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# A multi-objective robust optimization model for location-allocation decisions in two-stage supply chain network and solving it with non-dominated sorting ant colony optimization

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Uncertainty;
Non-dominated sorting ant colony optimization;
NSGA-II.

**Abstract.** This study proposes a new, robust multi-objective model for capacitated multi-vehicle allocation of customers to potential Distribution Centers (DCs) under uncertain environment. Uncertainty is defined by discrete scenarios on demands where occurrence probability of each scenario is known. The optimization objectives are to minimize transit time and total cost, including opening cost, assumed for opening potential DCs and shipping cost from DCs to the customers, where considering different types of vehicles leads to a more realistic model and causes more conflict in these two objectives. A swarm intelligence-based algorithm named Non-dominated Sorting Ant Colony Optimization (NSACO) is used as the optimization tool. The proposed methodology is based on a new variant of Ant Colony Optimization (ACO) customized in multi-objective optimization problem of this research. For ensuring the authenticity of the proposed method, the computational results are compared with those obtained by NSGA-II. Results show the advantages and the effectiveness of the used method in reporting the optimal Pareto front of the proposed model. Moreover, the optimal solutions of the robust optimization model are insensitive to the disturbance of parameters under different scenarios, thus the risk of decision can be effectively reduced.



## 1. Introduction

Supply Chain Management (SCM) is a set of approaches utilized to efficiently integrate the suppliers, manufacturers, warehouses, and stores, so that the merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements [1]. The above definition reveals that there are many independent entities in a supply chain, each of which tries to maximize its own inherent objective functions in business transactions. This is a complicated problem as too many factors are involved and need more than one objective to be satisfied, simultaneously. In traditional SCM, the focus of the designs of supply chain network is usually on single objective, minimum cost, or maximum profit. However, the design, planning, and scheduling projects usually involve trade-offs among different incompatible goals such as fair profit distribution among all members, customer service levels, fill-rates, safe inventory levels, volume flexibility, etc. Hence, real supply chains are to be optimized simultaneously considering more than one objective. Many of the problems that occur in

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supply chain optimization are combinatorial in nature, and picking a set of optimal solutions in the case of multi-objective formulations requires an algorithm that can efficiently search the entire objective space, using small amounts of computation time. Literature shows that evolutionary and swarm intelligence-based algorithms perform well in this respect and give good optimal results when applied to many combinatorial problems.

Efficient allocation of customers to Distribution Centers (DCs) always plays an important role in developing a flawless and reliable supply chain. In this paper, two-stage supply chain network, including the distribution centers and the customers are considered. There are customers with uncertain demands and potential places which are candidates to serve as distribution centers called potential DCs.

Each of the potential DCs can be shipped to any of the customers. This study proposes the utility of a new swarm intelligence-based algorithm called Non-dominated Sorting Ant Colony Optimization algorithm (NSACO) and the Non-dominated Sorting Genetic Algorithm (NSGA-II) for simultaneous robust optimization of two objectives, including minimizing the total transit time and total cost.

## 2. Prior related works

Since this research concentrates on location-allocation decisions, robust multi-objective optimization using ant colony optimization algorithm and NSGA-II, this section deals with prior works related to all these areas.

Many researchers worked on basic facility location problem formulations recognized as static and deterministic which take constant, known quantities as inputs and derive a single solution to be implemented at one point in time. These fundamental location problems are categorized into median problems [2], covering problems [3], center problems [3], etc. Later, focus was shifted to location-allocation problems which simultaneously locate facilities and dictate flows between facilities and demands. Warszawski and Peer (1973) [4] are among the first who studied the multi-commodity location problem. Their models consider fixed location costs and linear transportation costs and assume that each warehouse can be assigned at most one commodity.

In literature, another set of problems is called fixed charge facility location problems which consider fixed charge associated with locating at each potential facility site. There are two types of problems, including capacitated and uncapacitated plant location problems. Uncapacitated and capacitated plant location models are extensively dealt with in [5] and capacitated plant location models in [6]. Hajiaghaei-Keshteli (2011) [7] considered two stages of supply chain network including Distribution Centers (DCs) and customers. His proposed model selects some potential places as distribution centers in order to supply demands of all customers; and in order to solve the given problem, two algorithms, genetic algorithm and artificial immune algorithm, were developed.

Different methodologies are found in the literature for treating multi-objective optimization problems. These are the weighted-sum method, the $\varepsilon$-constraint method, the goal-programming method, fuzzy method, etc. [8]. Zhou et al. (2003) [9] proposed a mathematical model and an efficient solution procedure for the bi-criteria allocation problem involving multiple warehouses with different capacities. The Bi-criteria Multiple Warehouse Allocation Problem (BMWAP) is similar to the well-known generalized assignment problem, but it is more challenging to solve due to its multiple criteria structure.

Ordonez and Zhao (2007) [10] investigated the robust capacity expansion problem of network flows under demand and travel time uncertainty. They provided complexity results for the two-stage network flow and design problem. Further, the problem of locating a competitive facility in the plane in the presence of uncertain demand was studied in [11] with a deviation robustness criterion. Baron et al. (2011) [12] applied robust optimization to the problem of locating facilities in a network facing uncertain demand over multiple periods. They considered a multi-period fixed-charge network location problem for which they show that different models of uncertainty lead to very different solution network topologies, with the model with box uncertainty set opening fewer, larger facilities. Gabrel et al. (2011) [13] investigated a robust version of the location transportation problem with an uncertain demand using a two-stage formulation. The resulting robust formulation is a convex (nonlinear) program, and the authors apply a cutting plane algorithm to solve the problem exactly. Gulpinar et al. (2013) [14] considered a stochastic facility location problem in which multiple capacitated facilities serve customers with a single product, with uncertain customer demand and a constraint on the stock-out probability. Ghahtarani and Najafi (2013) [15] proposed a robust optimization model for the multi-objective portfolio selection problem that uses a Goal Programming (GP) approach.

Non-dominated Sorting Genetic Algorithm II (NSGA-II), multi-objective ACO (MOACO), and Multi-Objective PSO (MOPSO) are few examples of multi-objective metaheuristic optimization algorithms of this type [16]. Chan and Kumar (2009) [17] discussed a Multiple Ant Colony Optimization (MACO) approach in an effort to design a balanced and efficient supply chain network that maintains the best balance of transit time and customer service. The focus of their paper is on the effective allocation of the customers to the DCs with the two-fold objective of minimization of the transit time and degree of imbalance of the DCs. Kalhor et al. (2011) [18] proposed a non-dominated archiving ant colony approach to solve the stochastic time-cost trade-off optimization problem. Mostafavi and Afshar (2011) [19] used a powerful ant colony algorithm known as non-dominated archiving multi-colony ant algorithm (NA-ACO) to solve the optimal Waste Load Allocation as a multi-objective optimization problem. Srinivas and Deb (1994) [20] used the non-dominated sorting concept on the GA. Then, NSGA-II, which was proposed by Deb et al. (2000) [21], is one of the most efficient and famous multi-objective evolutionary algorithms. Bhattacharya and Bandyopadhyay (2010) [22] solved the conflicting bi-objective facility location problem with certain demand by NSGA II evolutionary algorithm. Shankar et al. (2013) [23] proposed a bi-objective optimization of supply chain design and distribution operations using Multi-Objective Hybrid Particle Swarm Optimization algorithm (MOHPSO). This heuristic incorporates non-dominated sorting procedure to achieve bi-objective optimization of two conflicting objectives. Sadeghi et al. (2014) [24] proposed a hybrid vendor managed inventory and redundancy allocation optimization problem in supply chain management, and they used NSGA-II for solving their problem.

The above literature review indicates that very little research has been carried out to implement swarm intelligence-based algorithms in robust multi-objective optimization for supply chain network. The purpose of this paper is to formulate and analyze a location-allocation model for a multi-vehicle single product in two-stage Supply Chain (SC) network with respect to the conflicting objectives including minimizing total transit time and total cost, using NSACO and NSGA-II algorithms. The total cost involves opening cost assumed for opening potential DCs and shipping cost from DCs to the customers. The proposed model should lead to a final two-stage SC design which would represent the desired compromise among the different objectives from the decision-maker's perspective.

## 3. Background

### 3.1. Multi-objective optimization

Multi-objective optimizations concerned with mathematical optimization problems involve more than one objective function to be optimized simultaneously. To obtain the optimal solution, there will be a set of optimal trade-offs between the conflicting objectives, where the set of optimal solution is known as Pareto front. A multi-objective optimization problem is defined as the maximization or the minimization of many objectives subject to equality and inequality constraints. The multi-objective optimization problem can be formulated as follows:

$$\text{Max./Min.} f_i(x), \quad i = 1, ..., N_{\text{obj}}. \tag{1}$$

Subject to constraints:

$$g_j(x) = 0, \quad j = 1, ..., M,$$

$$h_k(x) \leq 0, \quad k = 1, ..., K, \tag{2}$$

where $f_i$ is the $i$th objective function, $x$ is the decision vector, $N_{\text{obj}}$ is the number of objectives, $g_j$ is the $j$th equality constraint, and $h_k$ is the $k$th inequality constraint.

There are techniques such as weighting methods and $\varepsilon$-constraint method which transfer multi-objective problems to a single-objective one, using different combinations of a weighting vector and constraints. Thus, each optimal solution can be assigned to a specific combination of weighting vector and constraint. Hence, in each run of the algorithm, a single solution can be achieved. However, multi-objective metaheuristic algorithms are capable of finding almost all candidate solutions (Pareto) in a single run. Metaheuristic algorithms can perform optimal/near-optimal solutions in all types of problems (linear/nonlinear, discrete/continuous, convex/non-convex) especially with incomplete or imperfect information or limited computation capacity.

A set of solutions resulting from a program run, without using any techniques such as the weighting approach that are directly related to decision-makers' opinions, is the most important advantage of metaheuristic algorithms in the field of multi-objective optimization. In this paper, two multi-objective metaheuristic algorithms, the NSGA-II and NSACO algorithm are used as optimization tools in extraction solution of the developed deterministic and non-deterministic models.

