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Integrated forward-reverse logistics network design under uncertainty and reliability consideration

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Abstract. This paper proposes a robust optimization model for robust and reliable design of an integrated forward-reverse logistics network with hybrid facilities under uncertainty and random facility disruptions. The proposed model utilizes several effective reliability strategies to mitigate the impact of random facility disruptions. First, the proposed model allows two types of hybrid facilities, namely, reliable and unreliable, to be located in the concerned logistics network where unreliable ones may be partially or fully disrupted, and thus a percentage of their capacities may be lost. However, they can still serve their customers with the remaining of their available capacities. Furthermore, a sharing strategy is taken into account, in which goods can be shipped from reliable hybrid facilities to unreliable ones to compensate their lost capacity. A robust optimization approach is applied on the developed model to handle the uncertainties in the parameters of the concerned network. Finally, several numerical experiments along with a sensitivity analysis are conducted to illustrate the significance and applicability of the proposed model as well as the effectiveness of the robust optimization approach in this context.

1. Introduction

Recent studies in the supply chain literature pay a particular attention to designing integrated forward-reverse logistics networks to avoid sub-optimality resulting from the separated design of forward and reverse networks and, at the same time, reaching higher levels of productivity and customer satisfaction [1,2]. It has been recognized that robust and reliable design of such networks helps firms to maintain and enhance their competitive advantages, and assists them to cope with the growing environmental turbulence. A part of literature refers to configuration of integrated forward-reverse logistics networks, including both forward and reverse flows, because of the existing legal requirements, environmental protection, as well as related economic benefits [3,4]. The goal of a forward network is to provide a value for the end consumer in terms of product attributes like quality and cost, while the reverse network tries to recover the economic and environmental value from used products in a cost-effective manner. The processes and activities concerned with material supply, production, distribution, and consumption are embedded in the forward network, while reverse network encompasses the activities associated with collection, inspection/separation, recovery, and disposal of the used products [5,6].

Another attention of the recent research is paid to incorporate risk management into the design phase of global supply chains. There are two wide categories of risks that impress supply chain network design problem:

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1. The risks originating from the difficulties in coordinating supply and demand;
2. The risks originating from the threat of disruptions to normal activities, which include the issues concerned with natural disasters, strikes, and economic disruptions, terrorist attacks, etc. [7].

Consequently, designing reliable and robust networks is of particular interest. It is worth pointing out the difference between robustness and reliability terms when studying supply chain risks. A supply chain is robust if it performs well when facing the first category of risks; while it is reliable if it performs well when disruptions occur, for example, when parts of the supply chain system fail due to natural disasters [8].

A large body of the extant literature is assigned to the first category of risks in which some parameters of a logistics network such as demand, lead times, transportation costs, and quantity of returned products may be uncertain. Environmental and system uncertainties are two aspects of uncertainty, which drastically affect the overall performance of supply chains [9]. The environmental uncertainty addresses uncertainties concerned with demand and supply originated in the performance of suppliers/manufacturers and behavior of customers. Those uncertainties impressing production, distribution, collection, and recovery processes are called system uncertainties. Since the logistics network design problem has a strategic nature, it is very critical that uncertainties are to be incorporated in the problem [9].

Stochastic programming, robust optimization, and fuzzy mathematical programming are three powerful tools, which can tackle the existing uncertainties in the parameters of the model. Application of stochastic programming for modeling uncertain supply chain network design can be seen in [4,10-15]. However, in many real applications of stochastic programming, there is no enough historical data to estimate the probability distribution of uncertain parameters. As an alternative, robust optimization can be applied to handle uncertainty in a given bounded uncertainty set. Implementation of robust optimization approach to design closed-loop logistics networks can be traced in [16,17]. In this paper, we resort to the robust optimization approach to handle uncertainties in the parameters of an integrated forward-reverse logistics network design problem. Furthermore, fuzzy mathematical programming is a flexible tool for modeling epistemic uncertainty that comes from lack of information about the actual value of parameters [18-20]. For effective implementation of fuzzy mathematical programing to design supply chain networks with uncertain parameters, the reader may refer to [21-23].

The second type of risks (our concern in this paper) can disrupt any point of a supply chain network (e.g., facilities and/or transportation links) in a relatively random manner. As highlighted by Peng et al. [8], disruption risks may lead to both negative financial effects and serious operational consequences, i.e. higher transportation costs, delays in delivery of orders, inventory shortages, loss of market shares, etc. Therefore, it is important to incorporate disruption risks carefully when configuring supply chain networks. Most studies have focused on managing disruption risks in the facility location problems, while there is a less attention on incorporating this type of risks when designing a forward-reverse logistics network.

One of the first reliability models refers to the unreliable p-median and (p,q)-center location problem, in which suppliers may be inactive by a given probability [24]. Snyder and Daskin [25] formulate the random disruptions of a facility in a facility reliability problem by a r-level assignment approach. According to this approach, a customer will be served if and only if all of the assigned facilities at levels $0, 1, \cdots, r-1$ fail. In addition, the same disruption probability is considered for all distribution centers. Several reliability models, similar to that presented by Snyder and Daskin [25], have been developed in the literature, but the uniform-disruption-probability assumption is relaxed using a variety of modeling approaches [8,26-31].

Lim et al. [31] introduced a hardening strategy and incorporated it into a mixed integer programming model to hedge the impact of random facility disruptions. Two types of facilities, i.e. reliable facilities that are not subject to disruptions but are the more expensive and unreliable ones, which may be disrupted, are considered in their proposed model. In the hardening strategy, the reliable facilities are protected against random disruptions by a substantial investment and therefore, disruptions cannot affect them. Azad et al. [32] extend the hardening strategy introduced by Lim et al. [31] and propose the soft hardening strategy for a supply chain network design under random facility disruptions. Almost in all studies pertaining to reliable facility location problem, except Azad et al. [32], it is assumed that facilities may fully fail by a disruptive event and thus, they may not service their assigned customers. Furthermore, the capacity restrictions and disruptions are not considered in these studies. However, in real world, facilities may lose a portion of their capacities after disruptions. In this regard, Azad et al. [32] suppose that the capacity of unreliable facilities may be partially disrupted. Furthermore, it is assumed that reliable facilities have unlimited capacity while unreliable ones have finite capacities. Besides, Davarzani et al. [33] discussed the effect of single/dual/multiple sourcing to handle the potential disruptions occurred in supply chains. Lim et al. [34] consider a facility location problem in the presence of random facility disruptions where facilities can be protected with additional investments. Whereas most existing models in the literature implicitly assume that the disruption probability estimate is perfectly accurate, the authors investigated the impact of misestimating the disruption probability. Furthermore, Aboolian et al. [35] studied the reliable facility location problems in which facilities are subject to unexpected failures, and customers may be reassigned to facilities other than their regular facilities. The objective of these problems is to minimize the total expected costs in normal and failure scenarios.

