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Research Note

A mathematical model to evaluate knowledge in the knowledge-based organizations

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Abstract. Knowledge and its intangible appurtenances have not only resulted in movement in various businesses, but also, nowadays, have been viewed as the whole or a part of products of distributors' companies as well as service and military organizations. In recent years, estimation of knowledge level in organizations and industry companies has attracted considerable attentions. Contrary to a lot of prevalent models used for measuring efficiency, Data Envelopment Analysis (DEA) can take into account multiple inputs and outputs. In this regard, DEA was traditionally applied with crisp inputs and outputs, while in practical cases, we need to estimate organization efficiency in a different situation in which we have to deal with fuzzy or imprecise data. The aim of this paper is to present a DEA, employing fuzzy input and output data, toward assessing the knowledge levels established in a knowledge based-organization in various time intervals. In this case, the organization is able to define some areas in which it can improve its established knowledge level.

1. Introduction

Nowadays, companies feel more pressure resulted from competitive market than before. In the era in which economy is going to be smaller, they should employ fewer resources to do more work. In order to establish new knowledge, many organizations have recently been inclined to hire Knowledge Management (KM) systems. Some specialists believe that the recent obtained knowledge may only be the result of this competition. By using the obtained knowledge, a better knowledge management system is made, a more efficient team working is constituted, and finally learning is necessary. The concept of knowledge does not necessarily mean information transferring, but creativity and dynamism. In this case, a knowledge-based organization is idea maker, applies new ideas, and hence achieves competitive merit.

Shang & Sueyoshi [1] introduced DEA as a linear programming based technique to measure relative efficiency of an organization or different organizations, such as banking, post, hospitals, education centers, powerhouses, refineries, and so on. In addition to efficiency score, DEA can give very useful results to managers and determine pioneers for inefficient DMUs. This technique also provides strategic solution and efficiency improvement for developing the organization, and finally assists organizations to assign resources properly [2].

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This paper attempts to propose DEA-based approach with fuzzy data to estimate relative efficiency of knowledge-based organizations using knowledge management criteria. In this case, the organizations are able to detect their improvement areas.

The rest of this paper is organized as follows: Section 2 gives a brief literature review. The proposed mathematical model is introduced in Section 3. Inputs and outputs of the proposed DEA model are defined in Section 4. The solution procedure and computational results are given in Section 5, and finally Section 6 is devoted to some conclusion remarks.

2. Literature review

American Productivity and Quality Center (APQC) defines knowledge as strategies and processes for detecting, acquiring, and distributing knowledge to strengthen the competition power of the organization. Based on the idea of Gartner Group, many organizations are going to implement knowledge management systems and one third of the fortune 1000 companies apply knowledge management plans and strategies. Davenport & Prusak [3] introduces knowledge management as a structured approach containing procedures for detecting, evaluating, organizing, saving, and implementing knowledge toward covering requirements and objectives of organizations. Jakov Crnkovic et al. [4] proposed a comprehensive approach to KM which relates key processes of KM to critical factors for implementing it successfully. Using a matrix composing of the process and key success factors, they described an organization and estimated the KM criteria.

Efficiency evaluation has been comprehensively addressed by scholars because of its importance for performance evaluation of an organization.

Farrell [5] exploited a method which had been employed for efficiency evaluation, in engineering issues, to measure efficiency of a production trim. The study by Farrell involved just one input and one output. Farrell's study included measuring technical efficiency and derivative of efficient production function. Data Envelopment Analysis (DEA) is a mathematical programming based approach to estimate technical efficiency and inefficiencies in production. This method does not suppose any pre-determined shape for production function curve and estimates production or cost function applying mathematical models for actual input and output data of Decision Making Units (DMUs).

Charnes et al. [6] proposed the first mathematical DEA based on Farrell's definition of relative efficiency. The aim of this model was to measure and compare relative efficiency of Decision Making Units (DMUs) using multiple homogeneous inputs and outputs.

Banker et al. [7] proposed a new DEA model called BCC, which considered efficiency of DMUs with variable return to scale.

In fuzzy environment, Sengupta [8] proposed a fuzzy programming approach in which the objective function was crisply, not satisfied. He examined DEA model with multiple inputs and one output.

Kao and Liu [9] developed a method to find the membership functions of the fuzzy efficiency scores when some observations are in fuzzy numbers. Entani et al. [10] proposed interval efficiency obtained from the pessimistic and the optimistic viewpoints. Since Zadeh [11,12] initiated the possibility measure, many researchers such as Guo & Tanaka [13] and S. Lertworasirikul et al. [14] have introduced it into DEA.

