A two-stage stochastic programming model for value-based supply chain network design

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Abstract. Nowadays, Value-based Supply Chain Management (VbSCM) is considered a resource of competitive advantages, and companies with long term strategic plans find the VbSCM an effective factor in sustainability. In this context, supply chain network design has a significant impact on all value drivers (i.e. sales, supply chain costs, fixed assets and working capital). This paper proposes a stochastic mixed integer linear programming model for a value-based supply chain network design in which decisions on physical flow (raw materials and finished products) and financial flow are integrated. The proposed model is designed for a four-echelon, multi-commodity, multi-period supply chain, and it maximizes the value of the company, based on the economic value-added concept, by making some strategic and tactical decisions affecting the value drivers. Furthermore, a scenario-based two-stage stochastic programming model is developed with a scenario generation method based on Nataf transformation. Also, a computational analysis is undertaken to illustrate the performance of the proposed approach.

1. Introduction

In the past decade, the concept of Value-based Management (VbM), whose core objective is to increase the value of a firm, has been applied to supply chain management and is known as Value-based Supply Chain Management (VbSCM) [1-4]. In this approach, the value of a company is calculated by its ability to create future cash flow, which, in turn, is driven by profitability, capital efficiency and cost of capital [5,6]. Furthermore, profitability, capital efficiency and cost of capital are affected by management decisions of operations, investment and financing [1,7].

In Supply Chain Network Design (SCND), configuration of a supply chain requires different strategic and tactical decisions, such as facility location,

technology selection and production, and distribution planning. These decisions have a long-lasting effect on the survivability of a firm. Furthermore, most of these strategic and tactical decisions are highly correlated with financial flow in the supply chain.

In classical supply chain network design, it is usually assumed that the value of a company is affected only by its sales or costs [8-10]. Based on this assumption, other important value drivers, such as working capital and fixed assets, are ignored in the design and planning process. But, based on the VbSCM approach, all value drivers are taken into consideration. From this point of view, supply chain management influences company value via four financial drivers, including sales, cost, working capital and fixed assets. All decisions made by the management system in financing, investing and operating affect the value drivers.

In recent years, many conceptual frameworks have

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Table 1. Structure of the stochastic models in supply chain network design.

Paper	SCM level	Location level	Production stage		Time period		Final products		Objective function
			Single	Multiple	Single	Multiple	Single	Multiple	
Aghezzaf [8]	2	1	✓			✓	✓		TC[a]
Ambrosino and Scutell [9]	≥ 3	≥ 3	✓			✓	✓		TC
Chan et al. [18]	1	1	✓		✓		✓		TC
Melkote and Daskin [19]	2	1	✓		✓		✓		TC
Guillén et al. [16]	2	2	✓		✓			✓	MO
Hwang [20]	2	1	✓		✓		✓		TC
Lieckens and Vandaele [21]	2	1	✓		✓		✓		TNP[b]
Lowe et al. [22]	1	1	✓		✓		✓		TC
Miranda and Garrido [23]	2	1	✓		✓		✓		TC
Miranda and Garrido [24]	2	1	✓		✓		✓		TC
Pishvaee and Torabi [10]	≥ 3	≥ 3	✓			✓	✓		TC
Sabri and Beamon [17]	≥ 3	2	✓		✓			✓	MO[c]
Shen and Qi [25]	1	1	✓		✓		✓		TC
Snyder et al. [26]	1	1	✓		✓		✓		TC
Ommeren et al. [27]	2	1	✓		✓		✓		TC
This paper	≥ 3	2	✓			✓		✓	EVA

[a]TC: Total Cost; [b]TNP: Total Net Profit; [c]MO: Multiple Objective.

been proposed for VbSCM in which some elements, such as revenue growth, operating costs or operating capital, are considered value drivers [11-14]. In comparison to these conceptual models for VbSCM, a limited number of endeavours have been undertaken concentrating on quantitative models [1,3,15].

In the literature of SCND, there are several stochastic models developed to help managers in configuration of their supply chain. Most of the proposed stochastic models for SCND are categorized as static models in which decisions are made for a single period. In contrast, there are few multiple period models developed for SCND [8-10]. Production in all reviewed papers is supposed to be done in a single stage. Most of the reviewed papers have considered a single product, but a few have considered multiple products. When it comes to the objective function, total cost is the most prevalent objective function in the reviewed papers [16,17]. Also, some papers considered multiple objectives in their proposed model. Table 1 shows the structure of the reviewed stochastic models in the field of SCND.

Production planning, inventory management and capacity planning are the most prevalent decisions in the stochastic models of SCND. Also, routing has been considered in a few papers [9,18,20]. From four value drivers proposed by Rappaport [7], supply chain costs have been considered in all reviewed models; but none of the reviewed papers have considered three remaining value drivers, i.e. sales growth, working capital and fixed assets. Table 2 illustrates decisions in the reviewed models in the field of SCND.

This paper proposes a two-stage programming model for value-based supply chain network design.

Table 2. Decisions in the stochastic models in supply chain network design.

Paper	Decisions					
	Procurement	Production planning	Inventory management	Capacity planning	Finance, asset management, and pricing	Routing
Aghezzaf [8]			✓	✓		
Ambrosino and Scutell [9]			✓			✓
Chan et al. [18]						✓
Melkote and Daskin [19]			✓			
Guillén et al. [16]		✓	✓	✓		
Hwang [20]						✓
Lieckens and Vandaele [21]			✓	✓		
Lowe et al. [22]		✓		✓		
Miranda and Garrido [23]			✓			
Miranda and Garrido [24]			✓			
Pishvaee and Torabi [10]		✓				
Sabri and Beamon [17]		✓				
Shen and Qi [25]			✓			
Snyder et al. [26]			✓			
Ommeren et al. [27]			✓	✓		
This paper	✓	✓	✓	✓	✓	

The proposed model considers both strategic and tactical decisions in the supply chain. The objective of the proposed model is to maximize the value of the company during the planning horizon. For this purpose, four value drivers are considered: supply chain costs, sales growth, working capital, and fixed assets. The proposed model covers decisions in operations, financing and debt management.