### 3.2. Robust multi-objective optimization

Many real-world optimization problems are subject to uncertainties and noise. These uncertainties and noise are caused by manufacturing errors, measurement errors, external factors, and inability to predict the future events. The uncertainties emerge in different parts of the optimization process.

One of the basic assumptions in stochastic programming is that the probability distribution function of the uncertain parameter is known. The goal of the stochastic model is often to obtain an optimal solution, which can minimize the expected value of the objective. However, in robust optimization, the uncertain parameters are described by the discrete scenarios or a continuous range. Robust optimization is an approach that deals with the uncertainty parameters

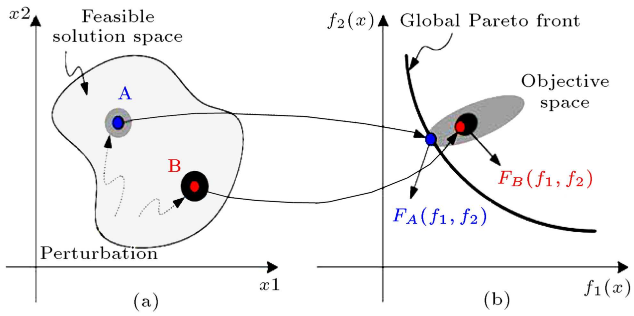


**Figure 1.** Effects of robust and sensitive solutions on objective functions.

in mathematical models and guarantees the feasibility of the solutions. The goal of this optimization method is to obtain an optimal solution, which is insensitive to almost all the samples of the uncertain parameters. Some minor deviations in the input variables of a system to be optimized may result in great deviations in the objective function values. The goal of robust optimization is not only to optimize the objectives, but also to take care of deviations of objective function values caused by small or large changes or fluctuations in the input variables. For multi-objective optimization, this means that, instead of looking for the global non-robust Pareto front, one is looking for the global robust Pareto front that means the Pareto fronts for different levels of uncertainty. Figure 1 illustrates the concept of the robustness of the solutions, where Figure 1(a) corresponds to the solution space ($x_1$ and $x_2$) and Figure 1(b) represents the objective space ($f_1$ and $f_2$).

For illustration purpose, it is assumed that we deal with minimization problem of two objective functions for which solutions A and B are found. As seen in Figure 1, solution A is better than solution B since both objective functions of solution A are smaller than those of solution B. However, let us assume that any fluctuation occurs in the solutions as depicted by a circle in Figure 1(a). The fluctuation of solution A is denoted by light gray circle and that for solution B is shaded by dark gray circle. Under these fluctuated conditions, the corresponding objective functions also show some perturbations which are depicted by ellipses in Figure 1(b). It is noteworthy that solution A shows large dispersion in objective space, whereas solution B is just perturbed by a small amount in objective space. Considering the worst condition, solution B is preferred to solution A, because the maximally perturbed objective functions of solution A are larger than those of solution B. Thus, solution A is inferior to the solution B in the worst case. In other words, solution B is more insensitive to the perturbation in terms of objective functions, and such insensitive solutions are said to be robust [25]. The robust optimization includes some formulations as *regret model*, *variability model*, and some other definitions such as the *worst case analysis*, which includes two principles named *minimax* and *maximin* [26].

To achieve robustness in the solutions, the regret model is used in this research. In regret model, the regret value of the scenario is described by the difference between the objective value of the feasible solution and the best objective value. It can be denoted by the absolute difference or relative difference. In the bi-objective problem, let $S$ denote the set of scenarios. For $\forall s \in S$, $x$ is the feasible solution of the deterministic programming model, $P_s$, while $Z_{1s}(x)$ and $Z_{2s}(x)$ are the objective values of $P_s$ with the solution $x$; $Z^*_{1s}$ and $Z^*_{2s}$ are the optimal objective values of $P_s$. Given a constant $\omega_1$, $\omega_2 > 0$, if $[Z_{1s}(x) - Z^*_{1s}]/Z^*_{1s} \leq \omega_1$ and $[Z_{2s}(x) - Z^*_{2s}]/Z^*_{2s} \leq \omega_2$ under every scenario $s \in S$, then $x$ is the robust solution of $P_s$. $(Z_{1s}(x) - Z^*_{1s})$ and $(Z_{2s}(x) - Z^*_{2s})$ are the absolute regret values and $[Z_{1s}(x) - Z^*_{1s}]/Z^*_{1s}$ and $[Z_{2s}(x) - Z^*_{2s}]/Z^*_{2s}$ are the relative regret values. $\omega_1$ and $\omega_2$ are the regret coefficients. There might be several robust solutions, and the best robust solution should be found out. Thus, the following model can be obtained:

$$P_{ro}: \ \min \sum_s \rho_s Z_{1s}(x),$$

$$\min \sum_s \rho_s Z_{2s}(x). \tag{3}$$

Subject to:

$$[Z_{1s}(x) - Z^*_{1s}]/Z^*_{1s} \leq \omega_1,$$

$$[Z_{2s}(x) - Z^*_{2s}]/Z^*_{2s} \leq \omega_2. \tag{4}$$

For $\forall s \in S$, $\rho_s$ denotes the probability of scenario $s$, where $\sum_{s=1}^{s} \rho_s = 1$. The optimal Pareto solutions of the above model are the best robust solutions (robust Pareto front) of the original problem [26].

## 4. Description of problem and model

In this paper, a location-allocation model for multi-vehicle single product in two-stage supply chain network is developed. This model includes distribution centers, and customers with respect to two conflicting objectives consist of minimizing total transit time and total cost. The total cost here involves opening cost, assumed for opening potential DCs, and shipping cost, from DCs to the customers. The proposed model selects some potential places as distribution centers in order to supply demands of all customers. It is assumed that distribution centers have unequal capacities, and each customer must be served from a single distribution center. Uncertainty is defined by discrete scenarios on demands where occurrence probability of each scenario is known. Considering different types of vehicles lead to a more realistic model and cause more conflict in the two objectives of the proposed problem, since a fast vehicle (because of high technology or having low capacity) has more cost, and a vehicle with low cost can lead to higher transit time.

According to Section 3.2, to solve the robust optimization model, we need the optimal objective values of the deterministic optimization model; we first define the multi-objective model with deterministic demand, and then, we formulate the robust multi-objective model with uncertain demand.

### *4.1. Multi-objective model with deterministic demand*

Let us denote $I$ as a set of nodes representing $m$ customers, $J$ as a set of nodes representing $p$ potential distribution centers (locations), $V$ as a set of types of vehicles for transferring process so that the number of vehicles is assumed to be unlimited, and $E$ as a set of edges representing a connection between customers and DCs. $d_i$ denotes the demand of customer $i$; $f_j$ is the fixed cost for opening a potential DC at site $j$; $q_v$ is the capacity of type of vehicle $v$, $v \in V$; and the associated capacity $q_j$ for such DC; $d_{ij}$ is the distance between DC $j$ and customer $i$; $c_{ij}^v$ is the cost of assigning customer $i$ to DC located at site $j$ with type of vehicle $v$; and $t_{ij}^v$ is the transit time between customer $i$ to DC located at site $j$ with type of vehicle $v$. All parameters introduced above are assumed to be non-negative. The binary variable $y_j$ is 1 if a DC is located at site $j$ and 0 otherwise. Similarly, binary variable $x_{ij}^v$ is equal to 1 if customer $i$ is served by the DC located at site $j$ with type of vehicle $v \in V$ and 0 otherwise. The bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem can be formulated as the following binary integer programming:

$$\min z_1 = \sum_{v=1}^{V}\sum_{j=1}^{p}\sum_{i=1}^{m} d_i d_{ij} c_{ij}^v x_{ij}^v + \sum_{j=1}^{p} f_j y_j, \tag{5}$$

$$\min z_2 = \sum_{v=1}^{V}\sum_{j=1}^{p}\sum_{i=1}^{m} t_{ij}^v x_{ij}^v. \tag{6}$$

Subject to:

$$\sum_{v=1}^{V}\sum_{j=1}^{p} x_{ij}^v = 1, \quad i = 1, ..., m, \tag{7}$$

$$\sum_{v=1}^{V}\sum_{i=1}^{m} d_i x_{ij}^v \leq q_j y_j, \quad j = 1, ..., p, \tag{8}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{p} d_i x_{ij}^v \leq q_v, \quad v = 1, ..., V, \tag{9}$$

$$x_{ij}^v, \quad y_j \in \{0,1\}, \quad \forall i = 1, ..., m, \quad \forall j = 1, ..., p, \quad \forall v = 1, ..., V. \tag{10}$$

The objective function (Eq. (5)) minimizes the total cost of opening distribution centers and assigning customers to such distribution centers, while objective function (Eq. (6)) minimizes total transit time between distribution centers and customers allocated to them. Constraints (Eq. (7)) guarantee that each customer is served by exactly one DC and also guarantee that each customer's demand is transferred by exactly one vehicle, while capacity constraints (Eq. (8)) ensure that the total demand assigned to a DC cannot exceed its capacity. The constraints (Eq. (9)) ensure that the total demand transferred by a vehicle cannot exceed its capacity. In this paper, capacity constraints of DCs have been relaxed through considering penalty function. In general, a penalty function approach is as follows. Given an optimization problem:

$$\begin{aligned} &\min f(X), \\ &\text{s.t.} \quad X \in A, \\ &\qquad\;\; X \in B, \end{aligned} \tag{11}$$

where $X$ is a vector of decision variables, the constraints "$X \in A$" are relatively easy to satisfy, and the constraints "$X \in B$" are relatively difficult to satisfy. The problem can be reformulated as:

$$\begin{aligned} &\min f(X) + p\left(d(X, B)\right), \\ &\text{s.t.} \quad X \in A, \end{aligned} \tag{12}$$

where $d(X, B)$ is a metric function describing the distance of the solution vector $X$ from the region $B$, and $p(0)$ is a monotonically non-decreasing penalty function such that $p(0) = 0$. Furthermore, any optimal solution of Eq. (12) will provide an upper bound on the optimum for Eq. (11), and this bound will be in general tighter than that obtained by simply optimizing $f(X)$ over $A$.

