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An inventory model for deteriorating items with inventory-dependent and linear trend demand under trade credit

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Abstract. One of the important issues in inventory management is permissible delay in payments. Previous inventory lot-size models allowing permissible delay in payments implicitly assumed that the demand rate is constant and inventory-dependent. However, this paper, unlike most existing models, this paper develops an Economic Order Quantity (EOQ) model for deteriorating items with a current inventory-dependent and linearly increasing time-varying demand under trade credit, which fits a more general inventory feature. An efficient solution procedure is shown to determine the optimal replenishment cycle of the model. Furthermore, this study deduces some previously published results as special cases of the proposed model. Finally, numerical examples are presented to illustrate the optimization procedure, and a sensitivity analysis is performed for changes in the parameters to obtain important and relevant findings on managerial implication.

1. Introduction

The traditional EOQ model considers a retailer to pay a purchasing cost for the items as soon as the items are received [1]. However, in today's competitive circumstance, the supplier may offer the retailer a delay period in settling the accounts. Trade credit often plays an important role in business transactions for several reasons. For the retailer, trade credit is an efficient method of binding the supplier when the retailer is at risk of receiving inferior quality goods and represents an effective means of reducing cost. For the supplier, it is an effective means of price discrimination that circumvents anti-trust measures and attracts new customers. The policy certainly adds extra cost and an extra dimension of default risk for the supplier [2]. Given the economic significance of trade credit, several papers have been published that probe inventory problems under varying conditions. Some notable papers are discussed below.

Goyal [1] presented an EOQ model where the supplier offers the retailer a permissible delay in payments. Chung [3] also discussed the economic quantity under permissible delay in payments. Aggarwal and Jaggi [4] then extended Goyal's model [1] to consider deteriorating items. Next, Jamal and Sarkar [5] further generalized Aggarwal and Jaggi's model [4] to allow for shortages. Chang and Dye [6] extended Jamal and Sarkar's model [5] to consider a varying deterioration rate and backlog rate. Later, Teng [7] further established an easy analytical closed-form solution drawing on Goyal's model [1]. In addition, Mohan et al. [8] investigated mathematical models for multi-item under the conditions of permissible delay in payment, a budget constraint and permissible partial payment at a penalty.

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These models assumed that the credit term is independent on the order quantity. Chang et al. [9] then extended Teng's model [7] and established an EOQ model for deteriorating items when credit policies are dependent on the order quantity. Chung et al. [10] then extended the model of Chang et al. [9] to consider cases where the retailer's capital is constrained. Ouyang et al. [11] proposed an EOQ model for deteriorating items with a partially permissible delay in payments relying on order quantity. Chen et al. [12] recently proposed an inventory model with conditionally trade credit link to order quantity.

Furthermore, Huang [13] first proposed an EOQ model under two levels of trade credit where the supplier permits delay in payments to the retailer, and the retailer provides its customer with the trade credit. Later, Teng and Goyal [14] improved the disadvantage of Huang's model [13]. Kreng and Tan [15] presented an EOQ model under two levels of trade credit when the order quantity is greater than or equal to the predetermined quantity. Other prominent works include those by Chang et al. [16], Chung and Cárdenas-Barrón [17], Ouyang et al. [18], Yadav et al. [19], Guchhait et al. [20], Chung et al. [21], and Wu et al. [22], among others.

However, in practice, for certain commodities, such as consumer goods and food grains, among others, the demand may depend on the quantity size on hand, which is usually influenced by some factors, such as advertisements. Levin et al. [23] indicated that *'It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more'*. Since the introduction of this problem, much work has considered inventory with inventory-dependent demand. Related articles include those by Gupta and Vrat [24], Baker and Urban [25], Su et al. [26], Widyadana [27], Cárdenas-Barrón [28], Balakrishnan et al. [29] and their references.

Liao et al. [30] first developed an inventory model with an inventory-dependent demand rate when a delay in payment is permissible. Sana and Chaudhuri [31] established an EOQ model with stock-dependent demand under delays in payments and price-discounts. Soni and Shah [32] and Min et al. [33] researched the optimal replenishment time with inventory-dependent demand under two level trade credits. Min et al. [34] further extended the model of Min et al. [33] to include cases with a finite replenishment rate. Sarkar [35] investigated the retailer's optimal order policy under permissible delay in payments with stock-dependent demand within the Economic Production Quantity (EPQ) framework.

However, in many real-life situations, the demand for items may always be in a dynamic state during the growth and decline phases of the product life cycle, where the demand is either increasing or decreasing with time. Many researchers have addressed time-dependent demand. Related articles include those by Silver and Meal [36], Wee [37], Omar and Smith [38], and Sarkar et al. [39] and their references. Chang et al. [40] first proposed an inventory model with a varying deterioration rate and a linear time-varying demand under trade credit. Sana and Chaudhuri [31] established an inventory model with a permissible delay in payments under time-quadratic demand and linear trend demand, respectively. Khanra et al. [41] recently extended the model of Chang et al. [40] to consider time-quadratic demand with a constant deterioration rate under trade credit. Teng et al. [42] extended the constant demand to a linear non-decreasing demand under permissible delay and established some fundamental theoretical results.

All of the above researchers established their inventory models under delay in payments by assuming that demand rate is constant, time-varying, or inventory-dependent. However, these assumptions are very restrictive in many real-life situations. In many real-life situations, for certain commodities, such as best selling consumer goods, the demand rate may depend on time and stock level. In addition, deterioration of items is a common phenomenon for some consumer goods. Therefore, this paper aims to develop an inventory model for deteriorating items with stock-dependent and linearly increasing demand under a permissible delay in payments, which will fit a more general inventory feature. This is the major contribution of the paper.

