, nor OP. To fill this gap, we model the joint profit of a two-stage SC for these shipment policies and develop their corresponding solution procedures to find the optimal solutions. Besides, as for some problem instances, total profit obtained by adopting EP is greater than that of GP and vice versa. Also, there is no analysis comparing optimal profits of EP and GP; we compare their optimal joint profits for price-sensitive demand and find the most important factor that affects break-even point of them.

The main contributions of this paper can be summarized as follows. First, for the joint profit model of GP, the paper determines the upper and the lower limits for the number of shipments and develops a search based procedure to find the optimal solution. Second, it determines the break-even point of EP and GP for price-dependent demand. Third, similar to GP, the optimal solution for the joint profit model is determined. Fourth, for OP, the optimal selling price, production batch size, and the number of shipments are obtained by the proposed solution procedure.

In all of the developed solution procedures, secant method is used to determine a well approximation of the global optimum. Convergence properties of the secant method are studied in some research like [38-39]. In the current paper, the performance of the proposed procedure is evaluated by comparison of its optimal solution with the optimal solution obtained by adopting a Simulated Annealing (SA) algorithm.

This paper is organized as follows. Section 2 defines the general JELS problem and its assumptions. In section 3, the JELS problem with GP is modeled and a solution procedure is developed to find the optimal solution of the model. Section 4 investigates the optimal solutions of the proposed procedure and the SA algorithm. Section 5 gives a comparison of EP and GP and also determines break-even point of these policies. Section 6 deals with the JELS problem for GEP. Section 7 models the JELS problem when OP is applied to the chain and develops its optimal solution procedure. Section 8 presents numerical results and sensitivity analysis of the problem parameters. Finally, Section 9 is devoted to the conclusions and recommendations for future research.

2. Problem definition and assumptions

This paper studies a single-vendor single-buyer SC of a single product. Similar to other research in the JELS literature, time horizon is assumed to be infinite, and shortage is not allowed. In addition, production rate is assumed to be finite and greater than the demand rate. The demand (D) for the final product is linearly dependent on selling price (δ). The dependency function is $D(\delta) = a - b\delta$. The buyer's inventory holding cost is greater than that of the vendor, and the buyer continuously reviews its inventory. The following notations are used in the paper:

P	Vendor's production rate;
D	Demand rate as a function of selling price;
δ	Buyer unit selling price (paid by final customer);
A_v	Vendor's setup cost;
A_b	Buyer's ordering cost;
h_v	Vendor's inventory holding cost per year per unit product;
h_b	Buyer's inventory holding cost per year per unit product;
n	Number of shipments;
m	Number of equal shipments in a cycle;
l	Geometric growth factor (p/D);
Q	Vendor production batch size;
q_1	First shipment size from the vendor to the buyer.

The JELS model has a general formulation as shown in Eq. (1). The objective of this model is to minimize total cost of the chain and it consists of ordering and inventory holding costs of the chain's members:

$$\mathbf{TC}_{\text{joint}} = \frac{(\mathbf{A}_v + \mathbf{N}\mathbf{A}_b)\mathbf{D}}{\mathbf{Q}} + \mathbf{h}_v\mathbf{I}_s + (\mathbf{h}_b - \mathbf{h}_v)\mathbf{I}_b, \quad (1)$$

where:

$$\mathbf{I}_s = \mathbf{q}_1\frac{\mathbf{D}}{\mathbf{p}} + \mathbf{Q}\frac{(\mathbf{p}-\mathbf{D})}{2\mathbf{p}}, \quad (2)$$

$$\mathbf{I}_b = \frac{\sum_1^n \mathbf{q}_i^2}{2\mathbf{Q}}. \quad (3)$$

3. Geometric shipment policy (GP)

The idea of GP was first introduced in 1995 [17]. In this policy, the size of each shipment is a multiplier of the first shipment size as presented in Eq. (4). The production batch size is equal to the sum of shipments in a cycle; its corresponding expression is shown in Eq. (5):

$$\mathbf{q}_i = \mathbf{l}^{i-1}\mathbf{q}_1 \ \forall\ \mathbf{i} = 1, ..., \mathbf{n}. \quad (4)$$

The size of the first shipment can be written as Eq. (6):

$$\mathbf{Q} = \frac{\mathbf{q}_1(\mathbf{l}^n - 1)}{\mathbf{l} - 1}, \tag{5}$$

$$\mathbf{q}_1 = \frac{\mathbf{Q}(\mathbf{l} - 1)}{\mathbf{l}^n - 1}. \tag{6}$$

The objective of the general JELS model is to minimize total joint cost of the chain, but the current paper maximizes the total joint profit of the chain. Therefore, the exchange price between the vendor and the buyer has no effect on the joint profit of the chain. Here, the chain revenue is achieved through selling the final products to the customers, $D\delta$, that can be rewritten as $\frac{D(a-D)}{b}$. By replacing inventories of the system and the buyer in the general JELS model, total joint profit of the chain for GP becomes as Eq. (7). Since this expression is concave in q_1, the optimal value of this variable, q_1^*, can be determined using the first derivative of the joint profit relative to q_1, as shown in Eq. (8):

$$TO_{\text{joint}}^{\text{GP}}(q_1, D, n) = \frac{D(a-D)}{b} - \frac{(A_v + nA_b)\left(\frac{p-D}{D}\right)D}{q_1\left(\frac{p^n - D^n}{D^n}\right)} - h_v\left[\frac{D}{p} + \frac{(p-D)\left(\frac{p^n-D^n}{D^n}\right)}{2p\left(\frac{p-D}{D}\right)}\right]q_1 - (h_b - h_v)\left[\frac{\frac{(p^n-D^n)}{D^n}}{2\left(\frac{p+D}{D}\right)}\right]q_1, \tag{7}$$

$$q_{1GP}^* = \sqrt{\frac{\frac{(A_v+nA_b)\left(\frac{p-D}{D}\right)D}{\left(\frac{p^n-D^n}{D^n}\right)}}{h_v\left[\frac{D}{p} + \frac{(p-D)\left(\frac{p^n-D^n}{D^n}\right)}{2p\left(\frac{p-D}{D}\right)}\right] + (h_b - h_v)\left[\frac{\left(\frac{p^n+D^n}{D^n}\right)}{2\left(\frac{p+D}{D}\right)}\right]}}. \tag{8}$$

