Frequency Response Function Based Model Updating Considering Soil-Structure Interaction

M. Vahedi[1] and F.Khoshnoudian[2]*

[1] *PhD candidate, Dept of Civil Engineering, AmirKabir University of Technology, Tehran, Iran*

[2] *prof., Dept of Civil Engineering, AmirKabir University of Technology, Tehran, Iran*

**corresponding author: khoshnoudian@gmail.com; khoshnud@aut.ac.ir*

ABSTRACT

Damage detection is recognized noteworthy in all fields of engineering to avoid sudden failure of structural components. Various methods have been developed for this purpose such as sensitivity based model updating methods utilizing frequency response function (FRF). These methods are based on the concept that an analogy between the test results and analytical data obtained from structural model can provide an indication of damage where discrepancy is apparent. This might be a complex procedure because the differences between analytical FRF of intact structure and FRF of existed structure may not be a direct consequence of structural damage but could be attributed to other factors including underlying soil effects. In this regard valid modelling of soil and foundation in analytical model or indication of soil effects on structural response must be addressed to achieve reliable model updating. In this paper a method is proposed to modify FRF of intact structure in order to include underlying soil effects without subjecting the analytical model to alteration. A plane truss is considered as numerical example to investigate efficiency of method. Results indicate improvement in damage detection results precision.

Key Words: Damage Detection, Model updating, Frequency Response Function, soil structure interaction

1. Introduction

Every structure may experience changes in structural properties during its design life due to the occurrence of several conditions such as extreme loading, earthquake, strong wind, erosion, fatigue, etc. Even if a structure is managed to withstand such changes, it is crucial to investigate its integrity. To ensure this objective, many approaches have recently been developed to detect the location and severity of structural damages and evaluation of its safety. The basic principle of damage detection algorithms is based on the fact that damage may cause observable changes in structural response.

A general classification of damage identification techniques refers to types of data used to detect damage including static and dynamic data. Regardless of measurement accuracy, dynamic based approaches have been used widely considering the advantage of more sensitivity to damage in comparison with static based methods. Most common dynamic features are modal data including mode shapes [1-4], natural frequencies [6, 7], strain energy [8], transfer function parameters [9], measured modal flexibility [10-12], frequency response function [13-15] or a combination of these features [16-18].

Damage detection by model updating is the process of modification of structural parameters to minimize discrepancy between response of structure and numerical model. Due to the fact that model updating is an inverse problem and the relationship between structural parameters and measured response is inherently nonlinear, methods known as sensitivity based approaches are developed to simplify solving this problem through linearization of equations. These methods can be developed based on various characteristics such as natural frequencies, mode shapes, modal curvature, frequency response function, etc [19]. Model updating using frequency response function (FRF) improves predictive capabilities of updating procedure through providing large amount of information. The fundamentals of such model updating methods rely on minimization of a residual function which avoids the identification errors that might be introduced in extraction of modal parameters [20-23].

FRF changes rapidly by changes of structural parameters and is an inherently nonlinear function. For model based techniques of damage detection, the adjustment of analytical model and existing structure must be considered to achieve reliable results. One important issue in this field is the need to assess soil structure interaction (SSI). In most of the cases ignoring SSI is conservative but in some cases it can be detrimental particularly when the underlying soil is soft. FRF could also be affected by soil behavior, particularly the resonances and anti-resonances of the FRF curve. The soft soils with low shear wave velocities can influence extensively on the first mode anti-resonance frequency of a shear building [24].

Hence ignoring SSI in analytical model leads to inaccurate model updating and damage detection results in many cases which originates from structural modelling errors. For instance, in damage detection using modal parameters, changes in the frequencies is not necessarily due to the failure but could be due to the behavior of underlying soil or non structural components.

Many researchers have addressed the issue of soil structure interaction in the fields of system identification and model updating. Luco introduced required input records for structural identification with fixed base and flexible base condition [25].Ghaghari et al. proposed a new output-only system identification method by which dynamic characteristics of the structure could be obtained. They proved that the proposed method is applicable for models with and without SSI [26]. Lin et al examined the effect of SSI on identification of dynamic structural parameters of irregular buildings [27]. Bayraktar et al investigated effect of model updating on seismic response of a dam-reservoir-foundation [28]. Butt and omenzetter updated the finite element model of a three storey building taking into account the effect of the SSI and non-structural elements through a sensitivity equation based technique [29].Wang utilized SSS elements to introduce SSI effects in model updating of a bridge based on cross mode cross model method [30]. As stated in mentioned researches, considering effect of soil behavior is noteworthy in model updating and system identification. Hence, the objective of this research is to investigate SSI effect on damage detection procedure.

In this study, a FRF-based finite element model updating algorithm is presented. Changes of the FRF of the structure are correlated to changes of stiffness parameters to construct sensitivity equation. The derived sensitivity equations are solved for finite element model updating by least square method. The robustness of the proposed finite element model updating method using FRFs has been examined through a truss model. Comparative study of results obtained using FRFs data based on the proposed formulation and basic method demonstrate significant improvement of model updating results. Obtained results indicate adequate accuracy of the proposed formulation considering measurement and mass modelling errors.

2. Effects of SSI on FRF

In structural dynamics, the frequency domain equation for a multi degrees of freedom (MDOF) structure is expressed as:

$$Z(\omega)X(\omega) = F(\omega) \tag{1}$$

Where ω is the frequency of excitation, X(ω) is the displacement response, F(ω) is the applied force and Z(ω) is the system impedance matrix which could be defined as

$$Z(\omega) = K - \omega^2 M + i\omega C \tag{2}$$

Where K, M and C stand for the finite element stiffness, mass and damping matrices respectively and $i = \sqrt{-1}$.

According to eq (1), the function which expresses the ratio of the structural response to applied force in frequency domain is denominated as frequency response function (FRF) or transfer function and can be written as

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = Z(\omega)^{-1} \tag{3}$$

Hence the transfer function could be represented in terms of system matrices as

$$H(\omega) = (K - \omega^2 M + i\omega C)^{-1} \tag{4}$$

For a fixed based structure with n degree of freedom, the transfer function matrix is a n×n matrix. Using modal characteristics of the structure (i.e. mode shapes and modal frequencies) FRF could be calculated in a decomposed form as

$$H(\omega) = \sum_{j=1}^{n} \frac{\varphi_j \varphi_j^T}{\Omega_j^2 - \omega^2 + 2i\xi_j \Omega_j \omega} \tag{5}$$

Where Ω_j, φ_j and ξ_j are the jth natural frequency, mode shape and damping loss factor respectively.