The remainder is organized as follows. In Section 2, the notations and assumptions are presented. In Section 3 and 4, the mathematical model is presented and an efficient solution procedure is developed, respectively. In Section 5, we develop numerical examples to illustrate the proposed model and their optimal solutions, analyze the effect of the optimal solution with respect to system parameters and obtain important and relevant conclusions on managerial phenomena. The last section summarizes the findings and implications and suggests areas for future research.

2. Notations and assumptions

The following notations and assumptions are used to develop the mathematical model in the paper. Some notations will be presented later when they are needed.

2.1. Notations

S	The ordering cost per order, \$/order;
P	The selling price per unit, \$/unit;
C	The purchasing cost per unit, \$/unit, with $C < P$;
a	The initial demand rate per year;

b	The increasing demand rate per year;
Q	The order quantity, which is a function of T;
h	The holding cost excluding interest charges, \$/unit/year;
I_e	The rate of interest that can be earned, \$/year;
I_c	The rate of interest charges that are invested in inventory, \$/year;
M	The retailer's trade credit period offered by the supplier, years;
θ	The constant inventory-dependent rate;
r	The constant deteriorating rate of items;
T	The length of inventory cycle;
$I(t)$	The level of inventory at time t, $0 \leq t \leq T$;
$TC(T)$	The retailer's annual total cost, which is a function of T;
T^*	The optimal replenishment interval of $TC(T)$;
D^*	The average optimal demand rate per year, which is equal to $Q(T^*)/T^*$.

2.2. Assumptions

The following assumptions are similar to those in EOQ models of Goyal [1] and Teng et al. [42]:

(i) Shortages are not permitted;

(ii) The inventory system involves only one item;

(iii) The replenishment occurs instantaneously at an infinite rate;

(iv) According to previous studies, such as those by Sana and Chaudhuri [31], Min et al. [34], Dye and Ouyang [43] and Koschat [44], among other authors, demand rate is assumed to be a linear function of the instantaneous inventory level $I(t)$. Meanwhile, during the growth stage of a product life cycle, especially for fashionable commodities, seasonal goods and state-of-the art computers, among others, the demand rate may be a linear function of t. This demand pattern may be found in models by Chang et al. [40], Teng et al. [42], and Dave and Patel [45], among others. In real-life situations, for certain commodities, such as seasonal consumer goods, the demand may depend on both inventory level and time.

Therefore, combined with the aforementioned assumptions, the demand rate $D(t)$ may be given by:

$$D(t) = a + bt + \theta I(t), \tag{1}$$

where, $a > 0$, $b \geq 0$, and t is within a positive time frame;

(v) The objective in this paper is to minimize the annual total cost for the first replenishment cycle. In practice, we make an initial solution under initial information and then change the solution whenever the demand information is changed. Therefore, after acquiring the initial optimal time interval t_1 based on $D(0) = a$, we reevaluate the growth situations of a product life cycle to determine whether the model continues to hold. If so, we may reuse the same method to obtain the next optimal cycle time t_2 based on the new $D(0) = a + bt_1$. Otherwise, we should use new demand rate, i.e. $D(t) = a' + \theta I(t)$. Since the problem with constant demand or stock-dependent has been solved in studies such as those by Teng [7], Sana and Chaudhuri [31], we focus on the problem with increasing demand and inventory-dependent for deteriorating items;

(vi) During the trade credit period, the account is not settled. The retailer may accumulate sales revenue, which is deposited in an interest-bearing account with I_e. At the end of the period M, the retailer pays off all units sold and keeps the profits. When $T \geq M$, the retailer begins paying the interest charges on those unsold items in stock at an interest rate of I_c;

(vii) To simplify the problem and acquire uniform results, we assume that $(\theta+r)C - \theta PMI_e \geq 0$ and $(\theta + r)a \geq rbM$, which is a rational assumption in practice.

3. Mathematical formulation of the model

The level of inventory $I(t)$ gradually decreases primarily to meet demands as well as the loss due to deterioration. Therefore, the change in the inventory level $I(t)$ may be described by the following differential equation:

$$dI(t)/dt = -a - bt - (\theta + r)I(t), \qquad 0 \leq t \leq T, \tag{2}$$

where the boundary conditions $I(T) = 0$ and $I(0) = Q$. Therefore, the solution of differential equation (2) is given by:

$$I(t) = \big\{[(\theta + r)(a + bT) - b]e^{(\theta+r)(T-t)} - (\theta + r)(a + bt) + b\big\}/(\theta + r)^2, \quad 0 \leq t \leq T, \tag{3}$$

and the order quantity is:

$$Q = I(0) = \big\{[(\theta + r)(a + bT) - b]e^{(\theta+r)T} - (\theta + r)a + b\big\}/(\theta + r)^2. \tag{4}$$

The total cost consists of the following: (a) ordering cost; (b) holding cost (excluding interest charges); (c) purchasing cost; (d) interest earned; and (e) interest payable. The elements comprising the retailer's total cost function per cycle are presented as follows:

(a) The ordering cost= S;

(b) The holding cost (excluding the interest charges) $= h\int_0^T I(t)dt$;

(c) The purchasing cost= CQ.

Regarding interest earned and payable (i.e., the costs of (d) and (e)), based on the length of the inventory cycle T, we have two alternative cases: (i) $T \leq M$; and (ii) $T \geq M$.

