Scientia Iranica E (2015) 22(6), 2571–2584

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
www.scientiairanica.com

A compromise decision-making model based on VIKOR for multi-objective large-scale nonlinear programming problems with a block angular structure under uncertainty

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Received 6 August 2013; received in revised form 7 July 2014; accepted 1 November 2014

KEYWORDS
VlseKriterijumska-Optimizacija I Kompromisno Resenje (VIKOR);
Multiple Criteria Decision Making (MCDM);
Multi-Objective Decision Making (MODM);
Multi-Objective Large-Scale Nonlinear Programming (MOLSNLP);
Block angular structure.

Abstract. This paper proposes a model on the basis of VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) methodology as a compromised method to solve the Multi-Objective Large-Scale Nonlinear Programming (MOLSNLP) problems with block angular structure involving fuzzy coefficients. The proposed method is introduced for solving large scale nonlinear programming in fuzzy environment for first time. The problem involves fuzzy coefficients in both objective functions and constraints. In this method, an aggregating function developed from LP- metric is based on the particular measure of "closeness" to the "ideal" solution. The solution process is composed of two steps: First, the decomposition algorithm is utilized to reduce the q-dimensional objective space into a one-dimensional space. Then a multi-objective identical crisp non-linear programming is derived from each fuzzy non-linear model for solving the problem. Second, for finding the final solution, a single-objective large-scale nonlinear programming problem is solved. In order to justify the proposed method, an illustrative example is presented and followed by description of the sensitivity analysis.

1. Introduction

Decision making is the processes by which a course of action is selected from among several alternatives on the basis of multiple criteria. Many decision-making problems in management and engineering involve multiple requirements which reflect technical and economical performance in selecting the course of action while which satisfies both environment and resources constraints. In other words, there are many decision problems with multiple objectives in a decision-making process [1-3]. The complexity of many real situation problems increases when the number of variables is very large. In other words there are various factors in the objective functions and constraints in such problems. Specially, the computational complexity increases sharply in nonlinear objectives and constraints with large variables. Therefore it becomes difficult to obtain efficient solutions for these problems in a short time and efficient manner. However, most of the real-world large scale programming problems of practical interest usually has some special structures that can be exploited. Block angular structure is one of such

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familiar structures [1,4-6]. The block angular structure problems are solved by a decomposition method [4].A decomposing algorithm is introduced for parametric space in large scale linear optimization problems with fuzzy parameter [4,7].Then this method is applied on large-scale nonlinear programming problems with block angular structure [6,8].

Recently, some compromise Multi-Criteria Decision-Making (MCDM) methods are extended and applied to find the suitable solution for MOLSNLP problems. TOPSIS method is utilized for solving multi-objective dynamics programming problems [9,10]. An effective approach is present based on TOPSIS for solving the inter-company comparison process problem [11]. TOPSIS is extended for solving multi-person multi-criteria decision-making problems in fuzzy environment [12]. An extended TOPSIS method is also presented for solving MODM problems [13]. TOPSIS is extended to solve MOLSNLP problems with block angular structure [1].

VIKOR is another compromise MCDM method that is extended for solving MOLSNLP problems [5,14,15]. The VIKOR method was proposed as a compromised approach to prioritize and select from among a set of alternatives on the basis of conflicting or non-commensurable criteria. The VIKOR is utilized to find suitable solution based on the particular measure of closeness to the ideal solution [14,15]. The VIKOR method was extended in 2007 [16-18]. This method is employed for making decision about effective information technology outsourcing management in a real-time decision situation [19]. Moreover, a systematic procedure is developed using MCDM compromise ranking method VIKOR to optimize the multi-response process [20]. The VIKOR method is also extended to prioritize alternatives with fuzzy parameter by many researchers. The fuzzy sets and VIKOR method is integrated to fuzzy VIKOR for solving the fuzzy MCDM programming problems [21]. Thus VIKOR is an interactive method in developing methods and its applications. Although, a large body of studies has utilized crisp and exact data, uncertainty and vagueness are the prominent characteristic of many real world situation decision making problems. In other words it is obvious that much knowledge in the real world is uncertain rather than crisp [22,23]. Fuzzy set theory is a valuable tool for describing this concept. Fuzzy set theory was proposed as a vagueness concept for decision-making problems with conflict of preferences involved in the selection process [22,24]. Moreover, the fuzzy set concept and the MCDM method were manipulated to consider the fuzziness in the decision making parameter and group decision-making process. A fuzzy MCDM process was introduced based on the fuzzy model and concepts of positive and negative ideal points for solving MCDM problems in a fuzzy environment [25,26].The studies also focused on applying MCDM methods for solving MOLSNLP problems with crisp parameters in objective functions and constrain [1]. The VIKOR method is utilized for solving MOLSNLP problems where the formulation of objective functions and constraints is introduced with crisp data whereas coefficient of objective function and constraint may not be exact and complete. Moreover, Abo-Sinna and Abou-El-Enien proposed a TOPSIS interactive algorithm to solve large scale multi-objective programming problems with fuzzy parameters and only for linear programming problems [27].

In this paper, a new extended VIKOR is proposed for solving MOLSNLP problems with block angular structure where the problem is formulated with fuzzy parameters in the objective functions and constraints. Since in real situations, the information of decision maker related to coefficient of objective function and constraint may not be exact and complete, a simple method is proposed which can be applied to formulate the equivalent crisp model of the fuzzy optimization problem. Moreover, the proposed method is utilized for solving nonlinear problems with fuzzy parameters, whereas the recent research studies focus only on the linear programming problems with fuzzy parameters. In the present study, first, the decomposition algorithm is used to reduce the q-dimensional objective space into a one-dimensional space. Then a multi-objective identical crisp non-linear programming is derived from each fuzzy non-linear model for solving the problem. Second, a model with fuzzy coefficients in objective function will be transferred to crisp model. Then, the method is applied for fuzzy constraints. Following that, a single-objective The logic of VIKOR method is utilized to aggregate the multi-objective programming problems into single-objective. In sum, it transfers n objectives, which are conflicting, into single-objectives involving the maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent", based on the shortest distance from the PIS and the longest distance from the NIS, which are commensurable and most of time conflicting. Following that, a single-objective large-scale nonlinear programming problem is solved to find the final solution. Finally, the Sensitivity analysis is described.

The remainder of this paper is organized as follows. The problem formulation is presented in the next section. In this section, the decomposed problem is introduced and then the parameters and variables are described. In Section 3, the VIKOR Solution method for fuzzy MOLSNLP is introduced. In Section 4, an example is provided to illustrate the process of proposed method step by step. Then, the Sensitivity analysis is described for each sub-problem. The last section is devoted to conclusion.

2. Problem formulation

The large-scale problems represent major companies that are composed of multiple units. The sub systems are almost independent with respect to each other. In other words the objective functions can be decomposed into some objectives.