A stochastic mathematical formulation is proposed for designing a network of multi-product supply chains comprising several capacitated production facilities, distribution centers, and retailers in markets under demand-side and supply-side uncertainties. The supply-side uncertainty includes possible disruptions in manufacturers, distribution centers, and the connecting links of the network [36]. Babazadeh and Razmi [37] present an efficient Mixed Integer Linear Programming (MILP) model that is able to consider the key characteristics of agile supply chain, which is the best competitive strategy for high turbulent environments, such as direct shipments, outsourcing, different transportation modes, discount, alliance (process and information integration) between the opened facilities and the maximum waiting time of customers for deliveries. Additionally, the robust stochastic programming approach is applied to handle both operational and disruption risks of the agile supply chain network. Garcia-Herreros et al. [38] proposed a two-stage stochastic programming framework to design supply chains under the risk of facility disruptions by simultaneously considering decisions on the facility location and the inventory management.

However, the current literature on forward-reverse logistics network design with reliability consideration is relatively limited. In this line of research, Vahdani et al. [39-41] propose various models for reliable design of a closed-loop logistics network in an iron and steel industry. Various solution approaches are proposed to solve the concerned problem. In these studies, the reliability concepts are considered for the collection centers with unlimited capacities. The r-level assignment approach introduced by Snyder and Daskin [25] is also adopted to deal with disruptions occurred at collection centers. Hatefi and Jolai [42] introduced a scenario based model for designing an integrated forward-reverse logistics network in which the customer demand is considered as an uncertain parameter and facilities are subject to the threat of disruptions. A scenario planning approach and a p-robustness criterion are developed to handle facility disruptions and control reliability of the network. Their proposed model can just protect the logistic networks against complete facility disruptions. To deal with this problem, Hatefi and Jolai [43] utilized a scenario planning approach and a robust optimization developed by Bertsimas and Sim [44,45] to model both partial and complete facility disruptions. In this paper, we introduce several reliability strategies to mitigate the impacts of disruptions.

This paper offers a mixed integer linear programming for robust and reliable design of an integrated forward-reverse logistics network where facilities may be randomly disrupted, and network parameters are uncertain. Our main contributions, which distinguish our work from those of relevant published works, are listed as follow:

- Offering a robust and reliable model for designing a capacitated forward-reverse logistics network with hybrid facilities, which can tackle random facility disruptions as well as the uncertainties embedded in the input data;
- Considering random disruptions at hybrid facilities, which play a critical role in the forward and reverse flows, concurrently;
- Imposing capacity restrictions on hybrid facilities and other facilities embedded in the concerned logistics network;
- Incorporating two reliability strategies in the developed model:
  - Locating two types of facilities, namely, reliable and unreliable hybrid ones;
  - Employing a sharing strategy which allows products to be shipped from reliable hybrid facilities to unreliable ones for their lost capacities to be compensated.
- Considering partial and complete capacity disruptions at unreliable hybrid facilities. The capacity of unreliable facilities may be lost partially due to the threat of disruptions. Therefore, they can serve their customers by the remaining of their available capacities;
- Applying robust optimization approach to handle uncertainties in input data, i.e. demands, returned products, fixed opening costs, and capacities.

The rest of the paper is organized as follows. In Section 2, the studied problem is defined and the proposed reliability-based model is elaborated. In Section 3, the robust optimization approach is briefly explained and the robust counterpart of the proposed reliability-based model is developed. Several computational experiments and related numerical results along with a sensitivity analysis are reported in Section 4. Finally, concluding remarks are discussed in Section 5.

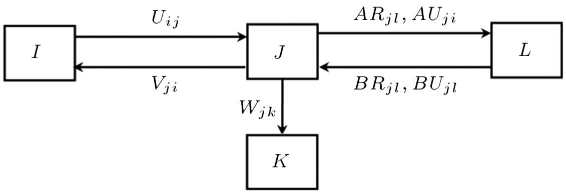

Figure 1. Integrated forward-reverse logistics network.

2. Problem definition and formulation

We consider a single product, multi-echelon forward-reverse logistics network consisting of production and distribution centers with limited capacities in the forward flow and collection, recovery and disposal centers with limited capacities in the reverse flow. The Hybrid Production-Recovery (HPR) and Hybrid Distribution-Collection (HDC) facilities are considered in the Integrated Forward-Reverse Logistics (IFRL) network, which has several advantages such as cost saving and pollution reduction [2,4,46]. The HPR facilities play the role of production centers in the forward flow and recovery centers in the reverse flow. Similarly, HDC facilities act as the distribution and collection centers in the forward and reverse flows, respectively. The structure of the concerned IFRL network is depicted in Figure 1.

As it is depicted in Figure 1, new products are shipped from HPR centers to customer zones through HDC centers in order to satisfy customer demands in the forward flow. In the reverse flow, the returned products are collected in the HDC centers for inspection purpose. After testing, they are divided into the recoverable and scrap products. The recoverable products are shipped to the HPR centers for recovery processes. Then, they are entered to the forward network as new products. The scrap products are shifted to the disposal centers. The average disposal rate reflects the quality of returned products; since high quality returns have a capability for recovery process (remanufacturing and de-manufacturing) and low quality returns must be entered to a safe disposal process. It is also assumed that the customer zones are predefined and fixed. The goal is finding the optimal number of required facilities (i.e., HPR, HDC, and disposal centers) and their best locations as well as the optimal quantity of material flows between them, while minimizing the total costs of the designed network. Gaining this goal highly depends on the way that uncertainty and reliability issues are handled when configuring the IFRL network. HPR and HDC facilities play an important role in both forward and reverse networks. However, for the sake of simplicity and without loss of generality, we assume that just HDC facilities are subject to random disruptions. Therefore, the reliability issues are taken into account for HDC facilities. However, similar reliability concepts can be considered for HPR facilities when they are subject to the threat of disruptions.