Furthermore, in the proposed model, a four-echelon supply chain is considered, including suppliers, production facilities, distribution centers and customer zones. All products are manufactured in a single stage. Also, two kinds of warehouse are considered, a warehouse inside the production facilities for storing raw materials, and a warehouse in distribution centers for the finished products. In this network, finished products should be shipped to the customer zones from the distribution centers, and they cannot be transported directly from the production facilities to the customer zones. Figure 1 illustrates the structure
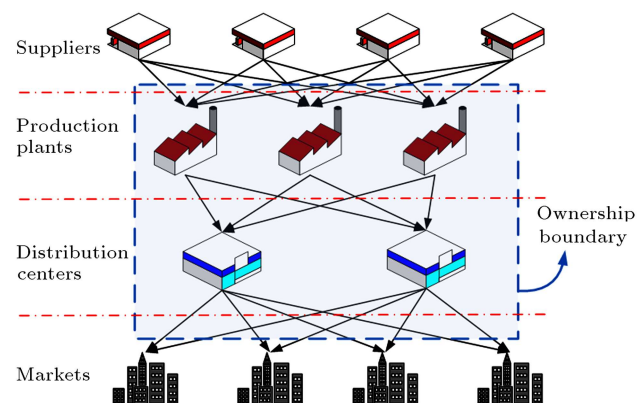
Figure 1. Structure of the considered supply chain.

of the supply chain considered in the proposed model. It is easily seen in this network that the company owns all production facilities and distribution centers.

The most important assumptions in the proposed model are as follows:

- Customer demand and associated prices are stochastic parameters with correlated behavior;

- The total number of distributed products in a market cannot violate the predicted demand for that time period;

- Short term debt is considered the main source of financing;

- An opened facility cannot be closed during the planning period;

- The capacity of manufacturing facilities is determined by the amount of installed manufacturing equipment;

- Transfers are not permitted between warehouses;

- All manufactured products are distributed through distribution centers;

- Depreciation is only considered for the manufacturing equipment;

- Safety stock is required for both raw materials and finished products;

- Raw materials are stored in manufacturing facilities, and finished products are stored in distribution centers.

Some of the most important decisions in the proposed model are as follows:

- Location and establishment time of facilities (production plant, warehouse);

- Amount of manufacturing equipment to be installed in each facility in each period;

- Total amount of each type of raw material to be supplied by each potential supplier;

- Total number of each product to be produced in each manufacturing facility;

- Total number of each finished product transported from each manufacturing facility to each distribution center;

- Financing decisions.

The remaining part of the paper is organized as follows: The next section includes the proposed mathematical programming model. Section 3 presents the two-stage stochastic programming approach. Computational analysis is presented in Section 4, and finally, conclusions are drawn is Section 5.

2. Mathematical programming model

This section proposes a Mixed Integer Linear Programming (MILP) model for the VbSCND in a multiple echelon, multiple commodity, and multiple period supply chain.

Notations

Sets

$\mathcal{T}(t \epsilon \mathcal{T}, t = t_0, ..., T)$ $(\tau \subset \mathcal{T})$ Set of time periods;

$\mathcal{S}(s \epsilon \mathcal{S})$	Set of suppliers;
$I(i \epsilon I)$	Set of production plants and distribution centers;
$\mathcal{M}(\mathcal{M} \subset i)$	Set of production plants;
$\mathcal{W}(\mathcal{W} \subset i)$	Set of distribution centers;
$\mathcal{C}(c \epsilon \mathcal{C})$	Set of customers;
$P(p \epsilon P)$	Set of products (raw material and finished products);
$P_r(P_r \subset P)$	Set of raw materials;
$P_f(P_f \subset P)$	Set of finished products.

Parameters

i^{wacc}	Weighted average cost of capital;
i^{tr}	Tax rate;
i^d	Interest rate for short term debts;
BM	A very large number;
$D_{p,c}^t$	Demand of finished product p in customer zone c;
COP_p^t	Cost price;
bv^t	Net book value at the end of lifetime period t;
sv	Salvage value of technical equipment;
$\mathcal{P}VC_{p,i}$	Production variable cost in facility i;
$\text{SVM}_{p,i}$	Storage variable cost for raw materials in production facilities;
$\text{SVW}_{p,i}$	Storage variable cost for finished products in distribution centers;
CAP	Capacity of each unit of manufacturing equipment;
Co_i^t	Fixed cost for opening facility i in period t;
CU_i	Fixed cost of operating facility i;
$B_{p,r}$	Quantity of raw material, r, necessary to produce a unit of product p (bill of materials);
ICAP_i	Capacity for storage at production facility/distribution center;
IS_p	Space occupied by unit raw material or finished product;
MSTD	Maximum available short term debts in each period;
$\mathcal{P}\mathcal{R}_{p,c}^t$	Price of a unit of finished product p;
$\mathcal{P}S_p^t$	Price of a unit of raw material p;
$\text{TC}_{p,s,i}$	Transportation variable cost for raw material p to be transported from supplier s to production facility i;

$TC_{p,i,i'}$ Transportation variable cost for finished product p to be transported from production facility i to distribution center i';

$TC_{p,i,c}$ Transportation variable cost for finished product p to be transported from distribution center i to customer zone c;

$A_{i,j}$ Number of deliveries from node i to node j in one period;

ICASH   Position of cash at period t_0;

Ih_p Position of inventory at period t_0;

α_p Coefficient for safety stock of raw material p;

β_p Coefficient for safety stock of finished product p;

SH^t Stockholder share.