A fuzzy MOLSNLP problem with the block angular structure can be stated as follows:

$p:$

$$\max(\min) f_1(x,\tilde{u}_1)=\sum_{j=1}^{N} f_{1j}(x_j,\tilde{u}_{1j})$$
$$\max(\min) f_2(x,\tilde{u}_2)=\sum_{j=1}^{N} f_{2j}(x_j,\tilde{u}_{2j})$$
$$\vdots$$
$$\max(\min) f_L(x,\tilde{u}_L)=\sum_{j=1}^{N} f_{Lj}(x_j,\tilde{u}_{Lj}),$$

s.t.:

$$FS=\begin{cases}\tilde{g}_m(x_1)\leq \tilde{B}_1\\ \quad m=1,2,\cdots,s_1\\ \tilde{g}_m(x_2)\leq \tilde{B}_2\\ \quad m=s_1,2,\cdots,s_2\\ \vdots\\ \tilde{g}_m(x_N)\leq \tilde{B}_N\\ \quad m=s_{r+1},2,\cdots,s_M\\ \tilde{H}_i(x)=\sum_{j=1}^{N}\tilde{h}_{ij}(x_j)\leq \tilde{B}\\ \quad i=1,2,\cdots,w\end{cases} \tag{1}$$

$$f_i(x,\tilde{u}_i)=\tilde{u}_i c_i x=\sum_{j=1}^{N} f_{ij}(x_j,\tilde{u}_i)=\sum_{j=1}^{N}\sum_{k=1}^{p}\tilde{u}_{ijk}c_{ijk}v_{ijk}(x), \tag{2}$$

where $V_{ijk}(X_j)$ is the kth function of jth variable in the ith objective function. This problem is a fuzzy MOLSNLP problem with the block angular structure as a big company which has q sub system. Moreover, there are N variables. Each sub problem has its own variables. For example the first sub problem has N_1 variables. Furthermore, the functions of each sub problem has several functions.

$\tilde{g}_i(x_j)=\tilde{u}_{ij}c_{ij}v_{ij}$ are the inequality constraint functions and $\tilde{H}_i(x)$ are the common constraint functions on R^n.

Model parameters:

L The number of objective functions;

q The number of sub problems;

N The number of variables;

N_i The set of variables of the ith sub problem, $i=1,2,\cdots,q$;

p_i ith sub problems;

p_{itj} The number of functions for tth function of jth variable in ith sub problem;

R The set of all real numbers;

c_i An $(N\times N)$ diagonal matrix for the ith function;

c_{itj} An $(N\times N)$ diagonal matrix for the kth function of the jth variable in the ith function;

c_{ij} An $(N\times N)$ diagonal matrix for the ith constraint of jth variable;

d_{ij} An $(N\times N)$ diagonal matrix for the ith common constraint for the jth variable;

$\tilde{U}_i$ An n-dimensional row vector of fuzzy parameters for the ith objective function;

$\tilde{U}_{ij}$ An n-dimensional row vector of fuzzy parameters for the ith constraint of jth variable;

$\tilde{U}_{ijk}$ An n-dimensional row vector of fuzzy parameters for the kth function of the jth variable in the ith function;

W The number of common constraints on R^N;

M The number of constraints;

S_i the number of constraints for the ith variable;

$\tilde{B}$ An w-dimensional column vector of right-hand sides of the common constraints whose elements are constants;

$\tilde{B}_j$ An S_i-dimensional column vector of independent constraints right-hand sides whose elements are fuzzy parameters for the ith subproblem, $i=1,2,\cdots,q$.

$X=(x_1,x_2,\cdots,x_N)$ is the N-dimensional decision vector. $f_i(x,\tilde{u}_i)$, $i=1,2,\cdots,L$ are the objective functions. It is assumed that the objective functions have an additively separable form. It is pointed out that any (or all) of the functions may be nonlinear. The fuzzy MOLSNLP problem can be decomposed into q sub-problems based on Dantzig-Wolfe decomposition algorithm. The objective functions break into q sub problems. The ith sub-problem for $i=1,\cdots,q$ is defined as:

$$P_i=\begin{cases}\max(\min)f_1(x,\tilde{u}_1)=\sum_{j\in N_i}\sum_{k=1}^{p_{i1j}}f_{1k}(x_j,\tilde{u}_{1k})\\ \qquad=\sum_{j\in N_i}\sum_{k=1}^{p_{i1j}}\tilde{u}_{1k}c_{1k}v_{1k}(x_j)\\ \max(\min)f_2(x,\tilde{u}_2)=\sum_{j\in N_i}\sum_{k=1}^{p_{i2j}}f_{1k}(x_j,\tilde{u}_{2k})\\ \qquad=\sum_{j\in N_i}\sum_{k=2}^{p_{i2j}}\tilde{u}_{2k}c_{2k}v_{2k}(x_j)\\ \vdots\\ \max(\min)f_L(x,\tilde{u}_L)=\sum_{j\in N_i}\sum_{k=1}^{p_{iLj}}f_{Lk}(x_j,\tilde{u}_{Lk})\\ \qquad=\sum_{j\in N_i}\sum_{k=1}^{p_{iLj}}\tilde{u}_{Lk}c_{Lk}v_{Lk}(x_j)\\ S.T.\\ FS_i=\begin{cases}\sum_{j\in N_i}\tilde{g}_j(x_j)\leq\tilde{B}_j\\ \tilde{H}_i(x)=\sum_{j=1}^{N}\tilde{h}_{ij}(x_j)\leq\tilde{B}\\ \qquad i=1,2,\cdots,w\end{cases}\end{cases}\tag{3}$$

As shown in Problem (3), the ith sub problem consists of L objective functions. There are P_{itj} functions for jth variable of tth objective function in ith sub problem. Moreover, $\tilde{h}_{ij}(X_j)=\tilde{U}_{ij}d_{ij}h_{ij}$, where h_{ij} is the function of jth variable in ith common constraint and $\tilde{U}$ is the coefficient of the objective function, $\tilde{C}$ and $\tilde{Z}$ are the coefficients of the left-hand side of constraints, and $\tilde{B}$ is the coefficient of the right-hand side of constraint in Problem (3). It is pointed out that all of the coefficients are presented as triangular fuzzy numbers.

