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Optimal multi-discount selling prices schedule for deteriorating product

A.A. Taleizadeh*, **F. Satarian and A. Jamili**

School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.

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Abstract. This paper investigates optimal multi discount price and order quantity for deteriorating product. We initially consider a time dependent demand function with two scenarios including positive exponential for the first interval and negative exponential for the second one, due to the obsolescent characteristic, without any exogenous factor. Then, we study the effect of changing selling price as an exogenous factor causing increase in demand. Finally, optimization model is formulated and the closed form solutions of the optimal prices are gained.

1. Introduction and literature review

One of the important assumptions in inventory management problem is the planning horizon of the product. There are two general categories for these problems: single period and multi period models. The first category, also known as the newsboy or newsvendor problem, is for items with a limited application time, such as those that must be prepared only at the beginning of each period and will be discarded at the end of the period or will be sold with special policies. In such a manner, the retailer adopts different policies, like pricing, to enhance sales and control inventories remaining at the end of the period. Therefore, we see many papers that have considered optimal Economic Order Quantity (EOQ) plus appropriate pricing policies as a new powerful approach in revenue management used for increasing the sales. Some of these works are mentioned in the following. Wang and Tung [1] proposed an EOQ model for gradual obsolescent products under multi-discount selling prices. Khouja [2] considered a newsboy problem with multiple discounts. Maximizing expected profit and the probability of achieving a target profit are two commonly used objectives of this work. Lau and Lau [3] developed a model based on the strategy of price decrease for demand increase. You and Hsieh [4] studied optimal selling strategies for a seasonal item when the inventory model is continuous. Urban and Baker [5] investigated pricing policy and economic order quantity for a problem with deterministic demand function depending on time, price, and level of the inventory. Dutta et al. [6] proposed an inventory model which had a fuzzy random demand variable. Lodree et al. [7] introduced a newsvendor problem with time and quantity dependent waiting cost in which all shortages are backlogged.

Inventory systems are always involved with the features of their items. Two imperative attributes considered in this paper are obsolescent and deterioration. Items such as festival decorating, seasonal clothes, etc. have the characteristic of obsolescence. In fact, as Cobbaert and Oudheusden [8] stated for obsolescent feature, their demand decreases after a certain point of time and they are not needed anymore, probably because they are replaced by a better substitute with higher performance, as Brown et al. [9] declared, or the

*. *Corresponding author. Tel.: +98 21 82084486;*
Fax: +98 21 88013102
E-mail addresses: Taleizadeh@ut.ac.ir, (A.A. Taleizadeh); neyreez@gmail.com (F. Satarian); ajamili@ut.ac.ir (A. Jamili)

environmental conditions, related to their application, changes. Delft and vital [10] considered a problem in which items become obsolescent with the probability of negative exponential distribution and investigated optimal inventory policies for it. Arcelus et al. [11] studied an inventory problem with both time and selling price dependent demand function involving a certain stochastic demand point. The demands immediately reached zero after this point. Ghare and Schrader [12] investigated exponential decay effect in standard EOQ model, considering the lifetime of a product as a random variable with negative exponential distribution. Cárdenas-Barrón et al. [13] studied the solution approaches of a multi-product Economic Production Quantity (EPQ) with budget constraint. They suggested an alternate heuristic algorithm which is more efficient in comparison with other existent algorithms in the literature. Maleki Vishkaei et al. [14] investigated economic order model under shortage and delay in payments. This study also considered a complete 100% screening process for defective items.

For deterioration issue, since it has a significant effect on the inventory model, the loss related to it cannot be ignored, and as it is a common phenomenon in real life for many items such as drugs, blood banks, etc., it has been widely studied in recent years. Firstly, Ghare and Schrader [15] discussed EOQ model in the case of continuous decaying inventory with constant demand. Shah and Jaiswal [16] investigated constant deterioration rate in a deterministic environment, whereas Covert and Philip [17] considered an inventory model with variable deterioration rate and constant demand. Philip [18] extended the previous work by involving three parameters of Weibull distribution for a variable deteriorating rate. Levin et al. [19] provided an exhaustive and detailed review for the deteriorating product in inventory management literature. Balkhi [20] developed an inventory control model for the deteriorating item under time value of money consideration. Taleizadeh et al. [21] proposed a new rough Economic Order Quantity (EOQ) model that was a mixed integer nonlinear programming type. Their model considered quantity discount and prepayment for the deteriorating items. Wu et al. [22] developed an economic order quantity model from the retailer's perspective which considered up and down stream trade credits and expiration dates for deteriorating items. Chung et al. [23] proposed an EPQ inventory model for an integrated supply chain system with three layers under two levels of trade credit. Widyadana et al. [24] developed an inventory control model for the deteriorating products with and without backorders. Chung and Cárdenas-Barrón [25] presented a simplified solution algorithm for an inventory - supply chain management problem. In this study, deterioration rate, stock dependent demand rate, and two levels of trade credit in the supply chain management are considered. In the field of time varying deterioration, Sett et al. [26] developed an inventory control model with two warehouse and quadratic increasing demand functions to determine the order quantity such that the total cost was minimized.