Decision variables

EVA   Economic Value Added;

NOPAT   Net Operating Profit After Tax;

NOA   Net Operating Assets;

TCM^t Total Contribution Margin in period t;

DEP^t Depreciation and capital loss of disinvestments in period t;

FA   Fixed Assets;

$MEQ_i^{t,\tau}$ Stock of manufacturing equipment at location i at the end of period t acquired in period τ;

$hm_{P_r,i}^t$ Inventory of raw material P_r in manufacturing facility i at the end of period t;

$hw_{P_f,j}^t$ Inventory of product P_f in distribution center j at the end of period t;

AR^t Account Receivable;

AP^t Account Payable;

$CASH^t$ Cash in period t;

CFO^t Cash Flow from Operations;

CFI^t Cash Flow from open Items;

CFS^t Cash Flow from Short term financial investment;

CFA^t Cash Flow from fixed Assets;

CFD^t Cash Flow from Debt management;

CA^t Current net Assets at the end of period t;

$f_{P_r,s,i}^t$ Flow of raw material from supplier s to manufacturing facility i at the beginning of period t;

$f_{P_f,i,j}^t$ Flow of raw material from manufacturing facility i to distribution center j at the beginning of period t;

$f_{P_f,j,c}^t$ Flow of raw material from distribution center j to customer c at the beginning of period t;

x_i^t Binary variable, 1, if a facility (production facility/ distribution center) is open in location i in period t, otherwise, 0.

Objective function

Shareholder value is created when earnings exceed total costs of invested capital [7]. Among different metrics, Economic Value Added (EVA) is the most prevalent metric of value-based performance [1,3,15]. Therefore, the objective function in the proposed model is to maximize the EVA over the time periods computed by Net Operating Profit After Tax (NOPAT) in period t minus total costs of invested capital in Net Operating Assets (NOA) at the end of the previous period, adjusted by the weighted average cost of capital (i^{wacc}) [28]. Eq. (1) shows the objective function in which economic value added is maximized:

$$\text{Maximize EVA} = \sum_{t=t_0}^{T}(TCM^t - DEP^t).(1 - i^{tr})$$

$$-\sum_{t=t_0}^{T}\left(fa^{t-1} + \sum_{i \in \mathcal{M}}\sum_{\tau=t_0}^{\tau=t-1} MEQ_i^{t,\tau}.(bv^{t-1-\tau} + sv)\right.$$

$$\left. + CA^{t-1}\right).i^{wacc}. \tag{1}$$

Constraints

Eqs. (2)-(6) define elements of the objective function. In Eq. (2), depreciation of the manufacturing equipment is calculated and invested capital in fixed assets is calculated in Eq. (3). Eq. (4) is related to current assets, including inventories, accounts receivable and cash, minus accounts payable. In Eq. (5), the total contribution margin is calculated by subtracting total costs from total sales. Cost elements in this equation are fixed cost of operating facilities, supply variable cost, production variable cost, storage variable cost for raw materials, storage variable cost for finished products, and transportation variable cost between supplier and production facilities, between production facilities and distribution centers and between distribution centers and customer zones.

$$DEP^t = \sum_{i \in \mathcal{M}}\sum_{\tau=t_0}^{t-1} MEQ_i^{t,\tau}.(bv^{t-1-\tau} - bv^{t-\tau}) \quad \forall t \epsilon \mathcal{T}, \tag{2}$$

$$\text{FA}^t = \sum_{\forall i \in \mathcal{M} \cup \mathcal{W}} Co_i^t . x_i^t \qquad \forall t \epsilon \mathcal{T}, \tag{3}$$

$$\text{CA}^t = \sum_{p \in P_f} \sum_{i \in \mathcal{W}} \text{COP}_p^t . h_{p,i}^t + \text{AR}^t - \text{AP}^t + \text{CASH}^t$$

$$\forall t \epsilon \mathcal{T}, \tag{4}$$

$$\text{TCM}^t = \sum_{p \in P_f} \sum_{i \in \mathcal{W}} \sum_{c \in \mathcal{C}} \mathcal{PR}_{p,c}^t . f_{p,i,c}^t - \sum_{i \in \mathcal{M} \cup \mathcal{W}} \text{CU}_i . x_i^t$$

$$- \sum_{p \in P_r} \sum_{s \in S} \sum_{i \in \mathcal{M}} \mathcal{PS}_p^t . f_{p,s,i}^t$$

$$- \sum_{p \in P_f} \sum_{i \in \mathcal{M}} \sum_{i' \in \mathcal{W}} \mathcal{PVC}_{p,i} . f_{p,i,i'}^t$$

$$- \sum_{p \in P_f} \sum_{i \in \mathcal{W}} \text{SVM}_{p,i} . \left(hw_{p,j}^t + \sum_{i' \in \mathcal{M}} \frac{1}{2A_{i',i}} . f_{p,i',i}^t \right)$$

$$- \sum_{p \in P_r} \sum_{i \in \mathcal{M}} \text{SVW}_{p,i} . \left(hm_{p,i}^t + \sum_{s \in S} \frac{1}{2A_{s,i}} . f_{p,s,i}^t \right)$$

$$- \sum_{p \in P_r} \sum_{s \in S} \sum_{i \in \mathcal{M}} \text{TC}_{p,s,i} . f_{p,s,i}^t$$

$$- \sum_{p \in P_f} \sum_{i \in \mathcal{M}} \sum_{i' \in \mathcal{W}} \text{TC}_{p,i,i'} . f_{p,i,i'}^t$$

$$- \sum_{p \in P_f} \sum_{i \in \mathcal{W}} \sum_{c \in \mathcal{C}} \text{TC}_{p,i,c} . f_{p,i,c}^t$$