3. The VIKOR solution method for fuzzy MOLSNLP

In this section, a compromised method based on VIKOR is presented for solving fuzzy MOLSNLP problems. The basic methodology is to decompose the original problem into smaller sub-problems. In other words this method is employed when the original problem is split into some sub-problems. In order to obtain a compromise solution, first, the MOLSNLP problem is decomposed into q sub-problems as shown in Eq. (3). Then by considering the individual minimum and maximum of each objective function, the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for jth sub-problem are computed. By Computing S_j,R_j,Q_j, the L-dimensional problem is transferred into a single-objective. The proposed method is administered through the following steps:

Step 1. Decompose the proposed problem into the q sub-problems based on the Dantzig-Wolfe decomposition algorithm for objective functions and constraints. Then transfer each fuzzy sub problem into three crisp sub problems as follows:

$$\tilde{U}_i=(a_i,b_i,c_i),$$

is a triangular fuzzy number.

$$f_i(x,\tilde{u}_i)=\tilde{u}_ic_ix=\sum_{j=1}^{N}\sum_{k=1}^{p_i}\tilde{u}_{ijk}c_{ijk}v_{ijk}(x_j)$$
$$=\sum_{j=1}^{N}\sum_{k=1}^{p_i}(a_{ijk},b_{ijk},c_{ijk})c_{ijk}v_{ijk}(x_j),\tag{4}$$
$$i=1,2,\cdots,L,$$

$$P_i=\begin{cases}\max(\min)f_1(x,\tilde{u}_1)=\sum_{j\in N_i}\sum_{k=1}^{p_{i1j}}\\ \qquad(a_{ijk},b_{ijk},c_{ijk})c_{ijk}v_{ijk}(x_j)\\ \max(\min)f_2(x,\tilde{u}_2)=\sum_{j\in N_i}\sum_{k=1}^{p_{i2j}}\\ \qquad(a_{ijk},b_{ijk},c_{ijk})c_{ijk}v_{ijk}(x_j)\\ \vdots\\ \max(\min)f_L(x,\tilde{u}_L)=\sum_{j\in N_i}\sum_{k=1}^{p_{iLj}}\\ \qquad(a_{ijk},b_{ijk},c_{ijk})c_{ijk}v_{ijk}(x_j)\\ \text{S.T.}\\ FS_i=\begin{cases}\sum_{j\in N_i}\tilde{g}_j(x_j)\leq\tilde{B}_j\\ \qquad m=s_{j-1}+1,\cdots,s_j\\ \tilde{H}_i(x)=\sum_{j=1}^{N}\tilde{h}_{ij}(x_j)\leq\tilde{B}\\ \qquad i=1,2,\cdots,w\end{cases}\end{cases}\tag{5}$$

s.t. $(x_1,x_2,\cdots,x_N)\in FS.$

The sub problems can be solved independently and their solution could be used to compute S_j,R_j,Q_j. So, using a simple approach which is adopted by some researchers [28-30], each fuzzy problem is converted to three crisp problems. In other words, we introduce three crisp sub problems (P_{ij}) instead of each fuzzy sub problem (P_i) as follows:

$$P_{i1}:\begin{cases}\min(\max)(b_1-a_1)(c_1v_1(x_j))\\ \min(\max)(b_2-a_2)(c_2v_2(x_j))\\ \vdots\\ \min(\max)(b_L-a_L)(c_Lv_L(x_j))\end{cases} \tag{6}$$

$$P_{i2}:\begin{cases}\max(\min)b_1(c_1v_1(x_j))\\ \max(\min)b_2(c_2v_2(x_j))\\ \vdots\\ \max(\min)b_L(c_Lv_L(x_j))\end{cases} \tag{7}$$

$$P_{i3}:\begin{cases}\max(\min)(c_1-b_1)(c_1v_1(x_j))\\ \max(\min)(c_2-b_2)(c_2v_2(x_j))\\ \vdots\\ \max(\min)(c_L-b_L)(c_Lv_L(x_j))\end{cases} \tag{8}$$

Moreover, transfer each fuzzy constraint into three crisp constraints as follow:

We can consider the fuzzy constraint as bellow:

$$\tilde{g}_m(x_j)\le\tilde{B}_m,\qquad m=s_{j-1}+1,\cdots,s_j,$$

$$j=1,2,\cdots,N,$$

$$\tilde{g}_m(x_j)=\tilde{c}_m g_i(x_j)=(c_{m1},c_{m2},c_{m3})g_m(x_j),$$

$$\tilde{c}_m=(c_{m1},c_{m2},c_{m3}),$$

$$\tilde{b}_m=(c_{m1},c_{m2},c_{m3}),$$

$$\begin{cases}\tilde{c}_{m1}g_m(x_j)\le\tilde{b}_{m1}\\ \tilde{c}_{m2}g_m(x_j)\le\tilde{b}_{m2}\\ \tilde{c}_{m3}g_m(x_j)\le\tilde{b}_{m3}\end{cases} \tag{9}$$

where $\tilde{c}_m$ is fuzzy coefficient of objective function and $\tilde{B}_m$ is fuzzy coefficient of constraints.

$$\tilde{H}_i(x)=\sum_{j=1}^{N}\tilde{h}_{ij}(x_j)\le\tilde{B}, \tag{10}$$

$$\tilde{h}_{ij}(x_j)=\tilde{z_{ij}}(x_j)h_{ij}(x_j), \tag{11}$$

$$\tilde{H}_i(x)=\sum_{j=1}^{N}\tilde{z}_{ij}(x_j)h_{ij}(x_j)\le\tilde{B}, \tag{12}$$

$$\tilde{z_{ij}}=(z_{ij1},z_{ij2},z_{ij3}), \tag{13}$$

$$\tilde{B}=(r_{ij},s_{ij},t_{ij}), \tag{14}$$

$$\sum_{j=1}^{N}z_{ij1}h_{ij}(x_j)\le r_i,\qquad\sum_{j=1}^{N}z_{ij2}h_{ij}(x_j)\le s_i,$$

$$\sum_{j=1}^{N}z_{ij3}h_{ij}(x_j)\le t_i,$$

$$i=1,2,...,w\qquad j=1,2,\cdots,N. \tag{15}$$

Therefore, the constraints of sub problem (P_i) are transferred to the following form:

$$\begin{cases}\begin{cases}r_{11}g_m(x_1)\le b_{11}\\ r_{12}g_m(x_1)\le b_{12}\quad m=1\cdots,s_1\\ r_{13}g_m(x_1)\le b_{13}\end{cases}\\ \begin{cases}r_{21}g_m(x_2)\le b_{21}\\ r_{22}g_m(x_2)\le b_{22}\quad m=s_1+1,\cdots,s_2\\ r_{23}g_m(x_2)\le b_{23}\end{cases}\\ \vdots\\ \begin{cases}r_{q1}g_m(x_q)\le b_{q1}\\ r_{q2}g_m(x_q)\le b_{q2}\quad m=s_r+1,\cdots,s_M\\ r_{q3}g_m(x_q)\le b_{q3}\end{cases}\\ \sum_{j=1}^{N}z_{i1}h_{ij}(x_j)\le r_i\\ \sum_{j=1}^{N}z_{i2}h_{ij}(x_j)\le s_i\quad i=1,2,\cdots,w\\ \sum_{j=1}^{N}z_{i3}h_{ij}(x_j)\le t_i\end{cases} \tag{16}$$

Step 2. Using VIKOR approach, first calculate the maximum f_{ij}^* value as Positive Ideal Solution (PIS) and the minimum $\tilde{f}_{ij}^-$ value as Negative Ideal Solution (NIS) of each objective function under the given constraints for variable x_j. The benefit and cost objectives are indexed as:

$$f_{bj}^*=\{\max(\min)f_{bj}(x_j)(f_{cj}(x_j)),\qquad\forall\ b(\forall\ c)\},$$

$$f_{bj}^-=\{\min(\max)f_{bj}(x_j)(f_{cj}(x_j)),\qquad\forall\ b(\forall\ c)\}, \tag{17}$$

$f_{bj}(x_j)$ Benefit objective for maximization,

$f_{cj}(x_j)$ Cost objective for maximization.