In recent years, the literature for the joint study of inventory models and pricing strategy in the case of deteriorating items has been extended vastly. Maihami and Kamalabadi [27] developed a joint pricing and inventory control system for the non-instantaneous deteriorating item with a price and time dependent demand function. Mishra and Mishra [28] analyzed price determination for an EOQ model of deteriorating items under perfect competition. Abad [29] investigated optimal pricing and lot sizing for perishable products when partial backordering is allowed. Dye [30] studied deteriorating inventory with partial backlogging, fond optimal pricing, and ordering policy. Dye et al. [31] discussed optimal pricing and inventory strategies for deteriorating items with shortage. It is assumed that demand and deterioration rates are continuous functions depending on time and price, correspondingly. Cai et al. [32] used game theory approach for the best possible pricing and ordering decisions with partial lost sale in two stages of the supply chain. Smith et al. [33] studied the benefits of optimal price and order quantity in the case, during a specific time. Yang et al. [34] studied pricing and replenishment strategies for a multi-market deteriorating product with time-varying and price-sensitive demand. Yang et al. [35] studied collaboration for a closed-loop deteriorating inventory supply chain with multi-retailer and price-sensitive demand.

The remainder of the paper is organized as follows. In Section 2, the problem is defined in details. In Section 3, the problem is modeled by defining notations, demand functions used in this study, the specific inventory model, and optimization of the maximum profit. Section 4 contains two numerical examples illustrating efficiency of the model. Section 5 presents a sensitivity analysis on the model parameters, and finally, the conclusion comes in Section 6.

2. Problem definition

We consider an inventory model of deteriorating gradual obsolescent product. Some items have a short lifecycle and they are needed for a short period of time; so, their demand first increases up to a peak point (maximum demand), and then, after a certain point, drops to reach zero. As an example of this type, we can mention a deteriorating obsolescent item related to only special occasions such as holidays. One of the effective marketing ways for increasing the demand is to decrease its price by multi-discount selling prices.

In this paper, we have some equal short time periods, and at each period, we change the price. The effect of changing the price is included in the demand model through adding the new term of $d(p_{i-1} - p_i)$ to the demand model which is related to, $D = \alpha - \beta P$, a linearly price dependent demand function. So the new demand function depends on two factors, both time and price. Now, the objective of this paper is to find optimal selling prices and order the quantity maximizing the total profit function. We assume that the deterioration rate is constant, shortage is not permitted, lead time is zero, and there is only one prospect to acquire items at the start of selling period. The whole order quantity is received at the same time.

As described above, this study proposes a new pricing model in the literature of inventory and pricing for deteriorating items with variable demand rates during different periods. In addition, since the total profit function is concave, closed form solutions for optimal price values, during different selling periods, are obtained. According to the above literature review section, there is no research in which all these factors are considered and it is a novel model for improving total profits of deteriorating products with multi-discount selling strategy.

3. Modeling

To model the problem, we first define the notations and then construct the new demand function with deteriorating and multi-selling price effect. Next, we gain total profit function and, at the end, closed form solution for optimal discount prices is presented.

3.1. Notations

To model the problem, the following notations are used, some of which are used from Wang and Tung [1]:

Parameters

μ	The time of peak demand;
U	The upper bound of the demand;
w, z	The given coefficients of the first demand function;
g, f	The given coefficients of the second demand function;
θ	The constant deterioration rate;
D_0	The initial demand;
L	The length of horizon planning;
n	The total number of subintervals;
T	The length of subintervals, $\frac{L-\mu}{n}$;
$I_0(t)$	The inventory level at time t, $0 \le t \le \mu$;
$I_i(t)$	The ith inventory level at time t, $0 \le t \le T$, $i = 1, 2, 3, \cdots, n$;
P_o	The regular selling price during interval $[0, \mu]$;
A	The fixed ordering cost;
c	The unit purchasing cost;
h	The unit holding cost per unit time,;
K	The cost incurred by each price change, including changing of price lists, tags, product catalogue.

Decision variables

P_i	The selling price during the ith interval, $i = 1, 2, 3, \cdots, n$;
Q	The order quantity.

Dependent variables

$E(P_1, P_2, P_3, \cdots, P_n)$ The retailer's total profit over the entire time span $[0, L]$.