Case 1: $T \leq M$. In this case, the replenishment time interval T is less than or equal to the credit period M. The retailer sells all units and receives total revenue at time T. Consequently, the cost of financing the inventory in stock is zero. However, the retailer may use the sales revenue to earn interest at an annual rate of I_e during the credit period. Therefore, the interest earned is $PI_e\int_0^T [a+bt+\theta I(t)](M-t)dt$. Therefore, we obtain the annual total cost $TC_1(T)$ for the retailer as follows:

$$TC_1(T) = \frac{1}{T}\Bigg\{S + h\int_0^T I(t)dt + CQ - PI_e\int_0^T [a+bt+\theta I(t)](M-t)dt\Bigg\}$$

$$= \frac{1}{T}\Bigg\{S + h\Bigg[\frac{a+bT}{(\theta+r)^2} - \frac{b}{(\theta+r)^3}\left(e^{(\theta+r)T}-1\right) - \frac{2aT+bT^2}{2(\theta+r)} + \frac{bT}{(\theta+r)^2}\Bigg]$$

$$+C\frac{[(\theta+r)(a+bT)-b]\,e^{(\theta+r)T}-(\theta+r)a+b}{(\theta+r^2)}$$

$$-PI_e\Bigg[\frac{r(6aTM+3bMT^2-3aT^2-2bT^3)}{6(\theta+r)} + \frac{\theta(\theta+r)(a+bT)-\theta b}{(\theta+r)^2}\left(\frac{T-M+Me^{(\theta+r)T}}{\theta+r} + \frac{1-e^{(\theta+r)T}}{(\theta+r)^2}\right) + \frac{\theta b(2MT-T^2)}{2(\theta+r)^2}\Bigg]\Bigg\}. \tag{5}$$

Case 2: $T \geq M$. In this case, the replenishment time interval is greater than or equal to the credit period. The retailer use the sales revenue to earn interest at an annual rate of I_e during $[0, M]$. The interest earned is $PI_e\int_0^M [a+bt+\theta I(t)](M-t)dt$. Beyond the credit period, the product still in stock is assumed to be financed at an annual rate of I_e and thus the interest payable is $CI_e\int_M^T I(t)dt$. As a result, the annual total cost $TC_2(T)$ is:

$$TC_2(T) = \frac{1}{T}\Bigg\{S + h\int_0^T I(t)dt + CQ + CI_c\int_M^T I(t)dt - PI_e\int_0^M [a+bt+\theta I(t)](M-t)dt\Bigg\}$$

$$= \frac{1}{T}\Bigg\{S + h\Bigg[\frac{a+bt}{(\theta+r)^2} - \frac{b}{(\theta+r)^3}\left(e^{(\theta+r)T}-1\right) - \frac{2aT+bT^2}{2(\theta+r)} + \frac{bT}{(\theta+r)^2}\Bigg]$$

$$+C\frac{[(\theta+r)(a+bT)-b]\,e^{(\theta+r)T}-(\theta+r)a+b}{(\theta+r)^2}$$

$$+CI_c\Bigg[\frac{\left((a+bT)(\theta+r)^2-b\right)\left(e^{(\theta+r)(T-M)}-1\right)}{(\theta+r)^3} - \frac{2aT+bT^2-2aM-bM^2}{2(\theta+r)} + \frac{b(T-M)}{(\theta+r)^2}\Bigg]$$

$$-PI_e\Bigg[\frac{r(3aM^2+bM^3)}{6(\theta+r)} + \frac{\theta(\theta+r)(a+bT)-\theta b}{(\theta+r)^2}\left(\frac{Me^{(\theta+r)T}}{\theta+r} + \frac{e^{(\theta+r)(T-M)}-e^{(\theta+r)T}}{(\theta+r)^2}\right) + \frac{\theta bM^2}{2(\theta+r)^2}\Bigg]\Bigg\}. \tag{6}$$

From the above results, the annual total cost $TC(T)$ is written as:

$$TC(T) = \begin{cases} TC_1(T) & \text{for } T \leq M \\ TC_2(T) & \text{for } T \geq M \end{cases} \tag{7}$$

It is clear that $TC_1(M) = TC_2(M)$.

In the next section, we characterize the retailer's optimal solution and determine its optimal cycle time T^* for both the cases of $T \leq M$ and $T \geq M$. For simplicity and convenience, the following some equations still include integral expression.

4. Determination of optimal policy

This paper does not use the traditional second-order derivative and convex analysis used in most previous studies on trade credit; it adopts the first-order derivative twice. The proof for this approach may be found in [34,46].

Case 1: $T \leq M$. The first-order condition for $TC_1(T)$ in Eq. (5) to be minimized is $dTC_1(T)/dT = 0$, and taking the first derivative of $TC_1(T)$ with respect to T will yield:

$$\frac{dTC_1(T)}{dT} = \frac{1}{T^2}\Big\{-S + h\int_0^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} -I(t)\Big]dt + C\left[(aT + bT^2)e^{(\theta+r)T} - Q\right] -PI_e\Big[(aT + bT^2)(M - T) + \theta\int_0^T (aT + bT^2) (M - t)e^{(\theta+r)(T-t)}dt - \int_0^T (a + bt + \theta I(t)) (M - t)dt\Big]\Big\}. \quad (8)$$

Furthermore, we let:

$$k_1(T) = -S + h\int_0^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} - I(t)\Big]dt +C\left[(aT + bT^2)e^{(\theta+r)T} - Q\right] -PI_e\Big[(aT + bT^2)(M - t) + \theta\int_0^T (aT + bT^2) (M - t)e^{(\theta+r)(T-t)}dt - \int_0^T (a + bt + \theta I(t)) (M - t)dt\Big]. \quad (9)$$

Consequently, $dTC_1(T)/d(T)$ and $k_1(T)$ have the same domain and sign. Furthermore, the derivative of $k_1(T)$ with respect to T is:

$$\frac{dk_1(T)}{dT} = h\Big\{\frac{bT}{\theta + r}[e^{(\theta+r)T} - 1] + (aT + bT^2)e^{(\theta+r)T}\Big\} + \left(C - \frac{\theta PMI_e}{\theta + r}\right)\left[bT + (\theta + r)(aT + bT^2)\right] e^{(\theta+r)T} + \frac{\theta PI_e}{(\theta + r)^2}\Big[bT + (\theta + r)(aT + bT^2)\Big] \Big[e^{(\theta+r)T} - 1\Big] + PI_e\Big\{[(\theta + r)a - rbM]\,T +(\theta + 2r)bT^2\Big\}. \quad (10)$$