At each node, j, an unreliable HDC facility can be located at a fixed cost of $\tilde{F}U_j$, which may fail with probability:

$$q_j (0 < q_j < 1)$$

or a reliable HDC facility at a cost of $\tilde{F}R_j$ which never fails. Disruptions occur at unreliable facilities, while reliable facilities are protected against disruptions. The reliable HDC facilities never fail, since we can strengthen them against disruptions by a financial investment and hence, disruptions do not affect them. In other words, the network can be made significantly more reliable to disruptions with additional investment in the infrastructure, which makes planning for disruptions more attractive from a managerial prospective. Obviously, the cost of opening reliable HDC facilities is more than that of unreliable ones (i.e., $\tilde{F}R_j > \tilde{F}U_j$; $\forall j$). At the HDC facility, j, the distribution and collection capacities are denoted by $\tilde{\gamma}_j$ and $\tilde{\eta}_j$, respectively. The previous studies addressing the facility reliability problem assume unlimited capacities, while we consider a capacitated IFRL network model. Furthermore, these studies suppose that the disrupted facilities completely fail after disruptions and cannot serve their assigned customers. However, we assume unreliable HDC facilities may lose a part of their distribution/collection capacities when a disruption strikes. Therefore, in the forward flow, they can serve the assigned customers with the remaining of their available distribution capacities. Notably, in the reverse flow, they can serve the HPR and disposal centers with the rest of their collection capacities. In this regard, the capacity failure fraction is defined for an unreliable HDC facility, which shows the percentage of the lost capacity during disruption situations. The notations p_j and p'_j, respectively, denote the percentages of distribution and collection capacities at unreliable HDC facility, j, which are lost as a result of disruption.

A sharing strategy is also considered in the forward flow, which allows reliable HDC facilities to share new products to unreliable HDC facilities to compensate their lost capacities. In this manner, new products can be trans-shipped from reliable HDC facilities to unreliable ones whose capacities have been partially disrupted. Therefore, customers of a disrupted HDC center are not necessarily assigned to other HDC centers, since the lost capacity of a disrupted HDC center will be amended by reliable HDC facilities. It is worthy to mention that without loss of generality, the sharing strategy is only applied on distribution capacity of HDC facilities in the forward flow. To develop the mathematical programming model, the following notations are used.

2.1. Notations

Sets:

I : Number of potential HPR centers, indexed by i;

J : Number of potential HDC centers, indexed by j;

K : Number of potential disposal centers, indexed by k;

L : Number of fixed customer zones, indexed by l.

Parameters:

d_l : Demand of customer zone l;

r_l : Amount of returned products from customer zone l;

Ad : Average disposal fraction;

c_{ab} : Transportation cost per unit of products from a center a to a center b or a customer zone b for $a, b \in I, J, K, L$;

φ_i : Production capacity of HPR center i in the forward flow;

τ_i : Recovery capacity of HPR center i in the reverse flow;

γ_j : Distribution capacity of HDC center j in the forward flow;

θ_j : Collection capacity of HDC center j in the reverse flow;

ω_k : Disposal capacity of disposal center k;

F_i : Fixed cost of opening HPR center i;

FR_j : Fixed cost of opening reliable HDC center j;

FU_j : Fixed cost of opening unreliable HDC center j;

FD_k : Fixed cost of opening disposal center k;

cpf_i : Production cost per unit of product at HPR center i;

cpr_i : Recovery cost per unit of product at HPR center i;

cdf_j : Distribution cost per unit of product at HDC center j;

cdr_j : Collection cost per unit of product at HDC center j;

cp_k : Disposal cost per unit of scrapped product at disposal center k;

q_j : Disruption probability in unreliable HDC center j;

p_j : Percentage of disrupted distribution capacity at opened unreliable HDC center j;

p'_j : Percentage of disrupted collection capacity at opened unreliable HDC center j.

Variables:

U_{ij} : Quantity of products shipped from HPR center i to HDC center j;

V_{ji} : Quantity of recoverable products shipped from HDC center j to HPR center i;

W_{jk} : Quantity of scrapped products shipped from HDC center j to disposal center k;

$T_{j'j}$: Quantity of products trans-shipped from reliable HDC center j' to unreliable HDC center j at a disrupted situation ($j' \neq j$);

X_i : Binary variable; equals 1 if HPR center i is opened, 0 otherwise;

YR_j : Binary variable; equals 1 if reliable HDC center j is opened, 0 otherwise;

YU_j : Binary variable; equals 1 if unreliable HDC center j is opened, 0 otherwise;

Z_k : Binary variable; equals 1 if disposal center k is opened, 0 otherwise;

AR_{jl} : Binary variable; equals 1 if customer zone l is assigned to reliable HDC center j in the forward flow, 0 otherwise;

AU_{jl} : Binary variable; equals 1 if customer zone l is assigned to unreliable HDC center j in the forward flow, 0 otherwise;

BR_{jl} : Binary variable; equals 1 if customer zone l is assigned to reliable HDC center j in the reverse flow, 0 otherwise;

BU_{jl} : Binary variable; equals 1 if customer zone l is assigned to unreliable HDC center j in the reverse flow, 0 otherwise.

2.2. Problem formulation

The developed model is a mixed integer linear programming model, which is formulated as follows:

$$P(\mathrm{I}) : \min \sum_i F_i X_i + \sum_j FR_j YR_j + \sum_j FU_j YU_j$$

$$+ \sum_k FD_k Z_k + \sum_i \sum_j (c_{ij} + cpf_i) U_{ij}$$

$$+ \sum_j \sum_l (c_{jl} + cdf_j) d_l (AR_{jl} + AU_{jl})$$

$$+\sum_l \sum_j (c_{lj} + cdr_j) r_l (BR_{lj} + BU_{lj})$$

$$+\sum_j \sum_k (c_{jk} + cp_k) W_{jk} + \sum_j \sum_i (c_{ji} + cpr_i) V_{ji}$$

$$+\sum_{j'} \sum_{j \neq j'} q_j c_{j'j} T_{j'j}, \qquad (1)$$

$$\text{s.t.} \quad \sum_j AR_{jl} + \sum_j AU_{jl} = 1 \qquad \forall l, \qquad (2)$$

$$\sum_j BR_{lj} + \sum_j BU_{lj} = 1 \qquad \forall l, \qquad (3)$$

$$\sum_j YR_j \geq 1, \qquad (4)$$

$$YR_j + YU_j \leq 1 \qquad \forall j, \qquad (5)$$

$$AR_{jl} \leq YR_j \qquad \forall j, l, \qquad (6)$$

$$BR_{lj} \leq YR_j \qquad \forall j, l, \qquad (7)$$

$$T_{j'j} \leq MYR_{j'} \qquad \forall j', j \neq j', \qquad (8)$$

$$T_{j'j} \leq \gamma_j YU_j \qquad \forall j', j \neq j', \qquad (9)$$

$$\sum_{j'} T_{j'j} + \gamma_j (1 - p_j) YU_j \geq \sum_l d_l AU_{jl} \qquad \forall j, \qquad (10)$$

$$\sum_j T_{j'j} + \sum_l d_l AR_{j'l} \leq \gamma_{j'} YR_{j'} \qquad \forall j', \qquad (11)$$