3.2. The demand function and the inventory model

The demand function for gradually obsolescent products, without consideration of exogenous effects, consists of two scenarios. The first one is related to interval time $t \le \mu$ with the demand $D_1(t)$ and upper bound of U such that when t increases, $D_1(t)$ increases too. Moreover the second scenario is for when $t > \mu$ in which $D_2(t)$ decreases when t increases. We assign $D_1(t)$ and $D_2(t)$ in this integrated demand model as follow:

$$D_1(t) = e^{w(t-z)}, \qquad 0 \le t \le \mu, \quad w(t-z) > 0, \quad (1)$$

$$D_2(t) = e^{g(t-f)}, \qquad t > \mu, \qquad g(t-f) < 0. \quad (2)$$

Commonly, the retailer initially sets a regular selling price for time interval $[0, \mu]$. But, since obsolescence and deterioration cause demand reduction, after point μ, the retailer intends to implement a multiple pricing strategy for sales promotion. So, during demand-declining time interval, meaning $[\mu, L]$, we divide that into n equal subintervals $[0, \frac{L-\mu}{n}]$. This means that the price reduction times for n discounts are $\mu, \mu + (L - \mu)/n, \mu + 2(L - \mu)/n, \cdots$, and $\mu + (n-1)(L-\mu)/n$. Consequently, the second demand function with $t \in [\mu, L]$ should be altered as $D_2(\mu + (i-1)T + t)$ $t \in [0, T]$ for $i = 1, 2, \cdots, n$, adapting to the change of variable t. Then, we add the new term of $d(p_{i-1} - p_i)$ with $d > 0$ into the demand model, denoting the effect of selling price differences as we consider that there is a linear correlation between changes of prior and present prices and there is an increase in the demand function. So, the final integrated demand function is presented as follows:

For regular selling prices p_0:

$$D(t, p_0) = D_1(t) = e^{w(t-z)}, \qquad 0 \le t \le \mu, \quad (3)$$

For regular selling prices p_0:

$$D(t,p_{i-1},p_i)=D_2(\mu+(i-1)T+t)+d(p_{i-1}-p_i)$$
$$=e^{g(\mu+(i-1)T+t-f)}+d(p_{i-1}-p_i),$$
$$0<t\leq T, \qquad i=1,2,3,\cdots,n, \tag{4}$$

where d is a positive constant and g is a negative one.

During interval $[0,\mu]$, demand rate is $D(t;p_0)$ with the constant deteriorating rate θ, thus the inventory level $I_0(t)$ at time t is gained by:

$$\frac{dI_0(t)}{dt}=-D_1(t)-\theta I_0(t)=-e^{w(t-z)}-\theta I_0(t),$$
$$0\leq t\leq \mu, \tag{5}$$

with the initial condition $I_0(0)=Q$.

Furthermore, during the ith subinterval, D (t, p_{i-1}, p_i) is the demand rate with the constant deteriorating rate θ, and the ith inventory level $I_i(t)$ at time t is:

$$\frac{dI_i(t)}{dt}=-(e^{g(\mu+(i-1)T+t-f)}+d(p_{i-1}-p_i))-\theta I_i(t),$$
$$0\leq t\leq T, \qquad i=1,2,3,\cdots,n, \tag{6}$$

such that $I_n(T)=0$. The solutions of differential Eqs. (5) and (6) are:

$$I_0(t)=Q-\frac{e^{w(t-z)}}{w+\theta}, \qquad 0\leq t\leq\mu, \tag{7}$$

$$I_i(t)=I_i(0^+)-\left[\frac{e^{g(\mu+(i-1)T+t-f)}}{g+\theta}+\frac{d(p_{i-1}-p_i)}{\theta}\right],$$
$$0\leq t\leq T, \qquad i=1,2,3,\cdots,n, \tag{8}$$

where $I_i(0^+)=\lim\limits_{t\to 0^+} I_i(t)$.

Solving the boundary condition $I_n(T)=0$ in Eq. (8), we have:

$$I_n(0^+)=\left[\frac{e^{g(\mu+nT-f)}}{g+\theta}+\frac{d(p_{n-1}-p_n)}{\theta}\right]. \tag{9}$$

By imposing continues conditions of $I_i(T)=I_{i+1}(0^+)$ and substituting $t=T$ into Eq. (8), we have:

$$I_i(0^+)=\frac{e^{g(\mu-f)}}{g+\theta}e^{gTi}\left(\frac{1-e^{gT(n-i+1)}}{1-e^{gT}}\right]$$
$$+\frac{d(p_{i-1}-p_n)}{\theta}, \qquad i=1,2,3,\cdots,n. \tag{10}$$

Now, by substituting Eq. (10) into Eq. (8), we have:

$$I_i(t)=\frac{e^{g(\mu+(i-1)T-f)}}{g+\theta}\left[e^{gT}\left(\frac{1-e^{gT(n-i+1)}}{1-e^{gT}}\right)-e^{gt}\right]$$
$$+\frac{d(p_i-p_n)}{\theta},$$
$$0\leq t\leq T, \qquad i=1,2,3,\cdots,n. \tag{11}$$