According to assumption (vii), it is easy to verify that $dk_1(T)/dT > 0$. Therefore, $k_1(T)$ is a strictly increasing function of T in $(0, M]$. Furthermore, it is easy to obtain that $\lim_{T\to 0} + k_1(T) = -S$. However, it is uncertain whether the value of $k_1(M)$ is negative or positive. As a result, if $k_1(M) > 0$, then the intermediate value theorem implies that $k_1(T) = 0$, i.e. $dTC_1(T)/dT = 0$ has a unique positive root T_1^* in $(0, M]$. Therefore, $k_1(T)$ is negative in $(0, T_1^*)$ and positive in $(T_1^*, M]$, which implies that $TC_1(t)$ is decreasing in $(0, T_1^*)$ and increasing in $(T_1^*, M]$. Therefore, T_1^* is the only optimal solution to $TC_1(T)$ in Eq. (8). However, if $k_1(M) \leq 0$, then $k_1(T)$ is non-positive for all T in $(0, M]$, and $TC_1(T)$ is decreasing in $(0, M]$. Therefore, the only optimal solution to $TC_1(T)$ is M.

From the above arguments, for $TC_1(T)$, the following theoretical result may be obtained.

Theorem 1. If $k_1(M) > 0$, then $TC_1(T)$ has the unique optimal solution T_1^*, which is less than M. Otherwise, if $k_1(M) \leq 0$, the optimal solution is $T_1^* = M$.

Proof. This theorem immediately follows from the above arguments.

Case 2: $T \geq M$. Likewise, the first-order condition for $TC_2(T)$, minimized in Eq. (6), is $dTC_2(T)/dT = 0$, and taking the first derivative of $TC_2(T)$ with respect to T will yield:

$$\frac{dTC_2(T)}{dT} = \frac{1}{T^2}\Big\{-S + h\int_0^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} -I(t)\Big]dt + CI_c\int_M^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} -I(t)\Big]dt + C\Big[(aT + bT^2)e^{(\theta+r)T} - Q\Big] -PI_e\Big[\theta\int_0^M (aT + bT^2)(M - t)e^{(\theta+r)(T-t)}dt - \int_0^M (a + bt + \theta I(t))\,(M - t)dt\Big]\Big\}. \quad (11)$$

Likewise, we let:

$$k_2(T) = -S + h\int_0^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} - I(t)\Big]dt +CI_c\int_M^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} - I(t)\Big]dt +C\Big[(aT + bT^2)e^{(\theta+r)T} - Q\Big]$$

$$-PI_e\left[\theta\int_0^M (aT+bT^2)(M-t)e^{(\theta+r)(T-t)}dt - \int_0^M (a+bt+\theta(t))(M-t)dt\right]. \quad (12)$$

Therefore, $dTC_2(T)/dT$ and $k_2(T)$ have the same domain and sign. Furthermore, the derivative of $k_2(T)$ with respect to T is obtained by Eq. (13), as shown in Box I.

According to Assumption (vii), it is easy to verify that $dk_2(T)/d(T)>0$. Therefore, $k_2(T)$ is a strictly increasing function about T in $[M,+\infty)$. Furthermore, we are able to prove that $\lim_{T\to+\infty} k_2(T) = +\infty$ (see Appendix A for proof). Likewise, it is uncertain whether the value of $k_2(M)$ is negative or positive. Therefore, if $k_2(M) < 0$, then the intermediate value theorem implies that $k_2(T) = 0$, i.e. $dTC_2(T)/d(T) = 0$ has a unique positive root T_2^* in $[M,+\infty)$. Therefore, $k_2(T)$ is negative in $[M,T_2^*)$ and positive in $(T_2^*,+\infty)$, which implies that $TC_2(T)$ is decreasing in $[M,T_2^*)$ and increasing in $(T_2^*,+\infty)$. Therefore, T_2^* is the only optimal solution to $TC_2(T)$ in Eq. (11). However, if $k_2(M) \geq 0$, then $k_2(T)$ is non-negative for all T in $[M,+\infty)$, $TC_2(T)$ is increasing in $[M,+\infty)$. Therefore, the only optimal solution to $TC_2(T)$ is M.

From the above arguments, for $TC_2(T)$, the following theoretical result is obtained.

Theorem 2. If $k_2(M) < 0$, then $TC_2(T)$ has the unique optimal solution T_2^*, which is greater than M. Otherwise, if $k_2(M) \geq 0$, the optimal solution is $T_2^* = M$.

Proof. This theorem immediately follows from the above arguments.

Based on the above two theorems and arguments, we have the following theorem about the optimal solution T^* of $TC(T)$. In addition, from Eqs. (9) and (12), one has $k_1(T) = k_2(T)$ if $T = M$. For convenience, let $\omega = k_1(T) = k_2(T)$, where $T = M$, i.e.:

$$\omega = -S + \frac{h}{(\theta+r)^2}\left\{\left[(\theta+r)(aM+bM^2) - a - bM + \frac{b}{\theta+r}\right]e^{(\theta+r)M} + a - \frac{(\theta+r)bM^2}{2} - \frac{b}{\theta+r}\right\} + \frac{C}{(\theta+r)^2}\left\{\left[(\theta+r)^2(aM+bM^2) - (\theta+r)(a+bM) + b\right]e^{(\theta+r)M} + (\theta+r)a - b\right\} - PI_e\left\{\left[aM+bM^2 - = \frac{a+bM}{\theta+r} + \frac{b}{(\theta+r)^2}\right]\left[\frac{\theta}{(\theta+r)^2} + \frac{\theta M e^{(\theta+r)M}}{\theta+r} - \frac{\theta e^{(\theta+r)M}}{(\theta+r)^2}\right] - \frac{aM^2}{2} - \frac{bM^3}{6} + \frac{3\theta aM^2 + \theta bM^3}{6(\theta+r)} - \frac{\theta bM^2}{2(\theta+r)^2}\right\}. \quad (14)$$

By comparing Theorems 1 and 2, we have the following theorem.

