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Research Note

Robust decentralized control of consensus-based formations of leader-follower networks with uncertain directed topologies on bounded velocity trajectories undefined for followers via backstepping method

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Abstract. In this paper, decentralized control of formation of a special category of leader-follower networks on bounded velocity trajectories is addressed. The network of the agents in this study is supposed to have a directed graph with a spanning tree rooted at the leader agent. Moreover, follower agents do not receive online or have offline velocity of the desired trajectory, such as in tracking problem of trajectories which are not predefined or when the total bandwidth is narrow. Furthermore, the leader does not receive any information from any agent and its control is fully centralized. In the present study, formation problem is considered a consensus problem. The controller is designed for integrator and double-integrator agents via backstepping. Furthermore, suitable condition of the robustness of the controller against the changes of the communication topology of the network is derived. Simulations verify the capability and robustness of the designed control law. In a simulation, formation keeping error reduction by tuning a gain of the controller, as claimed in the design procedure, is demonstrated.

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1. Introduction

In the recent decade, due to advancements in designing and producing high speed processors and wireless networks developments, researchers have been interested in performing complicated tasks by multi-agent systems of ordinary robots instead of designing super-mature robots. Therefore, various missions have been defined for multi-agent systems such as flocking [1], coverage [2], rendezvous [3], deployment [4],

formation [5], and, in a more general case, consensus [6,7]. Due to high bandwidth necessary for centralized controllers and failure of whole mission due to failure of a single-agent in centralized missions, decentralized control laws are preferred apparently. Formation methods, till now, can be classified to three approaches, including: behavior-based approach [8], virtual structure approach [9], and Leader-follower approach [10]. Robustness of the proposed formation algorithms against changes in communication topology, which are probable as a result of surrounded environmental terms and conditions, is so important. For example, in [11], a robust decentralized attitude formation controller against information flow changes and disturbances is designed via sliding mode method.

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In [12], by a method named leader-to-formation stability method, interconnection topology is correlated to stability and ways of improving robustness of formation in spite of changes of information flow topology have been suggested. In [13], suitable condition for robustness of formation of multi-agent systems in case of time varying communication graphs has been addressed.

The present paper deals with decentralized control of formation of leader-follower multi-agent system with directed networks having spanning tree on bounded velocity trajectories. The leader of the networks considered in this study is supposed not to be receiver of information from the followers. On the other hand, the followers do not know or receive any explicit information about the desired trajectory.

The first section of the present paper is about some of the needed preliminaries and theorems and lemmas about the networks. Control design section introduces the general properties of the considered category of networks in this study. So, control laws for integrator and double integrator agents are designed via backstepping method. Furthermore, robustness condition of the controller on the networks with uncertain topology of communications is derived. Finally, some simulations are performed to exhibit the capability, limitations, and robustness of the designed control law.

2. Preliminaries

For an index set $I = \{1, 2, \dots, n\}$, a digraph G consists of a triple (V, ε, A) in which $V = \{v_i | i \in I\}$ is a finite nonempty set of nodes and $\varepsilon = \{e_{ij} = (v_i, v_j) | i, j \in I\}$ is the edge set and $A = \{a_{ij} | a_{ij} \neq 0 \Leftrightarrow e_{ji} \in \varepsilon, a_{ij} = 0 \Leftrightarrow e_{ji} \notin \varepsilon\}$ is the adjacency matrix. v_i and v_j are called tail and head of the edge (v_i, v_j) . It is assumed that $a_{ii} = 0$ and $\forall a_{ij} \geq 0, i \neq j$. The adjacency matrix can be weighted or unweighted. The Laplacian matrix associated with graph G is defined as $L = \Delta - A$ in which $\Delta_{ii} = \deg_{in}(v_i)$ and $\Delta_{ij} = 0$ for all $i \neq j$ and $\deg_{in}(v_i) = \sum_{j=1}^n a_{ij}$. The Laplacian matrix always has a zero eigenvalue with the right eigenvector of one, because the sum of its columns is zero and so its determinant is zero, which means that the matrix has a zero eigenvalue with right eigenvector of one. These eigenvalue and eigenvector are denoted by $\lambda_1 = 0, w_r = 1 = [1, 1, \dots, 1]^T$. Moreover, it should be noted that a digraph has spanning tree if there is at least one node (named root) that can reach all the other nodes following the direction of information flow. In leader-follower multi-agent systems, the root of spanning tree is usually set at the leader node. A useful theorem about digraphs with spanning tree is expressed as follows.