Similarly, the continuous condition for the first scenario gives the following formula:

$$Q=\frac{e^{g(\mu-f)}}{g+\theta}e^{gT}\left(\frac{1-e^{gTn}}{1-e^{gT}}\right)+\frac{d(p_0-p_n)}{\theta}+\frac{e^{w(\mu-z)}}{w+\theta}. \tag{12}$$

3.3. Retailer's total profit function

The retailer's total profit function will be made of the ordering cost, price changing cost, purchasing cost, sale revenues, and holding cost. Let S be the sale revenues; then, we have:

$$S=P_0(Q-I_0(\mu))+\sum_{i=1}^{n}P_i(I_i(0^+)-I_i(T))$$
$$=P_0\frac{e^{w(\mu-z)}}{w+\theta}+\sum_{i=1}^{n}P_i\left(\frac{e^{g(\mu+iT-f)}}{g+\theta}+\frac{d(p_{i-1}-p_i)}{\theta}\right). \tag{13}$$

Also, the holding cost, denoted by H, is given as follows:

$$H=\int_0^{\mu}hI_0(t)dt+\sum_{i=1}^{n}\int_0^{T}hI_i(t)dt$$
$$=hQ\mu-h\int_0^{\mu}\frac{e^{w(t-z)}}{w+\theta}dt+\sum_{i=1}^{n}hTI_i(0^+)$$
$$-\sum_{i=1}^{n}\int_0^{T}h\frac{e^{g(\mu+(i-1)T+t-f)}}{g+\theta}-\sum_{i=1}^{n}h\frac{d}{\theta}T(P_{i-1}-P_i). \tag{14}$$

As a result, the retailer's total profit, represented by $TP(P_1,P_2,\cdots,P_n)$, is calculated by:

$$TP(Q,P_1,P_2,\cdots,P_n)=S-A-nk-cQ-H$$
$$=P_0\left(\frac{e^{w(\mu-z)}}{w+\theta}\right)+\sum_{i=1}^{n}P_i\left(\frac{e^{g(\mu+iT-f)}}{g+\theta}\right.$$
$$\left.+\frac{d(p_{i-1}-p_i)}{\theta}\right)-A-nK$$
$$-(c+h\mu)\left[\frac{e^{g(\mu-f)}}{g+\theta}e^{gT}\left(\frac{1-e^{gTn}}{1-e^{gT}}\right)\right.$$
$$\left.+\frac{d(p_0-p_n)}{\theta}+\frac{e^{w(\mu-z)}}{w+\theta}\right]+h\int_0^{\mu}\frac{e^{w(t-z)}}{w+\theta}dt$$

$$- hT\sum_{i=1}^{n}\left[\frac{e^{g(\mu-f)}}{g+\theta}e^{gTi}\left(\frac{1-e^{gT(n-i+1)}}{1-e^{gT}}\right)\right.$$

$$\left.+\frac{d(p_{i-1}-p_n)}{\theta}\right]+\sum_{i=1}^{n}\int_0^T h\frac{e^{g(\mu+(i-1)T+t-f)}}{g+\theta}$$

$$+\sum_{i=1}^{n}h\frac{d}{\theta}T(P_{i-1}-P_i). \tag{15}$$

Taking the first order partial derivative of Eq. (15) with respect to P_i, $i=1,2,3,\cdots,n$, we gain:

$$\frac{\partial TP}{\partial P_i}=\frac{e^{g(\mu+iT-f)}}{g+\theta}+\frac{d}{\theta}(P_{i-1}-2P_i+P_{i+1})-hT\frac{d}{\theta},$$

$$i=1,\cdots,n-1, \tag{16}$$

$$\frac{\partial TP}{\partial P_n}=\frac{e^{g(\mu+nT-f)}}{g+\theta}+\frac{d}{\theta}(P_{n-1}-2P_n)+\frac{cd}{\theta}$$

$$+\frac{h\mu d}{\theta}+\frac{hdT(n-1)}{\theta}. \tag{17}$$

Sequentially, for any $i,j=1,2,3,\cdots,n$, the second-order partial derivatives of E, with respect to P_i and P_j, are confirmed being reliable with the following characteristics:

$$\frac{\partial^2 TP}{\partial P_i\partial P_j}=\frac{-2d}{\theta},\qquad \text{if}\quad i=j, \tag{18}$$

$$\frac{\partial^2 TP}{\partial P_i\partial P_j}=\frac{d}{\theta},\qquad \text{if}\quad |j-i|=1, \tag{19}$$

$$\frac{\partial^2 TP}{\partial P_i\partial P_j}=0,\qquad \text{if}\quad |j-i|>1. \tag{20}$$