$$+ \sum_{p \in P_r} \sum_{i \in \mathcal{M}} \mathcal{PS}_p^t . (hm_{p,i}^t - hm_{p,i}^{t-1})$$

$$+ \sum_{p \in P_f} \sum_{i \in \mathcal{W}} \mathcal{PR}_{p,c}^t . (hw_{p,i}^t - hw_{p,i}^{t-1})$$

$$\forall t \epsilon \mathcal{T}. \tag{5}$$

Eqs. (6) and (7) define the position of accounts receivable and accounts payable in each period, respectively:

$$\sum_{p \in P_f} \sum_{i \in \mathcal{W}} \sum_{c \in \mathcal{C}} \mathcal{PR}_{p,c}^t . f_{p,i,c}^t - \text{AR}^t = 0$$

$$\forall t \epsilon \mathcal{T}, \tag{6}$$

$$\sum_{p \in P_f} \sum_{i \in \mathcal{W}} \sum_{c \in \mathcal{C}} \text{COP}_p^t . f_{p,i,c}^t - \text{AP}^t = 0$$

$$\forall t \epsilon \mathcal{T}. \tag{7}$$

Eq. (8) is to ensure the equilibrium of financial flow in the supply chain in each period:

$$\text{CASH}^t = \text{CASH}^{t-1} - \text{CFO}^t - \text{SH}^t + \text{CFI}^t$$

$$+ \text{CFS}^t + \text{CFA}^t + \text{CFD}^t$$

$$\forall t \epsilon \{t_0 + 1, \cdots, T\}. \tag{8}$$

In Eq. (9), operating cash flow in each period is calculated:

$$\text{CFO}^t = \sum_{i \in \mathcal{M} \cup \mathcal{w}} \text{CU}_i . x_i^t + \sum_{p \in P_r} \sum_{s \in S} \sum_{i \in \mathcal{M}} \mathcal{PS}_p^t . f_{p,s,i}^t$$

$$+ \sum_{p \in P_f} \sum_{i \in \mathcal{M}} \sum_{i' \in \mathcal{W}} \mathcal{PVC}_{p,i} . f_{p,i,i'}^t$$

$$+ \sum_{p \in P_f} \sum_{i \in \mathcal{W}} \text{SVM}_{p,i} . \left(hw_{p,i}^t + \sum_{i' \in \mathcal{M}} \frac{1}{2A_{i',i}} . f_{p,i',i}^t \right)$$

$$+ \sum_{p \in P_r} \sum_{i \in \mathcal{M}} \text{SVW}_{p,i} . \left(hm_{p,i}^t + \sum_{s \in S} \frac{1}{2A_{s,i}} . f_{p,s,i}^t \right)$$

$$+ \sum_{p \in P_r} \sum_{s \in S} \sum_{i \in \mathcal{M}} \text{TC}_{p,s,i} . f_{p,s,i}^t$$

$$+ \sum_{p \in P_f} \sum_{i \in \mathcal{M}} \sum_{i' \in \mathcal{W}} \text{TC}_{p,i,i'} . f_{p,i,i'}^t$$

$$+ \sum_{p \in P_f} \sum_{i' \in \mathcal{W}} \sum_{c \in \mathcal{C}} \text{TC}_{p,i',c} . f_{p,i',c}^t$$

$$+ \sum_{\tau = t_0}^{t-1} \sum_{i \in \mathcal{M}} \sum_{i' \in \mathcal{M}} \text{RLC}_{i,i'} . RL_{i,i'}^{t,\tau} = 0 \qquad \forall t \epsilon \mathcal{T}. \tag{9}$$

Eq. (10) defines cash flow from open items, which is the amount of accounts receivable minus the amount of accounts payable in the previous period:

$$\text{CFI}^t = \text{AR}^{t-1} - \text{AP}^{t-1}$$

$$\forall t \epsilon \{t_0 + 1, \cdots, T\}. \tag{10}$$

Cash flow from financing, based on short term debt, is calculated in Eq. (11). In Eq. (12) the amount of short term debt borrowed in each period is restricted to the maximum debts available:

$$\text{CFS}^t = \text{STD}^{t-1} . (1 + i^d) - \text{STD}^t$$

$$\forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{11}$$

$$\text{STD}^t \leq \text{MSTD} \qquad \forall t \epsilon \mathcal{T}. \tag{12}$$

Cash flow from fixed assets is calculated in Eq. (13):

$$\text{CFA}^t = - \sum_{i \in \mathcal{M}} \text{MEQ}_i^{t,t}.(bv_0 + sv)$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{W}} Co_i^t.(x_i^t - x_i^{t-1})$$

$$\forall t \epsilon \{t_0 + 1, \cdots, T\}. \tag{13}$$

Constraints (14)-(16) are related to the equilibrium of flows of raw materials and finished products in production facilities and distribution centers, respectively. The quantity of finished product stored at the beginning of the current period, plus the total quantity of the same product delivered to the distribution center during the current period, should be equal to the quantity of that product transported to customer zones plus the quantity stored at the beginning of the next period. Also, the quantity of raw material stored at the beginning of the current period, plus the total quantity of the same raw material delivered to a production facility during the current period, should be equal to the quantity of that raw material used to manufacture products during the current period plus the quantity stored at the beginning of the next period.

$$hw_{p,i'}^{t-1} + \sum_{i \in \mathcal{M}} f_{p,i,i'}^t - \sum_{c \in \mathcal{C}} f_{p,i',c}^t - hw_{p,i'}^t = 0$$

$$\forall i' \in \mathcal{W}, \forall p \in P_f, \forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{14}$$

$$h_{p,i}^{t-1} + \sum_{s \in S} f_{p,s,i}^t - \sum_{p \in P_f} \sum_{i' \in \mathcal{W}} B_{P_f,p}.f_{p,i,i'}^t - h_{p,i}^t = 0$$

$$\forall i \in \mathcal{M}, \forall p \in P_r, \forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{15}$$

$$h_{p,i}^t - \sum_{s \in S} f_{p,s,i}^t = 0 \qquad \forall i \in \mathcal{M}, \forall p \in P_r, \forall t = t_0. \tag{16}$$