Theorem

If a digraph has a spanning tree, then its Laplacian matrix has a simple zero eigenvalue associated with an eigenvector 1 and all of the other eigenvalues have positive real parts [14].

Laplacian matrix is used to model multi-agent system problems such as consensus, formation, etc. In fact, the problems of the multi-agent systems, such as formation, can be considered a consensus problem. Consensus simply means that values of a quantity of all agents in a system should be converged to a same value. This value can be speed of agents, position of agents, etc. If consensus is reached, then $(L \otimes I_m)r = 0$ by remembering that Laplacian matrix always has a zero eigenvalue with 1 vector as the right eigenvector. m is the quantity dimension of r , obviously. Also, if $(L \otimes I_m)r = 0$ is satisfied, then it will mean that consensus is accomplished due to:

$$(L \otimes I_m)r = 0 \Rightarrow r \in \text{span}(1 \otimes r_c), r_c \in \mathbb{R}^3 (L \otimes I_m)r = 0 \Rightarrow r \in \text{span}(1 \otimes r_c), r_c \in \mathbb{R}^3, \quad (1)$$

where, r_c is consensus value and depends on initial value of r . Now, by substituting r with $\delta = r - r_{\text{form}}$ in which r_{form} is reference vector of the desired formation, one can write the following equation:

$$(L \otimes I_m)\delta = 0 \Rightarrow \delta \in \text{span}(1 \otimes r_e), r_e \in \mathbb{R}^3 \Rightarrow r \in \text{span}(1 \otimes r_e) + r_f. \quad (2)$$

It means that formation is reached because summing formation reference vector with a vector does not affect formation as shown in Figure 1. So if a decentralized control law causes $(L \otimes I_m)\delta = 0$, then the desired formation will be achieved.

Lemma [15]

If a digraph has a spanning tree and the associated Laplacian graph is L , then there exists a symmetric positive definite matrix P satisfying the following equation:

$$PL + L^T P = Q, \quad (3)$$

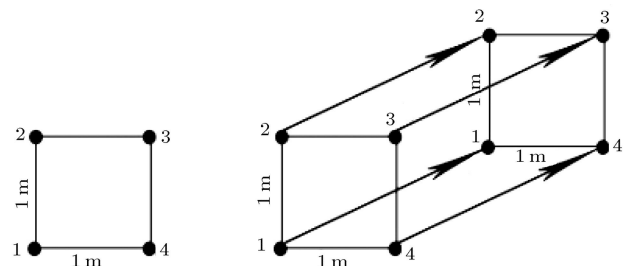


Figure 1. Summing formation reference vector of a square formation $([0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1]^T)$ with a vector that does not affect formation.

where, Q is a positive semi-definite matrix. This lemma has been proven in [15] using Theorems 4.6 and 4.29 from [16,17], respectively.

3. Designing decentralized controller via backstepping

Characteristics of the network of the system can affect the control law design. So, at first step of the controller design, general characteristics of the networks considered in the present work should be defined. After that, decentralized control laws are designed to track the velocity bounded trajectories by formation of integrator and double-integrator agents. Robust controller design is the final step of the controller design.

3.1. General properties of network

Narrow necessary total bandwidth to perform missions always is preferred. Decreasing this bandwidth even in case of the decentralized controllers is recommended because it results in ad-hoc communications with more narrow bandwidths. Therefore, using decentralized controllers on digraph networks is preferred to their usage in undirected graphs, which needs higher total bandwidth. So, the chosen proximity graph of this study is a digraph. In leader-follower approach, the desired trajectory of mission is sent from a station to the leader of the multi-agent system (in case of missions which are not predefined) or is set on the leader, previously, while followers communicate together and with the leader. In the mission of this study, it is assumed that the leader does not receive any information from other agents to reduce the required bandwidth. The digraph of this study has spanning tree with the leader as the root of the tree. Consequently, the general properties of the network of this study can be listed as follows:

- The proximity graph of the network is a directed graph;
- No follower sends information to the leader;
- The trajectory which should be tracked by the formation is sent only to the leader from the central station or is defined, previously, only for the leader;
- The digraph of the network has a spanning tree with the root located on the leader.