Theorem 3.

(a) If $\omega > 0$, then we have the optimal replenishment interval $T^* = T_1^* < M$;

(b) If $\omega < 0$, then we have the optimal replenishment interval $T^* = T_2^* > M$;

(c) If $\omega = 0$, then we have the optimal replenishment interval $T^* = M$.

Proof. See Appendix B.

Note that Theorem 3 is a generalization of the corresponding Theorem 3 of Teng et al. [42], in which the inventory-dependent rate and deteriorating rate both are zero. In addition, the equations $dTC_1(T)/dT = 0$ and $dTC_2(T)/dT = 0$ are solved with the help of MATLAB R2011a.

$$\frac{dk_2(T)}{dT} = h\left\{\frac{bT}{\theta+r}[e^{(\theta+r)T}-1] + (aT+bT^2)e^{(\theta+r)T}\right\} + \left(C - \frac{\theta PMI_e}{\theta+r}\right)\left[bT + (\theta+r)(aT+bT^2)\right]e^{(\theta+r)T} + CI_e\left\{\frac{bT[e^{(\theta+r)(T-M)}-1]}{\theta+r} + (aT+bT^2)e^{(\theta+r)(T-M)}\right\} + \frac{\theta PI_e\left[bT+(\theta+r)(aT+bT^2)\right]\left[e^{(\theta+r)T} - e^{(\theta+r)(T-M)}\right]}{(\theta+r)^2}. \quad (13)$$

Box I

Table 1. Sensitivity analysis.

% Change in parameter		Parameters										
		S	P	C	h	I_e	I_c	M	a	b	θ	r
$\Delta T/T^*\%$	-60	-35.90	2.03	36.84	5.60	2.03	0.01568	1.53	8.14	36.16	1.07	1.08
	-30	-15.86	1.02	14.12	2.67	1.02	0.00776	0.97	3.89	13.89	0.53	0.53
	30	14.04	-1.02	-9.53	-2.38	-1.02	-0.00762	0.18	-3.42	-9.38	-0.52	-0.53
	60	26.37	-1.96	-16.78	-4.55	-1.96	-0.01509	0.40	-6.49	-16.48	-1.03	-1.03
$\Delta Q/Q^*\%$	-60	-38.41	2.25	42.38	6.25	2.25	0.01739	1.70	-49.27	30.31	0.92	0.93
	-30	-17.31	1.13	15.89	2.97	1.13	0.00861	1.07	-23.70	11.69	0.46	0.46
	30	15.79	-1.13	-10.48	-2.63	-1.13	-0.00845	0.20	22.26	-7.83	-0.45	-0.46
	60	30.03	-2.17	-18.31	-5.02	-2.17	-0.01674	0.44	43.27	-13.77	-0.89	-0.89
$\Delta D/D^*\%$	-60	-3.91	0.22	4.05	0.61	0.22	0.00172	0.17	-53.09	-4.30	-0.14	-0.14
	-30	-1.73	0.11	1.55	0.29	0.11	0.00085	0.11	-26.56	-1.93	-0.07	-0.07
	30	1.54	-0.11	-1.04	-0.26	-0.11	-0.00083	0.02	26.59	1.71	0.07	0.07
	60	2.89	-0.21	-1.83	-0.50	-0.21	-0.00165	0.04	53.21	3.24	0.14	0.14
$\Delta TC/TC^*\%$	-60	-9.07	0.50	-52.84	-1.14	0.50	-0.00002	0.85	-47.27	-6.21	-0.22	-0.23
	-30	-4.05	0.25	-26.16	-0.56	0.25	-0.00001	0.48	-23.62	-2.86	-0.11	-0.11
	30	3.48	-0.26	25.83	0.55	-0.26	0.00001	-0.53	23.58	2.54	0.11	0.11
	60	6.58	-0.52	51.43	1.08	-0.52	0.00002	-1.06	47.14	4.84	0.22	0.23

5. Numerical example and sensitivity analysis

To illustrate the proposed method, the following two numerical examples are considered. These examples cover the two cases in the model. The sensitivity analysis of the present model is examined for changes in the parameters. In addition, it should be noted that the time units used for M and T in the model are in 'years', whereas the units used in the following examples are 'days'. The following examples use the assumptions mentioned in Section 2.

Example 1. Given that S = \$80/order, P = \$5/unit, C = \$2/unit, h = \$1/unit/year, $I_e = 0.1$/year, $I_c = 0.1$/year, $M = 30$ days= 30/365 years, $a = 3000$ units/year, $b = 8100$ units/year, $\theta = 0.1$ and $r = 0.1$, what will be the optimal results?

Using Eq. (14), we obtain that the value of ω is -1.279. Then, according to Part (b) of Theorem 3, we have $T^* = T_2^*$, $Q^* = Q(T_2^*)$, $D^* = Q(T_2^*)$, $TC^* = TC(T_2^*)$. Using corresponding equations and methods, we obtain that T^*=30.25 days, Q^*=278.8, D^*=3364.3, and TC^*=7771.679, respectively.

Example 2. Suppose that S = \$70/order, P = \$6/unit, C = \$2/unit, h = \$1/unit/year, I_e = 0.1/year, I_c=0.12/year, $M = 30$ days = 30/365 years, $a = 5000$ units/year, $b = 10000$ units/year, $\theta = 0.1$, $r = 0.06$.

Computing ω, the value of ω is 35.619. Then, according to Part (a) of Theorem 3, we have $T^* = T_1^*$, $Q^* = Q(T_1^*)$, $D^* = Q(T_1^*)/T_1^*$, $TC^* = TC(T_1^*)$. Using corresponding equations and methods, we obtain that T^* = 24.58 days, Q^* = 361.4, D^* = 5366.2, TC^* = 11802.199, respectively.

