Scientia Iranica D (2015) 22(6), 2457–2471

Sharif University of Technology
Scientia Iranica
Transactions D: Computer Science & Engineering and Electrical Engineering
www.scientiairanica.com

Opposition-based gravitational search algorithm for synthesis circular and concentric circular antenna arrays

G. Ram[a,*], D. Mandal[a], R. Kar[a] and S. Prasad Ghoshal[b]

a. *Department of Electronics and Communication Engineering, National Institute of Technology, Durgapur, Pin-713209, India.*
b. *Department of Electrical Engineering, National Institute of Technology Durgapur, Pin-713209, India.*

Received 22 August 2014; received in revised form 17 June 2015; accepted 5 September 2015

KEYWORDS
Circular arrays;
FNBW;
SLL;
OGSA;
Directivity.

Abstract. In this paper, a population based evolutionary optimization methodology, called Opposition-based Gravitational Search Algorithm (OGSA), is applied for optimal designs of three non-uniform single-ring Circular Antenna Arrays (CAA) of set 8, 10 and 12 elements and non-uniform 3-ring Concentric Circular Antenna Array (CCAA). Two 3-ring concentric circular antenna arrays having sets of 4-, 6-, 8- elements and 8-, 10-, 12- elements, with and without center element, are considered. The algorithm is used to determine an optimal set of current excitation weights and antenna inter-element separations for circular antenna array of 8, 10, and 12 elements and optimal current excitation weights for CCAA, respectively. OGSA provides optimal radiation pattern with maximum Side Lobe Level (SLL) reduction and First Null Beam Width (FNBW) reduction with improved directivity for CAA and maximum reduction of SLL for CCAA, respectively. OGSA is developed on the primary foundation of Gravitational Search Algorithm (GSA) blended with the concept of opposition based approach. Simulation results show a considerable improvement of radiation pattern with respect to the corresponding uniform cases of both the types of antenna array and those of some recent literature reported in this paper. Finally, comparison of accuracies of the proposed algorithm is performed by t-test calculation.

1. Introduction

The use of asymmetric current excitation weights and non-uniform inter-element spacing allows for increased degrees of freedom in design [1-3]. The increasing pollution of electromagnetic environment has prompted the study of antenna array pattern techniques for the suppression of Side Lobe Level (SLL) while preserving the beamwidth [4,5]. These techniques are very important in radar, sonar, and communication systems for minimizing degradation in signal-to-noise ratio performance due to undesired interference [6,7]. Uniformly excited and equally spaced antenna array [1,6] have high directivity, but they usually suffer from high side lobe level. To reduce the side lobe level further, the array is made aperiodic by altering the positions of the antenna elements with all excitation amplitudes being uniform [8]. A lot of research has been carried out in the past few decades on different antenna arrays in order to get improved radiation patterns [9,10]. In many applications, it is necessary to design antennas with very good directive

*. *Corresponding author.*
E-mail addresses: gopi203hardel@gmail.com (G. Ram); durbadal.bittu@gmail.com (D. Mandal); rajibkarece@gmail.com (R. Kar); spghoshalnitdgp@gmail.com (S. Prasad Ghoshal)

characteristics to meet the demand of long distance communication [1]. An antenna array is formed by assembly of radiating elements in an electrical or geometrical configuration. Total field of the antenna array is found by vector addition of the fields radiated by all individual elements [1,2]. To provide a good directive pattern, it is necessary that the fields from the array elements add constructively in some desired direction and destructively in the remaining space [10]. There are several parameters by varying which the radiation pattern can be modified [11-14]. These parameters are: geometrical configurations (e.g. linear, circular, planar, spherical etc.), inter-element spacing, individual excitation (amplitude and phase), and relative pattern of individual elements [1,6]. Circular arrays have become popular in recent years over other array geometries because they have the capability to perform the scan in all directions without a considerable change in the beam pattern and provide 360 degrees azimuth coverage [11-37]. Moreover, circular arrays have low sensitivity to mutual coupling as compared to linear and rectangular arrays, since these do not have edge elements [1]. Among the different types of antenna arrays, CCAA [15-34,37] has become the most popular in mobile and wireless communications. In this paper, optimization of CCAA design having a uniform element separation and a non-uniform excitation is performed with the help of evolutionary optimization techniques.