Constraints (17) and (18) are related to the storage capacity of raw material in the production facilities, as well as finished product in the distribution centers. Maximum production capacity, based on the number of installed manufacturing equipment, is calculated by Eq. (19). Also, the maximum amount of installable manufacturing equipment in each production facility is determined in Constraint (20):

$$\sum_{p \in P_f} IS_p. \left(hw_{p,i'}^t + \sum_{i \in \mathcal{M}} \frac{1}{A_{i,i'}}.f_{p,i,i'}^t \right) \leq \text{ICAP}_{i'}.x_{i'}^t$$

$$\forall i' \in \mathcal{W}, \forall t \epsilon \mathcal{T}, \tag{17}$$

$$\sum_{p \in P_r} IS_p. \left(hm_{p,i}^t + \sum_{s \in S} \frac{1}{A_{s,i}}.f_{p,s,i}^t \right) \leq \text{ICAP}_i.x_i^t$$

$$\forall i \in \mathcal{M}, \forall t \epsilon \mathcal{T}, \tag{18}$$

$$\sum_{p \in P_f} \sum_{i' \in \mathcal{W}} W L_{p,i}.f_{p,i,i'}^t \leq \sum_{\tau=t_0}^{t-1} \text{CAP.MEQ}_i^{t,\tau}$$

$$\forall i \in \mathcal{M}, \forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{19}$$

$$\sum_{\tau=t_0}^{t} \text{MEQ}_i^{t,\tau} \leq \text{MEQ}_i^{\text{MAX}} \qquad \forall i \in \mathcal{M}, \forall t \epsilon \mathcal{T}. \tag{20}$$

The safety stock of raw material and finished products are calculated by Constraints (21) and (22), respectively:

$$hm_{p,i}^{t-1} \geq \alpha_p. \sum_{p \in P_f} \sum_{i' \in \mathcal{W}} B_{p,P_r}.f_{p,i,i'}^t$$

$$\forall i \in \mathcal{M}, \forall p \in P_r, \forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{21}$$

$$hw_{p,i}^{t-1} \geq \beta_p. \sum_{c \in \mathcal{C}} f_{p,i,c}^t$$

$$\forall i \in \mathcal{W}, \forall p \in P_f, \forall t \epsilon \{t_0 + 1, \cdots, T\}. \tag{22}$$

Constraints (23)-(25) guarantee that only opened facilities receive or send products:

$$\sum_{p \in P_f} \sum_{i' \in \mathcal{W}} f_{p,i,i'}^t \leq \text{BM}.x_i^{t-1}$$

$$\forall i \in \mathcal{M}, \forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{23}$$

$$\sum_{p \in P_f} \sum_{i \in \mathcal{M}} f_{p,i,i'}^t \leq \text{BM}.x_{i'}^{t-1}$$

$$\forall i' \in \mathcal{W}, \forall t \epsilon \{t_0 + 1, \cdots, T\}, \tag{24}$$

$$\sum_{p \in P_f} \sum_{c \in \mathcal{C}} f_{p,i,c}^t \leq \text{BM}.x_i^{t-1}$$

$$\forall i \in \mathcal{W}, \forall t \epsilon \{t_0 + 1, \cdots, T\}. \tag{25}$$

Based on Constraint (26), the total number of finished products sent to a customer zone in each period is limited to the demand of that market:

$$\sum_{i' \in \mathcal{W}} f_{p,i',c}^t \leq D_{p,c}^t \qquad \forall c \in \mathcal{C}, \forall p \in P_f, \forall t \epsilon \mathcal{T}. \tag{26}$$

Constraint (27) ensures that the established facility will remain open till the end of the planning horizon:

$$x_i^{t-1} - x_i^t \leq 0 \qquad \forall i \in \mathcal{M} \cup \mathcal{W}, \forall t \epsilon \{t_0 + 1, \cdots, T\}. \tag{27}$$

Constraints (28) and (29) determine the initial value

of cash, as well as the inventory of raw material in the first period, respectively:

$$\text{CASH}^t - \text{ICASH} = 0 \qquad \forall t = t_0, \tag{28}$$

$$\sum_{\forall i \in \mathcal{W}} hw_{p,i}^t - Ih_p = 0 \qquad \forall p \in P_f, \forall t = t_0. \tag{29}$$

Constraint (30) requires that x_i^t be a binary variable. Constraints (31)-(34) restrict some variables from taking negative values. Constraint (35) attributes customer demand and the selling price of finished products to joint stochastic space:

$$x_i^t \epsilon \{0,1\} \qquad \forall i \in \mathcal{M} \cup \mathcal{W}, \forall t \epsilon \mathcal{T}, \tag{30}$$

$$\text{FA}^t, \text{DEP}^t, \text{AR}^t, \text{AP}^t, \text{CASH}^t, \text{CFO}^t, \text{TCM}^t,$$

$$\text{STD}^t, \text{CA}^t \geq 0 \qquad \forall t \epsilon \mathcal{T}, \tag{31}$$

$$\text{MEQ}_i^{t,\tau} \geq 0 \qquad \forall i \in \mathcal{M}, \forall t, \tau \epsilon \mathcal{T}, \tag{32}$$

$$f_{p,s,i}^t, f_{p,i,i'}^t, f_{p,i',c}^t \geq 0$$

$$\forall s \in S, \forall i \in \mathcal{M}, \forall i' \in \mathcal{W}, \forall c \in \mathcal{C}, \forall t \epsilon \mathcal{T}, \tag{33}$$