Wen & Li [15] proposed a fuzzy DEA model for ranking and evaluating. In this regard, they introduced a hybrid model with fuzzy simulation and genetic algorithm, in which fuzzy triangular and trapezoidal fuzzy number were exploited. They, then, transformed the hybrid model to a linear mathematical model and applied it to solve a simple numerical example.

Yuan Feng Wen [16] introduced criteria to measure knowledge management by reviewing literature, using scholars and practitioners in the areas of KM and AHP and a questionnaire. Leibowitz & Suen [17] and Bose [18] proposed more criteria to measure knowledge management. There exist many criteria to measure KM; the interesting point is that the majority of the proposed criteria are human based and can be utilized to quantify knowledge management plans and implementation. The criteria proposed by ICM group include value adding customer resource, infrastructure resource and human resource the proposed mathematical model.

In this section, a fuzzy DEA mathematical model is formulized to assess knowledge level of a knowledge-based organization in various time intervals, which can monitor the organization progress in terms of knowledge management. In this regard, the BCC model is developed by fuzzy input and output data using Zadeh's extensions principle. The following programming problem gives the upper band of α-level cut of fuzzy efficiency measure of a DMU understudy, in which the primal BCC model is applied:

$$\text{Max} Z_0 = \sum_{r=1}^{s} \mu_r y_{r0} + w',$$

$$\sum_{r=1}^{s} \mu_r y_{rj} + w' - \sum_{i=1}^{m} w_i x_{ij} \leq 0,$$

$$\sum_{i=1}^{m} w_i x_{i0} = 1,$$

$$(\tilde{y}_{rj})_\alpha^L \le y_{rj} \le (\tilde{y}_{rj})_\alpha^U,$$

$$(\tilde{x}_{ij})_\alpha^L \le x_{ij} \le (\tilde{x}_{ij})_\alpha^U,$$

$$\mu_r, w_i \ge 0, w' \text{ free.}$$

In the above-mentioned programming problem, x_{ij} denotes the ith input of the jth DMU. The weight assigned to the ith is represented by w_i. On the other hand, y_{rj} is the rth output from the jth DMU. The weight of rth output is described by μ_r. The variable return to scale in the aforementioned programming problem is represented by w', where if $w' < 0$, then it is said that return to scale is decreasing, and if $w' > 0$, then it is increasing, otherwise, it is constant. Since, in the present study, it is supposed that the input and output data are in the form of fuzzy numbers, α-level cut of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are represented as $(\tilde{x}_{ij})_\alpha = \left[(\tilde{x}_{ij})_\alpha^L, (\tilde{x}_{ij})_\alpha^U\right]$ and $(\tilde{y}_{rj})_\alpha = \left[(\tilde{y}_{rj})_\alpha^L, (\tilde{y}_{rj})_\alpha^U\right]$, respectively.

In order to obtain the lower bound of α-level cut of fuzzy efficiency measure, the dual form of BSS model is applied. In this case, using Zadeh's extension principle, we have:

$$\text{Min} y_0 = \theta,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{r0},$$

$$\theta x_{i0} - \sum_{j=1}^{n} \lambda_j x_{ij} \ge 0,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$(\tilde{y}_{rj})_\alpha^L \le y_{rj} \le (\tilde{y}_{rj})_\alpha^U,$$

$$(\tilde{x}_{ij})_\alpha^L \le x_{ij} \le (\tilde{x}_{ij})_\alpha^U,$$

$$\lambda_j \ge 0, \theta \text{ free.}$$

In the aforementioned programming problem, θ and λ_j, $j = 1, \cdots, n$, are the decision variables corresponding $=$ and $\le$ constraints, respectively, in primal BCC model. Other parameters and variables are the same as those in primal BCC model.

3. Introducing input and output in the proposal model

It should be pointed out that DEA models are sufficiently valid if their inputs and outputs are identified properly. This has a significant role for model's validation. In the present study, to assess knowledge level by the mathematical models, several criteria are defined using experts' opinion and reviewing the literature, which directly relate to knowledge management and influence knowledge level of the organization. In this regard, inputs and outputs to be used in the proposed fuzzy DEA model are defined as represented in Table 1.