In this paper, the objective functions are as follows:

$$\begin{aligned} &\min \hat{z}_1 = z_1 + \delta_1 . Vi, \\ &\min \hat{z}_2 = z_2 + \delta_2 . Vi, \end{aligned} \tag{13}$$

where $\delta_1.Vi$ and $\delta.Vi$ are penalty functions. $\delta_1$ and $\delta_2$ are two positive coefficients where they are usually considered greater than $\max(z_1)$ and $\max(z_2)$, respectively. Also, $Vi$ represents relatively the violation of the value of capacity constraints related to DCs (Eq. (8)):

$$Vi = \left(\sum_{v=1}^{V}\sum_{i=1}^{m} d_i x_{ij}^v - q_j y_j\right) / q_j y_j,$$

$$\text{if} \quad \sum_{v=1}^{V}\sum_{i=1}^{m} d_i x_{ij}^v > q_j y_j, \quad j = 1, ..., p. \tag{14}$$

And also:

$$Vi = 0, \quad \text{if} \quad \sum_{v=1}^{V}\sum_{i=1}^{m} d_i x_{ij}^v \leq q_j y_j, \ j = 1, ..., p.$$

Besides fulfilling other constraints (Eqs. (7) and (9)), the solutions with $Vi = 0$ are feasible, otherwise the solutions are infeasible.

### *4.2. The robust multi-objective model with uncertain demand*

As mentioned in Section 3.2, the formulation of the regret model is applied in this paper. It is assumed that the demand is uncertain in the future with several possible scenarios, while the other parameters are deterministic. The parameters in the robust model are all deterministic under a certain scenario $s$. Hence, in each scenario, the distribution center location and allocation problem can be described as a deterministic optimization model. In other words, for a non-deterministic model with $S$ scenarios on demand, $S$ deterministic models should be considered. For $\forall s \in S$, the optimal objective values of the deterministic optimization model is denoted by $Z_{1s}^*$ and $Z_{2s}^*$. $x$ is feasible under all scenarios, and $Z_{1s}(x)$ and $Z_{2s}(x)$ denote the objective values of $x$ under scenario $s$. Given the regret coefficients (confidence level), $\omega_1$, and $\omega_2 > 0$, if and only if $[z_{1s}(x) - z_{1s}^*]/z_{1s}^* \leq \omega_1$ and $[z_{2s}(x) - z_{2s}^*]/z_{2s}^* \leq \omega_2$ under every scenario $s \in S$, $x$ is the robust solution of this problem. There might be several robust solutions, and the best robust solutions should be found out. Thus, the robust multi-objective optimization model ($P_{ro}$) of capacitated multi-vehicle location and allocation problem can be formulated as follows:

$$P_{ro}: \ \min z_1 = \sum_{s=1}^{S} \rho_s z_{1s}(x), \tag{15}$$

$$\min z_2 = \sum_{s=1}^{S} \rho_s z_{2s}(x). \tag{16}$$

Subject to:

$$z_{1s}(x) = \sum_{v=1}^{V}\sum_{j=1}^{p}\sum_{i=1}^{m} d_{is} d_{ij} c_{ij}^v x_{ij}^v + \sum_{j=1}^{p} f_j y_j, \quad \forall s = 1, ..., S, \tag{17}$$

$$z_{2s}(x) = \sum_{v=1}^{V}\sum_{j=1}^{p}\sum_{i=1}^{m} t_{ij}^v x_{ij}^v, \quad \forall s = 1, ..., S, \tag{18}$$

$$\sum_{v=1}^{V}\sum_{j=1}^{p} x_{ij}^v = 1, \quad \forall i = 1, ..., m, \ \forall s = 1, ..., S, \tag{19}$$

$$\sum_{v=1}^{V}\sum_{i=1}^{m} d_{is} x_{ij}^v \leq q_j y_j, \quad \forall j = 1, ..., p, \ \forall s = 1, ..., S, \tag{20}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{p} d_{is} x_{ij}^v \leq q_v, \quad \forall v = 1, ..., V, \ \forall s = 1, ..., S, \tag{21}$$

$$[z_{1s}(x) - z_{1s}^*]/z_{1s}^* \leq \omega_1, \tag{22}$$

$$[z_{2s}(x) - z_{2s}^*]/z_{2s}^* \leq \omega_2, \tag{23}$$

$$x_{ij}^v, \quad y_j \in \{0,1\}, \ \forall i = 1, ..., m, \ \forall j = 1, ..., p, \quad \forall v = 1, ..., V. \tag{24}$$

In the above model, Eq. (15) is the first objective, which is aiming at the total average cost in all scenarios, while the second objective function Eq. (16) is aiming at the total average transit time in all scenarios. Eq. (22) and Eq. (23) ensure that the feasible solution of model $P_{ro}$ should meet the requirement of the robust solution. The optimal Pareto solutions of the above model are the best robust solutions (robust Pareto front) of the original problem.

## 5. Solution procedure

A main branch in the theory of computation, named computational complexity, considers classifying computational problems regarding their inherent difficulty. Some important complexity classes are P, NP, NP-complete, NP-hard, EXP-space, EXP-time, P-space, etc. Many real-world optimization problems belong to the class of NP-hard, and in order to solve NP-hard problems, there are no provably efficient algorithms, i.e. exact methods cannot solve the problems in normal time. According to the performed studies, metaheuristic algorithms are suitable tools to optimize this class of problems [27]. Mirchandani and Francis (1990) [28] showed that Capacitated Facility Location Problem (CFLP) is NP-hard. Minoux (2010) [29] proved that the robust network design problem with uncertain demand is NP-hard. Since the proposed bi-objective models known in Sections 4.1 and 4.2 consist of the above NP-hard problems, these are NP-hard as well. This justifies the use of a metaheuristic algorithm. In this section, the well-known Multi-Objective Evolutionary Algorithm (MOEA) of NSGA-II and a new swarm intelligence based algorithm called Non-dominated Sorting Ant Colony Optimization (NSACO) are presented to solve the problem.

### 5.1. Non-dominated Sorting Genetic Algorithm-II (NSGA-II)

A population-based search MOEA can present a set of Pareto optimal solutions of multi-objective optimization problems involving two or more conflicting objectives. One of these MOEAs that is frequently used in many optimization problems as the best technique to generate Pareto frontiers is the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) proposed by Deb et al. (2000) [21]. To start NSGA-II, one first randomly generates a population $P_1$ with size $nPop$ chromosomes (solutions) and then sorts the chromosomes in $P_1$ into several fronts of non-dominated solutions. All chromosomes in this population are sorted into different front levels based on the domination of pair comparison. Each front level is assigned a fitness (or a rank) which equals its non-domination level. Level 1 is the top level in which the individual is dominated by none of the other chromosomes; level 2 is the secondary level in which the chromosome is dominated by some chromosomes only in level 1, and so on. Considering the obtained chromosomes using the tournament selection operator for $P_1$, the offspring population $O_1$ is created with respect to the crossover probability ($P_c$) and the mutation probability ($P_m$). Moreover, the algorithm obtains the objective values of each chromosome in $P_1$ and $O_1$.

After merging $P_1$ and $O_1$ to form $R_t$, the algorithm sorts $R_t$ in several non-dominated fronts $F_i$, where the best $F_i s$ form the next population $P_{t+1}$. Since the size of $P_{t+1}$ is equal to the size of $P_t$, all of the elements from a front cannot be in $P_{t+1}$. Hence, when a front is added to $P_{t+1}$ incompletely, the crowding distance approach is applied. The crowding distance is an important concept proposed by Deb et al. (2000) [21] in his algorithm NSGA-II. It serves for getting an estimate of the density of solutions surrounding a particular solution in the population. Figure 2 shows the calculation of the crowding distance of point $i$ which is an estimate of the size of the largest cuboid enclosing $I$ without including any other points.

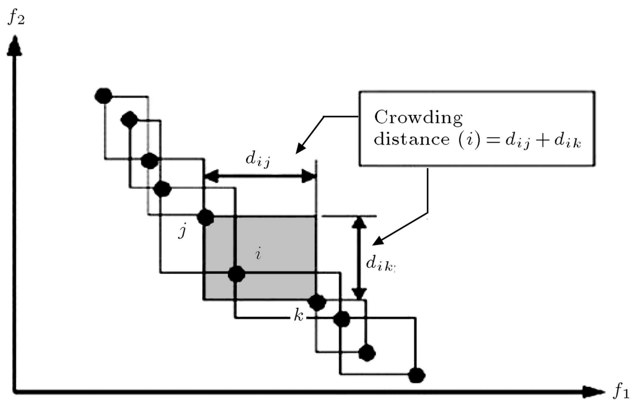


**Figure 2.** Crowding distance computation.

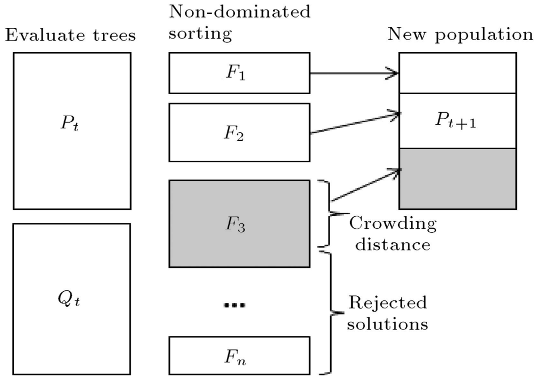


**Figure 3.** Graphical representation of NSGA-II.

In fact, the crowding distance is a measure of how close an individual is to its neighbors. Consequently, the required population is organized from the top elements of the front without losing good solutions (elitism). The algorithm creates $O_{t+1}$ from $P_{t+1}$ using a crowding distance method and crossover and mutation operators.