It is well recognized that SSI can affect the dynamic responses and dynamic properties of the structure including modal frequencies, mode shapes and frequency response function accordingly. One procedure to deal with this issue is to apply more accurate boundary conditions through utilizing springs in boundaries. Spring properties must be representative of soil and foundation properties such as shear wave velocity of the underlying soil and dimension specifications. Applying the superposition concept, the spring stiffness properties will be added to the correlated orthogonal elements of the stiffness matrix to obtain global system stiffness matrix as follows

$$\overline{K} = K + K_s \tag{6}$$

Where K_s is a n×n orthogonal matrix containing spring stiffness properties. In most of the cases FRF can be obtained through application of impact loading at different DOFs and measuring structural response in all DOFs. Ignoring damping of the soil due to the impact loading and neglecting the mass of the foundation, frequency response function of the soil-foundation-structure system can be written as

$$\bar{H}(\omega) = (\bar{K} - \omega^2 M + i\omega C)^{-1} \tag{7}$$

Where $\overline{\mathrm{H}}(\omega)$ represents frequency response function of soil structure system.

3. Effects of SSI on Damage Detection

Structural damage cause changes in the stiffness matrix, modal properties and dynamic response. In order to estimate structural damage, it is necessary to correlate the changes of structural response to the changes of unknown structural parameters.

Rearranging eq (3), the displacement response can be obtained as

$$X(\omega) = H(\omega)F(\omega) \tag{8}$$

Assuming applied force to be identical for both intact and damaged structure, the change of the displacement response $\delta X(\omega)$ can be defined as

$$\delta X(\omega) = \delta H(\omega)F(\omega) \tag{9}$$

Where $\delta H(\omega)$ is the change of frequency response function due to damage which is described as follows:

$$\delta H(\omega) = H_d(\omega) - H(\omega) \tag{10}$$

Based on eq (9), if the applied force is considered unit, one can calculate the displacement response changes through identifying changes of the FRF. Using measured natural frequencies and damping loss factors for a damaged structure with n degrees of freedom, Transfer function can be expressed as:

$$H_d(\omega) = \sum_{j=1}^{n} \frac{\varphi_{jd}\varphi_{jd}^T}{\Omega_{jd}^2 - \omega^2 + 2i\xi_{jd}\Omega_{jd}\omega} \tag{11}$$

The subscript d indicates the damaged state. From experimental point of view, for a structure with large number of degrees of freedom (DOFs), conducting measurement in all the extension or volume of the structure is impractical. Hence $H_d(\omega)$ must be approximated using a truncated definition of $H_d(\omega)$ for measured DOFs in addition to the analytical $H(\omega)$ of the intact model for unmeasured DOFs [31]:

$$H_d(\omega) = \sum_{j=1}^{m} \frac{\varphi_{jd}\varphi_{jd}^T}{\Omega_{jd}^2 - \omega^2 + 2i\xi_{jd}\Omega_{jd}\omega} + \sum_{j=m+1}^{n} \frac{\varphi_j\varphi_j^T}{\Omega_j^2 - \omega^2 + 2i\xi_j\Omega_j\omega} \tag{12}$$

Where m is the number of measured modes.

The fact that the dynamic response of a structure is a function of changes in the stiffness matrix is the basis of dynamic methods of model updating. The global stiffness matrix K of the finite element model is the sum of the element matrices:

$$K = \sum_{e=1}^{ne} K_e \tag{13}$$

Where ne is the number of elements and K_e is the contributions of the eth element to the global stiffness matrix of the model. Reductions in stiffness of elements are:

$$\delta K = \sum_{e=1}^{ne} \delta K_e = \sum_{e=1}^{ne} K_{ed} - K_e \tag{14}$$

Where δK_e is scalar multiplier representing proportional changes in stiffness parameters of the eth element in the damaged state from the corresponding values in the intact state. Therefore, variations in the stiffness matrix at the system level are expressed as the sum of the changes in stiffness matrices at the element level.

Existing FRF sensitivity approaches are based on the interpretation of changes in FRF through the sensitivity of reference FRF to local stiffness changes in an experimental model. For all measured responses at all excitation frequencies, sensitivity equations can be represented as:

$$\delta H(\omega) = S(\delta K, \omega)\, \delta K \tag{15}$$

Where $S(\delta K, \omega)$ is the sensitivity matrix.

There are several methods that can be used to solve sensitivity equation such as Least Square method, Non-Negative Least Square method, Singular value Decomposition, Bounded Value Least Square, etc. In this paper Least Square method is utilized to solve the sensitivity equation. Least squares allow the residuals to be treated as a continuous quantity where derivatives can be found. It must be mentioned that the sensitivity equation is a function of locations of measurement, excitation points and excitation frequency ranges for model updating. Accuracy of predicted parameters is depended on accuracy of the measured FRF data and homomorphy between model and system including assessment of underlying soil effects on model updating. For an existing structure constructed on soft soil, the stiffness matrix will be

$$\bar{K}_d = K + \delta K + K_s \tag{16}$$

In such cases the issue of soil structure interaction in analytical model must be addressed to achieve more reliable model updating results. Ignoring soil effects leads to elimination of K_s in stiffness matrix and error in damage prediction will be encountered. This fact necessitates modelling soil and foundation in analytical model. An alternative approach is the modification of FRF of the intact structure with fixed base condition to consider soil effects. In this paper, a modification method is presented to include underlying soil effects on FRF of fixed base structure.