The classical gradient search based optimization methods are not suitable for optimal design of complex, nonlinear, multimodal, non-differentiable antenna array design problem [38,39]. So, metaheuristic evolutionary methods have been employed for the optimal design of antenna array design problem [38,39]. Few such evolutionary optimization techniques used are as follows: Genetic Algorithm (GA) is inspired by the Darwin's "Survival of the Fittest" strategy [13,31,40-43]. Conventional PSO has mimicked the behaviour of bird flocking or fish schooling [14,23,44]. GA is a probabilistic heuristic search optimization technique developed by Holland [43]. PSO is a swarm intelligence based algorithm developed by Eberhart and Eberhart [44]. It has been realized that GA is incapable of local searching in a multidimensional search space. GA [13], PSO [14], IWO [12], and FA [25] suffer from premature convergence and get easily trapped to suboptimal solution.

So, for the improvement of optimization algorithm, the authors propose an alternative superior technique called Opposition Based Gravitational Search Algorithm (OGSA) in which solution is based on the consideration of a solution and its opposite solution for each object governed by the rules of GSA [44-52]. Computational results achieved with the proposed opposition based [53-63] gravitational search algorithm based technique are compared with those of algorithms in the reported literature to demonstrate the effectiveness and superior performance of OGSA for getting optimal radiation pattern of CCA. Statistical performance of the proposed algorithm and modified IWO algorithm were performed by t-test and p-value calculations [64-65].

Based upon OGSA, this paper presents good and comprehensive sets of results and states arguments for the superiority of the algorithm over IWO [12], CAA, EP [23], and FA [25] for CCAA, respectively.

The paper is ordered as follows: In Section 2, the design equations of CAA and CCAA are formulated. In Section 3, the design of cost function is formulated. Section 4 deals with employed algorithm for the design of circular antenna arrays. Section 5 describes the comparative simulation results obtained using OGSA and other approaches in the published literature. Section 6 shows the effective convergence of employed algorithms. Finally, Section 7 concludes the paper.

2. Design equation

2.1. Circular Antenna Array (CAA)

Figure 1 assumes the geometry of a circular array of N isotropic sources laid on $x-y$ plane having a radius 'a' and scanning at point PP in the far field [10,31]. The elements in the non-uniform circular antenna array are taken to be isotropic sources, so the radiation pattern of this array can be described by its array factor [1,31]. In the $x-y$ plane, the array factor for the circular array, shown in Figure 1, is given [1,31] as:

$$AF(\phi, I, d) = \sum_{n=1}^{N} A_n.\exp\left(jkr\sin\theta\cos(\phi-\phi_n)\right), \tag{1}$$

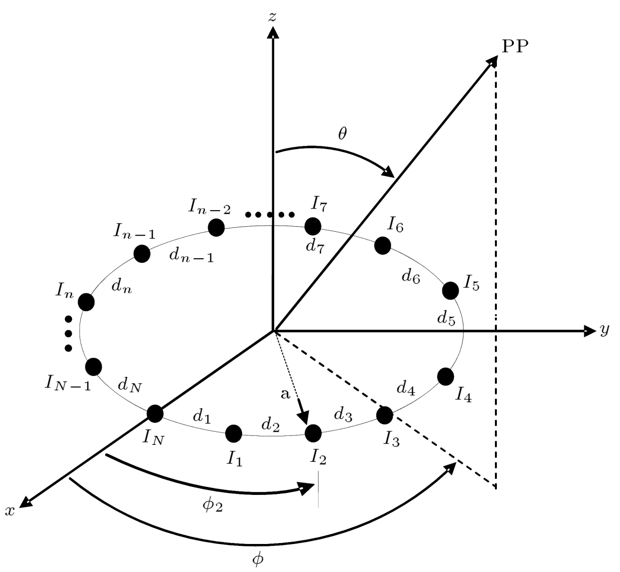

Figure 1. Geometry of non-uniform circular array laid on the $x-y$ plane with N isotropic elements scanning at a point PP in the far field.