Regarding the stopping criteria and iterating the above stages, the algorithm hopefully presents the best Pareto optimal solutions. Figure 3 shows a graphical representation of NSGA-II. For more details on the implementation of NSGA-II see [21,30].

### 5.2. Non-dominated sorting ant colony optimization method

Ant Colony Optimization (ACO) algorithms are the most successful and widely recognized algorithmic techniques based on real ant behaviors [31]. Several papers were proposed to extend the Ant Colony Optimization (ACO) method in order to handle a multi-objective optimization problem [17-19].

In this paper, a swarm intelligence-based algorithm named Non-dominated Sorting Ant Colony Optimization (NSACO) is proposed to tackle the bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem in uncertain environment. NSACO algorithm is based on the same non-dominated sorting concept used in NSGA-II. The proposed methodology is based on a new variant of ACO specialized in multi-objective optimization problem. Steps of the NSACO are as follows.

In the first step, a colony of ants with size $nAnt$ is considered. Then, ACO parameters such as $\alpha$, $\beta$, $\rho$, etc. are initialized, where $\alpha$ and $\beta$ are parameters used for controlling the exponential weight of the pheromone trail and the heuristic exponential weight, respectively, and $\rho$ is evaporation rate [31]. Also in this step, the value of the initial pheromone trail, $\tau_0$, is determined and the tabu lists of all ants are constructed, which contain all the unvisited nodes for each ant and the list of optimal paths traversed by the ants. The initial pheromone intensity, $\tau_{ij}$, or the path from nodes $i$ to $j$ is set equal to $\tau_0$, that is $\tau_{ij} = \tau_0$ and $\Delta\tau_{ij} = 0$.

In the second step, for each ant of the colony, a new solution using ACO probabilistic rule is created. It means that, for each ant, a DC vector, an allocation matrix and a vehicle vector are assigned. The DC vector is a binary vector that indicates the opening or not opening DCs, the allocation matrix is a binary matrix that indicates the allocation of customers to the located DCs, and the vehicle vector is an integer vector that indicates the type of vehicle for transferring customer's demand. The allocation matrix and vehicle vector form a three-dimensional decision variable named $x_{ij}^{v}$. Then, objective values for this solution are calculated and evaluated.

In order to construct the solution, ant $k$ currently at node $i$ determines the next node to visit, node $j$, by applying the sampling approach known as the Roulette Wheel Selection. For this purpose, first, movement probability for ant $k$ from node $i$ to other nodes including the neighbors of the node $i$, must be calculated. $S_k(i)$ is a Tabu list to avoid creating a loop, containing those unvisited nodes for ant $k$ currently at node $i$. Therefore, node $j \in S_k(i)$ is the node randomly chosen from the list $S_k(i)$ according to the pseudo random proportional distribution rule Eq. (25) and the Roulette wheel selection [31]:

$$P_{ij}^{k} = \tau_{ij}^{\alpha}\eta_{ij}^{\beta} / \sum_{u \in Sk(i)} \left(\tau_{iu}^{\alpha}\eta_{iu}^{\beta}\right),$$

$$\text{if } j \in S_k(i), \text{ otherwise } P_{ij}^{k} = 0, \tag{25}$$

where $P_{ij}^{k}$ is the probability that ant $k$ chooses to move from node $i$ to node $j$, and $\eta_{ij}$ is a heuristic value which equals to the inverse of the length from node $i$ to node $j$, $\tau_{ij}$ is the amount of pheromone trail of the path from node $i$ to node $j$, $\alpha$ and $\beta$ are two parameters used for controlling the exponential weight of the pheromone trail and the heuristic value. Then, after calculating the probability values, the Roulette wheel selection is used to select next node among these existing neighbor nodes [32]. In this paper, this process is occurred three times for constructing the DC vector, the allocation matrix, and the vehicle vector.

In the third step, after all the ants of the colony traversed their paths, the non-dominated sorting method is applied, where the entire population is sorted into various non-domination fronts. In a minimization problem, a vector $x^{(1)}$ is partially less than another vector $x^{(2)}$, $\left(x^{(1)} < x^{(2)}\right)$ when no value of $x^{(2)}$ is less than $x^{(1)}$ and at least one value of $x^{(2)}$ is strictly greater than $x^{(1)}$ [33]. A solution which is not partially less is a dominated solution and a solution which cannot be dominated throughout an existing solution set is called a non-dominated solution or Pareto front. The first front being completely a non-dominant set in the current population and the second front being dominated by the individuals in the first front only and the front goes so on. Each individual in each front is assigned fitness values or based on front in which they belong to. Individuals in the first front are given a fitness value of 1 and individuals in the second are assigned a fitness value of 2 and so on. A major difference of NSACO and NSGA-II is that in NSACO, an additional population because of operators like crossover and mutation is not generated, and population size always equals $nAnt$. Also, all ants of a colony are sorted based on quality and discipline factors, simultaneously. Therefore, in addition to the fitness value, a parameter called crowding distance is calculated for each ant to ensure the best distribution of the non-dominated solutions.

Once the non-dominated solutions are found, other (dominated) solutions are discarded and once again the pheromone trails are updated and evaporation process is occurred according to non-dominated solutions. In this paper, three pheromone trails matrix are designed for DC vector, allocation matrix and vehicle vector. The pheromone trails matrix for DC vector is a $2 \times p$ dimensions matrix, in which 2 is identified as open or closed state of each DC, that the first row and the second row are considered for closing and opening the DCs, respectively, and $p$ is identified as the number of DCs (Eq. (26)). The pheromone trails matrix for allocation matrix is a $p \times m$ dimensions matrix, in which $p$ and $m$ are identified as number of DCs and number of customers, respectively (Eq. (27)), and the pheromone trails matrix for vehicle vector is a $V \times m$ dimensions matrix, in which $V$ and $m$ are identified as types of vehicles and number of customers, respectively (Eq. (28)).

$$\tau 1 = \begin{bmatrix} \tau_{11} & . & . & \tau_{1p} \\ \tau_{21} & . & . & \tau_{2p} \end{bmatrix}, \tag{26}$$

$$\tau 2 = \begin{bmatrix} \tau_{11} & . & . & \tau_{1m} \\ \tau_{p1} & . & . & \tau_{pm} \end{bmatrix}, \tag{27}$$

$$\tau 3 = \begin{bmatrix} \tau_{11} & . & . & \tau_{1m} \\ \tau_{v1} & . & . & \tau_{vm} \end{bmatrix}. \tag{28}$$

The heuristic information matrix for DC vector is a $2 \times p$ dimensions matrix, in which 2 is identified as closed or open state of each DC, in which the first row and the second row are considered for fixed cost for opening potential DCs and inverse of fixed cost for opening potential DCs, respectively, and $p$ is identified as the number of DCs (Eq. (29)). The heuristic information matrix for allocation matrix is a $P \times m$ dimensions matrix, in which $p$ and $m$ are identified as number of DCs and number of customers, respectively, which contains inverse of distance values between customers and DCs (Eq. (30)), and the heuristic information matrix for vehicle vector is a $V \times m$ dimensions matrix, in which $V$ and $m$ are identified as types of vehicles and number of customers, respectively, which contains inverse of shipping cost from DCs to customers. There are three heuristic information matrices $\forall j = 1, ..., p$ (Eq. (31)).

$$\eta 1 = \begin{bmatrix} f_1 & .. & f_p \\ 1/f_1 & .. & 1/f_p \end{bmatrix}, \tag{29}$$

$$\eta 2 = \begin{bmatrix} 1/d_{11} & .. & 1/d_{1m} \\ . & .. & . \\ 1/d_{p1} & .. & 1/d_{pm} \end{bmatrix}, \tag{30}$$

$$\eta 3 = \begin{bmatrix} 1/c_{1j1} & .. & 1/c_{mj1} \\ . & .. & . \\ 1/c_{1jv} & .. & 1/c_{mjv} \end{bmatrix}. \tag{31}$$

The pheromone trails are updated according to the non-dominated solutions in the Pareto front, and in order to prevent unlimited accumulation of the pheromone trails and help the algorithm to forget bad decisions of formers, evaporation process is applied on pheromone trails. This updating process affects the selection of new solutions using ACO probabilistic rule in the next iteration. This cycle is repeated for a pre-defined number of iterations known as Cycle Iteration. At the end of running this algorithm, the present non-dominated solutions in the last iteration are the optimal solutions of the multi-objective problem. Figure 4 shows a graphical representation of NSACO.

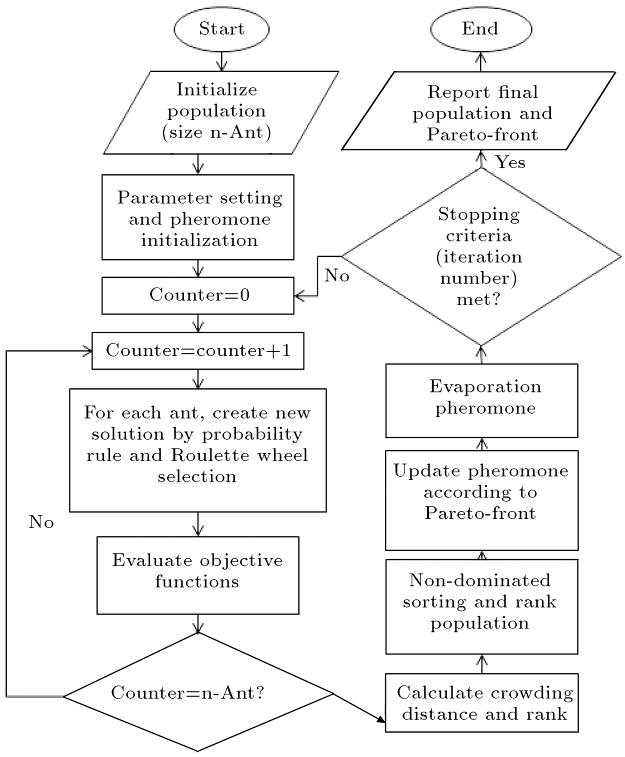


**Figure 4.** Graphical representation of NSACO.

### *5.3. Adaptive algorithms for solving the multi-objective robust optimization*

In this paper, two metaheuristic algorithms, NSGA-II and NSACO, are proposed as the optimization tools. The algorithms are coded in MATLAB software and tested on a Core 2 Duo/2.66 GHz processor. Steps of adaptive algorithms for solving the multi-objective robust optimization model by NSGA-II and NSACO are as follows:

Step 1. Considering a *for* loop over the number of scenarios for demand;

Step 2. Solving the bi-objective deterministic model in all scenarios by NSGA-II or NSACO and saving the objective values of Pareto front, $Z^*_{1s}$ and $Z^*_{2s}$, in memory of algorithms;

Step 3. After optimizing the deterministic models in all scenarios, the first three fronts of solutions for each model (each scenario) are considered together in a set named good solutions;

Step 4. For each solution in the set *good solutions*, feasibility survey is done according to the constraints (Eqs. (19), (20) and (21));

Step 5. The solutions that are feasible in all scenarios of demands simultaneously are stored in an archive named *feasible solutions*;

Step 6. For each solution in feasible solutions archive:

- Step 6.1. Calculating objective values for each scenario ($z_{1s}(x)$ and $z_{2s}(x)$) according to Eqs. (17) and (18);
- Step 6.2. Feasibility survey according to the regret constraints (Eqs. (22) and (23));
- Step 6.3. If all regret constraints are satisfied for a solution, that solution is stored in an archive named *robust solutions*, as a robust solution.