4. Description of proposed method to modify FRF

Some methods have been developed to investigate soil structure interaction. A simple technique to model the compliance of soil is application of springs in foundation DOFs. The values of the stiffness of the springs are dependent on the mechanical characteristics of the soil material (including shear wave velocity), the dimension of the foundation and the embedment depth of the foundation. The analytical frequency response function of a structure with springs applied in boundaries to simulate soil effects, is

$$\bar{H}(\omega) = ((K_s + K) - \omega^2 M + i\omega C)^{-1} = (K_s + Z(\omega))^{-1} \tag{17}$$

Using matrix block wise inversion, Woodbury formula could be derived which indicates that the inverse of a rank-k correction of some matrix can be computed by doing a rank-k correction to the inverse of the original matrix. Hence the last term in eq (17) can be calculated as

$$\bar{H}(\omega) = H(\omega) - H(\omega)(H(\omega) + K_s{}^{-1})^{-1} H(\omega) \tag{18}$$

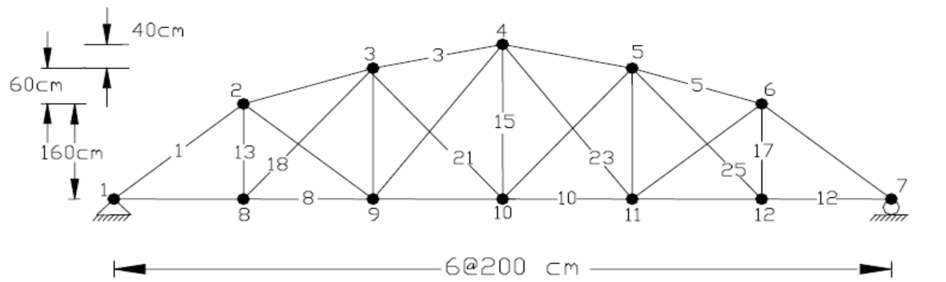
Using eq (18), frequency response function of fixed base structure will be modified to include underlying soil effects. The changes of the frequency response function due to damage in sensitivity equation will be

$$\delta H(\omega) = -H(\omega)(H(\omega) + K_s{}^{-1})^{-1} H(\omega) \tag{19}$$

5. Numerical Example and results

SSI can be significant for stiff structures founded on soft soils [32]. Hence the presented damage detection method was applied to a two-dimensional truss structure shown in Figure 1. The truss structure is modeled numerically using finite element method. The model of the truss structure has 25 elements and 24 degrees of freedom. Elements are made by steel material with Young's modulus of 200 Gpa, mass density of 7300 of kg/m3 and cross-sectional area as given in

Table 1.



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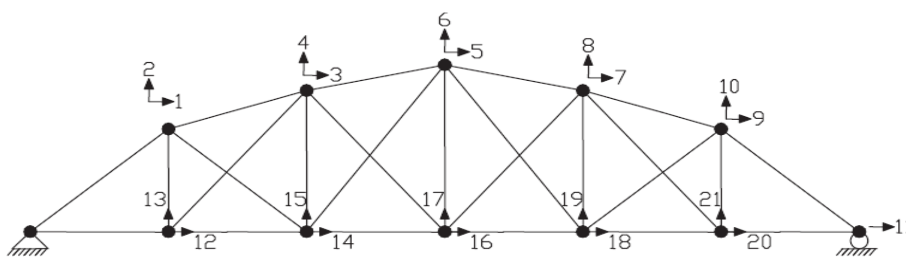

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Fig 1. plane truss model: a) geometry b) degrees of freedom

Table 1- Cross sectional area of truss members

Member	Area (cm^2)
1-6	18
7-12	15
13-17	10
18-25	12

Various base springs representing different soil and foundation stiffness are assumed to demonstrate SSI effect on FRF of structure. Three types of soil properties [33] as given in Table 2 are considered.

Table 2- Average shear wave velocity of soil

General Description	Average Shear Wave Velocity (m/s)
Firm to hard rock	1200
Stiff soil	700
Medium stiff soil	400
Soft soil	200

Figure 2 illustrates FRF of fixed base intact structure in comparison with FRF of intact structure with different flexible base conditions.

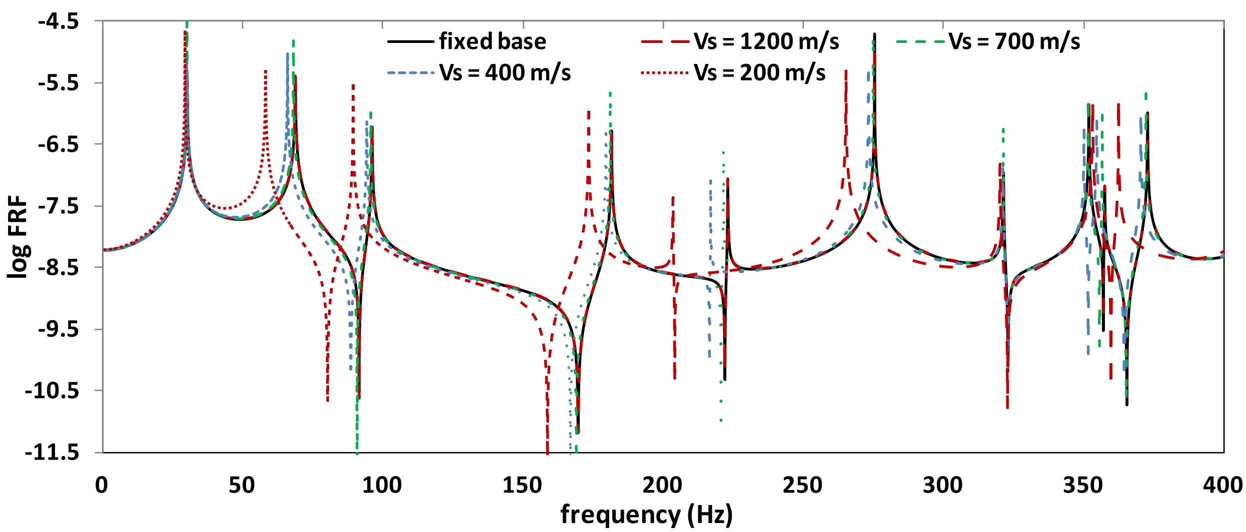


Fig 2. FRF of the truss model with different soil parameters

It can be observed that for very stiff type of soil, the changes in the FRF of structure is not significant relative to the FRF of the fixed base structure while disparities are more prominent for soft soils with smaller shear wave velocities. As mentioned before these discrepancies are related to SSI effects and may lead to inaccuracy of damage detection results especially when the soil is soft.

In order to investigate impacts of location and severity of damage on parameters estimation using the proposed method, several damage cases are assumed. These damage cases are considered for estimating parameters. The considered damage scenarios are given in Table 3.