3.2. Controller design for network of integrator agents

For an Integrator agent, equation of motion is $\dot{r}_i = u_i$, obviously. Therefore, proper control input of the leader agent can be adopted as $u_L = \dot{r}_d - K_L e_{r_L}$ in which K_L is a positive constant and r_d is the desired value of r_L and $e_{r_L} = r_L - r_d$. This form of input makes the tracking error for the leader exponentially stable

with zero equilibrium point, obviously. To design the decentralized controller of the system, a Lyapunov function, such as $V_f = \frac{1}{2} \delta^T (L^T P L \otimes I_m) \delta$, can be considered, in which matrix P is the matrix defined in the lemma. So, derivative of this Lyapunov function will be:

$$\begin{aligned} \dot{V}_f = & \frac{1}{2} (U^T (L^T P L \otimes I_m) \delta \\ & + \delta^T (L^T P L \otimes I_m) U). \end{aligned} \quad (4)$$

U is the input vector of all the agents in the recent equation. Without losing generality, it can be assumed that leader agent is the first agent. By choosing U as:

$$U = -K_0 (L \otimes I_m) \delta + \begin{bmatrix} u_L \\ 0_{(n-1) \times 1} \end{bmatrix}, \quad (5)$$

in which k_0 is a positive constant, it can be concluded that:

$$\begin{aligned} \dot{V}_f \leq & \frac{1}{2} (-k_0 \delta^T (L^T Q L \otimes I_m) \delta + \delta^T (L^T Q' L \otimes I_m) \delta) \\ & + \|\dot{r}_d\|^2 + K_L^2 \|e_{r_L}\|^2. \end{aligned} \quad (6)$$

The matrix Q in the recent equation is the same as the Q matrix in the lemma. In the derived inequality matrix, $Q' = P L^T L P$, which is a positive semi-definite matrix due to the definition of the P matrix. As explained previously, tracking error of the leader is exponentially stable with zero equilibrium point. So, a Lyapunov function for the leader tracking error can be found such that $\dot{V}_L \leq -K_L^2 \|e_{r_L}\|^2$. Consequently, one can have the following inequality:

$$\begin{aligned} \dot{V}_f + \dot{V}_L \leq & \frac{1}{2} (-k_0 \lambda_2(Q) + \lambda_{\max}(Q')) \|(L \otimes I_m) \delta\|^2 \\ & + \|\dot{r}_d\|^2. \end{aligned} \quad (7)$$

λ_2 denotes the second smallest eigenvalue (the first one is zero) and λ_{\max} is notation of the largest eigenvalue, definitely. If $k_0 \geq \frac{\lambda_{\max}(Q')}{\lambda_2(Q)}$, then it can be concluded that if the velocity of the trajectory is bounded then formation keeping error will be up-bounded with $\frac{2 \max(\|\dot{r}_d\|)^2}{(k_0 \lambda_2(Q) - \lambda_{\max}(Q'))}$. Therefore, increasing k_0 can increase formation precision. Notice that input is fully decentralized for the followers and fully centralized for the leader because the leader does not listen to the followers and the first row of L is zero, accordingly.

3.3. Controller design for network of double integrator agents

For double integrator agents, equation of motion is $\ddot{r}_i = u_i$, apparently. To perform Backstepping control design, a Lyapunov function should be candidate.

The candidate function is the same as the candidate function of the integrator agents, but the derivative of this Lyapunov function for double integrator agents will be:

$$\begin{aligned}\dot{V}_f = & \frac{1}{2} (\dot{r}^T (L^T P L \otimes I_m) \delta \\ & + \delta^T (L^T P L \otimes I_m) \dot{r}).\end{aligned}\quad (8)$$

By defining $Z = \dot{r} - \dot{r}_s$ in which \dot{r}_s is defined as:

$$\begin{aligned}\dot{r}_s = & -K_0(L \otimes I_m)\delta + \begin{bmatrix} \dot{r}_{sL} \\ 0_{m(n-1) \times 1} \end{bmatrix}, \\ \dot{r}_{sL} = & \dot{r}_d - k_L e_{rL}.\end{aligned}\quad (9)$$

Eq. (8) will result in the following inequality:

$$\begin{aligned}\dot{V}_f \leq & \frac{1}{2}(Z^T Z) + \frac{1}{2}(-k_0 \delta^T (L^T Q L \otimes I_m) \delta \\ & + 2\delta^T (L^T Q' L \otimes I_m) \delta) + \frac{\|\dot{r}_{sL}\|^2}{2}.\end{aligned}\quad (10)$$

Now, if $\dot{Z} = -K_Z Z$ in which K_Z is a positive constant, then Z will be exponentially stable with zero equilibrium point. Therefore, a Lyapunov function V_Z can be found such as $\dot{V}_Z \leq -\frac{1}{2} Z^T Z$. Consequently, the following inequality is correct due to definition of \dot{r}_{sL} :

$$\begin{aligned}\dot{V}_f + \dot{V}_Z \leq & \frac{1}{2}(-k_0 \delta^T (L^T Q L \otimes I_m) \delta \\ & + 2\delta^T (L^T Q' L \otimes I_m) \delta) + \|\dot{r}_d\|^2 \\ & + K_L^2 \|e_{rL}\|^2.\end{aligned}\quad (11)$$