To maximize the total profit $TP(Q,P_1,P_2,\cdots,P_n)$, we need to calculate the following matrix:

$$H=\begin{pmatrix}\frac{\partial^2TP}{\partial P_1^2} & \cdots & \frac{\partial^2TP}{\partial P_n\partial P_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial^2TP}{\partial P_1\partial P_n} & \cdots & \frac{\partial^2TP}{\partial P_n^2}\end{pmatrix}$$

$$=\begin{pmatrix}\frac{-2d}{\theta} & \frac{d}{\theta} & & 0\\ \frac{d}{\theta} & & & \\ & \ddots & & \\ & & & \frac{d}{\theta}\\ 0 & & \frac{d}{\theta} & \frac{-2d}{\theta}\end{pmatrix}. \tag{21}$$

Assume Δ_m represents the determinant of $m\times m$ matrix obtained by deleting the last n-m rows and columns of Hessian matrix shown in Eq. (21). Also, we have $\Delta_0=1$. Since the values of $\Delta_0,\Delta_1,\Delta_2,\cdots,\Delta_n$ are alternately positive and negative for all p_i, $i=1,\cdots,n$, $n\in N$, we can conclude that the total profit $TP(P_1,P_2,\cdots,P_n)$ has a global maximum point [1,36]. For obtaining the optimal selling prices, the necessary conditions:

$$\frac{\partial E}{\partial p_i}=0,\qquad i=1,\cdots,n,\qquad n\in N,$$

must be satisfied, simultaneously. These conditions result in the following system of linear equations:

$$\frac{\partial TP}{\partial P_i}=\frac{e^{g(\mu+iT-f)}}{g+\theta}+\frac{d}{\theta}(P_{i-1}-2P_i+P_{i+1})$$

$$-hT\frac{d}{\theta}=0,\qquad i=1,\cdots,n-1, \tag{22}$$

$$\frac{\partial TP}{\partial P_n}=\frac{e^{g(\mu+nT-f)}}{g+\theta}+\frac{d}{\theta}(P_{n-1}-2P_n)$$

$$+\frac{cd}{\theta}+\frac{h\mu d}{\theta}+\frac{hdT(n-1)}{\theta}=0. \tag{23}$$

Defining O_i as a constant for every specific i:

$$O_i=-\left[\frac{e^{g(\mu+i.T-f)}}{g+\theta}-hT\frac{d}{\theta}\right]\frac{\theta}{d},\quad i=1,\cdots,n-1, \tag{24}$$

$$O_n=-\left[\frac{e^{g(\mu+nT-f)}}{g+\theta}+(c+h\mu+hT(n-1))\frac{d}{\theta}\right]\frac{\theta}{d}, \tag{25}$$

which yield the following closed form solutions for the optimal prices:

$$P_1=\frac{-(O_n+2O_{n-1}+3O_{n-2}+\cdots+nO_1)+nP_0}{(n+1)}, \tag{26}$$

$$P_i=O_{i-1}+2O_{i-2}+3O_{i-3}+\cdots+(i-1)O_1$$

$$+iP_1-(i-1)P_0,\qquad i=2,\cdots,n. \tag{27}$$

4. Numerical example

In this section, we achieved optimal selling prices and maximal total profit for the same parameter values in the case of $n=1,2,\cdots,6$ to investigate its effect on the maximal total profit, since theoretical analysis for the optimal value of n is somehow difficult.

Example 1. Let $U=1000$, $A=1000$, $g=-7$, $f=3$, $p_0=100$, $\mu=2$, $L=3$, $k=200$, $h=0.5$, $D_0=90$, $d=50$, $c=30$, $\theta=0.05$, $w=1.2$ and $z=-3.7$.

For a single discount $n=1$, we have the optimal solution of $P_1=65.50$ at time $t_1=2$, $TP_1=$

1234177.8, and $Q_1 = 35274.92$. For two discounts $n = 2$, the optimal solution is $P_1 = 76.9$ at time $t_1 = 2$, $P_2 = 54.08$ at $t_2 = 2.5$, $TP_2 = 1639123.7$, and $Q_2 = 46688.6$. For three discounts $n = 3$, the optimal solution is $P_1 = 82.6$ at time $t_1 = 2$, $P_2 = 65.4$ at time $t_2 = 2.33$, $P_3 = 48.4$ at time $t_3 = 2.66$, $TP_3 = 1837788$, and $Q_3 = 52389.9$ (P_i= the optimal selling price at the ith period, TP= the maximal profit and Q= the optimal order quantity). In this case, the original demand function without the price change effects is shown in Figure 1.

Example 2. Consider the parameter values of the Example 1 to investigate the impact of n on maximal profit for which we can determine the maximal profit TP_n from $n = 1$ to $n = 6$. Table 1 shows that when n increases, maximal profit reduces, considerably, from 24.71% to 2.64%. As a result, too many discounts are not an efficient solution. As Wang and Tung [1] revealed, this can have negative effects on the consumer perception about the product from the managerial point of view. To explore this, more, a result analysis is presented in the next section.