Table 3- Considered damage cases

Damage Case	Element Number and Percentage of Damage				
1	Element Number	7	18		
	Damage Percent	40	50		
2	Element Number	7	14	18	
	Damage Percent	30	30	20	
3	Element Number	3	8	17	23
	Damage Percent	30	40	40	30

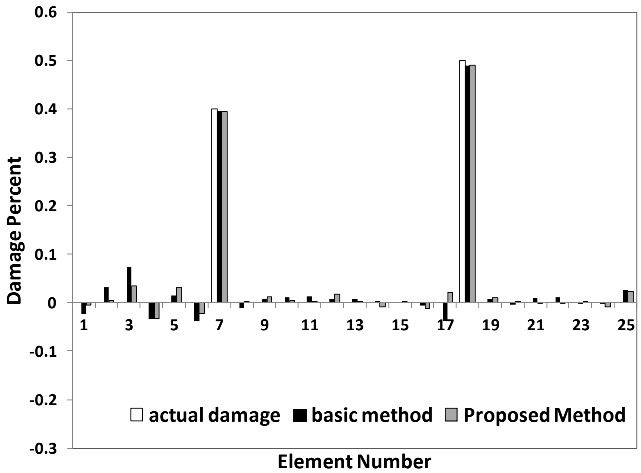
In this study FRF data has been simulated numerically using a finite element model. A single load is applied at DOF numbers 9, 12, 15, 17 and 19 for each load case and DOF numbers 3, 13,15,16,20 and 21are selected as measurement points.

Robust model updating against measurement errors, necessitates that the difference of $\overline{H}_d(\omega)$ and $H(\omega)$ must be large enough at selected excitation frequencies. By using close values of $\overline{H}_d(\omega)$ and $H(\omega)$ noise-induced errors in the parameter estimation results become more significant and model updating results show less stability and robustness against measurement errors. Therefore, selected excitation frequencies for model updating must be shifted to a higher frequency range where significant differences between $\overline{H}_d(\omega)$ and $H(\omega)$ are observable. Hence, excitation frequencies were selected in vicinity of natural frequencies of the damaged structure in a range far from resonances by a frequency band of 5 Hz.

There are unavoidable errors and deviations in finite element model updating results due to existence of errors such as environmental noise, FE modeling errors and measurement errors [34, 35], ideal boundary conditions, uncertainty of FE model and non-linearity in structural properties which adversely affect model updating results. Therefore, it is very important to be aware of adverse effects of these types of errors on damage identification results. In order to simulate these experimental inaccuracies, measurement errors are simulated by adding random values to numerically extracted FRFs by FEM. In this study a 10 percent of uniformly distributed random error is added to simulated FRFs as measurement error. As natural frequencies of low damped structures can be conducted noise-free or with high level of confidence [36], measured natural frequencies are considered to be noise-free.

Model updating is conducted using fifty sets of error contaminated data and mean values of estimated parameters are considered as model updating output. Results are presented in figures 3 to 5. The results entitled as Basic Method refer to parameter estimation using unreformed FRF, and the results obtained by proposed modification are entitled as proposed method.

Presented results prove that obtained results by proposed method are more accurate in compare with the results obtained by the Basic Method. Both methods are capable of identifying the location of damaged elements almost accurately, however Proposed method results exhibit more precision in identifying damage severity. It must be mentioned that a large number of intact elements are incorrectly detected as damaged elements by the Basic Method when the underlying soil is soft, while the proposed method accurately indicates them as intact elements.



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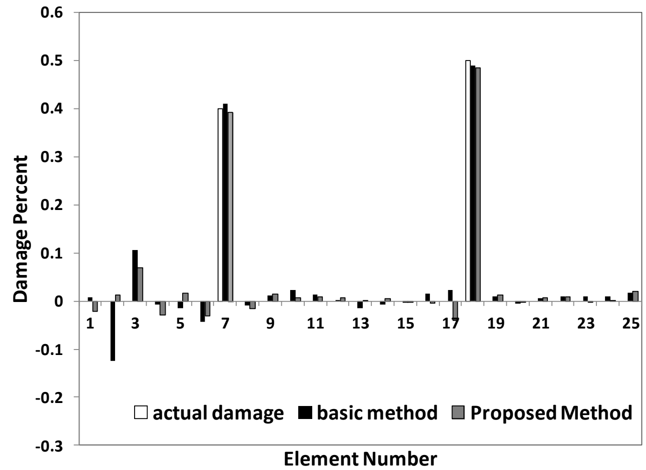

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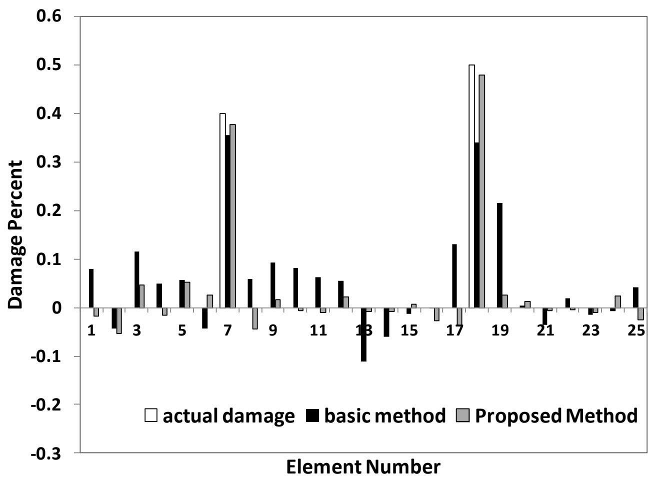

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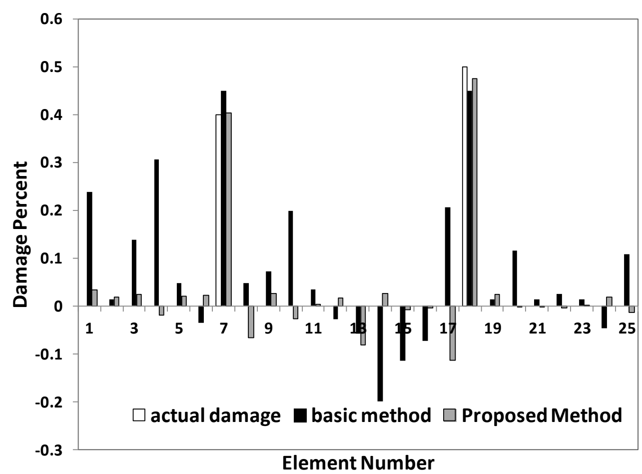