Step 7. After finding all robust solutions and saving them to robust solutions archive, the objective values of each robust solution are calculated according to Eqs. (15) and (16);

Step 8. Non-dominated sorting and crowding distance methods are done on robust solutions archive;

Step 9. The robust Pareto front is found.

In this paper, sixteen numerical examples, including eight cases in small scale and eight cases in large scale are considered for experimental study which presents different levels of difficulty for alternative solution approaches.

Initial population size is assumed 100 and 200 for small and large scales, respectively. Problem size differs from each other by changing DCs/customer's numbers, types of vehicles, and number of scenarios of demand.

## 6. Parameter tuning

In order to obtain solutions with better quality, the parameters of both algorithms are adjusted in this section using an auto tuning approach. For NSGA-II parameters, first, some random numbers, for example, 10 numbers in the range 0.55 to 0.85, are selected randomly for $P_c$. This range is considered according to existing literature in the field of genetic algorithms. It could be considered in 0 to 1 in the most pessimistic case. For each random number in the range, NSGA-II algorithm runs, and the results are saved. Then, by observing the best solutions, we tried the next random numbers that could be close to the $P_c$ of the best solutions. In fact, after observing the best solutions, lower and upper bound of the range are updated according to good values of $P_c$. Exactly the same procedure in the range 0 to 0.45 is repeated for $P_m$. This process is performed by an external NSGA-II program for auto tuning parameters. Figures 5 and 6 show that $P_c$ equals 0.73, and $P_m$ equals 0.37.

As shown in these figures, $P_c$ and $P_m$ are tuned in 15th iteration, approximately. If in each iteration, 10 random numbers are considered, $P_c$ and $P_m$ are tuned with considering 150 times running of NSGA-II algorithm. As previously mentioned, for NSACO

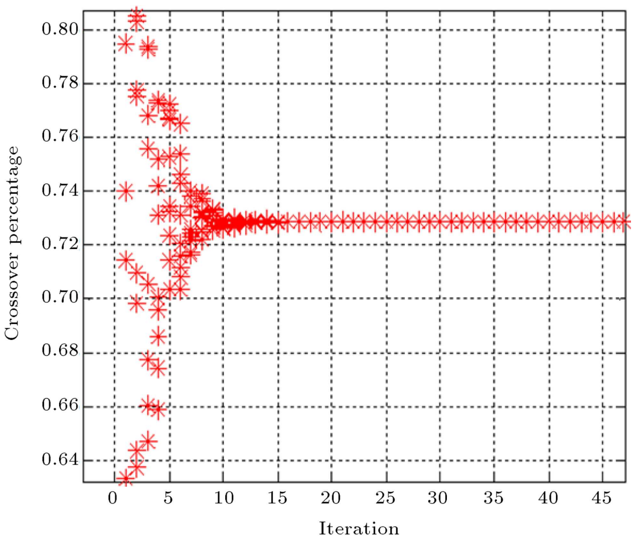


**Figure 5.** Auto tuning parameters (crossover probability).

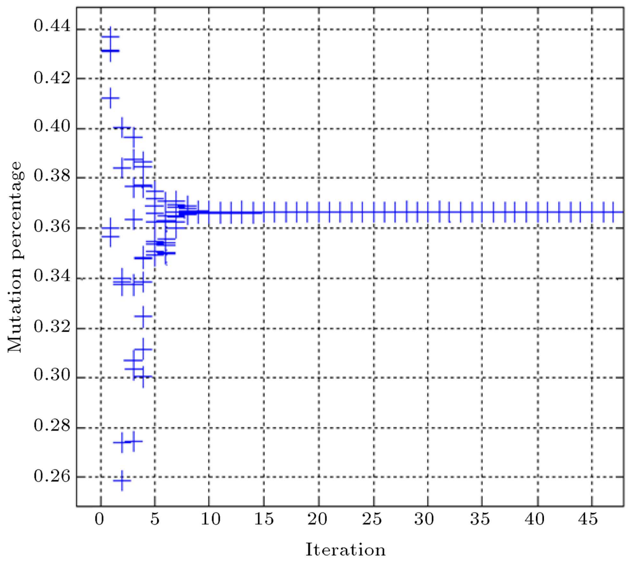


**Figure 6.** Auto tuning parameters (mutation probability).

parameters, some numbers, for example, 10 numbers in the range 0.8 to 1.8, are selected randomly for $\alpha_1$, *pheromone exponential weight for DC vector*, and then by observing the best solutions, we tried the next random numbers that could be close to the $\alpha_1$ of the best solutions.

Exactly the same procedure in the range 0.05 to 0.6 is repeated for $\beta_1$, *heuristic exponential weight for DC vector*. These initial ranges are considered according to both existing literature in the field of ACO algorithm and some tentative running of NSACO program. This procedure is repeated for other parameters. This process is performed by an external NSACO program for auto tuning parameters.

The parameters of NSACO for all optimization cases are summarized in Table 1.

## 7. Performance evaluation of the algorithms

To illustrate the performance of the used procedures to optimize the proposed models, problem 1 in small scale is considered as an example. As mentioned before, for solving the bi-objective robust model, first, the deterministic models $\forall s \in S$ should be solved. Figures 7 and 8 show the performance of the proposed

**Table 1.** NSACO parameters.

| | |
|---|---|
| $\alpha_1$ (Pheromone exponential weight for DC vector) | 1.30 |
| $\beta_1$ (Heuristic exponential weight for DC vector) | 0.40 |
| $\alpha_2$ (Pheromone exponential weight for allocation matrix) | 1.58 |
| $\beta_2$ (Heuristic exponential weight for allocation matrix) | 0.33 |
| $\alpha_3$ (Pheromone exponential weight for vehicle vector) | 1.34 |
| $\beta_3$ (Heuristic exponential weight for vehicle vector) | 0.52 |
| $\rho$ (Evaporation rate) | 0.05 |