Then Compute the amount of S_j, R_j and Q_j as follows:

$$S_j=\sum_{b\in B}w_b\left(\frac{f_{bj}^*-f_{bj}(X_j)}{f_{bj}^*-f_{bj}^-}\right)+\sum_{c\in B}w_c\left(\frac{f_{cj}(X_j)-f_{cj}^*}{f_{cj}^--f_{cj}^*}\right), \tag{18}$$

$$R_j = \max\left\{ w_b \left(\frac{f_{bj}^* - f_{bj}(x_j)}{f_{bj}^* - f_{bj}^-} \right), \right.$$
$$\left. w_c \left(\frac{f_{bj}(x_j) - f_{bj}^*}{f_{bj}^- - f_{bj}^*} \right) \right\}, \quad (19)$$

$$\min \alpha$$

s.t.

$$w_b \left(\frac{f_{bj}^* - f_{bj}(x_j)}{f_{bj}^* - f_{bj}^-} \right) \leq \alpha,$$
$$w_c \left(\frac{f_{bj}(x_j) - f_{bj}^*}{f_{bj}^- - f_{bj}^*} \right) \leq \alpha, \quad (20)$$

$$s_j^* = \min(s_j), \qquad s_j^- = \max(s_j),$$
$$R_j^* = \min(R_j), \qquad R_j^- = \max(R_j), \quad (21)$$

$$Q_j = \nu \left(\frac{s_j - s_j^*}{s_j^- - s_j^*} \right) + (1-\nu) \left(\frac{R_j - R_j^*}{R_j^- - R_j^*} \right), \quad (22)$$

where, $W_b(W_c)$ represents weights of objective functions, S_j represents the distance of the jth objective function achievement to the positive ideal solution, and R_j implies maximal regret of each objective function, and ν is a weight of the strategy for S_j and R_j. In other word, the decision making strategy can be used with maximum group utility ($\nu > 0.5$), with consensus ($\nu = 0.5$), or with minimum individual regret ($\nu < 0.5$) (Vahdani et al., 2010; Opricovic, 1998).

Step 3. From the results of Step 2, determine the constraints corresponding to the each Q_{ij}. Afterward construct the final single-objective problem according to the values of Q_{ij}, for each problem as will be shown in Eq. (30). Then solve it to obtain the final optimal solution.

$$\min \alpha_1 + \alpha_2 + \cdots + \alpha_q,$$

s.t.

$$\begin{array}{l} Q_{11} \leq \alpha_1 \\ Q_{12} \leq \alpha_1 \\ Q_{13} \leq \alpha_1 \\ \vdots \\ Q_{q1} \leq \alpha_q \\ Q_{q2} \leq \alpha_q \\ Q_{q3} \leq \alpha_q \\ X \in FS \end{array} \quad (23)$$

Find the optimal solution vector X^*, where $X^* = (x_1^*, x_2^*, \cdots, x_n^*)$ is the best value of the original MODM problem. Finally, the flowchart of the proposed VIKOR method for solving MOLSNLP problem is depicted in Figure 1. The proposed method is illustrated through a numerical example.

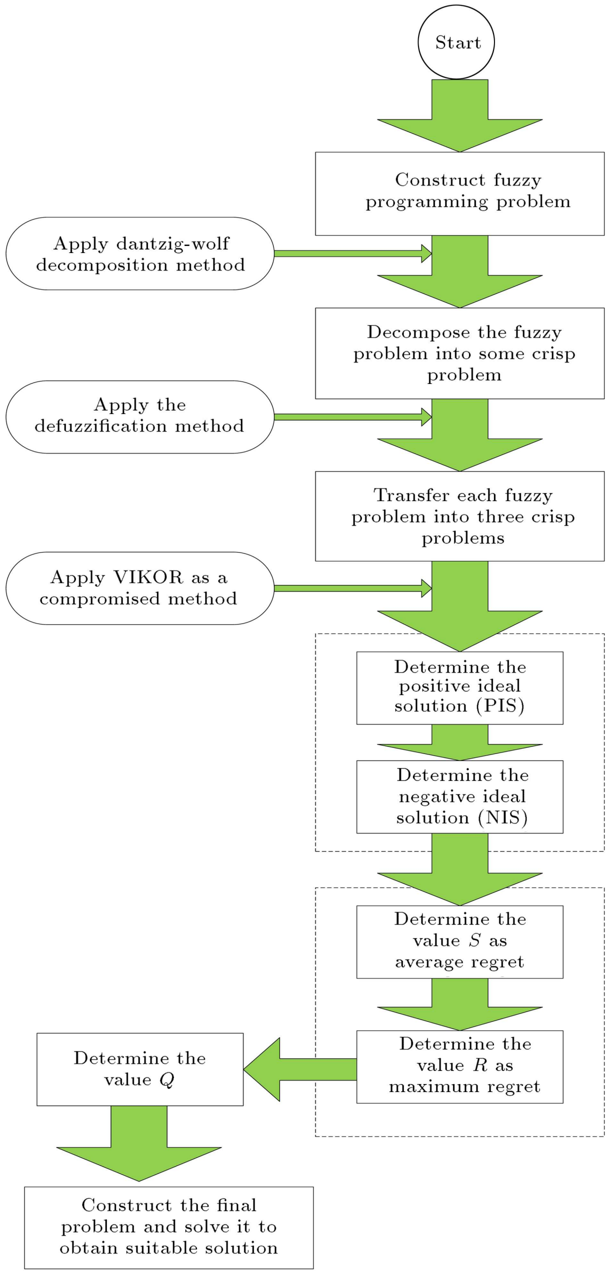

Figure 1. The flowchart of proposed VIKOR solution method.