$$\sum_i U_{ij} + \sum_{j'} T_{j'j} \geq \sum_l d_l AU_{jl} \qquad \forall j, \qquad (12)$$

$$\sum_i V_{ji} = \sum_l (1 - Ad) r_l (BR_{lj} + BU_{lj}) \qquad \forall j, \qquad (13)$$

$$\sum_k W_{jk} = \sum_l Ad r_l (BR_{lj} + BU_{lj}) \qquad \forall j, \qquad (14)$$

$$\sum_i \sum_j U_{ij} \geq \sum_l d_l, \qquad (15)$$

$$\sum_j U_{ij} \leq \varphi_i X_i \qquad \forall i, \qquad (16)$$

$$\sum_j V_{ji} \leq \tau_i X_i \qquad \forall i, \qquad (17)$$

$$\sum_i U_{ij} \leq \gamma_j (YU_j + YR_j) \qquad \forall j, \qquad (18)$$

$$\sum_l d_l AU_{jl} \leq \gamma_j YU_j \qquad \forall j, \qquad (19)$$

$$\sum_l r_l BU_{lj} \leq (1 - p'_j) \theta_j YU_j \qquad \forall j, \qquad (20)$$

$$\sum_l r_l BR_{lj} \leq \theta_j YR_j \qquad \forall j, \qquad (21)$$

$$\sum_j W_{jk} \leq \omega_k Z_k \qquad \forall k, \qquad (22)$$

$$X_i, YR_j, YU_j, Z_k, AR_{jl}, AU_{jl}, BR_{lj}, BU_{jl} \in \{0,1\},$$
$$\forall i \in I, \forall j \in J, \forall l \in L, \forall k \in K, \qquad (23)$$

$$U_{ij}, W_{jk}, V_{ji}, T_{j'j} \geq 0,$$
$$\forall i \in I, \forall j, j' \in J, \forall l \in L, \forall k \in K, \qquad (24)$$

where M is a large positive number. The aim of the objective function of Relation (1) is to minimize the total costs, including the fixed costs of opening facilities, processing and transportation costs, and the expected costs of sharing products from reliable HDC facilities to unreliable HDC facilities. The 1st to the 4th terms show the costs of locating HPR facilities, reliable and unreliable HDC facilities, and disposal centers, respectively. The 5th term represents transportation costs from HPR to HDC centers and production processing costs at HPR centers. The 6th term indicates the costs of assigning customers to reliable and unreliable HDC centers in the forward network, respectively, and distribution processing costs at HDC facilities. The 7th term presents the costs of assigning customers to reliable and unreliable HDC centers in the reverse flow, respectively, and collection processing costs at HDC facilities. The 8th and 9th terms show the transportation costs from HDC centers to disposal and HPR centers, and the disposal and recovery processing costs at disposal and HPR facilities, respectively. The last term presents the expected disruption costs, which is the expected cost of sharing products from reliable HDC centers to reliable HDC centers during disruptions.

Constraints (2) and (3) ensure that each customer zone should be exactly assigned to one HDC facility in the forward and reverse flows, respectively. Constraint (4) shows that at least one reliable HDC center must be opened to enforce the sharing strategy in a disruption situation. Constraint (5) indicates that both reliable and unreliable HDC facilities cannot be opened simultaneously at a potential node j. Constraints (6) and (7) enforce the creation of a reliable HDC center at potential node j, if a customer is assigned to it in both forward and reverse flows. Constraint (8) guarantees that in a disruption situation, if a reliable HDC facility is located at potential node j', products can be trans-shipped from it to unreliable HDC facilities. Constraint (9) ensures that during disruption situation, products cannot be shipped to potential node j, unless an unreliable HDC facility is located at it. Constraint (10) states that for an unreliable HDC center located at node j, the sum of products trans-shipped from reliable HDC facilities to it and its available capacity after disruption should be greater than or equal to the total demands of its assigned customers.

Constraint (11) indicates that for a reliable HDC center opened at potential node j', the sum of products trans-shipped from this facility to unreliable HDC facilities and the total demands of its assigned customers should not exceed its capacity. Constraint (12) states that the sum of flows entered to an unreliable HDC center located at node j cannot be lower than the total demands of its assigned customers. Constraints (13) to (15) address the balance constraints. Constraints (16) and (17) enforce restrictions on the production and recovery capacities at HPR centers, respectively. Constraints (18) to (21) enforce restrictions on the distribution and collection capacities at HDC facilities in the forward and reverse flows, respectively. Notably, the lost distribution capacity occurred at unreliable HDC centers can be amended by the means of sharing strategy which is integrated in the forward flow. This issue is reflected in Constraints (10) and (19). Constraint (22) enforces the capacity restrictions at disposal centers. Finally, Constraints (23) and (24) denote the binary and flow variables and their corresponding restrictions.

3. Robust optimization model

Soyster [47] is the first who introduces the idea of Robust Optimization (RO), but his idea turns to be very pessimistic, which makes it unfavorable among practitioners. Furthermore, the RO approach was developed independently by El-Ghaoui and Lebert [48], El-Ghaoui et al. [49], and Ben-Tal and Nemirovsky [50,51]. The Ben-Tal and Nemirovsky approach [50,51] provides less conservative solutions than earlier worst-case solutions provided by robust mathematical programming approaches (e.g., [47]) by trading off some of the conservatism for improvement in the objective function by bounding the set of values uncertain parameters could achieve. A key feature of the RO approach is its tractability, which depends on the structure of the uncertainty set. Bertsimas and Sim [44,45] develop different robust optimization techniques in an attempt to keep the structure of the original problem. The optimal solution is more optimistic than the robust optimization approach introduced by Soyster [47]. Furthermore, Bertsimas and Sim [44,45] develop a new robust methodology, wherein the optimal solution is more optimistic. In this paper, we follow the robust optimization approach developed by Ben-Tal and Nemirovski [50-52]. The significant advantages of this approach are as follow:

a) The robust formulation of the problem is tractable when the uncertainty set is described as a box or an ellipsoid. Furthermore, the structure of the final robust method remains the same as the nominal problem in the case of box uncertainty;

b) It is less conservative rather than the robust optimization introduced by Soyster [47] and Bertsimas and Sim [44,45] which makes it more favorable;

c) It allows us to control the level of conservatism in the solution set by changing uncertainty level.