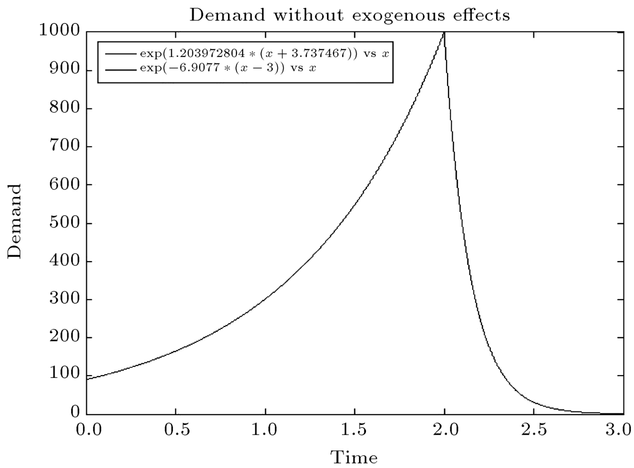

Figure 1. Demand function of examples.

5. Result analysis

Table 2 reveals some important results which can guide manager's inventory control as follow. With respect to high unit holding cost h, the retailer prefers to have a less amount Q for reducing the inventory level, thereby, avoiding high holding cost. Also, we know from the prior experiment that an approach to balance losses for high value h is to improve the selling price. We see that, except for the first period, the optimal selling prices increase as h increases, since Q decreases in this process.

In the case of high unit purchasing cost c, the retailer wants to order a less amount of order quantity to reduce a high purchasing cost, plus to rise the selling price offsetting negative effect due to high value of unit purchasing cost. So Q decreases, but P increases as c increases. As it is obvious, larger value of L means a longer selling price period. Experiment shows that it is much better for the retailer to choose the sale strategy which offers lower price to gain more sales instead of price consideration. In a longer selling period, priority of amount of sales overcomes selling price, equivalently. Adversely, for the case of a short selling period, selling price plays a key role in making profit, in which setting higher price is preferable to a long selling period, though it causes sale reduction. Apparently, in the case of high deterioration rate, we expect the retailer to want to order a less amount of q to reduce high losses cost due to deterioration of inventory.

Also, it is logical to offer lower price, since its demand is less than that of the case of low deterioration rate. So, we see in Table 2 that by increase in the

Table 1. Results of Examples 1 and 2.

$U = 1000$	$A = 1000$	$g = -6.90$	$P_0 = 100$	$\mu = 2$	$w = 1.20$	$L = 3$
$h = 0.5$	$D_0 = 90$	$d = 50$	$c = 50$	$f = 3$	$z = -3.73$	$K = 200$
$\theta = 0.05$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
Q	35274.92	46688.6	52389.9	55801.5	58070.8	59692.5
E	1234177.8	1639123.7	1837788.6	1947306.6	2032836.6	2087928.8
P_1	65.50	76.91	82.61	86.02	88.29	89.91
P_2		54.08	65.40	72.20	76.72	79.95
P_3			48.36	58.51	65.26	70.08
P_4				44.94	53.91	60.31
P_5					42.65	50.62
P_6						41.015
Difference of E		404946	198665	109518	85530	55093
Proportion (%)		24.71	10.81	5.62	4.21	2.64

Table 2. Result analysis.

$U = 1000$	$A = 1000$	$D_0 = 90$	$K = 200$	$w = 1.20$	$P_0 = 100$	$z = -3.73$
$g = -6.90$	$f = 3$	$\mu = 2$				
	P_1	P_2	P_3	Q	E	**Increment of E (%)**
	$L = 3$	$a = 30$	$C = 30$	$\theta = 0.05$		
$h = 0.3$	82.5551	82.5551	82.5551	31949.07	56953713	
$h = 0.5$	82.6056	82.6056	82.6056	31739.37	56707559	-0.4322
$h = 0.7$	82.6561	82.6561	82.6561	31529.67	56459547	-0.43735
	$L = 3$	$d = 30$	$H = 0.5$	$\theta = 0.05$		
$c = 30$	82.6057	65.40193	48.36583	31739.37	56707559	
$c = 40$	85.1057	70.40193	55.86583	27239.37	51966607	-8.3603
$c = 50$	87.6057	75.40193	63.36583	22739.37	46035655	-11.413
	$L = 3$	$c = 30$	$H = 0.5$	$\theta = 0.05$		
$d = 30$	82.4407	65.07193	47.87083	31739.37	1124809	
$d = 40$	82.4489	65.07816	47.874	42064.62	1483580	31.8962
$d = 50$	82.4524	65.08083	47.87536	52389.87	1838944	23.9531
	$d = 30$	$c = 30$	$H = 0.5$	$\theta = 0.05$		
$L = 2.5$	82.6133	65.3890	48.2729	31746.26	1122882	
$L = 3.0$	82.6057	65.4019	48.3658	31739.37	1124809	0.1716
$L = 3.5$	82.5581	65.3718	48.4334	31710.19	1125345	0.0477
	$L = 3$	$c = 30$	$H = 0.5$	$d = 30$		
$\theta = 0.03$	82.6139	65.4082	48.3690	52389.87	1836664	
$\theta = 0.05$	82.6057	65.4019	48.3658	31739.37	1122529	-38.8822
$\theta = 0.07$	82.5974	65.3957	48.3627	22889.08	816468.7	-27.2652

deterioration rate, order quantity, selling prices, and maximal profit, all, decrease and it has a great effect on the maximal profit.