Next, we will study the effects of changes in the values of parameters on the optimal values of T^*, Q^*, D^* and TC^*. The sensitivity analysis is examined by changing the value of each parameter by −60%, −30%, 30% and 60%, taking one parameter at a time and keeping the values of the remaining parameters. The sensitivity analysis is based on the Example 1. and the results are given in Table 1. In addition, in order to better understanding, the sensitivity analysis has been graphed in Figure 1 based on the data in Table 1.

The following points and inferences are obtained:

(i) T^*, Q^*, D^*, and TC^* increase with the increase in the value of parameter S. A simple economical interpretation is that the retailer will place orders with less frequency to avoid the larger ordering cost. Furthermore, T^* and Q^* are moderately sensitive to the parameter S; TC^* is lowly sensitive to the parameter S; and Q^* is insensitive to changes in S.

(ii) T^*, Q^*, D^*, and TC^* decrease as the value of parameter P or I_e increase. It indicates that a larger selling price or interest earned rate leads to a higher interest being earned during the trade credit period. Therefore, the retailer

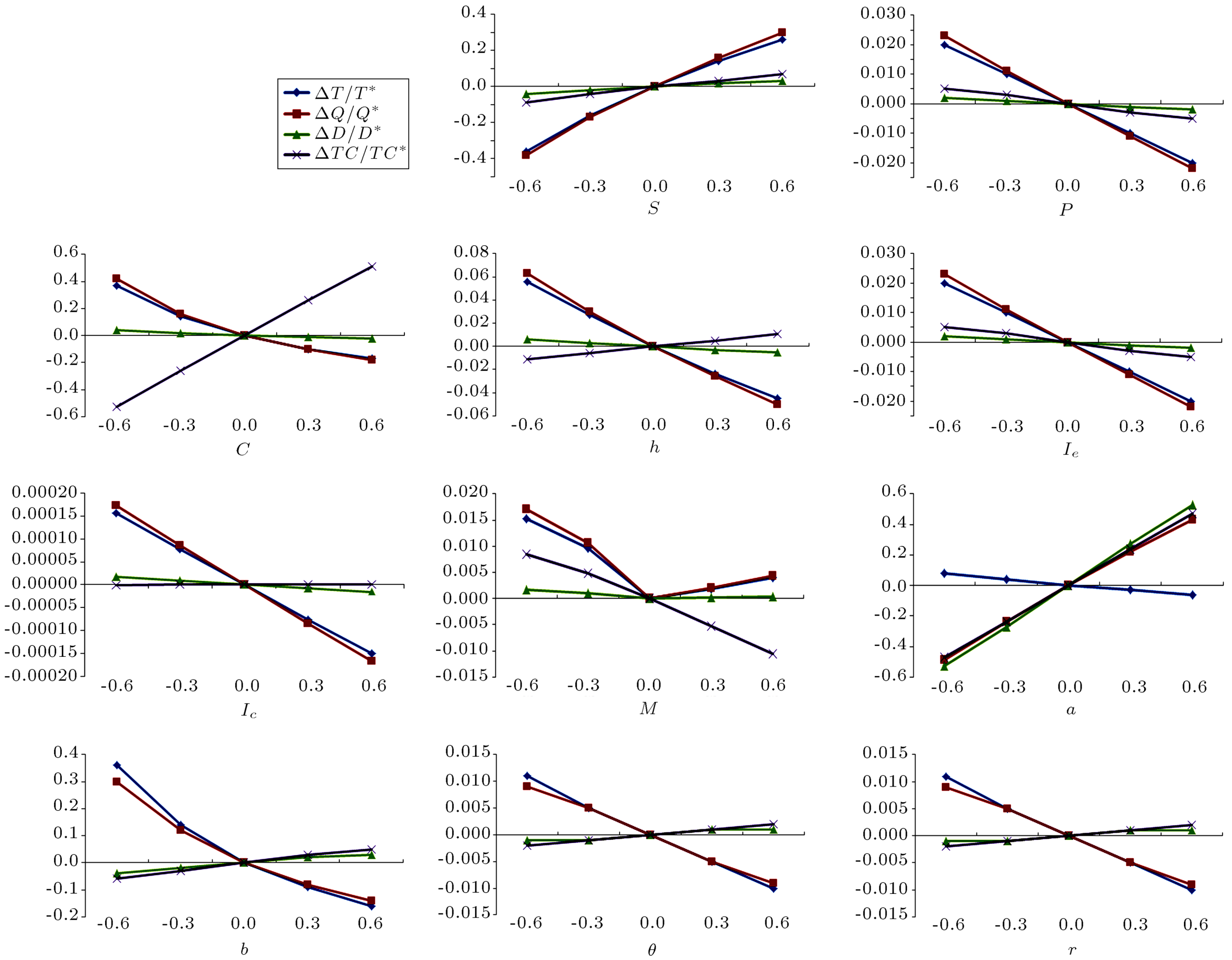

Figure 1. Sensitivity analysis of the parameters for the presented model.

places orders with more frequently. However, T^*, Q^*, D^* and TC^* are all insensitive to parameters P and I_e.

(iii) T^*, Q^*, and D^* decrease while TC^* increase with the increase in the value of parameter C. It shows that a higher purchasing cost leads to a higher inventory cost and then the retailer will order lower quantity to reduce the higher stock cost. T^* and Q^* are moderately sensitive while TC^* is highly sensitive to parameter C. D^* is insensitive to changes in C.

(iv) T^*, Q^*, and D^* decrease while TC^* increases with the increase in the value of parameter h. Its economical interpretation is similar to parameter C. Furthermore, T^* and Q^* are lowly sensitive while D^* and TC^* are insensitive to parameter h.

(v) T^*, Q^*, and D^* decrease while TC^* increase with the value of parameter I_c increasing. In addition, T^*, Q^*, D^* and TC^* are all insensitive to parameter I_c. Note that I_c is most insensitive, than the other parameters, to the all optimal values.