4. Illustrative numerical example

In this section, we give an example to illustrate the stages of proposed model. There are three objectives functions on R^3, where the coefficient of the objective functions and constraints are proposed as triangular fuzzy numbers. It is assumed that the importance of weight is the same ($w = 1/3$) among the objective functions of all sub problems. The original problem is proposed as:

$P:$

$$\max f_1(x) = (1,2,3)(x_1-1)^2 + (2,3,4)x_2^2 + (1,3,5)(x_3+1)^2,$$

$$\max f_2(x) = (2,4,6)x_1 + (1,2,3)x_2 + (1,3,5)(x_3)^2,$$

$$\min f_3(x) = (1,2,3)(x_1)^2 + (1,3,5)x_2 + (1,2,3)(x_3)^2,$$

s.t.

$$FS = \left\{ \begin{array}{c} (1,2,3)x_1 - (1,2,3)x_2 + (2,4,6)x_3 \\ \leq (6,7,8) \\ (1,2,3)x_1^2 + (1,3,5)x_2 + (1,2,3)x_3 \\ \leq (10,11,12) \\ (0,0,0) \leq (1,2,3)x_1 \leq (3,4,5) \\ (0,0,0) \leq (1,2,3)x_2 \leq (4,5,6) \\ (0,0,0) \leq (1,2,3)x_3 \leq (2,3,4) \end{array} \right\}. \quad (24)$$

The problem can be split into three sub-problems. Therefore the new method is exploited to obtain optimal solution in the following steps:

Step 1. In the first stage, consider problem (P) and decompose it into three fuzzy sub problems (P_1, P_2, P_3). Because the coefficient of the objective functions and constraints are proposed as triangular fuzzy numbers, each objective function is transferred into three crisp functions for each fuzzy sub problem. Moreover, each fuzzy constraint is transferred to three crisp constraints. Based on the proposed method, this problem can be decomposed as the following program. First sub problem (P_1) is proposed based on variable x_1.

$P_1:$

$$\max f_1(x) = (1,2,3)(x_1-1)^2,$$

$$\max f_2(x) = (2,4,6)x_1,$$

$$\min f_3(x) = (1,2,3)(x_1)^2,$$

s.t.

$$FS_1 = \left\{ \begin{array}{c} (1,2,3)x_1 - (1,2,3)x_2 + (2,4,6)x_3 \\ \leq (6,7,8) \\ (1,2,3)x_1^2 + (1,3,5)x_2 + (1,2,3)x_3 \\ \leq (10,11,12) \\ (0,0,0) \leq (1,2,3)x_1 \leq (3,4,5) \end{array} \right\}. \quad (25)$$

Similar to sub problem P_1, sub problems P_2 and P_3 can be formulated as:

$P_2:$

$$\max f_1(x) = (2,3,4)x_2^2,$$

$$\max f_2(x) = (1,2,3)x_2,$$

$$\min f_3(x) = (1,3,5)x_2,$$

s.t.

$$FS = \left\{ \begin{array}{c} (1,2,3)x_1 - (1,2,3)x_2 + (2,4,6)x_3 \\ \leq (6,7,8), \\ (1,2,3)x_1^2 + (1,3,5)x_2 + (1,2,3)x_3 \\ \leq (10,11,12) \\ (0,0,0) \leq (1,2,3)x_2 \leq (4,5,6) \end{array} \right\}, \quad (26)$$

$P_3:$

$$\max f_1(x) = (1,3,5)(x_3+1)^2,$$

$$\max f_2(x) = (1,3,5)(x_3)^2,$$

$$\min f_3(x) = (1,2,3)(x_3)^2,$$

s.t.

$$FS = \left\{ \begin{array}{c} (1,2,3)x_1 - (1,2,3)x_2 + (2,4,6)x_3 \\ \leq (6,7,8) \\ (1,2,3)x_1^2 + (1,3,5)x_2 + (1,2,3)x_3 \\ \leq (10,11,12) \\ (0,0,0) \leq (1,2,3)x_3 \leq (2,3,4) \end{array} \right\}. \quad (27)$$

Now, using Relations (6) and (20), convert each sub problem of fuzzy MONLFP (31) into its non-fuzzy version sub problem. As will be shown in Eqs. (35), (36), and (37), the sub problems P_{11}, P_{12}, and P_{13} are constructed as:

$P_1:$

$$P_{11}: \begin{cases} \min f_1(x) = (x_1-1)^2 \\ \min f_2(x) = 2x_1 \\ \max f_3(x) = (x_1)^2 \\ \text{s.t.} \\ X \in FS_1 \end{cases} \quad (28)$$

$$P_{12}: \begin{cases} \max f_1(x) = 2(x_1-1)^2 \\ \max f_2(x) = 4x_1 \\ \min f_3(x) = 2(x_1)^2 \\ \text{s.t.} \\ X \in FS_1 \end{cases} \quad (29)$$

$$P_{13}:\begin{cases}\max f_1(x)=2(x_1-1)^2\\ \max f_2(x)=4x_1\\ \min f_3(x)=2(x_1)^2\\ \text{s.t.}\\ X\in FS_1\end{cases}\tag{30}$$

The above three crisp objectives programming are equivalent to the fuzzy problem P_1. Similar to P_1, the above procedure is utilized to obtain P_2 as:

P_2 :

$$P_{21}:\begin{cases}\min f_1(x)=x_2^2\\ \min f_2(x)=x_2\\ \max f_3(x)=2x_2\\ \text{s.t.}\\ X\in FS_2\end{cases}\tag{31}$$

$$P_{22}:\begin{cases}\max f_1(x)=3x_2^2\\ \max f_2(x)=2x_2\\ \min f_3(x)=3x_2\\ \text{s.t.}\\ X\in FS_2\end{cases}\tag{32}$$

$$P_{23}:\begin{cases}\max f_1(x)=x_2^2\\ \max f_2(x)=x_2\\ \min f_3(x)=2x_2\\ \text{s.t.}\\ X\in FS_2\end{cases}\tag{33}$$

It is clear that by Eqs. (8) and (20), the fuzzy sub problem P_3 can be transferred to three crisp problem as bellow:

P_3 :

$$P_{31}:\begin{cases}\min f_1(x)=2(x_3+1)^2\\ \min f_2(x)=2(x_3)^2\\ \max f_3(x)=(x_3)^2\\ \text{s.t.}\\ X\in FS_3\end{cases}\tag{34}$$

$$P_{32}:\begin{cases}\max f_1(x)=3(x_3+1)^2\\ \max f_2(x)=3(x_3)^2\\ \min f_3(x)=2(x_3)^2\\ \text{s.t.}\\ X\in FS_3\end{cases}\tag{35}$$

$$P_{33}:\begin{cases}\max f_1(x)=2(x_3+1)^2\\ \max f_2(x)=2(x_3)^2\\ \min f_3(x)=(x_3)^2\\ \text{s.t.}\\ X\in FS_3\end{cases}\tag{36}$$

Step 2. Calculate the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) of each objective function for all sub problems of P_1, P_2, and P_3 as shown in Tables 1 and 2. Next, compute the amount of S_{ij}, R_{ij}, and Q_{ij} for all sub problems under the given constraints for all variables as follows:

$$\text{PIS}: f_{11}^*=(f_1^*,f_2^*,f_3^*)=(0,0,2.7778),$$

$$f_{12}^*=(f_1^*,f_2^*,f_3^*)=(2,6.6667,0),$$

$$f_{13}^*=(f_1^*,f_2^*,f_3^*)=(1,3.3334,0),$$

Table 1. PIS payoff table of (P_1).