$\dot{Z} = -K_Z Z$ will result in control input vector as:

$$\begin{aligned}U = & -k_0(L \otimes I_m)\dot{r} - K_Z(\dot{r} + k_0(L \otimes I_m)\delta) \\ & + \begin{bmatrix} \ddot{r}_d - K_L \dot{e}_{rL} + K_Z(\dot{r}_d - K_L e_{rL}) \\ 0_{m(n-1) \times 1} \end{bmatrix},\end{aligned}\quad (12)$$

which is fully decentralized for the followers and fully centralized for the leader because the first row of the L is zero. By Eq. (12), input of the leader will be $u_L = \ddot{r}_d - (K_L + K_Z)\dot{e}_{rL} - K_Z K_L e_{rL}$. This form of the control input of the leader makes trajectory tracking error of the leader exponentially stable with zero equilibrium point. So, a Lyapunov function for the leader can be found such that $\dot{V}_L \leq -K_L \|e_{rL}\|^2$. Then, Inequality (11) will be changed to:

$$\begin{aligned}\dot{V}_f + \dot{V}_Z + \dot{V}_L \leq & \frac{1}{2}(-k_0 \lambda_2(Q) \\ & + 2\lambda_{\max}(Q'))\|(L \otimes I_m)\delta\|^2 + \|\dot{r}_d\|^2.\end{aligned}\quad (13)$$

If one chooses k_0 such as $k_0 \geq \frac{2\lambda_{\max}(Q')}{\lambda_2(Q)}$ and velocity of the trajectory is bounded, then $\|(L \otimes I_m)\delta\|^2$ is up-bounded with $\frac{2\max(\|\dot{r}_d\|)^2}{(k_0 \lambda_2(Q) - 2\lambda_{\max}(Q'))}$. The error of the formation, while tracking the trajectory, can be reduced by increasing the gain k_0 as possible.

3.4. Robustness of formation under changing network topologies

Because of terms, conditions, and changes of surrounded environment, topology of networks during missions may be altered accidentally. So, robustness of multi-agent systems against the variations of the proximity graph of the system is too important. Therefore, robustness of the derived control law in the previous section should be guaranteed.

For a network of agents with the mentioned characteristics, finite forms of proximity graphs and associated unweighted Laplacian matrices can be assigned. Moreover, Laplacian matrices of some of them have the same eigenvalues and eigenvectors and so they are the same from controller design point of view. Consider that there are m possible independent proximity graphs with the explained characteristics on a network of agents denoted by $\Gamma = \{L_j | j = 1, 2, \dots, m\}$. For each member of Γ with a constant Q_1 , matrices P_j , Q_j , and Q'_j are calculated. The second smallest eigenvalues of all Q_j s, denoted by $\lambda_2(Q_j)$, and the largest eigenvalue of Q'_j , denoted by $\lambda_n(Q'_j)$, are derived accordingly. So, for the system of integrator or double integrators, sets Λ_1 and Λ_2 can be found, respectively, as:

$$\begin{aligned}\Lambda_1 = & \left\{ \frac{\lambda_{\max}(Q'_j)}{\lambda_2(Q_j)} \mid j = 1, 2, \dots, m \right\}, \\ \Lambda_2 = & \left\{ \frac{2\lambda_{\max}(Q'_j)}{\lambda_2(Q_j)} \mid j = 1, 2, \dots, m \right\}.\end{aligned}\quad (14)$$

So, to design robust formation controller under varying topology networks, it is sufficient that $k_0 \geq \max(\Lambda)$.

4. Simulation results

In this section, some simulations have been performed on a multi-agent system of four double integrator agents. The desired shape of the formation is the square of Figure 1 and reference vector of formation is the same, too. At first, effect of k_0 on the precision of the formation keeping while tracking trajectory of a path with bounded velocity is examined. So, in the next simulation, it is shown that the designed controller loses its capability in keeping formation while tracking trajectories with boundless velocities. In the other words, followers cannot be located on the formation shape around the leader while the leader tracks its desired trajectory. The last simulation is around the robustness capabilities of the designed control law

against the changes of the Laplacian matrix of the network and it is verified that the proposed controller exhibits enough robustness when it is subjected to the aforementioned surrounded environmental conditions which cause changes in the topology of the network. In all these three simulations, it is assumed that all agents start to move from stationary situation and their initial positions are:

$$\begin{aligned} r_0 &= [r_{0_1} \quad r_{0_2} \quad r_{0_3} \quad r_{0_4}]^T \\ &= [2 \quad 3 \quad 4 \quad 7 \quad 6 \quad 2 \quad 1 \quad 6]^T. \end{aligned} \quad (15)$$