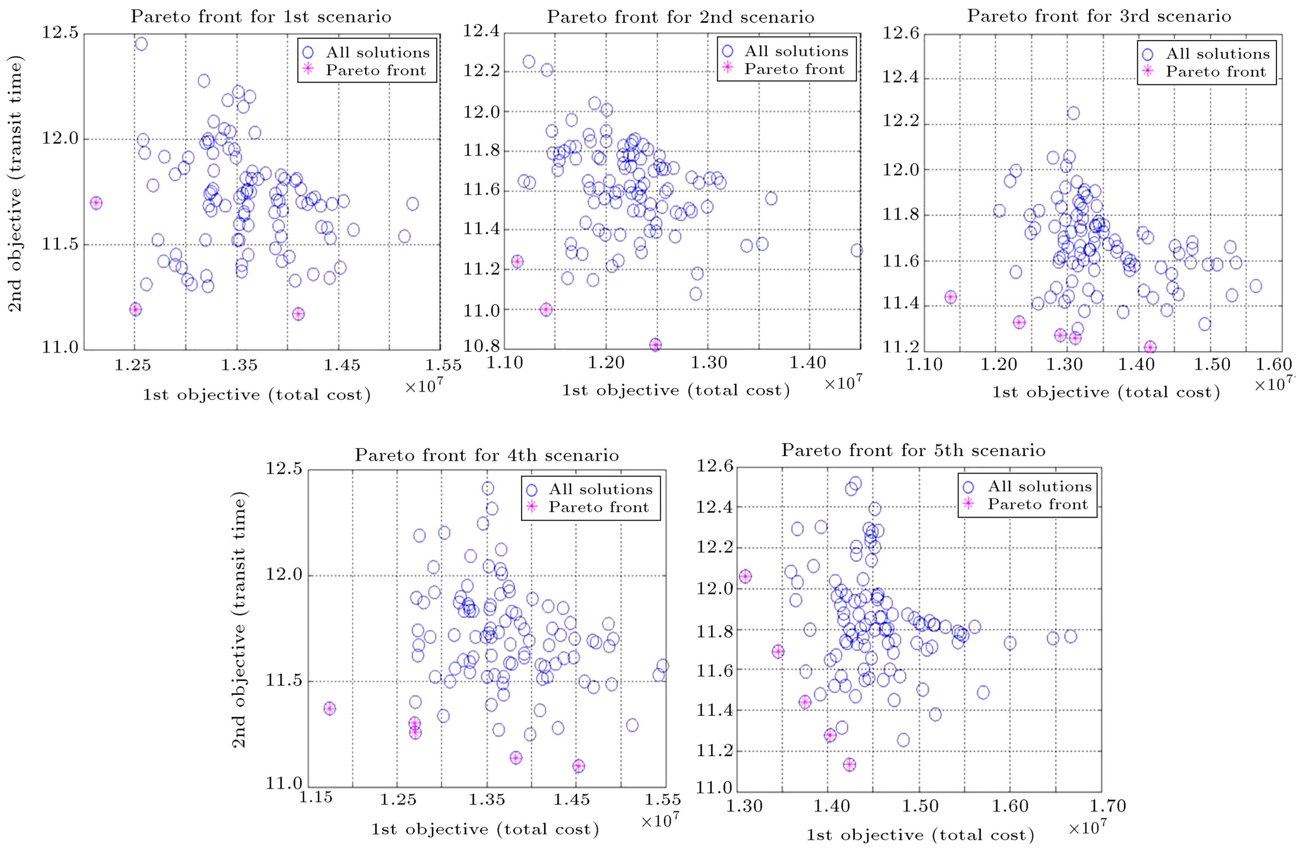


**Figure 7.** Pareto front of problem 1 in small scale by NSACO (3rd iteration with $nAnt = 100$).

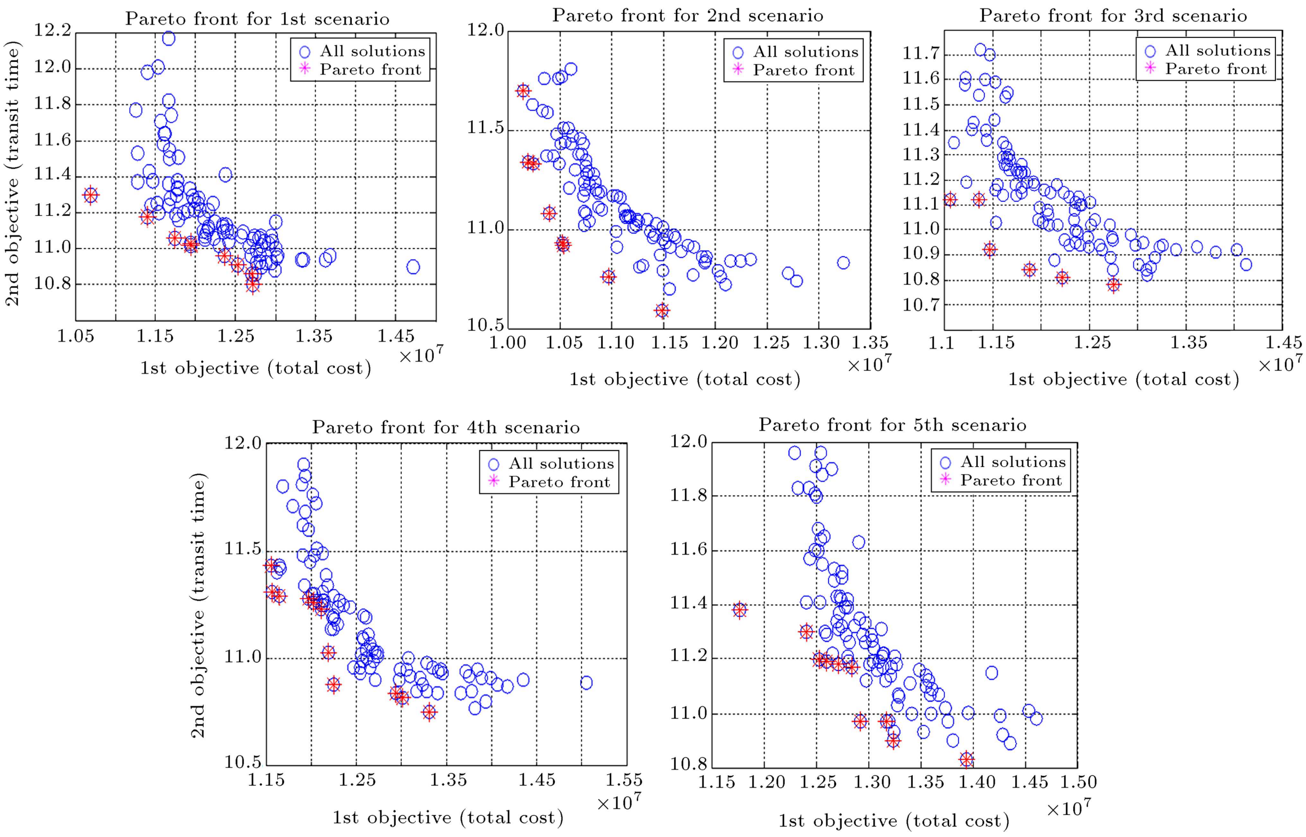


**Figure 8.** Pareto front of problem 1 in small scale by NSACO (200th iteration with $nAnt = 100$).

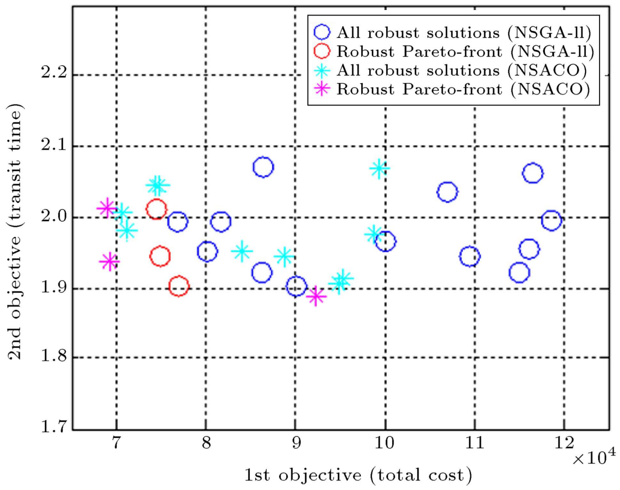


**Figure 9.** Robust Pareto solutions by NSACO & NSGA-II (problem 2 in small scale, in 200th iteration with $npop = 100$).

algorithm, NSACO, in the 3rd and 200th iterations with five scenarios on demand.

Figure 9 shows all robust solutions and robust Pareto solutions for problem 2 in small scale as an example. Also, to view the output of the decision variables, the robust Pareto solutions of problem 6 in small scale are given in the Appendix.

To check the quality of solutions obtained by the NSACO, four evaluation metrics including: (1) Number of Pareto solutions ($NOS$), (2) Maximum spread or diversity metric [34], (3) Mean Ideal Distance (MID) metric [35], and (4) time of running have been used. Diversity and MID metrics are formulated as follows:

$$\text{Diversity} = \sqrt{\sum_{j=1}^{m} \left( \max_{n} f_n^j - \min_{n} f_n^j \right)^2}, \tag{32}$$

$$\text{MID} = \sum_{i=1}^{n} \frac{C_i}{n}, \tag{33}$$

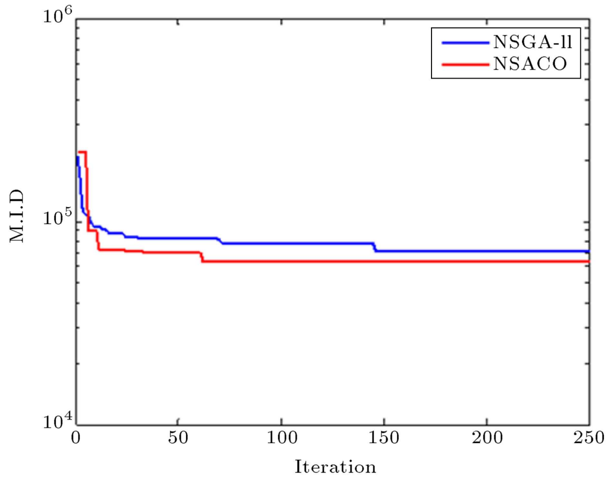


**Figure 10.** MID metric comparisons for problem 2 in small scale.

where in Eq. (32), $m$ is the number of objectives, $n$ is the number of Pareto solutions, and in Eq. (33), $n$ is the number of Pareto solutions and $C_i$ is the distance of $i$th Pareto solution from ideal point ((0,0) in bi-objective minimization).

Figure 10 shows $MID$ metrics comparison for problem 2 in small scale for first scenario (deterministic optimization). For better display, $MID$ axis is considered under logarithmic scale. As shown in Figure 10, in the first iterations, there are more infeasible solutions and they cause adding large penalty functions to objective values, but during the process of algorithm, the infeasible solutions, because of great objective values are discarded and objective values are more real and then convergence process goes smoothly.

Tables 2 and 3 show the algorithms comparison results for some small and large scale bi-objective robust optimization problems with iteration number 1000. From these results, it can be seen that the NSACO is more efficient than NSGA-II in the viewpoint of optimality, but, according to the *Diversity* and *NOS*,

**Table 2.** Algorithms comparison results for small scale cases.

| **Problems** | **Number of customers** | **Number of DCs** | **Types of vehicles** | **NSACO with 1000 iterations ($nAnt = 100$)** | | | **NSGA-II with 1000 iterations ($nPop = 100$)** | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **NOS** | **Diversity** | **Time (min)** | **NOS** | **Diversity** | **Time (min)** |
| Problem 1 | 21 | 7 | 3 | 2 | 105431.27 | 10.09 | 3 | 149360.85 | 15.93 |
| Problem 2 | 8 | 3 | 2 | 3 | 10942.25 | 5.65 | 3 | 21692.69 | 9.34 |
| Problem 3 | 15 | 3 | 2 | 2 | 19982.57 | 9.11 | 4 | 37299.6 | 12.15 |
| Problem 4 | 10 | 4 | 2 | 3 | 21503.86 | 7.03 | 4 | 34390 | 11.62 |
| Problem 5 | 12 | 5 | 2 | 3 | 18559.97 | 7.86 | 4 | 40290 | 13.74 |
| Problem 6 | 14 | 6 | 3 | 2 | 12982.57 | 8.69 | 3 | 22644.23 | 11.88 |
| Problem 7 | 20 | 5 | 2 | 3 | 1074.63 | 9.49 | 4 | 1474.63 | 14.06 |
| Problem 8 | 26 | 4 | 2 | 3 | 958.78 | 11.37 | 5 | 1548.52 | 17.09 |
| Average | - | - | - | 2.625 | 23929.49 | 8.66 | 3.725 | 38587.57 | 13.23 |