		f_1	f_2	f_3	x_1	x_2	x_3
	$\max f_1$	0^*	2	1	1	0	0
P_{11}	$\max f_1$	1	0^*	0	0	0	0
	$\max f_1$	0.4445	3.3333	2.7778^*	1.6667	0	0
	$\max f_1$	2^*	0	0	0	0	0
P_{12}	$\max f_1$	0.8890	6.6667^*	5.5558	1.6667	0	0
	$\max f_1$	2	0	0^*	0	0	0
	$\max f_1$	1^*	0	0	0	0	0
P_{13}	$\max f_1$	0.4445	3.3334^*	2.7779	1.6667	0	0
	$\max f_1$	1	0	0^*	0	0	0

Table 2. NIS payoff table of (P_1).

		f_1	f_2	f_3	x_1	x_2	x_3
	$\max f_1$	1^-	0	0	0	0	0
P_{11}	$\max f_1$	0.4445	3.3333^-	2.7779	1.6667	0	0
	$\max f_1$	1	0	0^-	0	0	0
	$\max f_1$	0^-	4	2	1	0	0
P_{12}	$\max f_1$	2	0^-	0	0	0	0
	$\max f_1$	0.8890	6.6667	5.5556^-	1.6667	0	0
	$\max f_1$	0^-	2	1	1	0	0
P_{13}	$\max f_1$	1	0^-	0	0	0	0
	$\max f_1$	0.4445	3.3333	2.7778^-	1.6667	0	0

Table 3. The values of S^*, S^-, R^* and R^- for (P_1).

	PIS	NIS	S^*	S^-	R^*	R^-
P_{11}	(0,0,2.7778)	(1,3.3333,0)	0.4114	0.6667	0	0.3333
P_{12}	(2,6.6667,0)	(0,0,5.5556)	-0.3333	-0.0781	0	0.3333
P_{13}	(1,3.3334,0)	(0,0,2.7778)	0.3333	0.4534	0	1

$$\text{NIS}: f_{11}^- = (f_1^-, f_2^-, f_3^-) = (1, 3.3333, 0),$$

$$f_{12}^- = (f_1^-, f_2^-, f_3^-) = (0, 0, 5.5556),$$

$$f_{13}^- = (f_1^-, f_2^-, f_3^-) = (0, 0, 2.7778).$$

The obtained PIS and NIS are shown in Table 3. Then S_{11} is obtained using Relation (23) as follows:

$$S_{11} = 0.2133(x_1)^2 - 0.4667X_1 + 0.6667. \qquad (37)$$

Moreover, R_{11} is obtained using Eqs. (24) and (26) as:

$$\min \lambda$$

s.t.

$$\frac{1}{3}\left(\frac{(x_1-1)^2-0}{1-0}\right) \leq \lambda,$$

$$\frac{1}{3}\left(\frac{2x_1-0}{3.3333-0}\right) \leq \lambda,$$

$$\frac{1}{3}\left(\frac{2.7778-(x_1)^2}{2.7778-0}\right) \leq \lambda,$$

$$X \in FS_1, \quad \lambda^* = 0.2060, \quad X^* = (1.030, 0, 0). \qquad (38)$$

The second and third constraints are active in point $x^* = (1.030, 0, 0)$. Moreover, the values of R^*, R^- for both constraints are the same. Therefore, each of the second and third constraints can be chosen anyas R_{11}. Here we choose the second constraint, so simplified R_{11} is as follows:

$$R_{11} = 0.2x_1. \qquad (39)$$

Suppose that the compromise is selected with "consensus" ($\nu = 0.5$). Then Q_{11} is obtained by computing Relation (29).The simplified result is as follows:

$$Q_{11} = 0.4177x_1^2 - 0.6140X_1 + 0.5. \qquad (40)$$

Similar to P_{11}, the values of S, R, and Q are obtained for problems P_{12} and P_{13}, as follow:

$$S_{12} = -0.2133x_1^2 + 0.4666X_1 - 0.3333, \qquad (41)$$

$$R_{12} = -0.3333x_1^2 + 0.6666X_1, \qquad (42)$$

$$Q_{12} = -0.9179x_1^2 + 1.9142X_1, \qquad (43)$$

$$S_{13} = -0.4533x_1^2 + 0.4666X_1 + 0.3333, \qquad (44)$$

$$R_{13} = -0.3333x_1^2 + 0.6666X_1, \qquad (45)$$

$$Q_{13} = -2.0539x_1^2 + 0.05667X_1. \qquad (46)$$

The amounts of S^*, S^-, R^*, and R^- are obtained for problems P_{11}, P_{12}, and P_{13}, as shown in Table 3, where:

$$S_{ij}^*(S_{ij}^-) = \max(\min S_{ij}),$$

$$R_{ij}^*(R_{ij}^-) = \max(\min R_{ij}),$$

$$\text{s.t.} \quad X \in FS_i.$$

Similar to P_1, the values of PIS and NIS of each objective function for all sub problems of P_2 are calculated in Tables 4 and 5.

Table 4. PIS payoff table of (P_2).

		f_1	f_2	f_3	x_1	x_2	x_3
P_{21}	$\max f_1$	0^*	0	0	0	0	0
	$\max f_1$	0	0^*	0	0	0	0
	$\max f_1$	4	2	4^*	0	2	0
P_{22}	$\max f_1$	12^*	4	6	0	2	0
	$\max f_1$	12	4^*	6	0	2	0
	$\max f_1$	0	0	0^*	0	0	0
P_{23}	$\max f_1$	4^*	2	4	0	2	0
	$\max f_1$	4	2^*	4	0	2	0
	$\max f_1$	0	0	0^*	0	0	0

Table 5. NIS payoff table of (P_2).

		f_1	f_2	f_3	x_1	x_2	x_3
P_{21}	$\max f_1$	4^*	2	4	0	2	0
	$\max f_1$	4	2^*	4	0	2	0
	$\max f_1$	0	0	0^*	0	0	0
P_{22}	$\max f_1$	0^*	0	0	0	0	0
	$\max f_1$	0	0^*	0	0	0	0
	$\max f_1$	12	4	6^*	0	2	0
P_{23}	$\max f_1$	0	0	0	0	0	0^*
	$\max f_1$	0^*	0	0	0	0	0
	$\max f_1$	2	4^*	0	2	0	4

Table 6. The values of S^*, S^-, R^* and R^- for (P_2).

	PIS	NIS	S^*	S^-	R^*	R^-
P_{21}	(0,0,4)	(4,2,0)	0.3333	0.6665	0	0.3333
P_{22}	(12,4,0)	(0,0,6)	0.3335	0.6667	0.2060	0.3333
P_{23}	(4,2,0)	(0,0,4)	0.3335	0.6667	0.2060	0.3333

The values of PIS and NIS are shown in Table 6. The values of S, R, and Q are obtained for problems P_2 as follow.