By replacing the optimal value of q_1 in $TP_{\text{joint}}^{\text{GP}}(q_1, D, n)$ and after some manipulation, the total joint profit of the chain becomes as Eq. (9). Constraints of this model are shown in Relations (10) and (11):

$$TP_{\text{joint}}^{\text{GP}}(D, n) = \frac{D(a-D)}{b} - 2\sqrt{\frac{(A_v + nA_b)D(Dh_v + ph_b)(p^n + D^n)(p-D)}{2p(p^n - D^n)(p+D)}}, \tag{9}$$

$$D \geq 0, \tag{10}$$

$$n: \text{ integer}. \tag{11}$$

Since there is no closed form for optimal D or n that can be determined by the corresponding first and second derivatives, the remaining analysis to find the optimal values of these variables should be continued based on the numerical methods.

As n is an integer variable, it is of great interest to determine its upper and lower limits and then search through the interval of these limits for optimal D. To find these limits, the objective function can be rewritten as a function of n as shown in Eq. (12):

$$TP'^{\text{GP}}_{\text{joint}}(n) = -\sqrt{\frac{(A_v + nA_b)(p^n + D^n)}{(p^n - D^n)}}. \tag{12}$$

The first derivative of the above expression relative to n is as follows:

$$\frac{\partial TP'^{\text{GP}}_{\text{joint}}(n)}{\partial n} = -\frac{A_b(p^{2n} - D^{2n}) - 2p^n D^n (A_v + nA_b)(\log(p) - \log(D))}{2(p^n - D^n)\sqrt{(A_v + nA_b)(p^{2n} - D^{2n})}}. \tag{13}$$

As $\delta \leq \frac{a}{b}$, D belongs to $(0, a)$. On the other hand $p > D$, therefore $\frac{\partial TP'^{\text{GP}}_{\text{joint}}(n)}{\partial n}$ is always less than zero and consequently $TP'^{\text{GP}}_{\text{joint}}(n)$ is a non-increasing function of D. Therefore, the upper and the lower limits of n can be found by the following two procedures:

- **Procedure 1:** Find the upper limit of n ($n_{\max}^{\text{GP}}$)

(a) Replace D with its lower limit, zero, in $TP_{\text{joint}}^{\text{GP}}(D, n)$;

(b) Determine the optimal n that maximizes $TP_{\text{joint}}^{\text{GP}}$ $(D = 0, n)$ using Secant method [39];

(c) The ceiling of the resultant n is the upper limit of this variable ($n_{\max}^{\text{GP}}$).

- **Procedure 2:** Find the lower limit of n ($n_{\min}^{\text{GP}}$)

(a) Replace D with its upper limit, a, in $TP_{\text{joint}}^{\text{GP}}(D, n)$;

(b) Determine optimal n that maximizes $TP_{\text{joint}}^{\text{GP}}(D = a, n)$ using Secant method;

(c) The floor of the resultant n is the lower limit of this variable ($n_{\min}^{\text{GP}}$).

Solution procedure

The following procedure is developed to solve the model.

- **Step 1.** Calculate $n_{\min}^{\text{GP}}$ and $n_{\max}^{\text{GP}}$;
- **Step 2.** Set TP_{opt}^{GP} and $n = n_{\min}^{\text{GP}}$;
- **Step 3.** Determine D that maximizes $TP_{\text{joint}}^{\text{GP}}(D, n)$ using Secant method and save the corresponding objective function value;
- **Step 4.** If $TP_{\text{joint}}^{\text{GP}}(D, n) > TP_{opt}^{\text{GP}}$, then set $n_{opt} = n$, $TP_{opt}^{\text{GP}} = TP_{\text{joint}}^{\text{GP}}(D, n)$, and $D_{opt} = D$;

- **Step 5.** Increase n by 1.;
- **Step 6.** If $n < n_{\max}^{\mathrm{GP}}$, go to Step 3; otherwise, go to Step 7;
- **Step 7.** The current solution is optimal.

4. Investigate performance of the proposed procedure vs. Simulated Annealing (SA) algorithm

To study the performance of the proposed procedure in Section 3, we solve the JELS problem for GP by implementing a Simulated Annealing (SA) algorithm. SA is a powerful algorithm commonly used for heuristic optimization due to its simplicity and effectiveness. Within this approach, variables to be optimized are viewed as the degrees of freedom of a physical system and the cost function of the optimization problem as the energy [40]. We implement an SA algorithm while the optimal solution obtained by the proposed procedure is considered as the initial solution to start the SA algorithm. The steps of the customized and tuned SA algorithm to solve the JELS problem are as follows. To reduce run time and enhance the performance of the SA algorithm, the maximum number of neighbors is linearly dependent on the temperature; more neighbors are generated for high temperatures, and as temperature decreases, less neighbors are considered to accelerate the convergence process.