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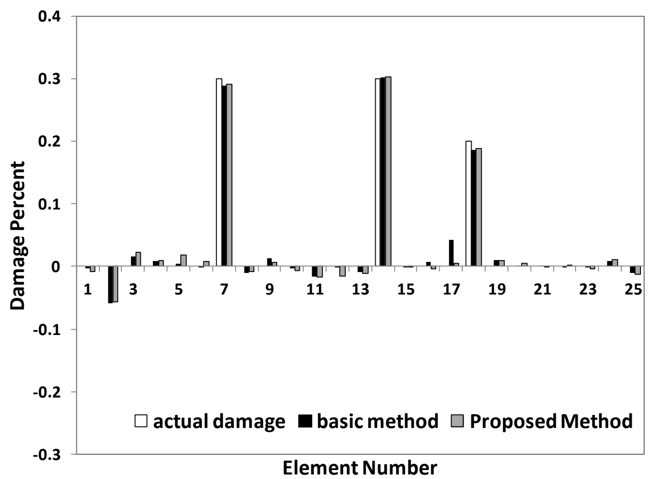
Fig. 3. Actual and Predicted Stiffness Parameters Changes in First Damage Case by **Basic and proposed Method** with Shear Wave Velocity: a) 1200, b)800,c)400 and d)200 (m/s)

Figure 4 illustrates damage prediction for second damage case. Results show that false positives are detected in all elements in basic method. Damage location and damage severity prediction is more reliable using the proposed method. Results prove that the proposed method is capable of more accurate estimation of structural parameters. Noting that all input data are the same for both methods, utilizing modified FRF by the proposed method improves model updating results.



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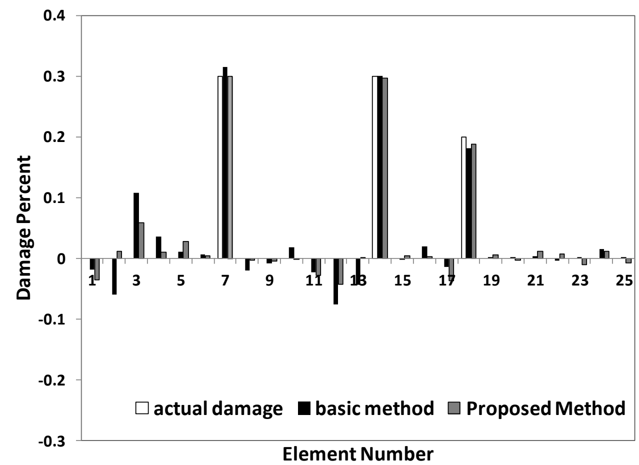

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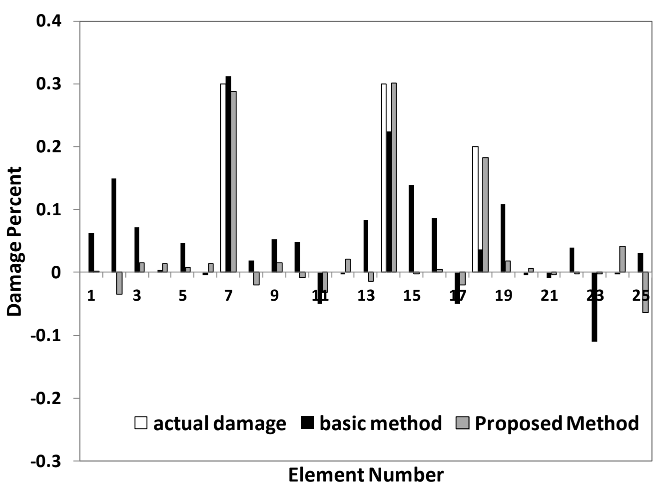

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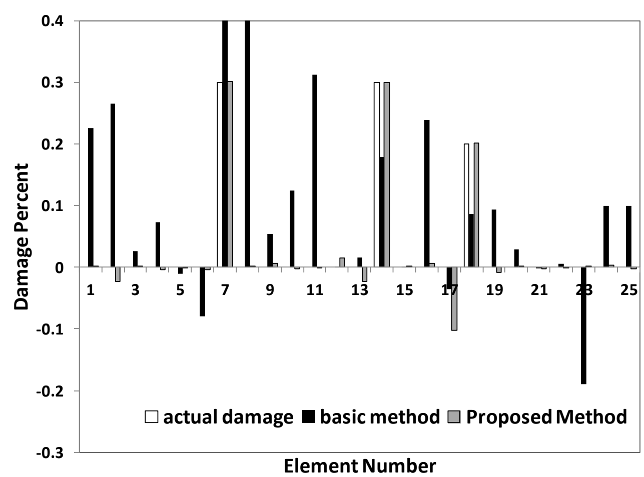

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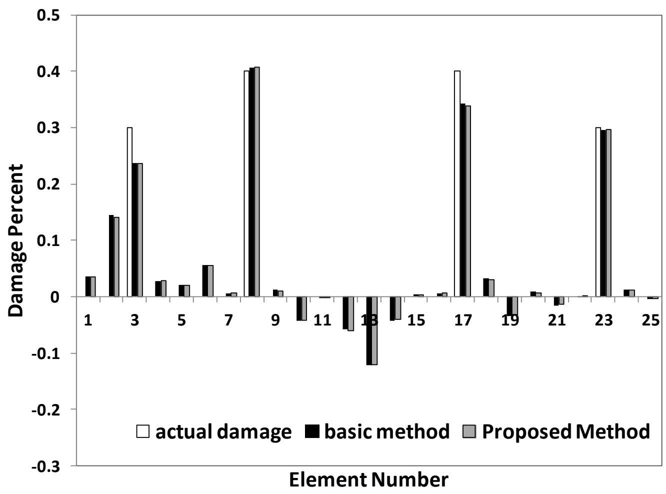
Fig. 4. Actual and Predicted Stiffness Parameters Changes in Second Damage Case by **Basic and proposed Method** with Shear Wave Velocity: a) 1200, b)800,c)400 and d)200 (m/s)



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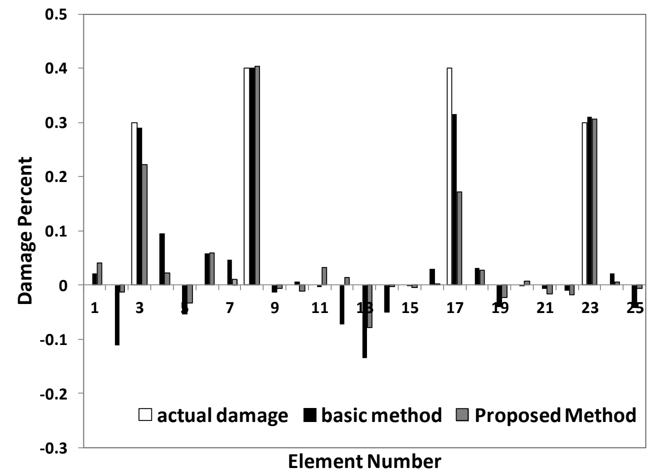