In the proposed model parameters, c, h, and θ have negative effect on the maximal profit, while the parameters L and d have a positive one. In addition, we see from Table 2 that the most influential parameters on maximal profit are θ, d, and c, respectively.

6. Conclusion

This study describes integrated pricing and ordering strategies for a deteriorating gradual obsolescent item by implementing multiple price discounts during the time when demand decreases. We derived the concavity of total profit function and closed form solution to determine the optimal values of price and order quantity. Example (1) shows that, in general, multiple discounts generate higher profit than that of a single discount, even though Example (2) demonstrates that too many discounts cannot increase the profit, significantly, and have a marginal rate of profit increase. Moreover, it causes negative consumer perception about the product. For more research in future, two practical assumptions, an unequal time subinterval for price discounts and uncertain demand, could be included in the model. Also, the model can be extended in the cases of multi-products, time varying deterioration, or considering expiration dates for deteriorating items.

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References

1. Wang, K.H. and Tung, C.T. "Construction of a model towards EOQ and pricing strategy for gradually obsolescent products", *Applied Mathematics and Computations*, **217**, pp. 6926-6933 (2011).
2. Khouja, M. "The newsboy problem under progressive multiple discounts", *European Journal of Operational Research*, **84**, pp. 458-466 (1995).
3. Lau, A. and Lau, H. "The newsboy problem with price-dependent demand distribution", *IIE Transaction*, **20**, pp. 168-175 (1998).
4. You, P.S. and Hsieh, Y.G. "An EOQ model with stock and price sensitive demand", *Mathematical and Computer Modelling*, **45**, pp. 933-942 (2007).

5. Urban, T.L. and Baker, R.C. "Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns", *European Journal of Operational Research*, **103**, pp. 573-583 (1997).

6. Dutta, P., Chakraborty, D. and Roy, A.R. "A single-period inventory model with fuzzy random variable demand", *Mathematical and Computer Modelling*, **41**, pp. 915-922 (2005).

7. Lodree Jr, E.J., Kim, Y. and Jang, W. "Time and quantity-dependent waiting costs in a newsvendor problem with backlogged shortages", *Mathematical and Computer Model*, **47**, pp. 60-71 (2008).

8. Cobbaert, K. and Oudheusden, D.V. "Inventory models for fast moving spare parts subject to "sudden death" obsolescence", *International Journal of Production Economic*, **44**, pp. 239-248 (1996).

9. Brown, G.W., Lu, J.Y. and Wolfson, R.J. "Dynamic modelling of inventories subject to obsolescence", *Management Science*, **11**, pp. 51-63, (1964).

10. Delft, C. and Vial, J.P. "Discount costs, obsolescence and planned stockouts with the EOQ formula", *International Journal of Production Economic*, **44**, pp. 255-265 (1996).

11. Arcelus, F.J., Pakkala, T.P.M. and Srinivasan, G. "A myopic policy for the gradual obsolescence problem with price-dependent", *Computer and Operation Research*, **29**, pp. 1115-1127 (2002).

12. Ghare, P. and Schrader, G. "A model for exponentially decaying inventories", *Journal of Industrial Engineering*, **14**, pp. 238-243 (1963).

13. Cárdenas-Barrón, L.E., Treviño-Garza, G., Widyadana, G.A. and Wee, H.M. "A constrained multi-products EPQ inventory model with discrete delivery order and lot size", *Applied Mathematics and Computation*, **230**, pp. 359-370 (2014).

14. Maleki Vishkaei, B., Pasandideh, S.H. and Farhangi, M., "The economic order quantity model under shortage and delay in payment", *Scientia Iranica*, Article in press.

15. Ghare, P.M. and Schrader, G.H. "A model for exponentially decaying inventory system", *International Journal of Production Research*, **21**, pp. 449-460 (1963).

16. Shah, Y.K. and Jaiswal, M.C. "An order-level model for a system with constant rate of deterioration", *Opsearch*, **14**, pp. 174-184 (1977).

17. Covert, R.B. and Philip, G.S. "An EOQ model with Weibull distribution deterioration", *IIE Transactions*, **5**, pp. 323-326 (1973).

18. Philip, G.C. "A generalized EOQ model for item with Weibull distribution", *IIE Transactions*, **16**, pp. 159-162 (1974).