Nowadays, robust optimization method is embedded into the mathematical programming problems to deal with the uncertainty in the model parameters. Robust optimization technique could provide a solution that is guaranteed to be good for all or most of the possible realizations of the uncertain parameters. To explain the robust optimization technique, consider the following linear programming problem, in which the parameters c, A, and b belong to a given uncertainty set U:

$$\begin{aligned} &\text{Min} \quad cx, \\ &\text{s.t.} \quad Ax \leq b, \end{aligned} \tag{25}$$

where x is the vector of decision variable. The concerned uncertain linear optimization model contains a series of linear optimization models whose parameters vary in the uncertainty set U [50-53]. According to Ben-Tal and Nemirovski [50-53], the robust counterpart (RO) of the model (Relation (25)) can be formulated as follows:

$$\text{Min}\left\{\sup_{(c,d,A,b\in U)}[cx] : Ax \leq b, \forall c,d,A,b \in U\right\}. \tag{26}$$

A vector x is a 'robust feasible' solution, if it satisfies all realizations of the constraints according to the uncertainty set U. Furthermore, it is a 'robust optimal' solution when there is no other feasible solution with better objective function value. In the case where uncertainty set is in the form of box uncertainty (U_{box}), the robust counterpart can be converted to a tractable equivalent model in which the extreme points of the box uncertainty are substituted instead of U_{box} [54]. Under box uncertainty, each uncertain parameter such as $\tilde{a}_{ij}$ is unknown, but bounded in a box of the form $U_{\text{box}} = \{a_{ij} \in R : |a_{ij} - \bar{a}_{ij}| \leq \rho_a G_{ij}^a\}$, where $\bar{a}_{ij}, \rho_a$, and G_{ij}^a denote the nominal value, the uncertainty level, and scale uncertainty, respectively. Notably, G_{ij}^a is a positive number and $0 < \rho_a \leq 1$. If we set $G_{ij}^a = a_{ij}$, then the relative deviation from the nominal value is at most ρ_a. According to Ben-Tal et al. [54]

and Pishvaee et al. [16], the tractable equivalent model of the RO problem (26) can be written as follows:

$$\text{Min} \quad z,$$

$$\text{s.t.} \quad \sum_j (\bar{c}_j x_j + \eta_j) \le z,$$

$$\rho_c G_j^c \le \eta_j, \qquad \forall j,$$

$$\rho_c G_j^c \ge -\eta_j, \qquad \forall j,$$

$$\sum_j (\bar{a}_{ij} + \rho_a G_{ij}^a) x_j \le \bar{b}_i - \rho_b G_i^b, \qquad \forall i. \tag{27}$$

The supply chain decisions can be categorized into three types according to their significance and the length of the planning horizon considered. First, decisions associated with the place, volume, and technology of facilities are often regarded as strategic with a planning horizon of several years. Second, supplier evaluation and selection, distribution channel, and transportation mode selection are the tactical decisions which can change every few months. Finally, operational decisions that are easily revised in the short term contain the decisions about raw material and semi-finished and finished product flows in the supply chain network. There are both single-period and multi-period supply chain network design problems in the concerned literature. Multi-period planning horizon models have been proposed for situations in which parameters change over time in a predictable way. Thereby, a planning horizon, divided into several time periods, is usually considered [55]. However, the proposed model is a single-period planning horizon model in which some parameters are uncertain. To develop the robust counterpart of the proposed reliability model and its tractable form, it is supposed that opening costs $(\tilde{F}_i, \tilde{F}R_j, \tilde{F}U_j, \tilde{F}D_k)$, demands $(\tilde{d}_l)$, returned products $(\tilde{r}_l)$, and capacities $(\tilde{\varphi}_i, \tilde{\tau}_i, \tilde{\gamma}_j, \tilde{\theta}_j, \tilde{\omega}_k)$ are uncertain and can vary in their corresponding box uncertainty sets.

Consequently, the robust counterpart of the proposed model can be formulated as the following equivalent tractable model:

$$P(\text{II}) : \min z \sum_i \left(\bar{F}_i X_i + \eta_i^F\right)$$

$$+ \sum_j \left(\bar{F}R_j YR_j + \eta_j^{FR}\right) + \sum_j \left(\bar{F}U_j YU_j + \eta_j^{FU}\right)$$

$$+ \sum_k \left(\bar{F}D_k Z_k + \eta_k^{FD}\right) + \sum_i \sum_j (c_{ij} + cpf_i) U_{ij}$$

$$+ \sum_j \sum_l (c_{jl} + cdf_j) \left[\bar{d}_l (AR_{jl} + AU_{jl}) + \eta_l^d\right]$$

$$+ \sum_l \sum_j (c_{lj} + cdr_j) \left[\bar{r}_l (BR_{lj} + BU_{lj}) + \eta_l^r\right]$$

$$+ \sum_j \sum_k (c_{jk} + cp_k) W_{jk} + \sum_j \sum_i (c_{ji} + cpr_i) V_{ji}$$

$$+ \sum_{j'} \sum_{j \neq j'} q_j c_{j'j} T_{j'j} \le z, \tag{28}$$

$$\text{s.t.} \quad \rho_F G_i^F X_i \le \eta_i^F \qquad \forall i, \tag{29}$$

$$\rho_F G_i^F X_i \ge -\eta_i^F \qquad \forall i, \tag{30}$$

$$\rho_{FR} G_j^{FR} YR_j \le \eta_j^{FR} \qquad \forall j, \tag{31}$$

$$\rho_{FR} G_j^{FR} YR_j \ge -\eta_j^{FR} \qquad \forall j, \tag{32}$$

$$\rho_{FU} G_j^{FU} YU_j \le \eta_j^{FU} \qquad \forall j, \tag{33}$$

$$\rho_{FU} G_j^{FU} YU_j \ge -\eta_j^{FU} \qquad \forall j, \tag{34}$$

$$\rho_{FD} G_k^{FD} Z_k \le \eta_k^{FD} \qquad \forall k, \tag{35}$$

$$\rho_{FD} G_k^{FD} Z_k \ge -\eta_k^{FD} \qquad \forall k, \tag{36}$$

$$\rho_d G_l^d (AR_{jl} + AU_{jl}) \le \eta_l^d \qquad \forall j, l, \tag{37}$$

$$\rho_d G_l^d (AR_{jl} + AU_{jl}) \ge -\eta_l^d \qquad \forall j, l, \tag{38}$$

$$\rho_r G_l^r (BR_{lj} + BU_{lj}) \le \eta_l^r \qquad \forall l, j, \tag{39}$$

$$\rho_r G_l^r (BR_{lj} + BU_{lj}) \ge -\eta_l^r \qquad \forall l, j, \tag{40}$$

$$\sum_j AR_{jl} + \sum_j AU_{jl} = 1 \qquad \forall l, \tag{41}$$

$$\sum_j BR_{lj} + \sum_j BU_{lj} = 1 \qquad \forall l, \tag{42}$$

$$\sum_j YR_j \ge 1, \tag{43}$$

$$YR_j + YU_j \le 1 \qquad \forall j, \tag{44}$$

$$AR_{jl} \le YR_j \qquad \forall j, l, \tag{45}$$

$$BR_{lj} \le YR_j \qquad \forall j, l, \tag{46}$$

$$T_{j'j} \le M YR_{j'} \qquad \forall j', j \neq j', \tag{47}$$

$$T_{j'j} \le \left(\bar{\gamma}_j - \rho_\gamma G_j^\gamma\right) YU_j \qquad \forall j', j \neq j', \tag{48}$$