4.1. Effect of k_0 value on precision of formation

In this simulation, it is verified that increasing k_0 will reduce the error of the formation, as claimed. Desired trajectory of leader is defined by a velocity bounded trajectory as $r_d = [t \sin(t)]^T$. Therefore, the designed control law can keep formation precision while tracking the trajectory. The proximity graph of the network is assumed to be a chain graph like the left side graph in Figure 2 having all the considered characteristics. By considering Q_1 of lemma as $(0.01)I_{3 \times 3}$, one can obtain $\lambda_2(Q) = 0.0034$, $\lambda_n(Q') = 0.0037$. Therefore, from the condition found while backstepping, k_0 should be adopted such that $k_0 \geq 2.13$. Consequently, for the first simulation, k_0 and all gains are set to 5. Values of other gains are assumed 5, too. In Figure 3, X and Y coordinates of agents during mission have been

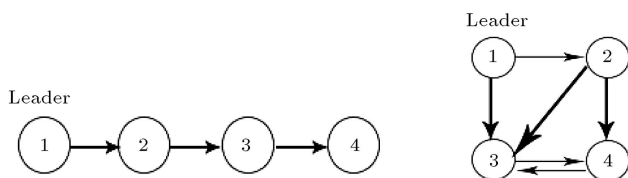


Figure 2. Proximity graphs of network in simulations.

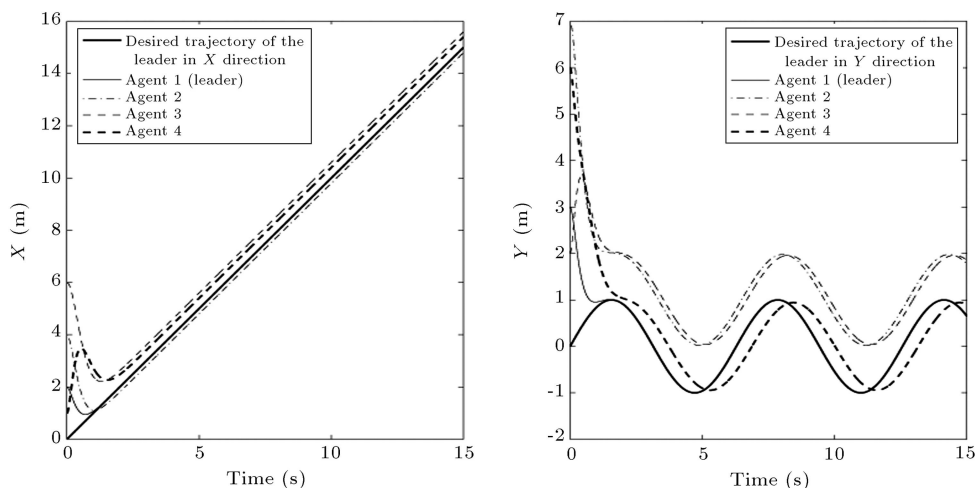


Figure 3. Coordinates of agents during mission in the first simulation.

demonstrated and it is obvious that the formation has been kept with a bounded error, as claimed formerly. From Figure 4, it is obvious that velocities of agents in X direction have converged to a same value because the acceleration of the desired trajectory in X direction is zero. As depicted in Figure 4, consensus on velocity in Y direction has a bounded error because acceleration in Y direction is bounded. Also, in Figure 5, $\|(L \otimes I_2)\delta\|$ as formation keeping error is plotted versus time and it has been verified that error of formation demonstrated by a thick line is under the up-bound derived in backstepping. Moreover, Figure 5 demonstrates that the shape of the formation is too different from the desired square shape.

In the next simulation, only k_0 is changed to 20 to verify that if the formation precision is improved or not. Plot of $\|(L \otimes I_2)\delta\|$ versus time depicts degradation of formation error and its bound in comparison with previous simulation and the square formation has been achieved well as depicted in Figure 6. Therefore, as proved, higher k_0 values increase precision of formation and decrease error of formation. Of course, increasing k_0 in real applications may cause saturation in actuators of robots.