- **Step 1.** Set the optimal solution of the proposed procedure as the initial solution and its objective function value as Z_{opt};
- **Step 2.** Set initial temperature $(t) = 100$ and final temperature $= 10^{-200}$;
- **Step 3.** While $t < 10^{-200}$:
  - 3.1. Set maximum number of neighbors ($n_{\max}$) as $70t$;
  - 3.2. While number of neighbors is less than $n_{\max}$:
    - 3.2.1 Randomly select between neighborhood generation methods A and B:
      - A. Assign a random number to one of the variables regarding its constraint;
      - B. Swap values of two variables regarding their constraints.
    - 3.2.2. Calculate objective function of the neighbors (Z);
    - 3.2.3. If $Z > Z_{opt}$, update the optimal solution found so far and increase number of neighbors by one. Otherwise, calculate acceptance probability, $p = \frac{Z - Z\_opt}{t}$. Generate a random number; if p is greater than it, increase number of neighbors by one.
  - 3.3. Set the best solution found in this t as optimal and its objective function value as Z_{opt};
  - 3.4. Schedule cooling by updating temperature as $0.9t$.
- **Step 4.** The best solution obtained in all temperatures is the optimal solution. The comparison between the optimal joint profits of the SA algorithm and the proposed procedure is shown in Table 1 for the benchmark problem of the JELS literature. The parameters of this problem are as follows:

$$p = 3200/\text{year}, \quad h_v = \$4/\text{unit/year},$$

$$h_b = \$5/\text{unit/year} \quad A_v = \$400/\text{setup},$$

$$A_b = \$25/\text{setup}, \quad a = 1500, \quad b = 50.$$

As presented in the table, for 10 runs of the algorithms, there is just a small difference between their optimal profits. Therefore, we can claim that

Table 1. Comparison of optimal profits obtained by the SA algorithm and the proposed procedure.

Run #	Optimal profit obtained by SA	Optimal profit obtained by the proposed procedure	% difference in optimal profit
1	9,617.502511572	9,617.519119639	0.000172686
2	9,617.504099580	9,617.519705279	0.000162263
3	9,617.517440652	9,617.519798804	0.000024519
4	9,617.507101522	9,617.518986796	0.000123579
5	9,617.513974360	9,617.518703130	0.000049168
6	9,617.519540566	9,617.519799290	0.000002690
7	9,617.513821152	9,617.519181529	0.000055736
8	9,617.519625431	9,617.519790744	0.000001719
9	9,617.508403004	9,617.519788001	0.000118378
10	9,617.513707257	9,617.519779509	0.000063137

the proposed procedure to solve the JELS problem has a good performance and its solution is a well approximation of the global optimum of the problem. It should be notified that the proposed algorithm is preferred due to its short run time compared to the SA algorithm while its performance is approximately the same. Besides, the proposed procedure uses secant method that is a traditional method and it can be easily implemented.

5. Comparison of equal shipment policy with geometric shipment policy

In the EP, all the orders are equal; so ordering and shipment management of this policy is simple for partners. On the other hand, in GP, the size of orders increases geometrically; so the time between two orders and the transportation capacities should be altered for each order. The main concentration of EP besides the simplicity of management is to minimize average inventory of the buyer while the main concentration of GP is to minimize the vendor's average inventory. These facts are presented in Figures 1 and 2 for the benchmark problem of the JELS literature for different values of the buyer's inventory holding cost near its value in the benchmark problem.

Therefore, it seems that the inventory holding cost of the vendor and the buyer may affect the efficiency of these two policies. To study this issue, the benchmark problem of JELS literature is considered.

Figure 3 illustrates the effect of h_b/h_v on the total joint profit of the chain. It can be inferred from the figure that there is a Break-Even Point (BEP) of EP and GP. Analyzing sensitivity of the total joint profit to other parameters of the problem and determining the BEP for some problem instances based on all parameters confirm the major effect of inventory holding cost

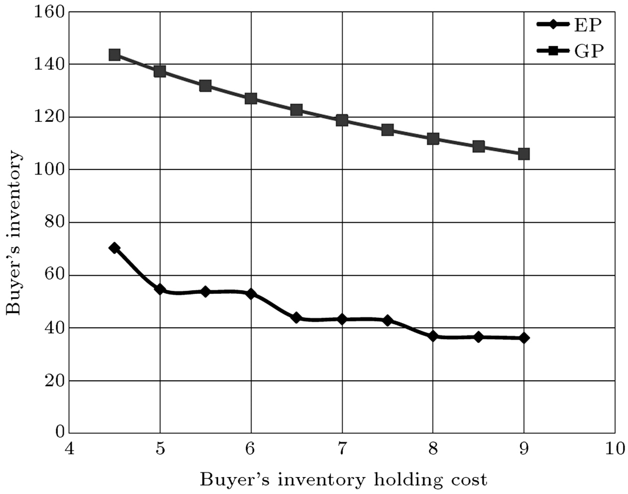

Figure 1. Effect of buyer's inventory holding cost on buyer's inventory.

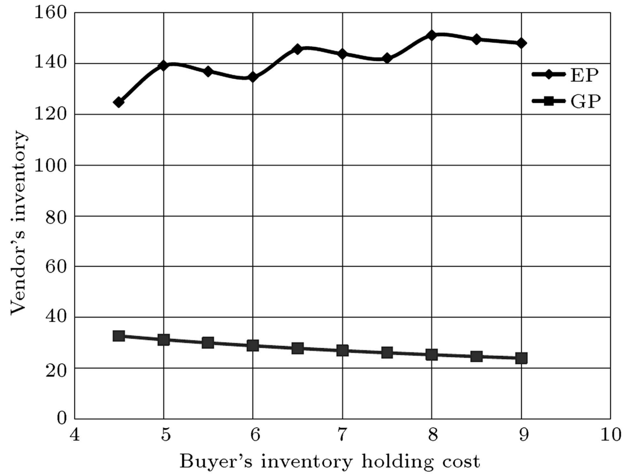

Figure 2. Effect of buyer's inventory holding cost on vendor's inventory.