$$\sum_{j'} T_{j'j} + \left(\bar{\gamma}_j - \rho_\gamma G_j^\gamma\right) (1 - p_j) YU_j$$

$$\ge \sum_l \left(\bar{d}_l + \rho_d G_l^d\right) AU_{jl} \qquad \forall j, \tag{49}$$

$$\sum_j T_{j'j} + \sum_l \left(\bar{d}_l + \rho_d G_l^d\right) AR_{j'l} \leq \left(\bar{\gamma}_{j'} - \rho_\gamma G_{j'}^\gamma\right) YR_{j'} \qquad \forall j' \qquad (50)$$

$$\sum_i U_{ij} + \sum_{j'} T_{j'j} \geq \sum_l \left(\bar{d}_l + \rho_d G_l^d\right) AU_{jl} \qquad \forall j, \qquad (51)$$

$$\sum_i V_{ji} = \sum_l (1 - Ad)(\bar{r}_l + \rho_r G_l^r)(BR_{lj} + BU_{lj}) \qquad \forall j, \qquad (52)$$

$$\sum_k W_{jk} = \sum_l Ad\left(\bar{r}_l + \rho_r G_l^r\right)(BR_{lj} + BU_{lj}) \qquad \forall j, \qquad (53)$$

$$\sum_i \sum_j U_{ij} \geq \sum_l \left(\bar{d}_l + \rho_d G_l^d\right), \qquad (54)$$

$$\sum_j U_{ij} \leq \left(\bar{\varphi}_i - \rho_\varphi G_i^\varphi\right) X_i \qquad \forall i, \qquad (55)$$

$$\sum_j V_{ji} \leq \left(\bar{\tau}_i - \rho_\tau G_i^\tau\right) X_i \qquad \forall i, \qquad (56)$$

$$\sum_i U_{ij} \leq \left(\bar{\gamma}_j - \rho_\gamma G_j^\gamma\right)(YU_j + YR_j) \qquad \forall j, \qquad (57)$$

$$\sum_l \left(\bar{d}_l + \rho_d G_l^d\right) AU_{jl} \leq \left(\bar{\gamma}_j - \rho_\gamma G_j^\gamma\right) YU_j \qquad \forall j, \qquad (58)$$

$$\sum_l \left(\bar{r} + \rho_r G_l^r\right) BU_{lj} \leq (1 - p_j')\left(\bar{\theta}_j - \rho_\theta G_j^\theta\right) YU_j \ \forall j, \qquad (59)$$

$$\sum_l \left(\bar{r}_l + \rho_r G_l^r\right) BR_{lj} \leq \left(\bar{\theta}_j - \rho_\theta G_j^\theta\right) YR_j \ \forall j, \qquad (60)$$

$$\sum_j W_{jk} \leq \left(\bar{\omega}_k - \rho_\omega G_k^\omega\right) Z_k \qquad \forall k, \qquad (61)$$

$$X_i, YR_j, YU_j, Z_k, AR_{jl}, AU_{jl}, BR_{lj}, BU_{jl} \in \{0,1\}, \quad \forall i \in I, \forall j \in J, \forall l \in L, \forall k \in K, \qquad (62)$$

$$U_{ij}, W_{jk}, V_{ji}, T_{j'j}, \eta_i^F, \eta_j^{FR}, \eta_j^{FU}, \eta_k^{FD}, \eta_l^d, \eta_l^r \geq 0, \quad \forall i \in I, \forall j, j' \in J, \forall l \in L, \forall k \in K. \qquad (63)$$

4. Computational experiments and sensitivity analysis results

In this section, several numerical experiments are conducted to show the significance and performance of the proposed deterministic and robust models, i.e. P(I) and P(II), respectively. To this end, two test problems, whose details are presented in Table 1, are taken into account and the results are provided under four uncertainty levels, i.e. $\rho = 0.25, 0.5, 0.75, 1$. Furthermore, nominal data is randomly generated from the uniform distributions presented in Table 2.

To provide numerical results, the deterministic and robust models are first solved under nominal data. Then, under each uncertainty level, five random realizations are generated from the corresponding uncertainty set (i.e., $\sim$ [nominal value $-\rho_\bullet G_\bullet^*$, nominal value $+\rho_\bullet G_\bullet^*$]) to investigate the behavior of the solutions provided by the proposed deterministic and robust models. The models can update their tactical decision variables, namely, flow quantities between facilities, i.e. the continuous variables, and assignment variables indicating the assignment of customer zones to the hybrid facilities under each realization. Due to the strategic nature of the decisions regarding the number and location of facilities, and since they cannot be changed in the short time [16,56], the corresponding location variables are fixed and cannot be changed under various realizations. However, the violation of chance constraints under realizations should be considered as a penalty in the objective function of the deterministic and robust models [57]. Both models are coded in GAMS 23.5/CPLEX 12.2 optimization software and all numerical experiments are solved using a Pentium dual-core 2.10 GHz computer with 3 GB RAM.

The deterministic and robust models are first solved under nominal data. Notably, all uncertainty levels, i.e. $\rho_F = \rho_{FR} = \rho_{FU} = \rho_{FD} = \rho_d = \rho_r = \rho_\gamma = \rho_\varphi = \rho_\tau = \rho_\theta = \rho_\omega$, are equally varied. The respected results are reported in the third and fourth columns of Table 3. Furthermore, the computational times are also reported for two test problems in Table 3.

According to these results, it can be affirmed that the total cost of the concerned forward-reverse network problem under uncertainty is greater than that of the respective deterministic model as expected. On the other hand, with additional costs in the infrastructure, the forward-reverse network will be significantly more stable against uncertainties. Furthermore, by augmenting the supply chain uncertainty level, total costs of the supply chain network increase due to the conservative nature of the robust optimization approach to uncertainty level.

Both deterministic and robust models are also solved under random realizations data. The mean and standard deviation of the objective function values under various realizations are considered as two performance criteria to evaluate these models. The computational results under random realizations are reported in the fifth to the eighth columns of Table 3. The entire results are also shown in Table 4. The results presented in Table 3 affirm that the robust model generates the solution with both higher quality

Table 1. Details of numerical experiments.