4.2. Controller failure in tracking trajectories with unbound velocities

In this simulation, the desired trajectory of leader is assumed to be $r_d = [t^2 \sin(t)]^T$. Since $\dot{r}_d = [2t \cos(t)]^T$, the velocity of the desired trajectory is unbounded. So, the formation error will be unbounded by the designed controller. The proximity graph of the system and Q_1 matrix are the same as those in previous simulations. The gains of the system by considering the condition derived in Backstepping are set to 5. In Figure 7, X and Y coordinates of the agents during mission are demonstrated. As demonstrated in Figure 7 and proved in Backstepping,

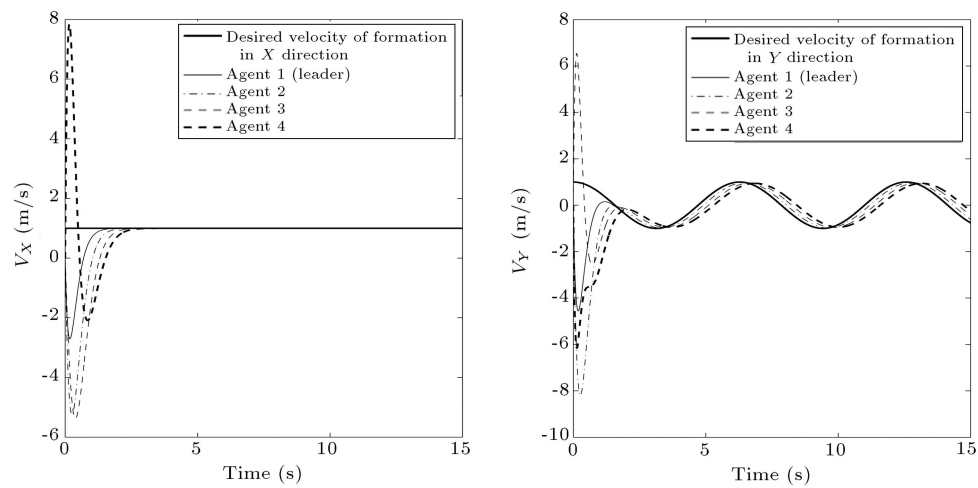


Figure 4. Velocity of agents in X direction during mission in the first simulation.

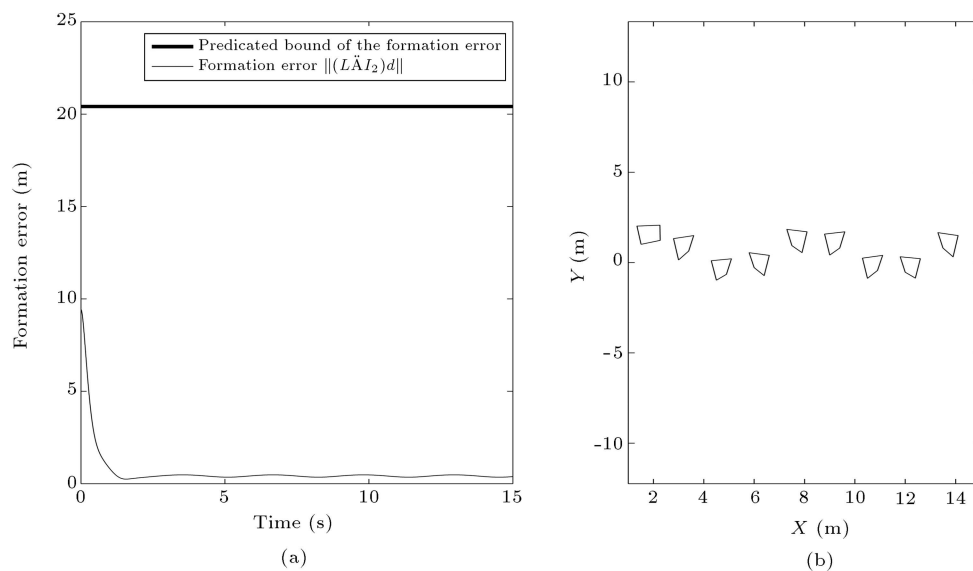


Figure 5. Error of formation and bound of error (a) and shape of formation during mission for $k_0 = 5$ (b).