19. Levin, R.I., McLaughlin, C.P., Lamone, R.P. and Kottas, J.F., *Productions/Operations Management: Contemporary Policy for Managing Operating Systems*, McGraw-Hill, New york (1972).

20. Balkhi, Z. "Optimal economic ordering policy with deteriorating items under different supplier trade credits for finite horizon case", *International Journal of Production Economics*, **133**, pp. 216-223 (2011).

21. Taleizadeh, A.A., Wee, H.M. and Jolai, F. "Revisiting a fuzzy rough economic order quantity model for deteriorating items considering quantity discount and prepayment", *Mathematical and Computer Modelling*, **57**, pp. 1466-1479 (2013).

22. Wu, J., Ouyang, L.Y., Cárdenas-Barrón, L.E. and Goyal, S.K. "Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing", *European Journal of Operational Research*, **237**, pp. 898-908 (2014).

23. Chung, K.J., Cárdenas-Barrón, L.E. and Ting, P.S. "An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit", *International Journal of Production Economics*, **55**, pp. 310-317 (2014).

24. Widyadana, G.A., Cárdenas-Barrón, L.E. and Wee, H.M. "Economic order quantity model for deteriorating items with planned backorder level", *Mathematical and Computer Modelling*, **54**, pp. 1569-1575 (2011).

25. Chung, K.J., Cárdenas-Barrón, L.E. "The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade credit in the supply chain management", *Applied Mathematical Modelling*, **37**, pp. 4653-4660 (2013).

26. Sett, B.K., Sarkar, B. and Goswami, A. "A two-warehouse inventory model with increasing demand and time varying deterioration", *Scientia Iranica*, **9**, pp. 1969-1977 (2012).

27. Maihami, R. and Kamalabadi, N. "A joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand", *International Journal of Production Economics*, **136**, pp. 116-122 (2012).

28. Mishra, S.S. and Mishra, P.P. "Price determination for an EOQ model for deteriorating items under perfect competition", *Computer and Mathematics with Applications*, **56**, pp. 1082-1101 (2008).

29. Abad, P.L. "Optimal pricing and lot sizing under conditions of perishability and partial backordering", *Management Science*, **42**, pp. 1093-1104 (1996).

30. Dye, C.Y. "Joint pricing and ordering policy for a deteriorating inventory with partial backlogging", *Omega*, **35**, pp. 184-189 (2007).

31. Dye, C.Y., Quyang, L.Y. and Hsieh, T.P. "Inventory and pricing strategy for deteriorating items with shortages: a discounted cash flow approach", *Computer and Industrial Engineering*, **52**, pp. 29-40 (2007).

32. Cai, G., Chiang, W.C. and Chen, X. "Game theoretic pricing and ordering decisions with partial lost sales in two stage supply chains", *International Journal of Production Economics*, **130**, pp. 175-185 (2011).

33. Smith, N.R., Martinez-Flores, J.L. and Cardenas-Barron, L.E. "Analysis of the benefits of joint price and order quantity optimization using a deterministic profit maximization model", *Production Planning and Control*, **18**, pp. 310-318 (2007).

34. Yang, P.C., Wee, H.M., Chung, S.L. and Huang, Y.Y. "Pricing and replenishment strategy for a multi-market deteriorating product with time-varying and price-sensitive demand", *Journal of Industrial and Management Optimization*, **9**, pp. 769-787 (2013).

35. Yang, P.C., Chung, S.L., Wee, H.M., Zahara, E. and Peng, C.Y. "Collaboration for a closed-loop deteriorating inventory supply chain with multi-retailer and price-sensitive demand", *International Journal of Production Economics*, **143**, pp. 557-566 (2013).

36. Apostal, T.M., *Mathematical Analysis*, 2nd Ed., Addison-Wesley Publishing Company, Massachusetts (1997).

Biographies

Ata Allah Taleizadeh is an Assistant Professor at the School of Industrial Engineering in University of Tehran, Iran. He received his PhD in Industrial Engineering from Iran University of Science and Technology. Moreover, he received his BSc and MSc degrees, both in Industrial Engineering, from Azad University of Qazvin and Iran University of Science and Technology, respectively. His research interest areas include inventory control and production planning, pricing and revenue optimization, and uncertain programming. He has published several book-chapters and papers in reputable journals and, nowadays, he serves as the editor/editorial board member for a number of international journals.

Fatima Satarian received her MSc degree in Industrial Engineering from University of Tehran and BSc degree in Industrial Engineering from Alzahra University. Her research interests are in supply chain management, inventory control, and operation research.

Amin Jamili is an Assistant Professor at the School of Industrial Engineering in University of Tehran, Iran. He received his PhD and MSc in Industrial Engineering and Transportation Engineering, respectively, from Iran University of Science and Technology. Moreover, he received his BSc degree in Industrial Engineering from Sharif University of Technology. His research interest areas include transportation research, logistic systems, and operation management. He has published several papers in reputable journals.