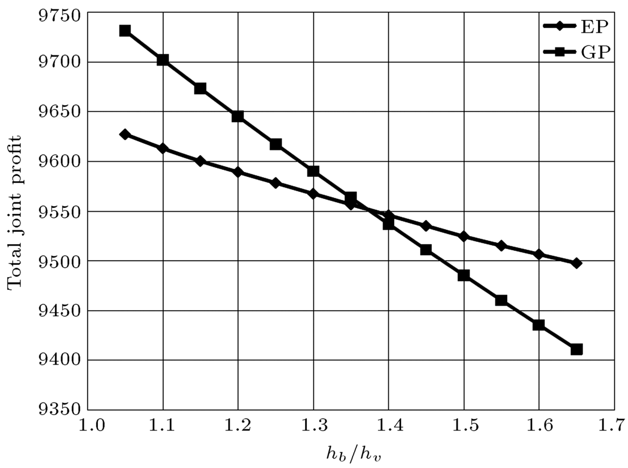

Figure 3. Effect of h_b/h_v on the total joint profit.

on the BEP of EP and GP. Indeed, when the range of parameters of a problem does not thoroughly change, as in practice happens, the main factor that affects this point is the ratio of h_b to h_v. When this ratio is greater than BEP, less inventory of buyer is desired, so EP shows better performance; but when this ratio is less than BEP, less inventory of vendor is desired, so GP is preferred. As shown in Figure 3, the value of h_b/h_v at the BEP of the benchmark problem is equal to 1.37. Therefore, when $h_v = 4$, if h_b is smaller than 5.48, GP shows better performance; otherwise, EP is preferable. Determination of this point can be helpful when the chain coordinator decides to choose between EP and GP. The BEP can be determined using the interpolation method to find the ratio of h_b to h_v.

6. Geometric-then-equal size shipment policy (GEP)

As explained in Section 5, EP or GP may yield more joint profit compared to each other. Therefore, it is of great interest to combine these policies and simultaneously benefit from their advantages. Ben-Daya et al. [1] generalize the proposed policy of [20] called geometric-then-equal size shipment policy. The size of shipments in this combined policy is according to Eq. (14):

$$q_i = \begin{cases} l^{i-1} * q_1 & i = 2, 3, ..., m \\ l^{m-1} * q_1 & i = m+1, ..., n \end{cases} \tag{14}$$

Based on the shipments' size, the production batch size is shown in Eq. (15).

$$\begin{aligned} Q &= \sum_{i=1}^{m} \left(\frac{p}{D}\right)^{i-1} q_1 + (n-m)\left(\frac{p}{D}\right)^{m-1} q_1 \\ &= q_1 \left[\frac{\left(\frac{p^m - D^m}{D^m}\right)}{\frac{p-D}{D}} + (n-m)\left(\frac{p}{D}^{m-1}\right) \right] \\ &= q_1 \varphi_q(D, m, n). \end{aligned} \tag{15}$$

The average inventories of the system and the buyer are also obtained as follows:

$$\begin{aligned} I_s &= \left[\frac{D}{p} + \frac{(p-D)\left(\frac{\left(\frac{p^m - D^m}{D^m}\right)}{\frac{p-D}{D}} + (n-m)\left(\frac{p}{D}\right)^{m-1} \right)}{2p} \right] q_1 \\ &= q_1 \varphi_s(D, m, n), \end{aligned} \tag{16}$$

$$\begin{aligned} I_b &= \frac{q_1}{2} \left[\frac{\frac{p^{2m} - D^{2m}}{D^{2m}} + (n-m)\left(\frac{p}{D}\right)^{2m-2}\left(\frac{p^2 - D^2}{D^2}\right)}{\left(\frac{p+D}{D}\right)\left[\left(\frac{p^m - D^m}{D^m}\right) + (n-m)\left(\frac{p}{D}\right)^{m-1}\left(\frac{p-D}{D}\right)\right]} \right] \\ &= q_1 \varphi_b(D, m, n). \end{aligned} \tag{17}$$

Using the above expressions, the total joint profit of the chain for GEP is presented in Eq. (18). Again, this equation is concave in q_1 and by a similar analysis, the optimal value of this variable is as Eq. (19) and the modified objective function with its 3 variables is show in Eq. (20). It should be notified that $1 \leq m \leq n$.

$$\begin{aligned} TP_{\text{joint}}^{\text{GEP}}(q_1, D, m, n) &= \frac{D(a-D)}{b} - \frac{(A_v + nA_b)D}{q_1 \varphi_q(D, m, n)} \\ &\quad - h_v q_1 \varphi_s(D, m, n) - (h_b \\ &\quad - h_v) q_1 \varphi_b(D, m, n), \end{aligned} \tag{18}$$

$$q_{1\text{GEP}}^{*} = \sqrt{\frac{(A_v + nA_b)D}{\varphi_q(D,m,n)[h_v \varphi_s(D,m,n) + (h_b - h_v)\varphi_b(D,m,n)]}} \tag{19}$$

$$TP_{\text{joint}}^{\text{GEP}}(D, m, n) = \frac{D(a-D)}{b} - 2 \sqrt{\frac{(A_v + nA_b)D}{\varphi_q(D, m, n)}[h_v \varphi_s(D, m, n) + (h_b - h_v)\varphi_b(D, m, n)]} \tag{20}$$

Similar to GP, determining the upper and the lower limits of n can be useful here. Since GEP is a combination of GP and EP, finding the upper and the lower limits of n is straightforward. According to [24], the lower and the upper limits of n for EP are as Eqs. (21) and (22), respectively;

$$n_{\min}^{\text{EP}} = \max\left\{ \sqrt{\frac{A_v(h_b - h_v)}{A_b h_v}}, 1 \right\}, \tag{21}$$

$$n_{\max}^{\text{EP}} = \sqrt{\frac{A_v p(h_b - h_v) + 2h_v A_v a}{A_b h_v (p-a)}}. \tag{22}$$

Since the optimal value of n is not out of the range of this variable in its parent policies, GP and EP, the limits of n in GEP, are as Eqs. (23) and (24).

$$n_{\min}^{\text{GEP}} = \min\left\{ n_{\min}^{\text{EP}}, n_{\min}^{\text{GP}} \right\}, \tag{23}$$

$$n_{\max}^{\text{GEP}} = \max\left\{ n_{\max}^{\text{EP}}, n_{\max}^{\text{GP}} \right\}. \tag{24}$$

To find the optimal values of variables with respect to the constraints, the following algorithm depicted in Figure 4 is developed. The proposed procedure to solve the model is similar to what was developed for GP as shown in Figure 4. According to Section 4, the final solution is a well approximation of the global optimal solution.