4. Solution procedure

In this section, input and output data collected from reviewing literature are introduced to be used in the proposed fuzzy BCC model. Table 2 represents these data for four various time intervals that includes the input data of the processes (min) and the amount of investment (million), respectively. The fourth, fifth, and sixth columns are the output data which

Table 1. Inputs and outputs of the proposed model.

Inputs	Outputs
Amount of investment	Knowledge sharing among personnel
Average time of each process	The number of software and new plans

Table 2. The inputs and outputs data of the problem.

Time interval	The average time of processes	The amount of investment	The number of processes without error	Knowledge sharing	The number of software
1	(20,24,27)	(120,148,155)	(14,17,22)	(46,53,55)	(7,9,11)
2	(18,21,24)	(132,140,146)	(14,18,20)	(44,48,51)	(5,7,11)
3	(20,26,28)	(134,144,150)	(10,14,17)	(47,56,59)	(6,10,13)
4	(15,20,26)	(145,151,153)	(16,20,23)	(47,50,56)	(6,13,19)

indicate the number of processes without error, knowledge sharing, and the number of software, respectively.

5. Computational result

In this section, the proposed fuzzy BCC model is solved for a knowledge based organization with the data given in Table 2 and Lingo 12 solver. The outcome of the proposed model, in fact α-level cut of fuzzy efficiency measure of the organization under study, is applied to construct membership functions, depicted in Figures 1 to 4.

After deriving the membership functions fuzzy efficiency measures as described before, it is the turn to describe a procedure to be used for making a decision about whether or not the capability of the variable measurement system under study with fuzzy data is acceptable. To do this, the abovementioned criteria should be compared with their corresponding critical values in fuzzy environment. In this regard, it necessitates a method to be applied for ranking fuzzy numbers. There are a lot of methods in the literature in this area [19-23]. However, most of them need membership functions of the fuzzy numbers to be ranked. The method proposed by Chen and Klein [20], on the other hand, is very appropriate for the present study, because it is based on α-level cuts.

To describe the method proposed by Chen and Klein [20], suppose the fuzzy numbers $\tilde{A}_j$; $j = 1, \cdots, m$, are going to be ranked. Let h be the maximum height of $\mu_{\tilde{A}_j}$; $j = 1, \cdots, m$. Assume h is split into n equal intervals such that $\alpha_i = ih/n$; $i = 0, \cdots, n$. Chen and Klein [20] proposed the following index to be used for ranking fuzzy numbers:

$$I_j = \sum_{i=1}^{n} \left[\left(\tilde{A}_j\right)_{\alpha_i}^{U} - c \right] \Bigg/ \left[\sum_{i=1}^{n} \left[\left(\tilde{A}_j\right)_{\alpha_i}^{U} - c \right] - \sum_{i=1}^{n} \left[\left(\tilde{A}_j\right)_{\alpha_i}^{L} - d \right] \right]$$

$$n \to \infty,$$

where:

$$c = \min_{i,j} \left\{ \left(\tilde{A}_j\right)_{\alpha_i}^{L} \right\},$$

$$d = \max_{i,j} \left\{ \left(\tilde{A}_j\right)_{\alpha_i}^{U} \right\}.$$

The larger (smaller) value of index I_j, the larger (smaller) the fuzzy number $\tilde{A}_j$ is. While this method is

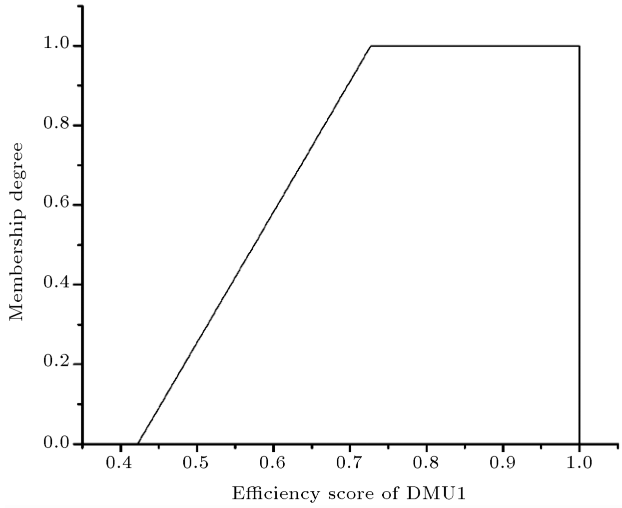

Figure 1. Membership function of fuzzy efficiency score in the first time interval.