Problem no.	No. of potential HPR centers	No. of potential HDC centers	No. of potential disposal centers	No. of customer zones	No. of binary variables	No. of constraints
1	5	5	3	8	218	225
2	7	10	5	15	632	631

Table 2. The sources of random generation of model parameters.

Parameter	Related random distribution	Parameter	Related random distribution
$\tilde{d}_l$	$\sim$ Uniform (150, 220)	$c\tilde{p}f_i$	$\sim$ Uniform (3, 6)
$\tilde{r}_l$	$\sim$ Uniform (90, 140)	$c\tilde{p}r_i$	$\sim$ Uniform (3, 5)
$\tilde{\varphi}_i$	$\sim$ Uniform (550, 800)	$c\tilde{d}f_j$	$\sim$ Uniform (1.5, 4)
$\tilde{\tau}_i$	$\sim$ Uniform (300, 400)	$c\tilde{d}r_j$	$\sim$ Uniform (1.5, 3)
$\tilde{\gamma}_j$	$\sim$ Uniform (350, 550)	$\tilde{c}p_k$	$\sim$ Uniform (2, 4)
$\tilde{\eta}_j$	$\sim$ Uniform (280, 400)	$\tilde{F}_i$	$\sim$ Uniform (320000, 480000)
$\tilde{\rho}_k$	$\sim$ Uniform (150, 250)	$\tilde{F}U_j$	$\sim$ Uniform (180000, 260000)
q_j	$\sim$ Uniform (0.025, 0.15)	$\tilde{F}D_k$	$\sim$ Uniform (150000, 220000)
p_j, p'_j	$\sim$ Uniform (0.1, 0.5)	G_l^d, G_l^r	$\sim$ Uniform (10, 15)
Ad	0.2	$G_i^\varphi, G_i^\tau, G_j^\gamma, G_j^\theta, G_k^\omega$	$\sim$ Uniform (15, 25)
$\tilde{d}_{ab}$	$\sim$ Uniform (4, 10)	$G_i^F, G_j^{FR}, G_j^{FU}, G_k^{FD}$	$\sim$ Uniform (5000, 10000)
		$\tilde{F}R_j = 1.2 * \tilde{F}U_j$	

Table 3. Computational results of comparing performances of deterministic and robust models.

Test problem no.	Uncertainty level	Objective function value under nominal data		Mean of objective function values under realizations		Standard deviation of objective function values under realizations	
		Deterministic (CPU time)	Robust (CPU time)	Deterministic	Robust	Deterministic	Robust
1	0.25	2065415.7 (1.19)	2112869.3 (1.41)	2068861.6	2096922.0	5903.1	2127.9
	0.5		2129360.6 (1.63)	2068132.0	2093077.5	5381.8	2240.6
	0.75		2146656.8 (1.56)	2077768.0	2099923.3	14351.3	6589.4
	1		2163368.6 (0.90)	2078139.7	2099571.9	25035.0	8727.8
2	0.25	3413102.3 (3.01)	3482435.8 (3.25)	3392808.6	3408827.1	12223.6	6741.5
	0.5		3674012.4 (3.42)	3416929.0	3568150.6	8209.6	5049.9
	0.75		3707211.2 (3.21)	3457480.9	3563381.9	32151.6	11353.6
	1		3780839.3 (3.37)	3505673.8	3656574.5	38294.0	3101.5

and lower standard deviation. Furthermore, in two test problems except test problem 2 with uncertainty level of 0.25, the robust model dominates the deterministic one in terms of the mean of objective function values. Moreover, with respect to standard deviation, the robust approach dominates the deterministic one with a high difference in two test problems. Finally, by comparing the columns 4 and 6 in Table 3, it can be concluded that the mean values of the objective function obtained by the robust optimization model under realizations are lower than those provided under nominal values. The reason for this matter can be interpreted as follows. The robust optimization protects the network against the worst case values of uncertain input data. In this manner, the total cost of the network (i.e., the objective function value) significantly increases. Therefore, the objective function values obtained under nominal data (i.e., column 4) are larger

Table 4. Computational results of solving deterministic and robust models under realizations.

Uncertainty level	Objective function value of test problem 1		Objective function value of test problem 2	
	Deterministic model	Robust model	Deterministic model	Robust model
0.25	2073941.8	2095163.0	3397128.0	3407425.5
	2068080.8	2099367.6	3377570.2	3400200.4
	2075620.7	2097972.3	3382655.1	3405412.3
	2065084.6	2097846.2	3400052.8	3416850.5
	2061580.3	2094261.2	3406637.0	3414246.7
0.5	2076900.1	2092975.6	3420669.6	3569850.7
	2067783.7	2094582.5	3405743.9	3565026.9
	2067095.7	2093771.0	3417193.4	3564497.6
	2066733.7	2094779.4	3427752.4	3576307.1
	2062146.7	2089279.0	3413285.6	3565070.5
0.75	2100149.6	2107830.0	3427752.4	3580027.7
	2062487.0	2090989.2	3451445.0	3553243.1
	2078472.1	2098781.2	3430072.4	3560741.5
	2068460.3	2097287.2	3473769.4	3553660.9
	2079271.0	2104729.0	3504365.3	3569236.5
1	2110251.8	2107153.4	3536774.0	3654595.0
	2055914.5	2094387.1	3526927.1	3660368.6
	2094812.8	2107041.2	3462342.7	3659533.4
	2078110.2	2102250.6	3465606.4	3654270.8
	2051609.2	2087027.0	3536719.0	3654104.6

than those obtained under realizations (i.e., column 6).

The computational times are reported in the third and fourth columns in Table 3. They show the computational time in seconds for solving the deterministic and robust models. According to these results, it can be affirmed that both deterministic and robust models are solved in a reasonable time.