**Table 3.** Algorithms comparison results for large scale cases.

| Problems | Number of customers | Number of DCs | Types of vehicles | NSACO with 1000 iterations ($nAnt = 200$) | | | NSGA-II with 1000 iterations ($nPop = 200$) | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | NOS | Diversity | Time (min) | NOS | Diversity | Time (min) |
| Problem 1 | 32 | 7 | 3 | 3 | 33058.6 | 34.77 | 4 | 52176 | 47.96 |
| Problem 2 | 40 | 11 | 3 | 2 | 24265.1 | 43.25 | 4 | 46206 | 68.26 |
| Problem 3 | 24 | 6 | 3 | 2 | 34265.05 | 27.86 | 5 | 43890.5 | 38.32 |
| Problem 4 | 70 | 9 | 3 | 3 | 26744.7 | 65.13 | 4 | 38730 | 71.4 |
| Problem 5 | 62 | 9 | 3 | 2 | 10300 | 57.56 | 3 | 37996 | 68.23 |
| Problem 6 | 80 | 7 | 3 | 3 | 115782.5 | 70. 3 | 5 | 223900 | 88.2 |
| Problem 7 | 68 | 11 | 3 | 2 | 10256 | 61.68 | 3 | 18957 | 76.9 |
| Problem 8 | 60 | 10 | 3 | 3 | 11541 | 55.23 | 4 | 13628 | 73.2 |
| Average | - | - | - | 2.5 | 33276.62 | 51.97 | 4 | 59435.44 | 66.56 |

**Table 4.** Statistical comparison results ($\alpha = 5\%$).

| | Mann-Whitney test | | | |
|---|---|---|---|---|
| | Small scale cases | | Large scale cases | |
| | ***P*-value** | **results** | ***P*-value** | **results** |
| **NOS** | 0.005 | NSGA-II is preferred to NSACO | 0.002 | NSGA-II is preferred to NSACO |
| **Diversity** | 0.115 | There were no significant differences | 0.074 | There were no significant differences |
| **MID** | 0.048 | NSACO is preferred to NSGA-II | 0.041 | NSACO is preferred to NSGA-II |
| **Time** | 0.002 | NSACO is preferred to NSGA-II | 0.046 | NSACO is preferred to NSGA-II |

the NSGA-II has better distribution of solutions in the trade-off surface.

In this paper, in order to evaluate the performance of the two algorithms, the *Mann-Whitney* test is done by Statistical Package for the Social Sciences (SPSS 16.0) software (Table 4).

The regret coefficients (confidence level), $\omega_1$ and $\omega_2$, are assumed the same ($\omega$, in this paper), where it takes values between 0 and 100%. Figure 11 depicts the robust multi-objective results for capacitated multi-vehicle allocation of customers to DCs problem with considering four values for $\omega$, including 5%, 10%, 15%, and 20%. When looking at the robust results, it is clear that the Pareto front shifts to higher values for both objectives when $\omega$ increases. The nominal case is related to the Pareto solutions of deterministic models in all scenarios.

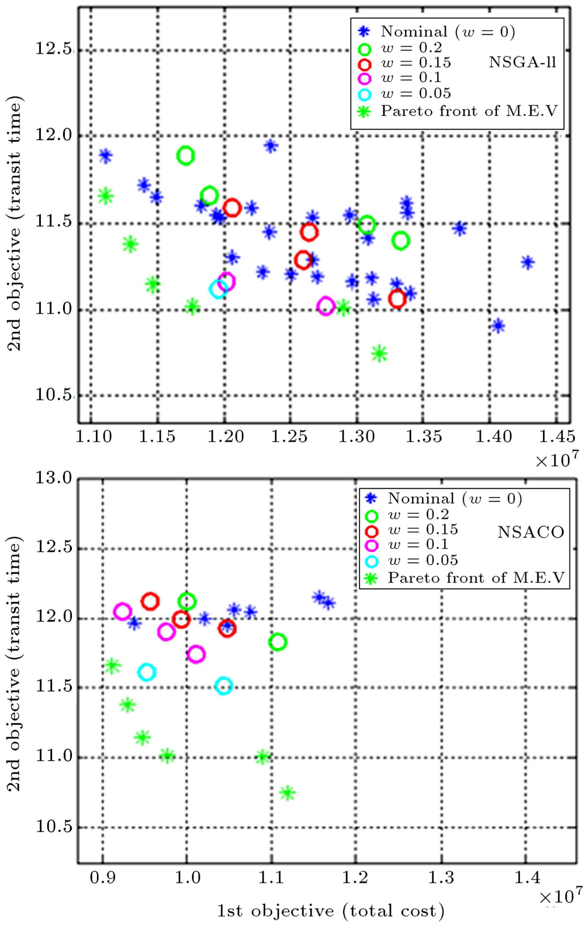


**Figure 11.** Bi-objective location allocation: Pareto set by NSGA-II & NSACO (problem 1 in small scale in 500th iteration with $npop = 100$).

It has to be mentioned that the robust optimization model can obtain more insensitive solutions than the stochastic optimization like mean expected value model (M.E.V model, with considering mean expected values of demand and solving with deterministic

**Table 5.** Comparison between the results of stochastic optimization model and robust optimization model for problem 5 in small scale.

| Scenario | Objective functions | $Z_s^*$ | $Z_s^{ro}$ | $Z_s^{so}$ | $\varepsilon_{ro-d}$ (%) | $\varepsilon_{so-d}$ (%) |
|---|---|---|---|---|---|---|
| 1 | Total cost | 287055.47 | 289889.31 | 313891.44 | 0.99 | 9.35 |
| 2 | | 264934.74 | 309011.22 | 320197.39 | 16.64 | 20.86 |
| 3 | | 265999.90 | 334929.45 | 301672.11 | 25.91 | 13.41 |
| 4 | | 296305.81 | 296418.94 | 296496.34 | 0.038 | 0.06 |
| 5 | | 353423.26 | 355750.82 | 496563.70 | 0.66 | 40.50 |
| 1 | Total transit time | 1.68 | 1.88 | 1.71 | 11.90 | 1.78 |
| 2 | | 1.73 | 1.88 | 1.75 | 8.67 | 1.16 |
| 3 | | 1.77 | 1.88 | 1.79 | 6.21 | 1.13 |
| 4 | | 1.67 | 1.88 | 1.69 | 12.57 | 1.19 |
| 5 | | 1.71 | 1.88 | 2.38 | 9.94 | 39.00 |

model), especially when the data distribution is large compared to the average. The robust optimization model in the both objectives does not change a lot under all scenarios, thus the risk of decision can be effectively reduced.

Table 5 shows the comparison between the results of stochastic optimization model (M.E.V model) and robust optimization model. As an example, the first member of Pareto front in each scenario for problem 5 in small scale is considered. The relative difference between $Z_s^{so}$ and $Z_s^*$ can be obtained by:

$$\varepsilon_{so-d} = \{(Z_s^{so} - Z_s^*)/Z_s^*\} \times 100\%.$$

The relative difference between $Z_s^{ro}$ and $Z_s^*$ can be obtained by:

$$\varepsilon_{ro-d} = \{(Z_s^{ro} - Z_s^*)/Z_s^*\} \times 100\%,$$

where $Z_s^*$, $Z_s^{so}$ and $Z_s^{ro}$ are the objective values of deterministic model, stochastic optimization model (M.E.V model), and robust optimization model, respectively.

It can be concluded from Table 5 that the $\varepsilon_{so-d}$ for 1st objective and 2nd objective is fluctuating from 0.06 to 40.5% and 1.13 to 39%, respectively, while $\varepsilon_{ro-d}$ for 1st objective and 2nd objective is fluctuating from 0.038 to 25.91% and 6.21 to 12.57%, respectively. The latter is more stable.

## 8. Discussion and conclusion

Nowadays, the competition is vital for the firms' survival in SCs. Then, the basic priority for supply chain management should be designing the SC network properly, to gain competitive advantage. In this paper, a multi-objective robust optimization model for capacitated multi-vehicle allocation of customers to DCs in two-stage SC considering distribution centers and customers is proposed. The optimization objectives are to minimize transit time and total cost. Results show the trade-off between total transit time and total cost, since the different types of vehicles used in the model cause more conflict in these two objectives.

In this paper, swarm intelligence-based algorithm named Non-dominated Sorting Ant Colony Optimization (NSACO) is presented to find Pareto fronts. The proposed methodology is based on a new variant of Ant Colony Optimization (ACO) customized in multi-objective optimization problem. The crowding distance technique is used to ensure the best distribution of the non-dominated solutions.

For ensuring the robustness of the proposed method and giving a practical sense of this study, the computational results are compared with those obtained by Non-dominated Sorting Genetic Algorithms (NSGA-II). Results show the advantages and effectiveness of the used procedures in reporting the optimal Pareto front of the proposed deterministic and non-deterministic models.

Moreover, it can be seen that the NSACO is more efficient than NSGA-II in the viewpoint of optimality and running time saving, but the NSGA-II has better distribution of solutions in the trade-off surface. Also, the optimal solutions of the robust optimization model are insensitive to the disturbance of parameters under different scenarios, and the robust optimization model can obtain better solutions than the stochastic optimization model, thus the risk of decision can be effectively reduced.

Future research may develop the NSACO to increase the diversity of solutions. Additionally, it may be combined routing with location-allocation problem. Furthermore, given the successful application of a NSACO to the bi-objective warehouse allocation problem, the used algorithm can be modified to obtain non-dominated solutions for warehouse allocation problems with more than two objectives.