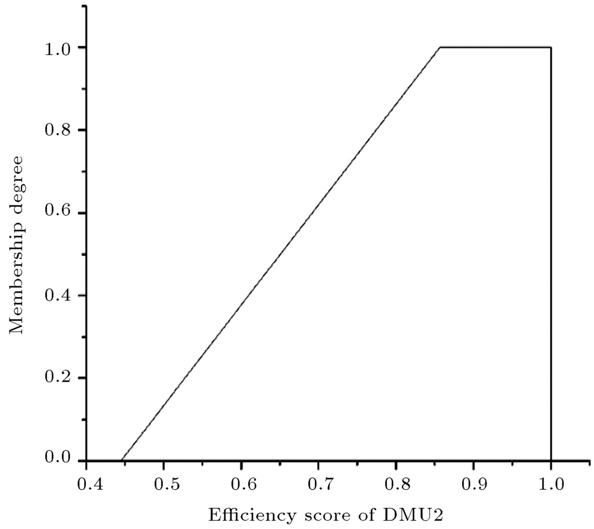

Figure 2. Membership function of fuzzy efficiency score in the second time interval.

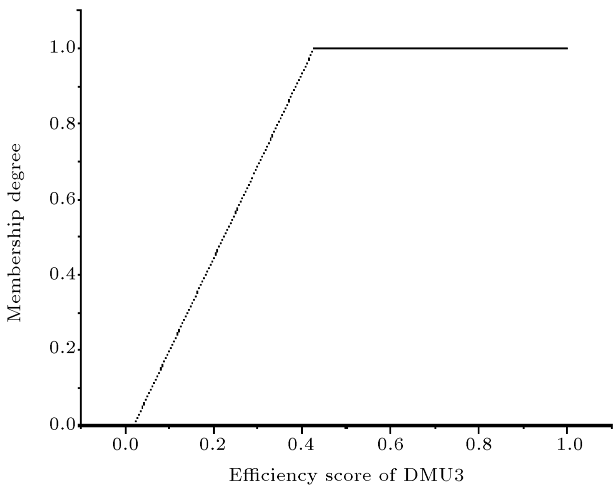

Figure 3. Membership function of fuzzy efficiency score in the third time interval.

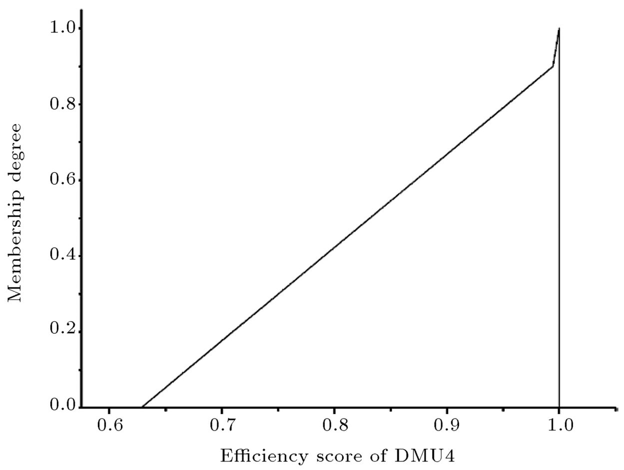

Figure 4. Membership function of fuzzy efficiency score in the fourth time interval.

authentic when n advances infinity, Chen and Klein [20] proposed that $n = 3$ or 4 suffices to discriminate the differences.

Using the abovementioned procedure for ranking fuzzy efficiency measure of the organization in four successive time intervals with the membership functions delineated in Figures 1-4, we have $c = 0.0189$ and $d = 1$. In this case, Chen and Klein's index for obtained fuzzy efficiency measures is corrupted as follow:

$$I_{\text{DMU1}} = 0.6976, \qquad I_{\text{DMU2}} = 0.7374,$$

$$I_{\text{DMU3}} = 0.5579, \qquad I_{\text{DMU4}} = 0.8535.$$

As it is observed, the organization is the most and least efficient in the fourth and third time intervals, respectively.

6. Conclusion

This paper was an effort to propose a fuzzy BCC model to assess knowledge level of a knowledge-based organization, wherein it was supposed that input and output data are fuzzy numbers. The proposed approach can also be used to rank similar organizations in term of knowledge management criteria. The approach is specifically applicable for organizations which want to compare themselves with similar organizations in terms of knowledge management and select the most efficient benchmark.

In this case, the inefficient organizations try to improve their performance in the area of knowledge management toward being assessed as efficient.

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