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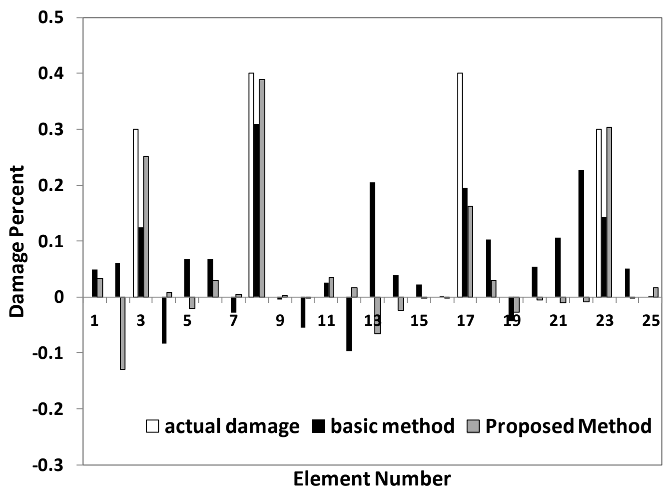

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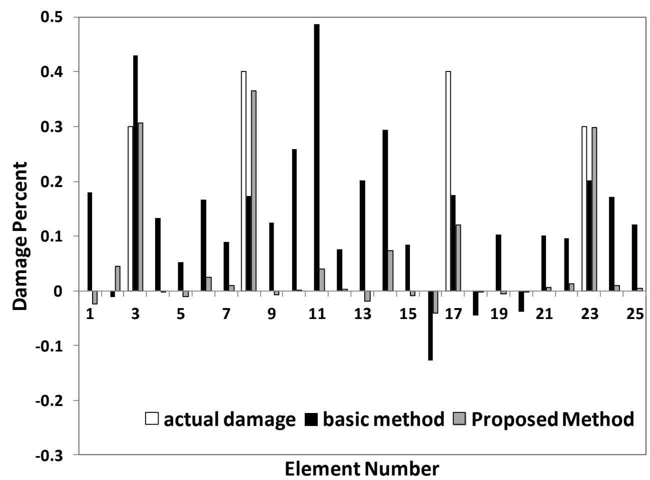

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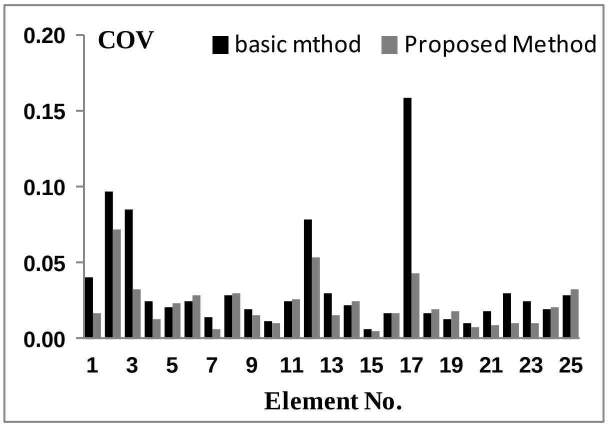
Fig. 5. Actual and Predicted Stiffness Parameters Changes in Third Damage Case by **Basic and proposed Method** with Shear Wave Velocity: a) 1200, b)800,c)400 and d)200 (m/s)

Average of estimated parameters does not reflect the robustness of the parameters estimation process. Scattering of predicted stiffness parameters around mean values is studied through evaluating standard deviations of predicted stiffness parameters. Coefficient of variation (COV) for each of the predicted unknown parameters is evaluated by normalization of standard deviation with respect to its mean value. Therefore, COV is a dimensionless parameter representing scattering of the predicted parameters. Coefficients of variations of estimated unknown parameters for the considered damage cases are plotted in figures 6 to 8. Low COVs indicate less scattering around mean values.



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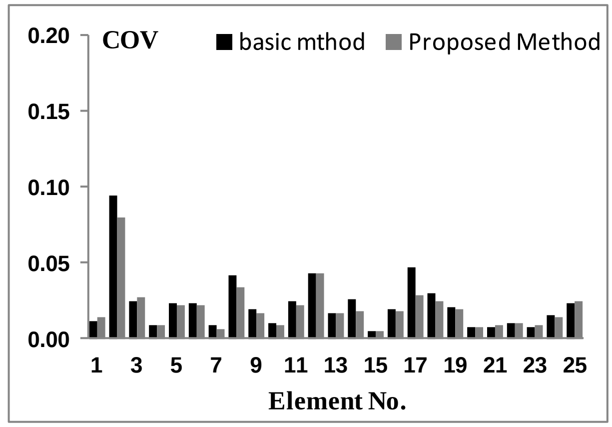

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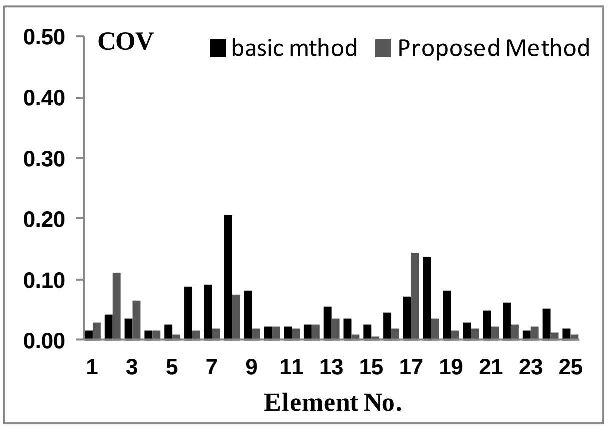

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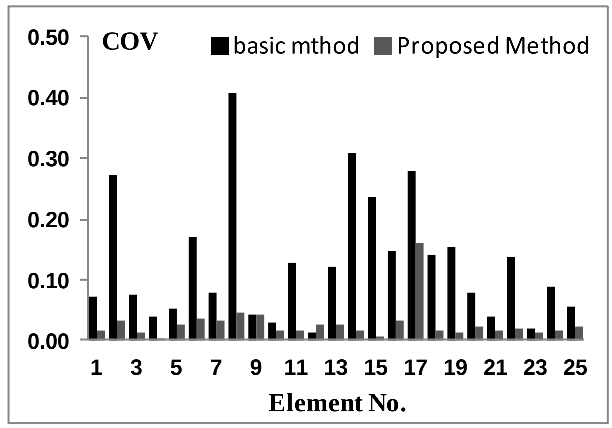