$$h_{P_r,i}^t, h_{P_f,i'}^t \geq 0 \qquad \forall i \in \mathcal{M}, \forall i' \in \mathcal{W}, \forall t \epsilon \mathcal{T}, \tag{34}$$

$$(D_{p,c}^t, \mathcal{PR}_{p,c}^t) \epsilon \Omega. \tag{35}$$

3. Stochastic programming approach

This paper applies an approach based on stochastic programming to consider the underlying uncertainty of the proposed model. Stochastic programming is a well-grounded approach to make proper decisions in a stochastic environment. Stochastic problems have some stochastic parameters with known or empirical distributions. One way to handle this problem is to work with some scenarios obtained from the population of stochastic parameters. The stochastic factors usually appear at some stage or time-period in the decision process. Accordingly, the optimal decision should be made at each stage, depending on all scenarios occurred before the current stage. To reach this, a negative correlation coefficient has been considered between the price of finished products and the associated demand [29]. The following variables are considered to be scenario dependent:

- Inventory of product P_f in distribution center j at the end of period t: $hw_{P_f,j}^{t,\vartheta}$;
- Flow of raw material from distribution center j to customer c at the beginning of period t: $f_{P_f,j,c}^{t,\vartheta}$;
- Total contribution margin in period t: $\text{TCM}^{t,\vartheta}$;

- Cash in period t: $\text{CASH}^{t,\vartheta}$;
- Cash flow from operations: $\text{CFO}^{t,\vartheta}$
- Current net assets at the end of period t: $\text{CA}^{t,\vartheta}$.

where ϑ is the scenario index. Accordingly, constraints (4)-(9), (14), (17), (22), (26), (28), and (29) are written for each scenario, as well.

If stochastic factors can be represented by some random variables with known or estimable probability functions, scenario-based stochastic programming requires samples from their parametric or empirical distribution functions. Several theorems have been presented to analyze multivariate distributions. However, most of them consider an identical distribution function for the multiple correlated variables [30,31]. In some cases, there are several coupled random variables with non-identical probability patterns and generating random samples for them might be of interest. Copula theory and Nataf transformation are two powerful methods introduced for this purpose [32,33].

3.1. Scenario generation method

Stochastic programming requires some samples from stochastic parameters that are combined into the scenarios. Next, each scenario is applied to develop the final stochastic programming model. In this paper, a method, based on the Nataf transformation, is proposed to generate correlated continuous random variables. The basics of the Nataf method are given below.

Let $X = (x_1, \cdots, x_n)$ be random variables with marginal probability distribution functions, $f(x_i)$, $i = 1, 2, \cdots, n$, and correlation matrix, ρ'. This vector of random variables can be transformed to a vector of multivariate normal distribution, $Y = (y_1, \cdots, y_n)' \sim N_n(0, \rho)$ by the following equations:

$$\Phi(y_i) = F(x_i) \qquad \forall i = 1, 2, \cdots, n, \tag{36}$$

where $\Phi(.)$ denotes cumulative normal probability function and $F(.)$ is the cumulative distribution function of random variables. The elements of ρ are obtained by the following equations:

$$\rho_{ij}' = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\frac{x_i - \mu_i}{\sigma_i} \right) \left(\frac{x_j - \mu_j}{\sigma_j} \right) f_{x_i x_j}(x_i, x_j) dx_i dx_j$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\frac{F_i^{-1}(\Phi(y_i)) - \mu_i}{\sigma_i} \right) \left(\frac{F_j^{-1}(\Phi(y_j)) - \mu_j}{\sigma_j} \right)$$

$$\varphi(y_i, y_j, \rho_{ij}) dy_i dy_j,$$

$$\forall i = 1, 2, \cdots, n, \quad \forall j = i, i+1, \cdots, n. \tag{37}$$

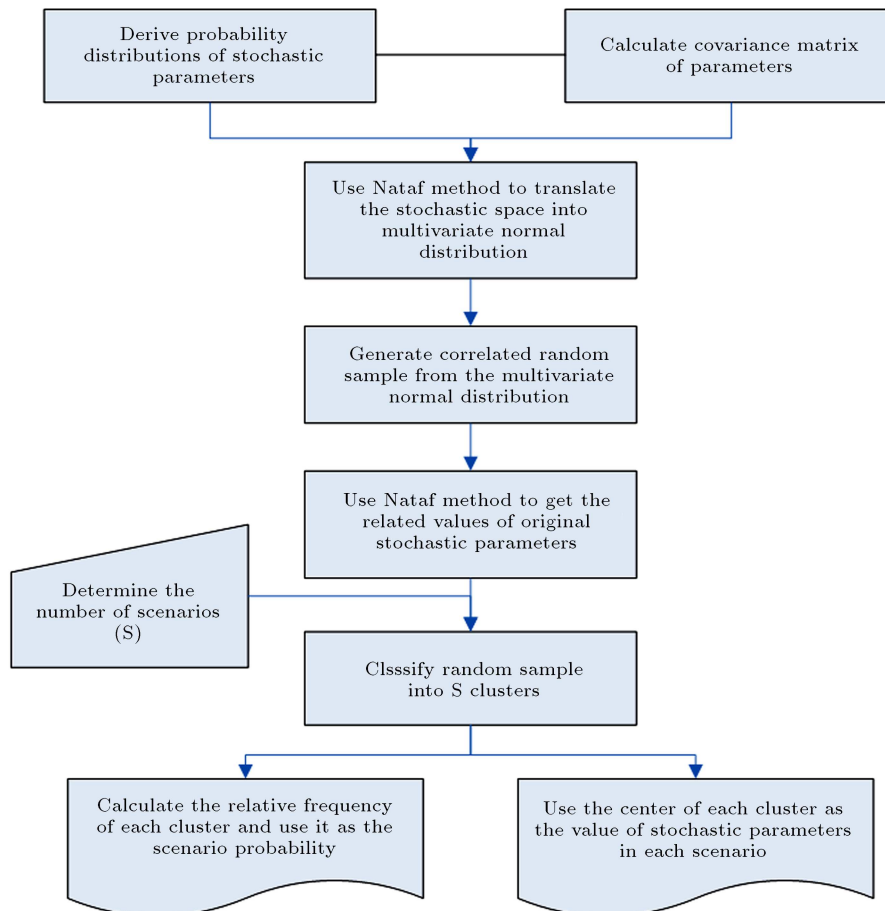Therefore, the resultant multivariate normal distribu-

Figure 2. Method of scenario generation.

tion can be considered for sample generation. After-
wards, the normal variables are again converted to the
original variables (X). Figure 2 shows the proposed
method of the scenario generation method proposed in
this paper.