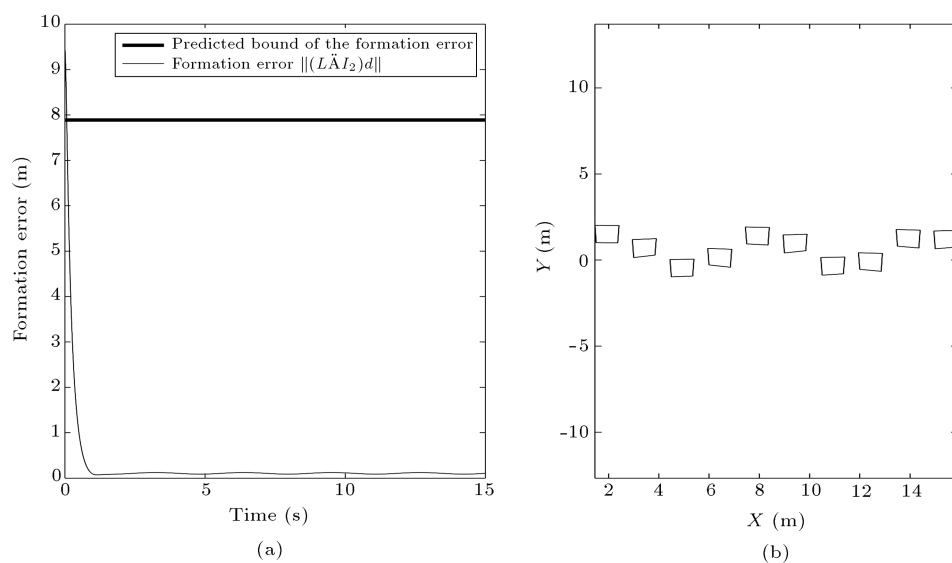


Figure 6. Error of formation and bound of error (a) and shape of formation during mission for $k_0 = 20$ (b).

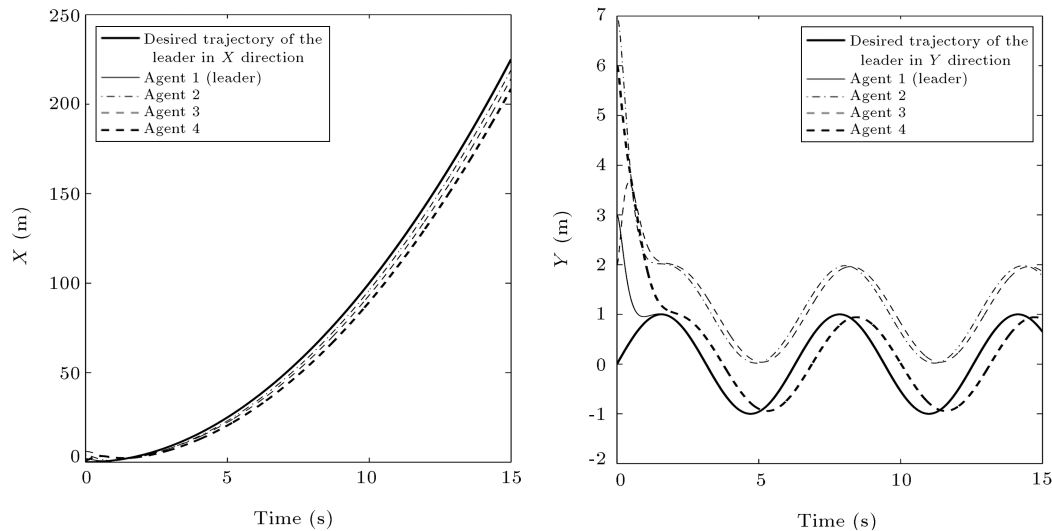


Figure 7. Coordinates of agents during mission in case of tracking trajectory with unbounded velocity.

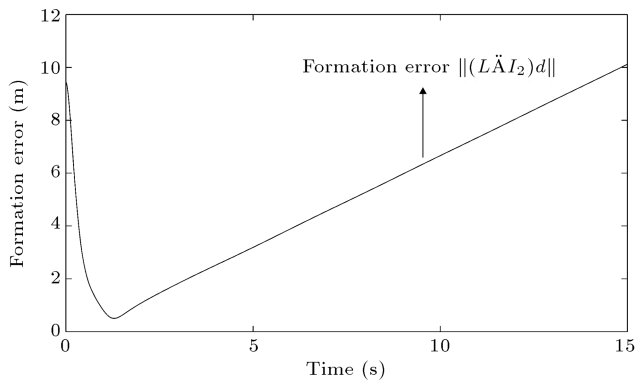


Figure 8. Error of formation during mission in case of tracking trajectory with unbounded velocity.

the error of the formation in X direction increases during the mission because the velocity of the desired trajectory in X direction increases by time. Nevertheless, the formation along Y axis has bounded error as proven and depicted in Figure 7. So, as proven formerly, $\|(L \otimes I_2)\delta\|$ would increase and the error of the formation would not be bounded, as depicted in Figure 8. Consequently, in case of the trajectories with unbounded velocities, the designed controller lost its capability.