(vi) TC^* decreases while T^*, Q^*, and D^* decrease at beginning and then increase with the increase in the value of parameter M. However, T^*, Q^*, D^*, and TC^* are all insensitive to parameter M.

(vii) T^* is decreasing while Q^*, D^*, and TC^* are increasing with the increase in the value of parameter a, which is consistent with the economic sense. In addition, Q^*, D^*, and TC^* are highly sensitive, while T^* is lowly sensitive to parameter a.

(viii) T^* and Q^* decrease while D^* and TC^* increase with the increase in the value of parameter b. Its economical interpretation is similar to parameters P and I_e. T^* and Q^* are moderately sensitive while TC^* is lowly sensitive to parameter C. Additionally, D^* is insensitive to changes in b.

(ix) T^* and Q^* decrease, while D^* and TC^* increase with the increasing value of parameter θ or

Table 2. Sensitivity hierarchies.

Parameters	S	P	C	h	I_e	I_c	M	a	b	θ	r
T^*	M	I	M	L	I	I	I	L	M	I	I
Q^*	M	I	M	L	I	I	I	H	M	I	I
D^*	I	I	I	I	I	I	I	H	I	I	I
TC^*	L	I	H	I	I	I	I	H	L	I	I

H-highly sensitive; M-moderately sensitive;
L-lowly sensitive; I-insensitive.

r. Their economical interpretation is similar to parameter C. Nevertheless, T^*, Q^*, D^*, and TC^* are all insensitive to parameters θ and r. It implies that some errors in estimating θ or r may result in little deviation from the optimal results.

According to the above analysis and description, we may obtain sensitivity hierarchies between parameters and the optimal values. The results are shown in Table 2.

6. Conclusions

Most of the previous inventory models allowing a permissible delay in payments have usually assumed that the demand is constant or merely dependent on the inventory level or others. In this paper, we develop an inventory model with inventory-dependent and linear trend demand under a permissible delay in payments for deterioration items. The main difference of this model compared to most existing studies is that we propose a more general inventory model by considering the following aspects: (i) The linear trend demand rate increases significantly during the growth stage of the product life cycle; (ii) The demand is also linearly dependent on the retailer's instantaneous inventory level; (iii) Deterioration rate is constant; and (iv) The supplier offers the retailer trade credit financing.

We adopt cost minimization as our objective to determine the optimal order policy. In addition, the conditions of the existence and uniqueness of optimal solution are discussed in detail and an efficient solution is developed to solve the proposed problem. Furthermore, we establish Theorem 3 by which we find a way to obtain the optimal ordering policies by examining the explicit conditions. Whereas Assumption (vii) restricts the application scope of the model to some extent, this assumption is rational in reality. Finally, we provide some numerical examples to illustrate the proposed model and their optimal solutions, and we also obtain some main managerial insights through the sensitivity analysis. Meanwhile, we obtain sensitivity hierarchies between parameters and the optimal values.

When there is no demand stimulation (i.e., θ=0), no deterioration rate (i.e., r=0), no increasing demand (i.e., b=0) and no trade credit (i.e., M=0), the proposed model may be reduced to the standard EOQ model. Furthermore, as shown in Appendix C, Teng et al. [42] may be viewed as special case.

The present model may be extended in several ways. For instance, we may extend the model to allow for a varying rate of inventory-dependent demand. In addition, we could consider steady decrease demand rate or quadratic time-varying demand rate. Furthermore, we could generalize the model to allow for shortages and partial backlogging, partially permissible delay in payments, quantity discounts, relaxation terminal condition of zero-ending inventory and inflation rates, among others. Therefore, the effects of all of these may be incorporated into future research.

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Appendix A

Proof of $\lim_{T\to\infty} k_2(T) = +\infty$.

$$k_2(T) = -S + h\int_0^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} - I(t)\Big]dt + CI_c\int_M^T \Big[(aT + bT^2)e^{(\theta+r)(T-t)} - I(t)\Big]dt + C\Big[(aT + bT^2)e^{(\theta+r)T} - Q\Big] - PI_e\Big[\theta\int_0^M (aT + bT^2)(M-t)e^{(\theta+r)(T-t)}dt - \int_0^M (a + bt + \theta I(t))(M-t)dt\Big].$$

By using L'Hospital rule and assumption (vii), it is easy to obtain Eqs. (A.1) to (A.3) as shown in Box II. From the analysis conducted so far, we can conclude that $\lim_{T\to+\infty} k_2(T) = +\infty$.

Appendix B

If $\omega>0$, according to Theorem 1, we obtain $TC_1(T_1^*) > TC_1(M)$. In addition, using $TC_1(M) = TC_2(M)$, and Theorem 2, we have $TC_1(T_1^*) > TC_1(M) = TC_2(M) > TC_2(T)$ for all $T > M$. It thus proves Part (a) of Theorem 3. Likewise, Parts (b) and (c) of Theorem 3 may be proved in a similar manner.

Appendix C

Special case of the presented model. When $\theta \to 0^+$, $r \to 0^+$, the proposed model may be reduced to the model of Teng et al. [42]. By using L'Hospital rule, from Eqs. (3), (4), (5) and (6), we obtain:

$$I(t) = \lim_{\theta\to 0^+}\lim_{r\to 0^+}\Big\{[(\theta+r)(a+bT) - b]\,e^{(\theta+r)(T-t)} - (\theta+r)(a+bt) + b\Big\}/(\theta+r)^2 = a(T-t) + \frac{1}{2}b(T^2 - t^2), \quad 0 \le t \le T, \tag{C.1}$$

$$Q = I(0) = \lim_{\theta\to 0^+}\lim_{r\to 0^+} + \Big\{[(\theta+r)(a+bT) - b]\,e^{(\theta+r)T} - (\theta+r)a + b\Big\}/(\theta+r)^2 = aT + \frac{1}{2}bT^2, \tag{C.2}$$