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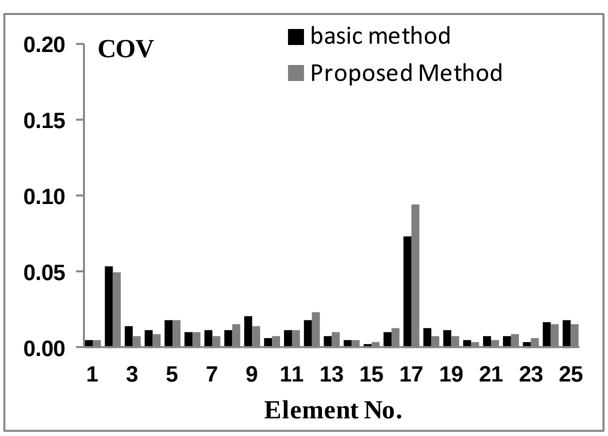
Fig.6 COV of the Predicted Damage Ratio of The Truss Model for First Damage Case with Shear Wave Velocity: a) 1200, b)800,c)400 and d)200 (m/s)

Figure 6 illustrates model updating results COVs for the first damage case. Reduction in COV values of damage detection using proposed method is obvious, which is a consequence of FRF modification to include SSI of intact structure model.



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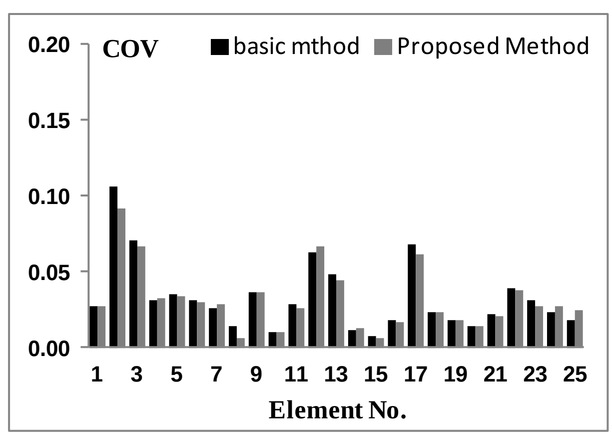

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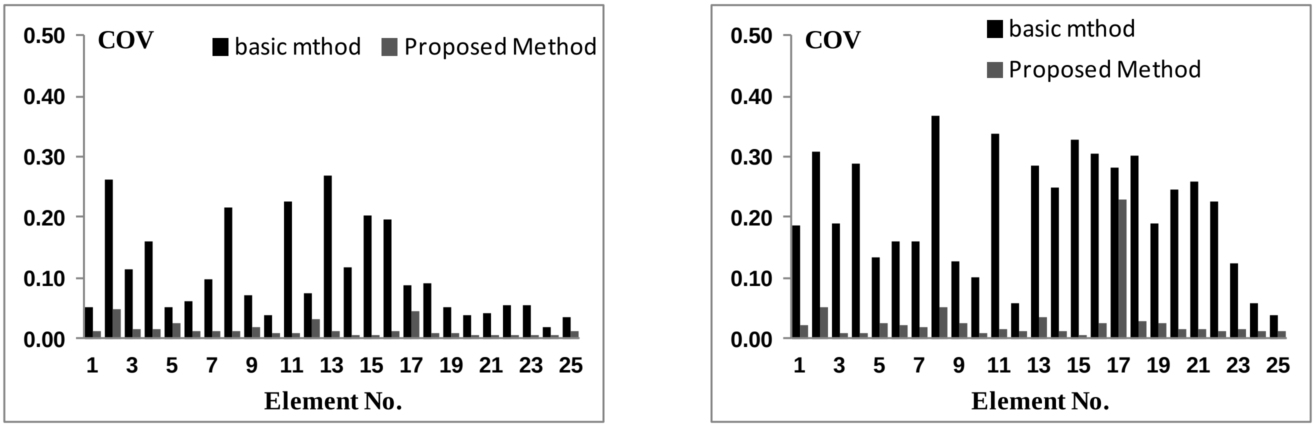
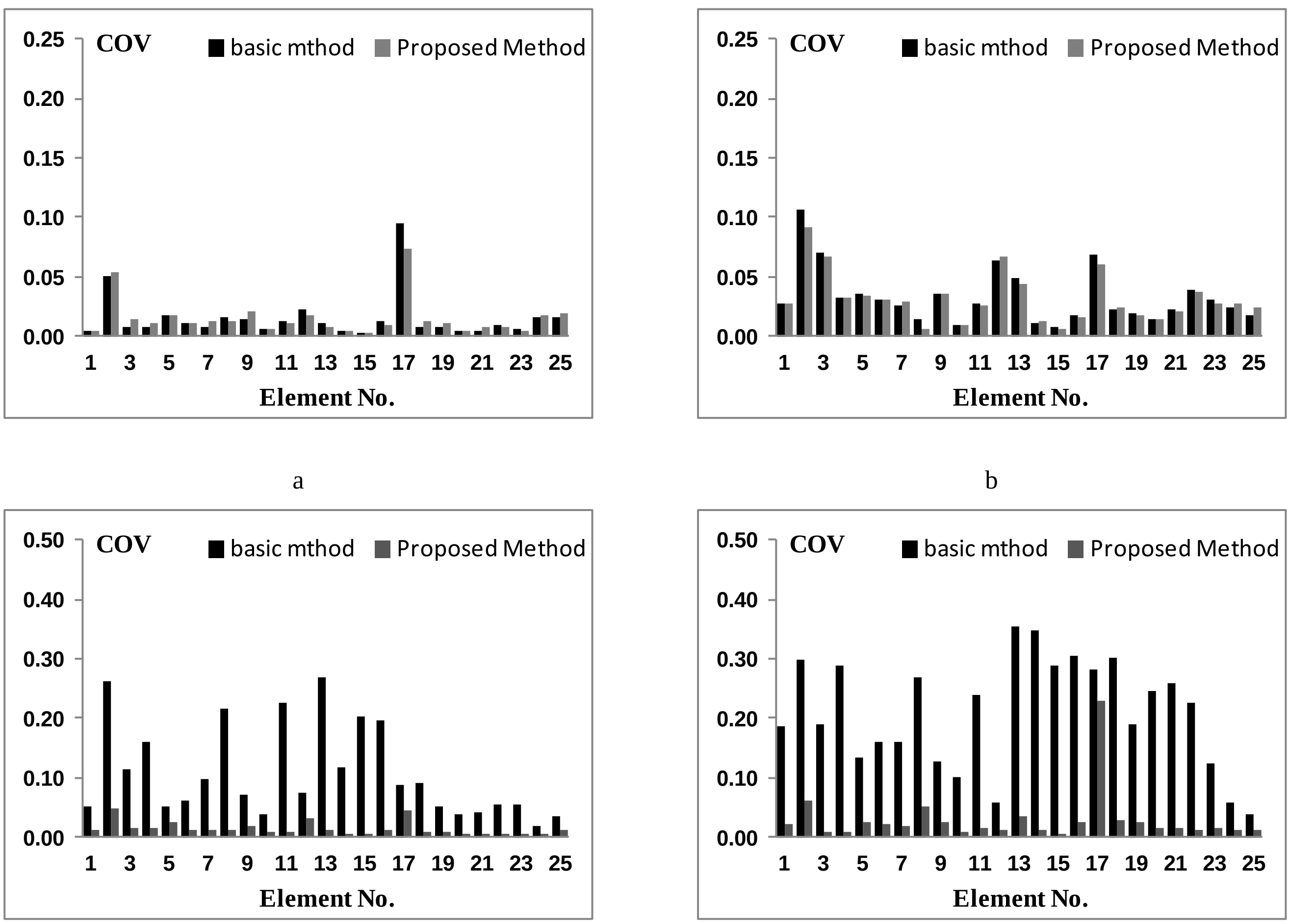