7. Optimal shipment policy (OP)

The last shipment policy introduced in the JELS literature is OP. The size of shipments in this policy is very similar to that in GEP. Using Lagrangian multipliers recursively, Hill [41] introduced this policy with the following size of shipments:

$$q_i = \begin{cases} l^{i-1} * q_1 & i = 1, 2, ..., m \\ \frac{Q - \sum_{i=1}^{m} q_i}{n-m} & i = m+1, ..., n \end{cases} \tag{25}$$

Hill [41] defines c as $\frac{h_v}{h_b - h_v}$ and shows that if there is no positive integer $m < n$, for which the following constraint holds, the OP changes to GP. By careful checking of this constraint, we found that there was a mistake in his expression. In other words, when we tried to obtain the constraint showed in Eq. (26) using the steps explained in [41], the resultant constraint was different. Further efforts ensured us that the

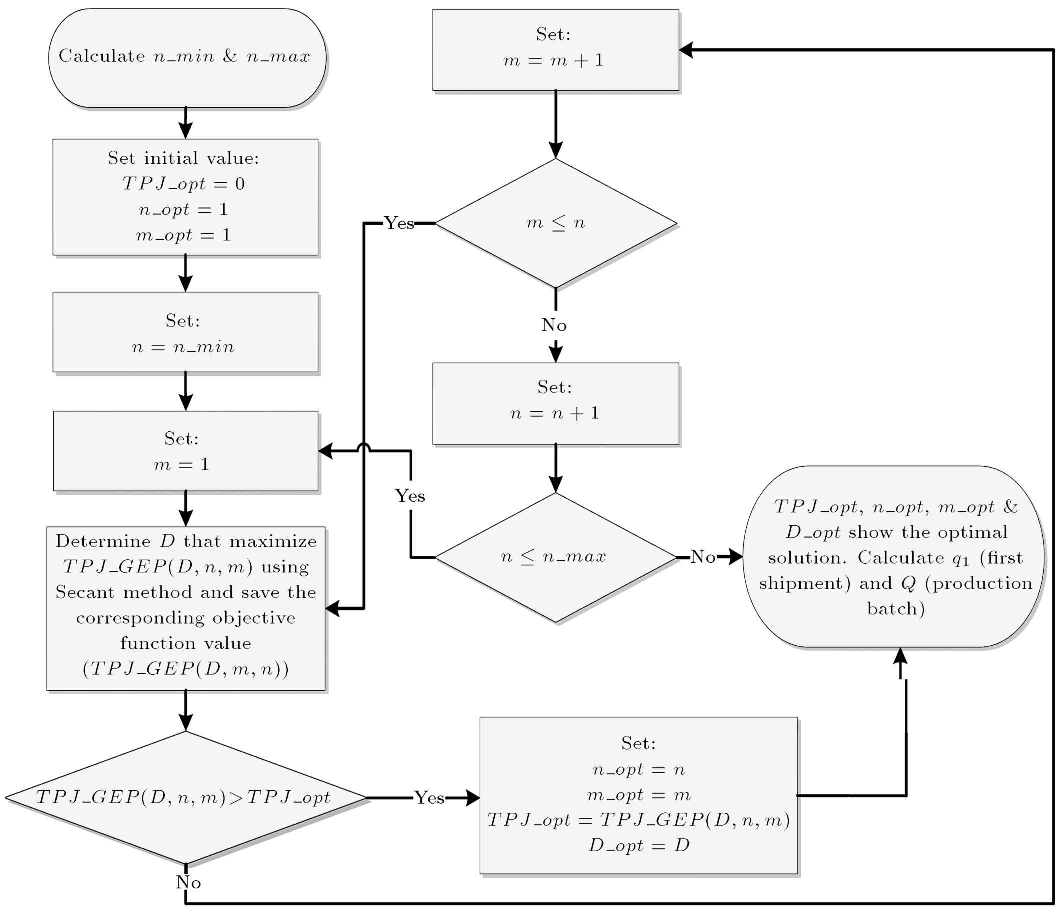

Figure 4. Solution procedure for JELS model when geometric-then-equal size shipment policy is applied.

correct constraint was as Eq. (27); therefore, we use this equation in the remaining analysis:

$$c \leq \frac{l(l^m - 1)}{(n-m)(l-1)} + \frac{l\left(\frac{(l^{2m}-1)}{(l^2-1)} + \frac{(l^m-1)^2}{(n-m)(l-1)^2}\right)}{(n-m)l^m + \frac{l^m-1}{l-1}}, \tag{26}$$

$$c \leq \frac{l(l^m - 1)}{(n-m)(l-1)} - \frac{l\left(\frac{(l^{2m}-1)}{(l^2-1)} + \frac{(l^m-1)^2}{(n-m)(l-1)^2}\right)}{(n-m)l^m + \frac{l^m-1}{l-1}}$$

$$= \theta(D, m, n). \tag{27}$$

It should be notified that l in the above constraint is equal to $\frac{p}{D}$ and can be replaced with this value. According to [41], we can rewrite the size of the first shipment as shown in Eq. (28). In a similar way, average inventories of the system and the buyer are presented in Eqs. (29) and (30), respectively:

$$q_1 = Q\frac{\frac{\left(\frac{p^m-D^m}{D^m}\right)}{(n-m)\left(\frac{p-D}{D}\right)} - \frac{cD}{p}}{\frac{\left(\frac{p^{2m}-D^{2m}}{D^{2m}}\right)}{\left(\frac{p^2-D^2}{D^2}\right)} + \frac{\left(\frac{p^m-D^m}{D^m}\right)^2}{(n-m)\left(\frac{p-D}{D}\right)^2}} = Q\theta_q(D, m, n), \tag{28}$$

$$I_s = \left[\frac{\tau_q(D, m, n)D}{p} + \frac{(p-D)}{2p}\right] Q = Q\theta_s(D, m, n), \tag{29}$$

$$I_b = \left[\frac{\frac{\left(\frac{p^{2m}-D^{2m}}{D^{2m}}\right)}{(n-m)\left(\frac{p^2-D^2}{D^2}\right)} + \left(\frac{cD}{p}\right)^2}{2\left(\frac{\left(\frac{p^{2m}-D^{2m}}{D^{2m}}\right)}{\left(\frac{p^2-D^2}{D^2}\right)} + \frac{\left(\frac{p^m-D^m}{D^m}\right)^2}{(n-m)\left(\frac{p-D}{D}\right)^2}\right)}\right] Q$$

$$= Q\theta_b(D, m, n). \tag{30}$$

Here, size of the production batch is considered as a decision variable and again the total joint profit is concave in Q. Therefore, the optimal production batch size, Q^*_{OP}, and modified objective function are presented in Eqs. (32) and (33):

$$TP^{OP}_{\text{joint}}(Q, D, m, n) = \frac{D(a-D)}{b} - \frac{(A_v + nA_b)D}{Q}$$

$$- h_v Q\theta_s(D, m, n) - (h_b - h_v)Q\theta_b(D, m, n), \tag{31}$$

$$Q^*_{OP} = \sqrt{\frac{(A_v + nA_b)D}{h_v\theta_s(D, m, n) + (h_b - h_v)\theta_b(D, m, n)}}, \tag{32}$$

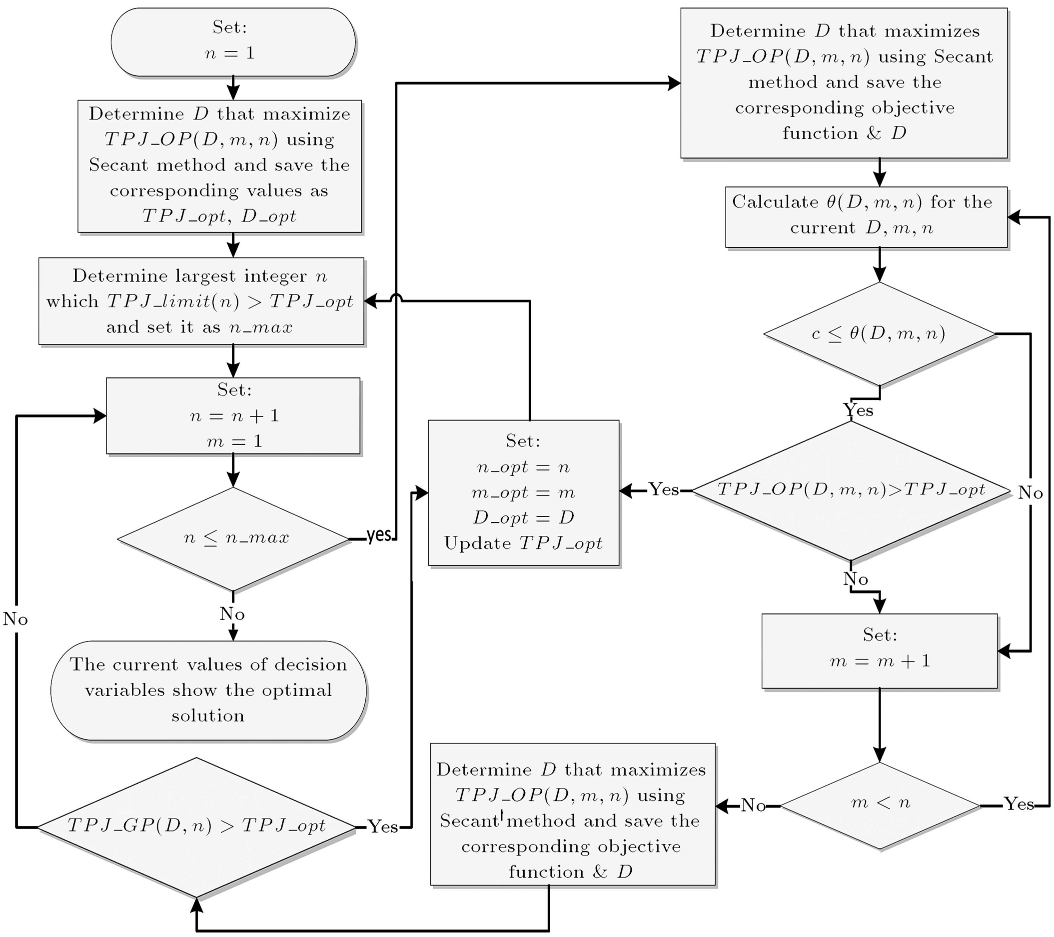

Figure 5. Solution procedure for JELS model when optimal shipment policy is applied.

$$TP^{OP}_{\text{joint}}(D, m, n) = \frac{D(a - D)}{b}$$
$$-2\sqrt{(A_v + nA_b)D[h_v\theta_s(D, m, n) + (h_b - h_v)\theta_b(D, m, n)]}. \tag{33}$$

As stated in [41], the minimum total cost of the chain is achieved when $h_b = h_v$. In this condition, the objective function becomes minimization of the total stock in the system and this was used to determine the maximum number of shipments, n.