4.1. Sensitivity analysis

After validating the proposed robust model, we conducted a sensitivity analysis to show the usefulness of incorporating the reliability concepts into the proposed model to mitigate the impacts of disruptions. We study the impact of the size of capacity disruptions (i.e., by changing the corresponding capacity failure fraction at unreliable HDC facilities) on the location of reliable and unreliable HDC facilities and their numbers, total network costs, transportation costs, costs of sharing strategy, and the amount of products trans-shipped from reliable HDC facilities to unreliable ones after disruption. To do so, the capacity failure fractions (i.e., $p_j = p'_j$) are equally varied. It should be mentioned

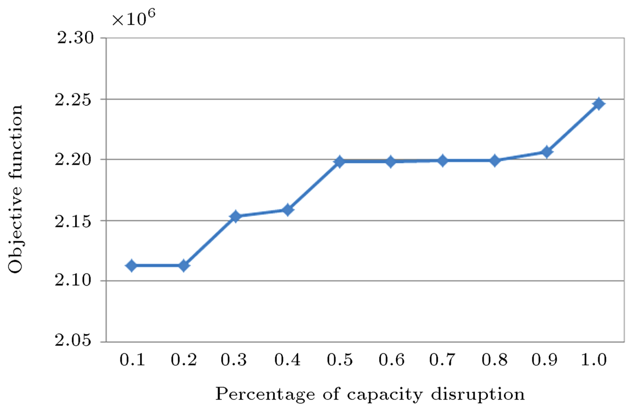

Figure 2. Capacity disruptions vs objective function.

that the sensitivity analysis is carried out on test problem 1 with uncertainty level of 0.25. The details of the results are reported in Table 5 and graphically depicted in Figures 2-4.

The first column of Table 5 presents the percentage of capacity disrupted at unreliable HDC facilities. The second and the fifth columns illustrate the objective function value and the fixed opening

Table 5. Results of sensitivity analysis.

$p_j = p'_j$	Objective function	$YR(j) = 1$	$YU(j) = 1$	Opening costs	Transportation costs	Disruption costs	Amount of shipped products
0.1	2112723.6	2	3 4 5	2071621	41042.6	747.0	124.5
0.2	2112723.6	2	3 4 5	2071621	41042.6	747.0	124.5
0.3	2153180.2	2 3	4 5	2112164	40991.7	414.8	48.5
0.4	2158605.7	2 5	3 4	2116685	41828.3	1542.2	219.7
0.5	2198695.8	2 3 5	4	2157228	41437.6	999.5	126.6
0.6	2198712.3	2 3 5	4	2157228	41441.9	1407.2	175.1
0.7	2199148.9	2 3 5	4	2157228	41865.3	1843.4	223.6
0.8	2199469.7	2 3 5	4	2157228	42173.1	2279.7	272.1
0.9	2206403.5	2 4 5	3	2164957	41227.2	2738.8	335.3
1	2246208.5	2 3 4 5	-	2205500	40708.2	0	0

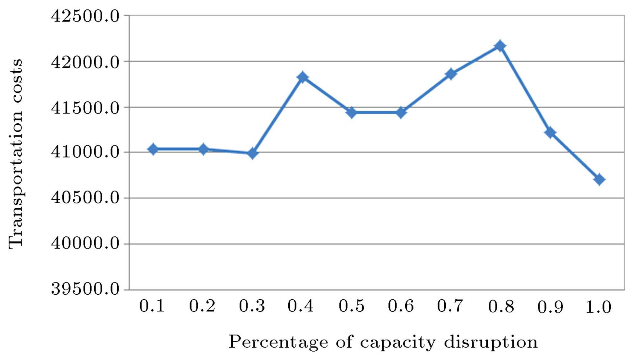

Figure 3. Capacity disruptions vs transportation costs.

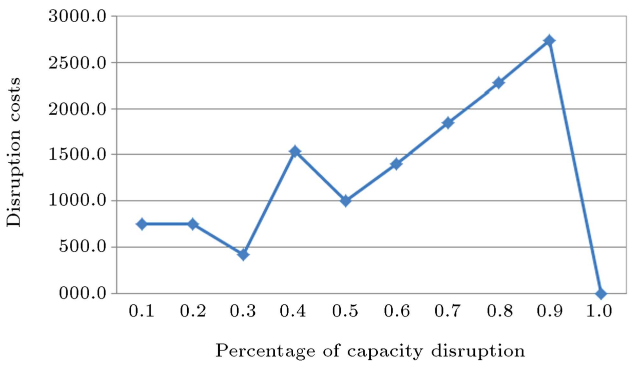

Figure 4. Capacity disruptions vs disruption costs.

costs, respectively. The third and fourth columns show the location of the reliable and unreliable HDC facilities and their opened numbers. The sixth column indicates the transportation costs between facilities and customers. The seventh column reports the disruption costs ($\sum_{j'} \sum_{j \neq j'} \tilde{d}_{j'j} T_{j'j}$) associated with the sharing strategy, which are the costs of trans-shipping products from reliable HDC facilities to unreliable ones in a disruption situation. Furthermore, the last column shows the amounts of trans-shipped products between reliable and unreliable HDC facilities ($\sum_{j'} \sum_{j \neq j'} T_{j'j}$) when capacities are disrupted.

According to the results reported in the first two columns of Table 5, the objective function value increases, when the percentage of capacity disruption is increased. This issue is also depicted in Figure 2. Furthermore, by increasing the capacity failure fractions, the model determines that more reliable HDC facilities must be located. However, when the capacity disruptions are small, most of the located HDC facilities are unreliable. This matter illustrates the impact of capacity disruptions on the location of reliable and unreliable HDC facilities as well as their numbers (see columns 3 and 4 of Table 5).

The fixed opening costs are not changed or increased when capacities are increasingly disrupted. In the cases where the location of reliable and unreliable HDC facilities is not changed by increasing the percentage of capacity disruptions (for example, see rows 6-9 in Table 5), the amount of products trans-shipped from reliable HDC facilities to unreliable ones and the corresponding costs, i.e. disruption costs, are increased. In these cases, the transportation costs are also increased. Figures 3 and 4 depict the behavior of the disruption costs and transportation costs at different levels of capacity disruptions, respectively. The above discussions approve the application of capacity disruptions, sharing strategy, and other reliability concepts in our proposed model.

5. Concluding remarks

This paper offers a robust and reliable model to protect an integrated forward-reverse logistics network against random facility disruptions and, at the same time, to cope with existing uncertainties in the model parameters. To capture random facility disruptions, several reliability strategies and assumptions are taken into account. Random disruption at hybrid HDC facilities is taken into consideration. In this manner, two types of facilities, reliable or unreliable HDC facilities, are allowed to be located in the concerned network. Furthermore, partial and complete capacity disruptions and a sharing strategy are also considered, which can improve the service level after occurrence of disruptions. To deal with the uncertainty in the parameters of the network, a robust optimization approach is applied to the original deterministic model. The effect of capacity disruptions on the objective function, opening and transportation costs, disruption costs, and the amount of shared products between HDC facilities are also investigated through a sensitivity analysis.

Finally, some directions are stated for future research. It is possible to incorporate the reliability concepts into the transportation and inventory decisions to design a more reliable supply chain network. Modeling the different types of disruption (caused by natural, man-made, or technological threats) and their impacts on facilities and/or transportation links through a scenario-based approach would be of particular interest.

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