## References


1. Simchi-Levi, D. Kaminsky, P. and Simchi-Levi, E., *Designing and Managing the Supply Chain*, New York: Irwin McGraw-Hill (2000).
2. ReVelle, C. "The maximum capture or 'sphere of influence' location problem: Hoteling revisited on a network", *Journal of Regional Science*, **26**(2), pp. 343-358 (1986).
3. Daskin, M.S., *Network and Discrete Location: Models Algorithms and Applications*, New York: Wiley (1995).
4. Warszawski, A. and Peer, S. "Optimizing the location of facilities on a building site", *Operational Research Quarterly*, **24**, pp. 35-44 (1973).
5. ReVelle, C.S., Eiselt, H.A. and Daskin, M.S. "A bibliography for some fundamental problem categories in discrete location science", *European Journal of Operational Research*, **184**, pp. 817-848 (2008).
6. Sridharan, R. "The capacitated plant location problem", *European Journal of Operational Research*, **87**, pp. 203-213 (1995).
7. Hajiaghaei-Keshteli, M. "The allocation of customers to potential distribution centers in supply chain networks: GA and AIA approaches", *Applied Soft Computing*, **11**, pp. 2069-2078 (2011).
8. Chen, C.L. and Lee, W.C. "Multi-objective optimization of multi-echelon supply chain networks, with uncertain product demands and prices", *Computers and Chemical Engineering*, **28**, pp. 1131-1144 (2004).
9. Zhou, G., Min, H. and Gen, M. "A genetic algorithm approach to the bi-criteria allocation of customers to warehouses", *International Journal of Production Economics*, **86**, pp. 35-45 (2003).
10. Ordonez, F. and Zhao, J. "Robust capacity expansion of network flows", *Networks*, **50**, pp. 136-145 (2007).
11. Blanquero, R., Carrizosa, E. and Hendrix, E.M.T. "Locating a competitive facility in the plane with a robustness criterion", *European Journal of Operational Research*, **215**, pp. 21-24 (2011).
12. Baron, O., Milner, J. and Naseraldin, H. "Facility location: A robust optimization approach", *Production and Operations Management*, **20**, pp. 772-785 (2011).
13. Gabrel, V., Lacroix, M., Murat, C. and Remli "Robust location transportation problems under uncertain demands", *Combinatorial Optimization, Discrete Applied Mathematics*, **164**(1), pp. 100-111 (2014).
14. Gulpinar, N., Pachamanova, D. and Canakoglu, E. "Robust strategies for facility location under uncertainty", *European Journal of Operational Research*, **225**, pp. 21-35 (2013).
15. Ghahtarani, A. and Najafi, A.A. "Robust goal programming for multi-objective portfolio selection problem", *Economic Modeling*, **33**, pp. 588-592 (2013).
16. Xing, H. and Qu, R. "A non-dominated sorting genetic algorithm for bi objective network coding based multicast routing problems", *Information Sciences.*, **233**, pp. 36-53 (2013).
17. Chan, F.T.S. and Kumar, N. "Effective allocation of customers to distribution centers: A multiple ant colony optimization approach", *Robotics and Computer-Integrated Manufacturing*, **25**(1), pp. 1-12 (2009).
18. Kalhor, E., Khanzadi, M., Eshtehardian, E. and Afshar, A. "Stochastic time-cost optimization using non-dominated archiving ant colony approach", *Automation in Construction*, **20**, pp. 1193-1203 (2011).
19. Mostafavi, S.A. and Afshar, A. "Waste load allocation using non-dominated archiving multi-colony ant algorithm", *Procedia Computer Science*, **3**, pp. 64-69 (2011).
20. Srinivas, N. and Deb, K. "Multi-objective optimization using non-dominated sorting genetic algorithms", *Evolutionary Computation*, **2**(3), pp. 221-248 (1994).
21. Deb, K., Agrawal, S., Pratap, A. and Meyarivan, T. "A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II", in *Proceedings of the 6th International Conference on Parallel Problem Solving from Nature*, pp. 849-58 (2000).
22. Bhattacharya, R. and Bandyopadhyay, S. "Solving conflicting bi-objective facility location problem by NSGA II evolutionary algorithm", *The International Journal of Advanced Manufacturing Technology*, **51**, Issue 1-4, pp. 397-414 (2010).
23. Shankar, B.L., Basavarajappa, S., Chen, J.C.H. and Kadadevaramath, R.S. "Location and allocation decisions for multi-echelon supply chain network -A multi-objective evolutionary approach", *Expert Systems with Applications*, **40**, pp. 551-562 (2013).
24. Sadeghi, J., Sadeghi, S. and Akhavan Niaki, S.T. "A hybrid vendor managed inventory and redundancy allocation optimization problem in supply chain management: An NSGA-II with tuned parameters", *Computers & Operations Research*, **41**, pp. 53-64 (2014).
25. Ok, S.Y., Lee, S.Y. and Park, W. "Robust multi-objective maintenance planning of deteriorating bridges against uncertainty in performance model", *Advances in Engineering Software*, **65**, pp. 32-42 (2013).
26. Baohua, W. and Shiwei, H. "Robust optimization model and algorithm for logistics center location and allocation under uncertain environment", *Journal of Transportation Systems Engineering and Information Technology*, **9**(2), Online English edition of the Chinese language journal (2009).
27. Talbi, E.G., *Meta-Heuristics: From Design to Implementation*, New York: Wiley (2009).
28. Mirchandani, P.B. and Francis, R.L., *Discrete Location Theory*, New York: Wiley (1990).
29. Minoux, M. "Robust network optimization under-polyhedral demand uncertainty is NP-hard", *Discrete Applied Mathematics*, **158**, pp. 597-603 (2010).

30. Deb, K., *Multi-Objective Optimization Using Evolutionary Algorithms*, England: John Wiley and Sons, Ltd (2001).
31. Dorigo, M. and Stutzle, T., *Ant Colony Optimization*, Cambridge, MA: MIT Press (2004).
32. Xia, X. "Particle swarm optimization method based on chaotic local search and roulette wheel mechanism", *International Conference on Applied Physics and Industrial Engineering, Physics Procedia*, **24**(A), pp. 269-275 (2012).
33. Tamura, K. and Miura, S. "Necessary and sufficient conditions for local and global non-dominated solutions in decision problems with multi-objectives", *Journal of Optimization Theory and Applications.*, **28**(4), pp. 501-523 (1979).
34. Zitzler, E. "Evolutionary algorithms for multi-objective optimization: Methods and applications", PhD. Thesis, Dissertation ETH No. 13398, Swiss Federal Institute of Technology (ETH) (1999).
35. Karimi, N., Zandieh, M. and Karamooz, H.R. "Bi-objective group scheduling in hybrid flexible flow shop: A multi-phase approach", *Expert Systems with Applications*, **37**, pp. 4024-4032 (2010).


## Appendix A

The robust Pareto solutions for problem 6 in small scale in 200th iteration by NSACO algorithm are as follows (where number of customers = 14, number of DCs = 6, types of vehicles = 3):

Number of robust Pareto front members = 2.

Robust Pareto front:

For the 1st element of Robust Pareto front, depot vector is: $y = 1\ 1\ 1\ 1\ 1\ 1$.
Allocation matrix for the 1st element of Robust Pareto front is shown in Table A.1.

For the 2nd element of robust Pareto front, depot vector is: $y = 1\ 1\ 1\ 1\ 0\ 1$.
Allocation matrix for the 2nd element of Robust Pareto front is shown in Table A.2.

Final objective values:
For the 1st element of robust Pareto front, objective values are:

Total cost = 1.1391e+006,
Transit time = 3.2200.

For the 2nd element of robust Pareto front, objective values are:

Total Cost = 1.1747e+006,
Transit Time = 3.1300.

**Table A.1.** Allocation matrix for the 1st element of robust Pareto front.

| | $x =$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | DC1 | DC2 | DC3 | DC4 | DC5 | DC6 | Types of vehicles |
| Customer 1 | 0 | 0 | 1 | 0 | 0 | 0 | 3 |
| Customer 2 | 0 | 0 | 1 | 0 | 0 | 0 | 2 |
| Customer 3 | 0 | 0 | 0 | 1 | 0 | 0 | 2 |
| Customer 4 | 0 | 0 | 1 | 0 | 0 | 0 | 3 |
| Customer 5 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Customer 6 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| Customer 7 | 1 | 0 | 0 | 0 | 0 | 0 | 3 |
| Customer 8 | 0 | 1 | 0 | 0 | 0 | 0 | 2 |
| Customer 9 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| Customer 10 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| Customer 11 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| Customer 12 | 0 | 1 | 0 | 0 | 0 | 0 | 2 |
| Customer 13 | 0 | 0 | 0 | 1 | 0 | 0 | 3 |
| Customer 14 | 0 | 0 | 0 | 0 | 1 | 0 | 3 |

**Table A.2.** Allocation matrix for the 2nd element of robust Pareto front.

| | $x =$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | DC1 | DC2 | DC3 | DC4 | DC5 | DC6 | Types of vehicles |
| Customer 1 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| Customer 2 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| Customer 3 | 0 | 0 | 0 | 1 | 0 | 0 | 3 |
| Customer 4 | 0 | 1 | 0 | 0 | 0 | 0 | 2 |
| Customer 5 | 0 | 0 | 1 | 0 | 0 | 0 | 3 |
| Customer 6 | 0 | 1 | 0 | 0 | 0 | 0 | 3 |
| Customer 7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Customer 8 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| Customer 9 | 0 | 0 | 1 | 0 | 0 | 0 | 3 |
| Customer 10 | 1 | 0 | 0 | 0 | 0 | 0 | 2 |
| Customer 11 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| Customer 12 | 1 | 0 | 0 | 0 | 0 | 0 | 2 |
| Customer 13 | 0 | 0 | 1 | 0 | 0 | 0 | 3 |
| Customer 14 | 1 | 0 | 0 | 0 | 0 | 0 | 3 |

## Biographies

**Jafar Bagherinejad** received his PhD degree from Bradford University, UK (1997), and is an Associate Professor of the Industrial Engineering Department, Faculty of Engineering and Technology, Alzahra University, Tehran, Iran. His research interest areas are location-allocation problems including queuing models,

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