First, the values of S_{21} and R_{21} are obtained as:

$$S_{21} = -0.0833x_2^2 + 0.3333, \tag{47}$$

$$\min \lambda,$$

s.t.

$$\frac{1}{3}\left(\frac{(x_2)^2}{4-0}\right) \le \lambda,$$

$$\frac{1}{3}\left(\frac{x_2}{2-0}\right) \le \lambda,$$

$$\frac{1}{3}\left(\frac{4-2x_2}{4-0}\right) \le \lambda,$$

$$X \in FS_2,$$

$$\lambda^* = 0.1667, \quad X^* = (1,0,0), \quad R_{21} = 0.1667X_2. \tag{48}$$

The simplified result of Q_{21} is as follow:

$$Q_{21} = 0.375X_2^2 - 0.5002. \tag{49}$$

Similar to sub problem P_{21}, S^*, S^-, R^* and R^- are obtained for problems P_{22} and P_{23}. The values of S_{22} and R_{22} are obtained similar to privies steps as:

$$S_{22} = -0.0833X_2^2 + 0.6667, \tag{50}$$

$$R_{22} = 0.6667X_2. \tag{51}$$

The value of Q_{22} is calculated as:

$$Q_{22} = -0.125X_2^2 + 0.6546X_2 - 0.3091. \tag{52}$$

In last step for sub problem P_2, the values of S_{23} and R_{23} are calculated as:

$$S_{23} = -0.0833X_2^2 + 0.6667, \tag{53}$$

$$R_{23} = 0.6667X_2, \tag{54}$$

$$Q_{23} = -0.125X_2^2 + 0.6546X_2 - 0.3091. \tag{55}$$

S^*, S^-, R^*, and R^- are obtained for problems P_{21}, P_{22}, and P_{23} as shown in Table 6.

Table 7. PIS payoff table of (P_3).

		f_1	f_2	f_3	x_1	x_2	x_3
P_{31}	max f_1	2*	0	0	0	0	0
	max f_1	2	0*	0	0	0	0
	max f_1	10.8889	3.5556	1.7778*	0	0	1.3333
P_{32}	max f_1	16.3333*	5.3333	3.5556	0	0	1.3333
	max f_1	16.3333	5.3333*	3.5556	0	0	1.3333
	max f_1	3	0	0*	0	0	0
P_{33}	max f_1	10.8889*	3.5556	1.7778	0	0	1.3333
	max f_1	10.8889	3.5556*	1.7778	0	0	1.3333
	max f_1	2	0	0*	0	0	0

Table 8. NIS payoff table of (P_3).

		f_1	f_2	f_3	x_1	x_2	x_3
P_{31}	max f_1	10.8889*	3.5556	1.7778	0	0	1.3333
	max f_1	10.8889	3.5556*	1.7778	0	0	1.3333
	max f_1	2	0	0*	0	0	0
P_{32}	max f_1	3*	0	0	0	0	0
	max f_1	3	0*	0	0	0	0
	max f_1	16.3333	5.3333	3.5556*	0	0	1.3333
P_{33}	max f_1	0	0	0	0	0	2*
	max f_1	0*	0	0	0	0	2
	max f_1	3.5556	1.7778*	0	0	1.3333	10.8889

Consequently, the values of PIS and NIS of each objective function for all sub problems of P_3 are calculated in Tables 7 and 8.

The values of PIS and NIS are shown in Tables 7 and 8.

$$S_{31} = 0.075X_3^2 + 0.075X_3 + 0.3333, \tag{56}$$

$$\min \lambda,$$

s.t.

$$\frac{1}{3}\left(\frac{2(x_3+1)^2-2}{10.8889-2}\right) \le \lambda,$$

$$\frac{1}{3}\left(\frac{2x_3^2-0}{3.5556-0}\right) \le \lambda,$$

$$\frac{1}{3}\left(\frac{1.7778-(x_3)^2}{1.7778-0}\right) \le \lambda,$$

$$X \in FS_2,$$

$$\lambda^* = 0.2081, \qquad X^* = (0,0,0.9428), \tag{57}$$

Table 9. The values of S^*, S^-, R^* and R^- for (P_3).

	PIS	NIS	S^*	S^-	R^*	R^-
P_{31}	(2,0,1.7778)	(10.8889,3.5556,0)	0.3333	0.6667	0	0.3333
P_{32}	(16.3333,5.3333,0)	(3,0,3.5556)	0.3333	0.6666	0	0.3333
P_{33}	(10.8889,3.5556,0)	(2,0,1.7778)	0.3333	0.6666	0	0

$$R_{31} = 0.0375X_3^2 + 0.075X_3. \tag{58}$$

Similar to sub problem P_{21}, S^*, S^-, R^*, and R^- are obtained for problems P_{22} and P_{23}, as shown in Table 9.

$$Q_{31} = 0.15X_3^2 + 0.3X_3. \tag{59}$$

Also similar to sub problem P_{31}, the values for S_{32}, R_{32}, and Q_{32} are obtained as:

$$S_{32} = -0.075X_3^2 - 0.15X_3 + 0.6667, \tag{60}$$

$$R_{32} = 0.1875X_3^2, \tag{61}$$

$$Q_{32} = 0.0.3938X_3^2 - 0.2250X_3 + 0.5. \tag{62}$$

In last step for sub problem P_3, the values of S_{33}, R_{33}, and Q_{33} are calculated as:

$$S_{33} = -0.075X_3^2 - 0.15X_3 + 0.6667, \tag{63}$$

$$R_{33} = 0.1875x_3^2, \tag{64}$$

$$Q_{33} = 0.1688X_3^2 - 0.2250X_3 + 0.5. \tag{65}$$

S^*, S^-, R^*, and R^- are obtained for problems P_{31}, P_{32}, and P_{33} as shown in Table 9.

Step 3. From the results of Step 2 determine the constraints corresponding to the each Q_{ij}. Afterward construct the final single-objective problem according to the values of Q_{ij} for each problems shown in Eq. (74). Then solve it to obtain the final optimal solution. The crisp single-objective problem for the numerical example is as follows:

$$\min \alpha 1 + \alpha 2 + \alpha 3,$$

$$0.4177x_1^2 - 0.6140x_1 + 0.5 \leq \alpha_1,$$

$$-0.9179x_1^2 + 1.9142x_1 \leq \alpha_1,$$

$$-2.0539x_1^2 + 0.5667x_1 \leq \alpha_1,$$

$$0.375x_2^2 - 0.5002x_2 \leq \alpha_2,$$

$$-0.125x_2^2 + 0.6546x_2 - 0.3091 \leq \alpha_2,$$

$$-0.0833x_2^2 + 0.6667 \leq \alpha_2,$$

$$0.15x_3^2 + 0.3x_3 + 0.5 \leq \alpha_3,$$

$$0.3938x_3^2 - 0.2250x_3 \leq \alpha_3,$$

$$0.1688x_3^2 - 0.2250x_3 \leq \alpha_3,$$

$$x_1^2 - x_2 + 2x_3 \leq 6, \qquad 2x_1 - 2x_2 + 4x_3 \leq 7,$$

$$3x_1 - 3x_2 + 6x_3 \leq 8, \qquad x_1^2 + x_2 + x_3 \leq 10,$$

$$2x_1^2 + 3x_2 + 2x_3 \leq 11, \qquad 3x_1^2 + 5x_2 + 3x_3 \leq 12,$$

$$0 \leq x_1 \leq 3, \qquad 0 \leq 2x_1 \leq 4, \qquad 0 \leq 3x_1 \leq 5,$$

$$0 \leq x_2 \leq 4, \qquad 0 \leq 2x_2 \leq 5, \qquad 0 \leq 3x_2 \leq 6,$$

$$0 \leq x_3 \leq 2, \qquad 0 \leq 2x_3 \leq 3, \qquad 0 \leq 3x_3 \leq 4. \tag{66}$$