$$TC_3(T) = \lim_{\theta\to 0^+}\lim_{r\to 0^+} TC_1(T) = \lim_{\theta\to 0^+}\lim_{r\to 0^+}\frac{1}{T}\Big\{S$$

$$\lim_{T\to+\infty}\frac{(aT+bT^2)\int_0^T e^{(\theta+r)(T-t)}dt}{\int_0^T I(t)dt}$$
$$=\lim_{T\to+\infty}\frac{(a+2bT)\int_0^T e^{(\theta+r)(T-t)}dt+(aT+bT^2)+(\theta+r)(aT+bT^2)\int_0^T e^{(\theta+r)(T-t)}dt}{(a+bT)\int_0^T e^{(\theta+r)(T-t)}dt}$$
$$>\lim_{T\to+\infty}1+(\theta+r)T=+\infty \tag{A.1}$$

$$\lim_{T\to+\infty}\frac{C\left[(aT+bT^2)e^{(\theta+r)T}-Q\right]}{PI_e\left[\theta\int_0^M(aT+bT^2)(M-t)e^{(\theta+r)(T-t)}dt-\int_0^M(a+bt+\theta I(t))(M-t)dt\right]}$$
$$=\lim_{T\to+\infty}\frac{C(\theta+r)^2}{\theta PI_e\left[(\theta+r)M+e^{-(\theta+r)M}-1\right]}\geq 1, \tag{A.2}$$

$$\lim_{T\to+\infty}\frac{\int_M^T(aT+bT^2)e^{(\theta+r)(T-t)}dt}{\int_M^T I(t)dt}$$
$$=\lim_{T\to+\infty}\frac{\int_M^T(a+2bT)e^{(\theta+r)(T-t)}dt+(aT+bT^2)+(\theta+r)(aT+bT^2)\int_M^T e^{(\theta+r)(T-t)}dt}{\int_M^T(a+bT)e^{(\theta+r)(T-t)}dt}$$
$$>\lim_{T\to+\infty}1+(\theta+r)T=+\infty. \tag{A.3}$$

Box II

$$+h\left[\frac{a+bT}{(\theta+r)^2}-\frac{b}{(\theta+r)^3}(e^{(\theta+r)T}-1)\right.$$
$$\left.-\frac{2aT+bT^2}{2(\theta+r)}+\frac{bT}{(\theta+r)^2}\right]$$
$$+C\frac{\left[(\theta+r)(a+bT)-b\right]e^{(\theta+r)T}-(\theta+r)a+b}{(\theta+r)^2}$$
$$-PI_e\left[\frac{r\left(6aTM+3bMT^2-3aT^2-2bT^3\right)}{6(\theta+r)}\right.$$
$$+\frac{\theta(\theta+r)(a+bT)-\theta b}{(\theta+r)^2}\left(\frac{T-M+Me^{(\theta+r)T}}{\theta+r}\right.$$
$$\left.\left.+\frac{1-e^{(\theta+r)T}}{(\theta+r)^2}\right)+\frac{\theta b(2MT-T^2)}{2(\theta+r)^2}\right]\Bigg\}$$
$$=\frac{1}{T}\left[S+(h+PI_e)\left(\frac{1}{2}aT^2+\frac{1}{3}bT^3\right)\right.$$
$$\left.+(C-PMI_e)\left(aT+\frac{1}{2}bT^2\right)\right],\quad \text{if}\quad T\leq M, \tag{C.3}$$

$$TC_4(T)=\lim_{\theta\to0^+}\lim_{r\to0^+}TC_2(T)=\lim_{\theta\to0^+}\lim_{r\to0^+}\frac{1}{T}\Bigg\{S$$
$$+h\left[\frac{a+bT}{(\theta+r)^2}-\frac{b}{(\theta+r)^3}(e^{(\theta+r)T}-1)\right.$$
$$\left.-\frac{2aT+bT^2}{2(\theta+r)}+\frac{bT}{(\theta+r)^2}\right]$$
$$+C\frac{\left[(\theta+r)(a+bT)-b\right]e^{(\theta+r)T}-(\theta+r)a+b}{(\theta+r)^2}$$
$$+CI_e\left[\frac{\left((a+bT)(\theta+r)^2-b\right)\left(e^{(\theta+r)(T-M)}-1\right)}{(\theta+r)^3}\right.$$
$$\left.-\frac{2aT+bT^2-2aM-bM^2}{2(\theta+r)}+\frac{b(T-M)}{(\theta+r)^2}\right]$$
$$-PI_e\left[\frac{r(3aM^2+bM^3)}{6(\theta+r)}+\frac{\theta(\theta+r)(a+bT)-\theta b}{(\theta+r)^2}\right.$$
$$\left(\frac{Me^{(\theta+r)T}}{\theta+r}+\frac{e^{(\theta+r)(T-M)}-e^{(\theta+r)T}}{(\theta+r)^2}\right)$$
$$\left.+\frac{\theta bM^2}{2(\theta+r)^2}\right]\Bigg\}=\frac{1}{T}\Big[S+(h+CI_c)$$

$$\times\left(\frac{1}{2}aT^2+\frac{1}{3}bT^3\right)+(C-CMI_c)(aT+\frac{1}{2}bT^2)$$
$$+(CI_c-PI_e)(\frac{1}{2}aM^2+\frac{1}{6}bM^3)\Bigg] \quad \text{if} \quad T\geq M. \tag{C.4}$$

Then, Eqs. (7) may be reduced as follows:

$$TC(T)=\begin{cases}TC_3(T) & T\leq M\\ TC_4(T) & T\geq M\end{cases} \tag{C.5}$$

Eqs. (C.5) are consistent with Eqs. (6) and (7), respectively, in Teng et al.'s model [42]. Therefore, the model of Teng et al. is a special case of this paper. Note that Teng et al.'s model is a profit function.

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