There are two differences between the objective functions of the current paper and the model considered in [41]. First, the objective of our model is to maximize the total profit of the chain while minimizing total cost is the objective of [41]; second, since we consider D as a decision variable, the current paper has a further variable compared to the model studied in [41]. In the current paper, the upper limit of the total joint profit is obtained by considering $h_b - h_v$ and it can be used to find the upper limit of n as presented in Eq. (34):

$$TP^{OP}_{\text{limit}}(D, m, n) = \frac{D(a - D)}{b}$$
$$-2\sqrt{(A_v + nA_b)Dh_v\theta_s(D, m, n)}. \tag{34}$$

The solution procedure for the joint profit of the chain when applying OP is depicted in Figure 5. As shown, if there is no integer $m < n$ that can pass the constraint, the objective function of GP will be applied that is referred to in the figure as $TPJ\_GP(D, n)$. Again, according to Section 4, the final solution is a well approximation of the global optimal solution of the problem.

8. Numerical results and sensitivity analysis

To analyze and compare all the shipment policies introduced in the literature for price-sensitive demand, we first implement the solution algorithm of the equal-size policy introduced by Sajadieh and Akbari Jokar [24] in Matlab Software. Comparison of the results of the implemented algorithm with numerical results of [24] confirmed the validity of this implementation. In the current paper, GP, GEP, and OP are studied for dependent demand.

The effect of different values of b on the optimal solution of the benchmark problem is shown in Table 2

Table 2. Optimal solutions of equal, geometric, and geometric-then-equal shipment policies.

Shipment policy	b	n	m	D	δ	q_1	Q	Total joint profit	Improvement percentage relative to equal policy	Improvement percentage relative to geometric policy	Improvement percentage relative to geometric-then-equal policy
Equal shipment policy [24]	10	4	—	745.1	75.5	110.9	443.7	54568			
	50	4	—	724.8	15.5	109.2	436.9	9578.4			
	100	4	—	698.3	8.02	107	427.9	3966.4			
	200	4	—	640.1	4.3	101.9	407.7	1182.3			
	300	3	—	564.3	3.12	120.6	361.7	277.8			
Geometric shipment policy	10	3	—	745.9	75.41	18.26	432.7	54611	0.079%		
	50	3	—	729.3	15.41	17.31	426.6	9617.5	0.408%		
	100	3	—	707.4	7.93	16.11	418.6	4001.3	0.880%		
	200	3	—	659.5	4.2	13.63	400.7	1208.1	2.182%		
	300	3	—	603.5	2.99	11.03	379.5	292.6	5.328%		
Geometric-then-equal shipment policy	10	3	2	745.5	75.45	45.8	439	54635	0.123%	0.044%	
	50	3	2	727.2	15.46	44.14	432.7	9643	0.674%	0.265%	
	100	3	2	703.24	7.97	42.01	424.31	4028.6	1.568%	0.682%	
	200	3	2	651.3	4.24	37.48	405.8	1239.2	4.813%	2.574%	
	300	3	2	591.6	3.03	32.49	384	327.7	17.963%	11.996%	
Optimal shipment policy	10	4	2	745.8	75.42	23.74	462.94	54637	0.126%	0.048%	0.004%
	50	4	2	728.5	15.43	23.39	456.40	9644.5	0.690%	0.281%	0.016%
	100	3	2	703.8	7.96	37.81	424.80	4029.8	1.598%	0.712%	0.030%
	200	3	2	652.0	4.24	34.49	406.29	1239.9	4.872%	2.632%	0.056%
	300	3	2	592.4	3.03	30.58	384.37	328.07	18.096%	12.123%	0.113%

for all shipment policies. As the ratio of h_b to h_v for this problem is smaller than this ratio in the corresponding BEP, it is expected that GP yields more joint profit than EP.

According to the definition of b, as this parameter increases, dependency of the demand on the selling price increases. Therefore, determining the optimal selling price is of great importance. According to Table 2, as b increases, the total joint profit of the chain deceases. A part of this decrease can be compensated by using OP instead of the other policies. Using GEP also yields more joint profit than its parents' policies. For example, when $b = 300$, if the chain coordinator applies GEP instead of EP, the total joint profit increases by 17.96%; The resultant profit improvement of applying OP instead of EP also increases by 18.09%. These improvements can be very interesting for SC members, especially for centralized chains as the chain coordinator can decide how to ship the products between the chain members.

Figure 6 illustrates the effect of 50% increase in the value of a, b, a_1, a_2, and p for the benchmark problem. The effect of h_b and h_v on the joint profit was previously studied in Section 9.

As expected, among these parameters, a has the most impact and p has the least impact on the joint profit of the policies. Since a is the potential demand of a product, it has major influence on the joint profit; but it is beneficial to study the effect of b on the objective function value. Figure 7 depicts the effect of different values of b on the optimal profit of the chain for four shipment policies. The exponentially decreasing behavior of the joint profit is similar for the policies. The behavior can be explained by considerable reduction in actual demand, $a - b\delta$, as sensitivity to the selling price, b, increases slowly.