Find the optimal solution vector X^*, where $X^* = (x_1^*, x_2^*, \cdots, x_n^*)$ is the best value of the original MODM problem. By solving Problem (74), we obtain the optimum minimum value of α_1, α_2, and α_3, as follows:

$$z^* = 0.4679, \qquad X^* = (0.2244, 1.5957, 0.2857),$$

$$\alpha_1 = 0.3833, \qquad \alpha_2 = 0.4546, \qquad \alpha_3 = 0.2857.$$

4.1. Sensitivity analysis

In this example, as it was observed, there are three objectives on R^3. Moreover, the optimal solution vector $X^* = (x_1^*, x_2^*, \cdots, x_n^*)$ where x_1^*, x_2^*, and x_3^* are obtained from sub problems P_1, P_2, and P_3, respectively. Considering Problem (74), the inequality constraint is proposed in three categories. First group of them are constructed based on the Q_{11}, Q_{12}, and Q_{13} where Q_{ij} is applied as functions of the left-hand side of the inequality constraints. The amount of α_1^* is determined according to objective function and constraints. When x_1 increases from 0.2244, the values of functions Q_{12} and Q_{13} will be decreased but the first inequality is impossible because the amount of Q_{11} is more than right-hand side of constraint. Therefore, simultaneously according to the objective function and constraint, $x_1 = 0.2244$, is optimal solution for x_1. Figure 2 represents the behavior of Q_{11}, Q_{12}, and Q_{13} based on x_1.

Similar to P_1, the problems P_2 and P_3 are solved. When x_2 increases from 1.5957 the values of functions

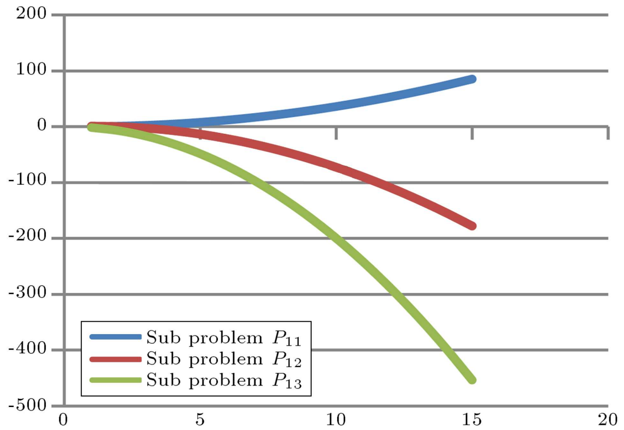

Figure 2. The values of function Q_{ij} for problem P_1.

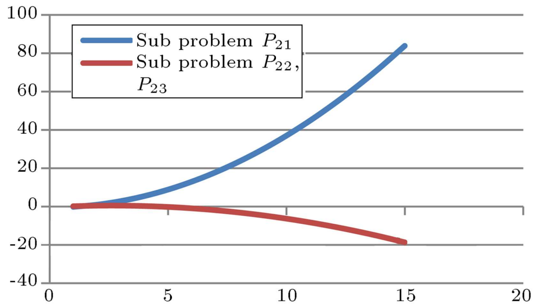

Figure 3. The values of function Q_{ij} for problem P_2.

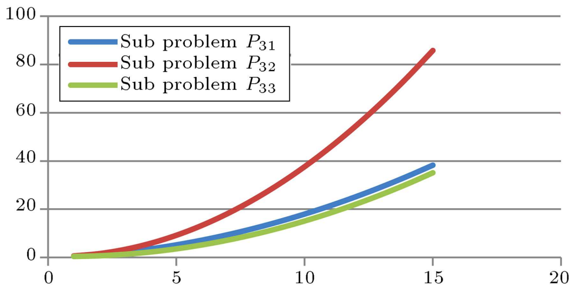

Figure 4. The values of function Q_{ij} for problem P_3.

Q_{22} and Q_{23} will be decreased but the amount of Q_{21} is more than right-hand side of first constraint. Moreover, When x_2 decreases from 1.5957 the amount of Q_{21} will be decreased but the values of functions Q_{22} and Q_{23} is more than right-hand side of first and second constraints, respectively, as shown in Figure 3. Therefore, $x_2 = 1.5957$ is the best solution of problem P_2.

Also similar to the problems P_1 and P_2, the optimal solution of P_3 is $x_3 = 0.2857$, as shown in Figure 4.

5. Conclusion

In this paper, the focus was on extending and applying a VIKOR approach as a compromise decision making method to deal with MOLSNLP problems with block angular structure under uncertainty. The proposed method was introduced for solving large scale nonlinear programming in fuzzy environment for first time. The new method employed the advantages of VIKOR as a compromised method for solving nonlinear problems. First, Dantzig-Wolfe decomposing algorithm was applied to decompose the n-dimensional space fuzzy MOLSNLP into n sub problems. In the proposed approach, the sub problems in fuzzy environment were solved by converting them into crisp environment. In other words, each fuzzy problem can lead to three crisp problems. Then the proposed VIKOR method was applied to obtain an equation for each sub problem in a crisp single-objective problem. Therefore, it can be argued that this method combines LSMONLP and VIKOR approach to obtain a compromise solution of the problem. In sum, it transfers n objectives, which are conflicting, into single-objectives involving the maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent", based on the shortest distance from the PIS and the longest distance from the NIS, which are commensurable and most of time conflicting. In other words, the VIKOR has been applied in MADM for ranking the alternatives versus some criteria whereas this paper applied VIKOR in MODM problems. The logic of VIKOR method was utilized to aggregate the multi-objective programming problems into single-objective. The MODM problems were considered with fuzzy parameters in objective function and constraints. Moreover, the constraints could be considered as non-linear equation. Finally, to justify the proposed method, an illustrative example was provided. The numerical example has three sub problems. The new method is utilized to solve each problem. The optimum solution and satisfaction value of each sub problem was proposed in sensitivity analysis. The optimum value of objective function is $Z^* = 0.4679$. Moreover the amounts of variables are $x^* = (0.2244, 1.5957, 0.2857)$ and the satisfaction values of each sub problem are $\alpha_1^* = 0.3833$, $\alpha_2^* = 0.4546$, and $\alpha_3^* = 0.2857$. For the future research, an MCDM method can be presented with interval data for solving the multi-objective nonlinear programming problems in large scale context.

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