c d

Fig.7 COV of the Predicted Damage Ratio of The Truss Model for Second Damage Case with Shear Wave Velocity: a) 1200, b)800,c)400 and d)200 (m/s)

Figure 7 shows model updating results COVs for the second damage case. In almost all of the truss elements COV values of proposed method results are smaller in compare with basic method results. Hence proposed modification method will lead to more precise model updating results.



a b

c d

Fig.8 c) COV of the Predicted Damage Ratio of The Truss Model for Third Damage Case with Shear Wave Velocity: a) 1200, b)800,c)400 and d)200 (m/s)

In figure 8 model updating results COVs for the third damage case is demonstrated. A noticeable issue in figures 6 to 8 is the improvement of COV values which is less for stiff soil. This is substantially related to reliable damage detection for a structure constructed on stiff soil even when the SSI is ignored. Reduction in COV of the results obtained from truss structure on soft soil is more obvious for all damage cases.

In order to have a quantitative comparison of the accuracy of results, some indices can be used. Accuracy of the results can be assessed by closeness index (CI) based on the difference between the actual and estimated damage vectors as:

$$\mathrm{CI} = 1 - \frac{|\delta p^p - \delta p^a|}{|\delta p^a|} \tag{19}$$

Where δp^a is the actual damage parameters vector and δp^p is the vector of predicted damage parameters.

For an accurate evaluation of the damaged parameters, CI value is one. CI values for the considered damaged cases are presented in Tables 4. Comparisons of the calculated CIs prove that proposed method is more accurate than the Basic Method.

Table 4- Closeness Index of Estimated Parameters by Basic and Modified Methods

		Vs = 1200 (m/s)	Vs = 800 (m/s)	Vs = 400 (m/s)	Vs = 200 (m/s)
Case 1	Basic method	0.824	0.722	0.364	0.064
	Proposed method	0.883	0.839	0.795	0.722
Case 2	Basic method	0.823	0.647	0.211	0.010
	Proposed method	0.840	0.783	0.773	0.767
Case 3	Basic method	0.657	0.626	0.270	0.012
	Proposed method	0.661	0.614	0.584	0.571

According to Table 4, CIs decrease as the number of damaged elements increase. For less values of shear wave velocity, CI values indicate more obvious improvement of results using proposed method. The calculated CI indexes emphasize on more accuracy of the proposed method for estimation of damage location and damage severity. In conclusion, more accurate FRF will guaranty robustness of a model updating procedure.

Another indicator of accuracy is mean sizing error (MSE). The mean sizing error defines an average value of the absolute discrepancies between the actual damage parameters δp^a and the predicted damage parameters δp^p:

$$MSE = \frac{1}{ne}\sum_{i=1}^{ne}\left|\delta p^{a} - \delta p^{p}\right| \quad (20)$$

MSE of damage detection results by basic method and proposed method is presented in Table 5.

Table 5- Mean Sizing Error of Estimated Parameters by Basic and Modified Methods

		Vs = 1200 (m/s)	Vs = 800 (m/s)	Vs = 400 (m/s)	Vs = 200 (m/s)
Case 1	Basic method	0.016	0.021	0.064	0.090
	Proposed method	0.011	0.014	0.022	0.024
Case 2	Basic method	0.010	0.021	0.057	0.113
	Proposed method	0.010	0.014	0.016	0.009
Case 3	Basic method	0.033	0.038	0.081	0.146
	Proposed method	0.032	0.030	0.030	0.027

According to Table 5, MSE of proposed method is less than MSE of basic method for all damage cases with different base conditions which proves more accuracy of results by proposed method.

4. Conclusions

Damage detection through model updating method necessitates proper finite element modelling of the structure to achieve precise results. One important issue is the effect of soil and foundation on the behaviour of structure. In many cases underlying soil can cause alteration of dynamic characteristics of a structure such as natural frequencies. Due to the fact that damage detection procedures are developed based on the investigation of dynamic changes in the structure, considering soil structure interaction effects is of important issue.

Modelling soil and foundation system in analytical model utilizing common methods can reduce influences of soil on dynamic characteristics. An alternative technique is modification of analytical FRF. In this paper a method is proposed to modify FRF of intact structure to eliminate effects of underlying soil on model updating process. This method was successfully applied to a truss model. Effect of existence of errors including measurement error is also considered. A comparison between the results using proposed method and the results based on existing methods show more accurate results by implementation of the proposed modification on modal method. Mean sizing error indicator and closeness index of results are presented to prove that Predicted damage ratios are more accurate and reliable by means of proposed method in compare with basic method.

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