4.3. Robust formation control against network topology changes

In this simulation, a formation controller, which is robust against the network topology changes, has been designed for a leader-follower multi-agent system of four double-integrator agents. A Leader-follower multi-agent system with four agents has 144 independent forms of unweighted Laplacian matrices. Set Λ_2 is calculated by the Q_1 matrix of the previous simulations and its maximum is 34.08. Therefore, if $k_0 \geq 34.08$, then the controller will be robust

against the changes of the proximity graph of the network. Two simulations have been performed to analyze effect of k_0 value on the robustness of the formation against the changes of proximity graphs. In one of the simulations, $k_0 < 34.08$ and in another, $k_0 \geq 34.08$, while other gains are set to 5. In both of the simulations, the simulation starts with proximity graph and unweighted Laplacian matrix of left graph of Figure 2 and at $t = 5.3$ seconds, the proximity graph changes to right graph of this figure, suddenly. The desired trajectory of formation in both of the simulations is same as that of the first simulation. In Figure 9, the error of the formation during the mission for $k_0 = 20$ and $k_0 = 40$ is demonstrated. Comparing these two plots represents that the deflection of the formation from its steady state value in the case of $k_0 = 40$ is less than when $k_0 = 20$, as predicted.

5. Conclusion

This paper introduces a robust decentralized control law to guide formations of special category of leader-follower multi-agent systems on bounded velocity trajectories which are not predefined or only defined for the leader agent. By implementing the proposed control laws, formation keeping will have bounded error while the desired trajectory is tracked by leader. This bound can be decreased by increasing a gain of the controller. Control laws are redesigned to be robust against the proximity graph changes. Simulations verified the capabilities of the designed control laws and their robustness, appropriately. Designing observer for agents to estimate the velocity of the trajectory may decrease the formation error. Moreover, using this control law, which enters the saturation problem, in real networks is recommended.

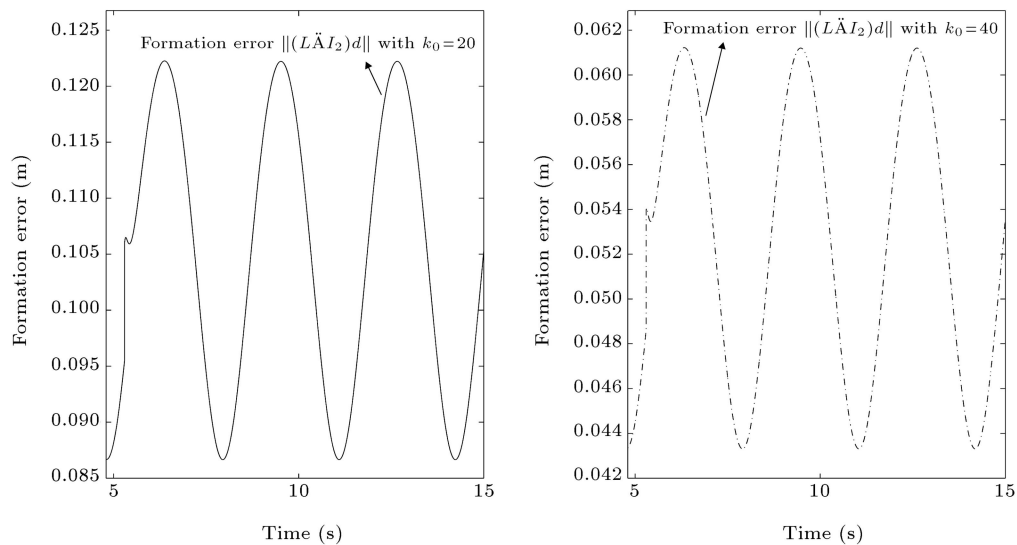


Figure 9. Error of formation with uncertain network topology during mission for $k_0 = 20$ (a) and $k_0 = 40$ (b); less deviation around $t = 5.3$ s is obvious when $k_0 = 40$.

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Biographies

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Ali Soltani received his BS and MS degrees from Sharif University of Technology, Tehran, Iran, in 2007 and 2010, respectively and he started PhD course in the same university in 2011, all in Mechanical Engineering. Now, he is PhD candidate student and progressing his PhD thesis. His research is all about multi-agent robotic systems and quadrotors.