9. Conclusion and recommendations for future studies

This paper analyzed the Joint Economic Lot-Sizing (JELS) model for a two-stage supply chain while demand for the final product was linearly dependent on the selling price. The main contributions of this paper can be summarized as follows. First, for the joint profit model of geometric shipment policy (GP), it determines the upper and the lower limits for the number of shipments and develops a search based algorithm to

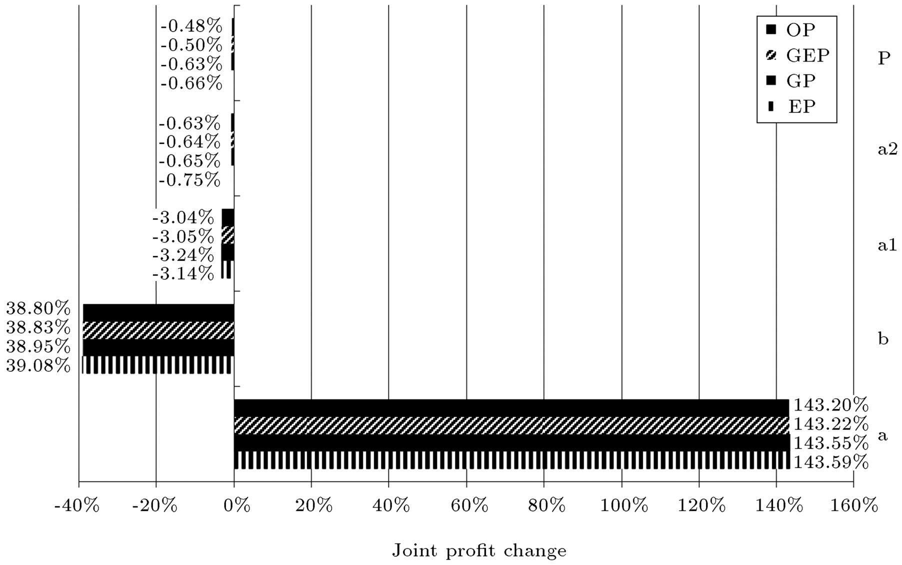

Figure 6. Joint profit change for 50% increase in the values of parameters.

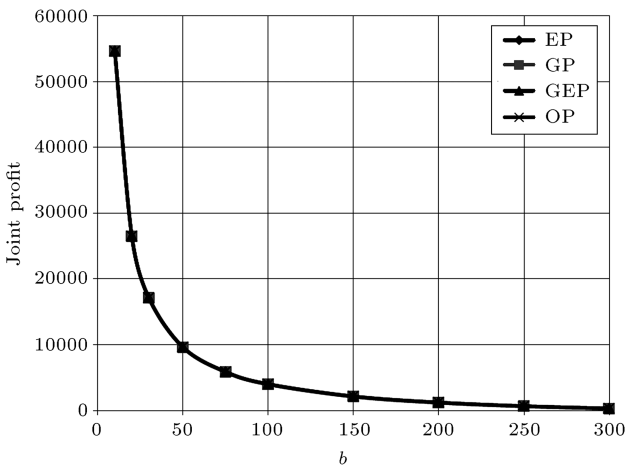

Figure 7. Effect of b on joint profit of shipment policies.

find the optimal solution. Second, it determines the break-even point of equal-size shipment policy (EP) and GP for price-dependent demand. Third, similar to GP, the optimal solution for the joint profit model is determined. Fourth, for optimal shipment policy (OP), the optimal selling price, production batch size, and the number of shipments are obtained by the proposed solution procedure. In all of the developed solution procedures, secant method was used to determine a well approximation of the global optimal solution. The performance of the proposed procedure was also verified by investigating its optimal solution vs. the one obtained by the simulated annealing algorithm.

To explain the second contribution of the paper, it should be notified that comparison of the optimal solutions of EP and GP for different instances proved that choosing the best policy between them was dependent on the parameters' values of the problem. As shown in the paper, when the range of parameters does not thoroughly change, as in practice happens, the ratio of vendor's inventory holding cost to buyer's inventory holding cost is the most important factor affecting Break-Even Point (BEP) of these two policies. When this ratio is greater than BEP, less buyer inventory is desired, so EP has better performance than GP; but when this ratio is less than BEP, less vendor inventory is desired, so GP is preferred. To determine this point, an interpolation method was applied.

Sensitivity analysis of the problem's parameters confirmed the major effect of demand dependency function on the joint profit of the chain. The analysis also showed an exponentially decreasing behavior of the joint profit as the slope of demand function (b) increased. Therefore, optimal solutions of the joint model were studied for different values of b. Numerical results suggest when demand is price-sensitive, applying OP, GEP, and GP, respectively, yields more joint profit than EP. Numerical results also proved that as b increased, more improvement would be achieved compared to EP. As an example, if the vendor ships the products to the buyer according to OP, the total joint profit of the chain for the benchmark problem of the JELS literature increases by 18.096% compared to EP when $b = 300$. From the managerial insight for products with high demand dependency on selling price, selecting GP, GEP, or OP instead of adopting lot-for-lot or equal shipment policies can significantly increase the joint profit of the chain. It should be notified that achieving such a percentage of improvement by changing the shipment policy is straightforward and does not need any more investment; it just needs a good planning for shipments, while implementing other ideas to increase joint profit is not simple or needs considerable investment, such as investing in advertisement to affect the parameters of demand dependency function. This significant profit improvement can be very interesting for supply chain coordinators as well as the chain members, especially in centralized chains that the joint profit is the main aim of all the members.

When adopting GP, GEP, or OP for the chain, various sizes of shipments make the implementation of these policies difficult and need a good planning and cooperation between the chain members to utilize its more joint profit. On the other hand, if we want to apply these policies for non-coordinated supply chains, developing incentives and profit sharing mechanisms are needed to persuade the members for joint commitment to implement these policies. In this study, the geometric growth factor considered in GP, GEP, and OP is equal to production rate to demand rate. Assuming it as a decision variable would be interesting for future research. Studying non-linear dependency of the demand on the selling price could be interesting for more research. Considering imperfect quality of the products and adding capacity constraint of the transportation equipment to the model is recommended for future studies. Integrating inbound logistics of the vendor to the JELS model and optimizing its corresponding costs are other areas for future researches. Investigating more retailers by equal or different ordering cycles and different demand rates are also recommended for more research.

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