The main steps of the proposed scenario genera-
tion method can be described as follows:

Step 1. Derive marginal distribution functions for
original stochastic variables and calculate
their covariance matrix. This can be done by
gathering historical data from each variable
and conducting some non-parametric statis-
tical hypothesis tests on goodness of fit that
can evaluate and select a proper probability
distribution function. Chi-square, Anderson-
Darling, and Kolmogorov-Smirnov tests are
the main methods in attaining this goal [34];

Step 2. Apply marginal distributions and the co-
variance matrix in the Nataf transformation
algorithm to construct a multivariate normal
distribution;

Step 3. Generate samples from the correlated nor-
mal distribution; methods such as Cholesky
and eigenvector decomposition could be ap-

plied [34]. Such methods are available in
most statistical software packages such as
Minitab and SAS;

Step 4. Invert the generated normal variables to the
original values using Nataf transformation
equations.

4. Computational analysis

This section conducts some computational experiments
to evaluate the performance of the proposed solution
approach. Different test problems were designed in
three classes, with five test problems $(S_1$-$S_5)$ in the
small class (S), three test problems $(M_1$-$M_5)$ in the
medium class (M), and six test problems $(L_1$ and $L_6)$ in
the large class (L). In the small and medium classes of
the test problem, five instances, and, in the large class
of the test problems, six test problems with a different
number of scenarios (5-30), are generated. Table 3
illustrates the structure of the test problems. For all
generated instances, the planning horizon was fixed to
five periods, and the number of customers was fixed to
10. Demand and price were assumed to be uniformly
distributed with known correlation, which could be

Table 3. Structure of the test problems.

Instances		Suppliers	Manufacturers	Warehouses	Row material	Scenarios
Class	ID					
Small	S_1	2	2	1	10	5
	S_2	2	2	1	10	15
	S_3	2	2	1	10	20
	S_4	2	2	1	10	25
	S_5	2	2	1	10	30
Medium	M_1	5	5	5	20	5
	M_2	5	5	5	20	15
	M_3	5	5	5	20	20
	M_4	5	5	5	20	25
	M_5	5	5	5	20	30
Large	L_1	10	10	10	30	5
	L_2	10	10	10	30	10
	L_3	10	10	10	30	15
	L_4	10	10	10	30	20
	L_5	10	10	10	30	25
	L_6	10	10	10	30	30

Table 4. Computational results.

Instances		Single equations	Single variables	Discrete variables	CPU (s)
Class	ID				
Small	S_1	1,351	1,344	15	0.206
	S_2	3,521	3,194	15	0.557
	S_3	4,606	4,119	15	0.818
	S_4	5,691	5,044	15	0.697
	S_5	6,776	5,969	15	1.746
Medium	M_1	2,856	7,778	50	13.978
	M_2	6,386	16,228	50	103.534
	M_3	8,151	20,453	50	186.441
	M_4	9,916	24,678	50	302.192
	M_5	11,681	28,903	50	3897.987
Large	L_1	5,666	26,668	100	56.263
	L_2	8,281	35,018	100	91.675
	L_3	10,896	43,368	100	581.896
	L_4	13,511	51,718	100	444.101
	L_5	16,126	60,068	100	> 3 hrs
	L_6	18,741	68,418	100	> 3 hrs

obtained even from the slope of linear relationships (details are provided in the appendix).

These instances were solved using the CPLEX MIP solver. The CPLEX MIP solver was run on a Dual core 2.26 GHz processor with 2 GHz of RAM.

Computational results are shown in Table 4. From the results, we observe that in the small class of instances, the total number of extended equations varies from 1,351 to 6,776, while the total number of variables falls between 1,344 and 5,969. For all instances in this
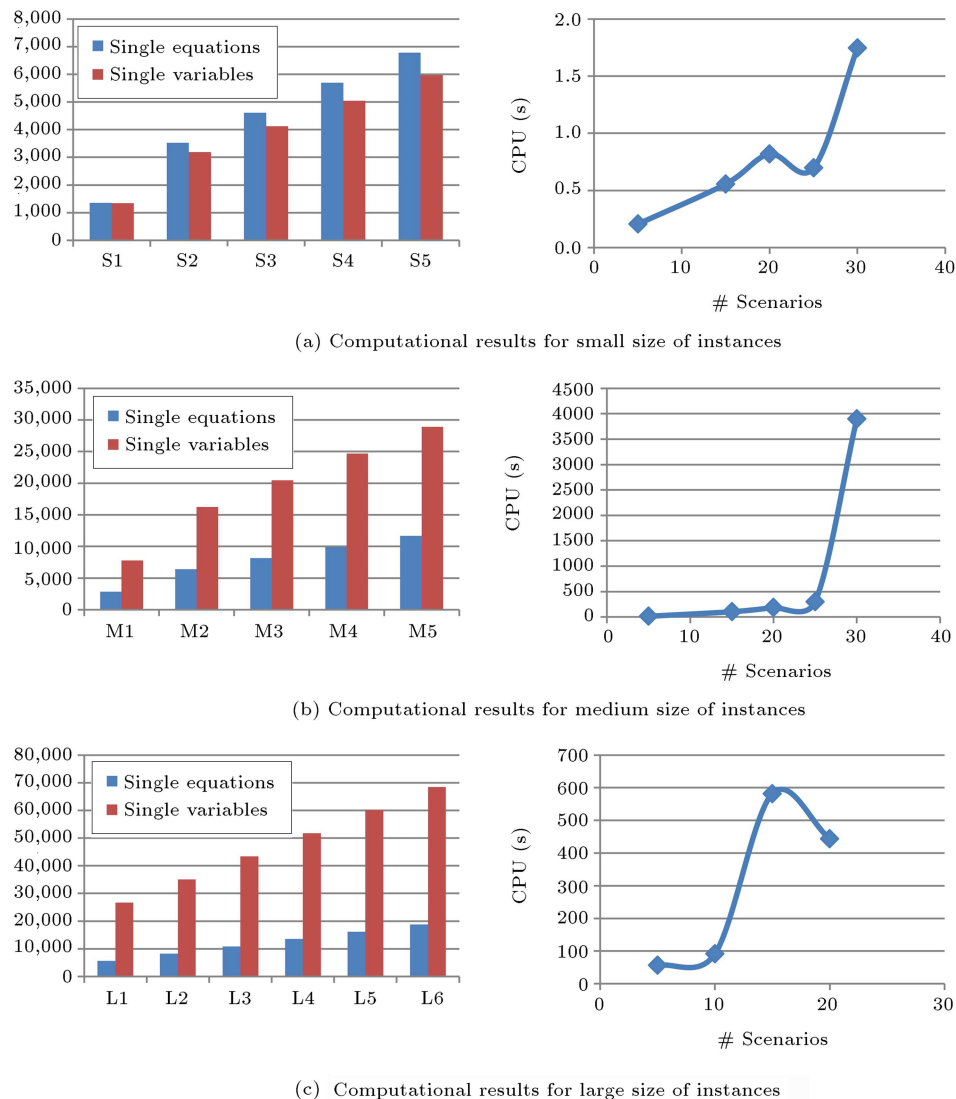
(a) Computational results for small size of instances

(b) Computational results for medium size of instances

(c) Computational results for large size of instances

Figure 3. Computational results for different classes of instances.

class, the total number of discrete variables is 15. The CPU time for these instances varies from 0.2 s to 1.7 s, indicating a good performance for this class of problem (see Figure 3(a)).

In the medium class of instances, the total number of equations starts from 2,856 (M1) and reaches 11,681 (M2). Also, the number of single variables for this class of test problem varies from 7,778 to 28,903. Furthermore, the total number of discrete variables for all generated instances in this class is 50. In this class of problems, the increasing trend in the CPU time is significant, varying from 13.9 s to 3897 s (see Figure 3(b)).

In the large class of instances, the total number of single equations exceeds 18,000. Also, while the minimum number of discrete variables is 26,668 in instance L1, it reaches 68,418 in instance L6. The total number of discrete variables in this class of problem is 100. The CPU time is 56 s for the smallest test problem

in this class (L1), 91 s for L2, 581 s for L3, and 444 for L4. But, the CPU time in instances L_5 and L_6 exceeds 3 h (see Figure 3(c)).

Based on computational results, the adopted approach is efficient for all test problems in the small size class, all test problems in the medium size, and some test problems in the large size. But, for the large size problems with more than 40 nodes (suppliers, manufacturing facilities, distribution centers, and customers) and more than 25 scenarios, the current approach cannot find the optimum solution in a reasonable amount of time (3 h).

5. Conclusion

This paper proposes a new stochastic mixed integer linear programming model for the supply chain network problem from a value based approach. To illustrate the position of this approach in the literature and

investigate to what extent the researchers have taken the value based concepts into consideration, an extensive review was conducted on papers in the field of supply chain network design. Based on the value-based approach, Supply Chain (SC) configuration and planning is undertaken in such a way that the value of SC is maximized. Also, SC costs, sales growth, working capital, and fixed assets are supposed to be the main value drivers. Furthermore, it is assumed that customer demand and the selling price of the finished products are stochastic parameters with correlated behavior.

Also a scenario-based two-stage programming approach is developed to consider the underlying uncertainty of the proposed model. Within the scenario generation process, Nataf transformation was applied to generate correlated continuous random variables. To evaluate the performance of the developed solution approach, some numerical instances are generated and solved. The results indicate that the developed solution procedure could find a robust solution in a reasonable amount of time.

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Appendix: A procedure for Nataf transformation

Parameters

$\mathcal{PR} = [pr]$: Prices;

$D = [d]$: Demands;

$\Phi(.)$: Cumulative distribution function of the normal variable.

Inputs

$F_{pr}(pr) = \frac{pr - L_{pr}}{U_{pr} - L_{pr}}$: Cumulative distribution function of the uniform variable for the price;

$F_d(d) = \frac{d - L_d}{U_d - L_d}$: Cumulative distribution function of the uniform variable for the demand;

P': Correlation matrix between $\mathcal{PR}$ and D.

Nataf transformations

Find new normally distributed variables, so that;

1. $M = 0$; vector of means;

2. $P = g(P')$; correlation matrix, where $g(.)$ is the function mentioned before by Eq. (37).

Generate random samples from the multivariate normal distribution; $Z \sim N(M, P)$.

　　Convert the generated samples to original variables using Eq. (36).

Biographies

Hossein Badri is a PhD degree candidate in the Department of Industrial Engineering at Amirkabir University of Technology, Tehran, Iran. His research interests are in supply chain network design, multiple criteria decision making, and combinatorial optimization methods. He is author and co-author of more than 20 technical papers in these fields.

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