where A_n is excitation coefficient (amplitude and phase) of the nth element. In general, excitation coefficient of the nth element can be written as:

$$A_n = I - ne^{j\delta_n}, \tag{2}$$

where δ_n is phase excitation (relative to the array centre) of the nth element. So Eq. (1) can be written as:

$$AF(\phi, I, d) = \sum_{n=1}^{N} I_n.\exp\left(j\left[kr\sin\theta \cos(\phi - \phi_n) + \delta_n\right]\right), \tag{3}$$

where:

$$kr = 2\pi r/\lambda = \sum_{i=1}^{N} d_i, \tag{4}$$

$$\phi_n = (2\pi/kr)\sum_{i=1}^{n} d_i, \tag{5}$$

$k = 2\pi/\lambda, \lambda,$ being the wavelength of operation;
θ Elevation angle;
ϕ Azimuth angle;
r Radius of the circular array;
ϕ_n Angular location of nth element along the $x-y$ plane.

Peak of the main beam is in the (θ_0, ϕ_0) direction; the phase excitation of the nth element can be given as:

$$\delta_n = -kr.\sin\theta_0\cos(\phi_0 - \phi_n). \tag{6}$$

In the present design, the following changes are assumed.

$$\theta = \pi/2; \qquad \delta_n = -kr.\cos(\phi_0 - \phi_n). \tag{7}$$

Thus, Eq. (3) can be rewritten as Eq. (8).

$$AF(\varphi_n, I, d) = \sum_{n=1}^{N} I_n \exp\left\{jkr\left[\cos(\phi - \phi_n) - \cos(\phi_0 - \phi_n)\right]\right\}. \tag{8}$$

The expression for the normalized array factor can be given as:

$$AF_n(\phi_n, I, d) = 20\log_{10}\left[\frac{|AF(\phi_n, I, d)|}{|AF(\phi_n, I, d)|_{\max}}\right], \tag{9}$$

where $I = [I_1, I_2, ..., I_N]$; I_N represents the excitation of the nth element of the array; $d = [d_1, d_2, ..., d_N]$, d_n represents the distance from nth element to $(n+1)$th element. ϕ_0 is the angle where global maximum is attained in $\phi = [-\pi, \pi]$. Further, ϕ_0 is chosen as 0, i.e. ϕ_0 is the maximum radiation angle. The design goal in the paper is to find the optimum set of values of I_n and d_n in order to get the improved optimal side lobe reduction in the radiation pattern in the desired direction ϕ. All the antenna elements are assumed isotropic. Amplitude excitation of each element and inter-element separations are used to change the antenna pattern. The directivity of circular antenna arrays is given as follows [1]:

$$D = \frac{|AF_0(\theta_0, \phi_0)|^2}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}|AF(\theta, \phi)|^2\sin\theta d\theta d\phi}. \tag{10}$$

2.2. *Concentric Circular Antenna Array (CCAA)*

Geometrical configuration is a key factor in the design process of an antenna array [5,27-30]. For CCAA, the elements are arranged in such a way that all the antenna elements are placed in multiple concentric rings, which differ in radii and number of elements [5,27-30]. Figure 2 shows the general configuration of CCAA with P concentric circular rings, where the pth ($p = 1, 2, ..., P$) ring has a radius, r_p, and the corresponding number of elements in each ring is N_p [26-30]. If all the elements in all the rings are assumed to be isotopic sources, the radiation pattern of this array can be written in terms of its array factor only [5,24,27-30].

Referring to Figure 2, the array factor, $AF(\theta, \phi, I)$, for the CCAA in $x-y$ plane may be written as [5,27-30]:

$$AF(\theta, \phi, I) = 1 + \sum_{p=1}^{P}\sum_{i=1}^{N_p} I_{pi} \exp\left[j\left(kr_p\sin\theta\cos(\phi - \phi_{pi}) + \alpha_{pi}\right)\right], \tag{11}$$

where I_p is the excitation amplitude of ith element in the pth ring; $k = 2\pi/\lambda$; λ is the signal wave-length.

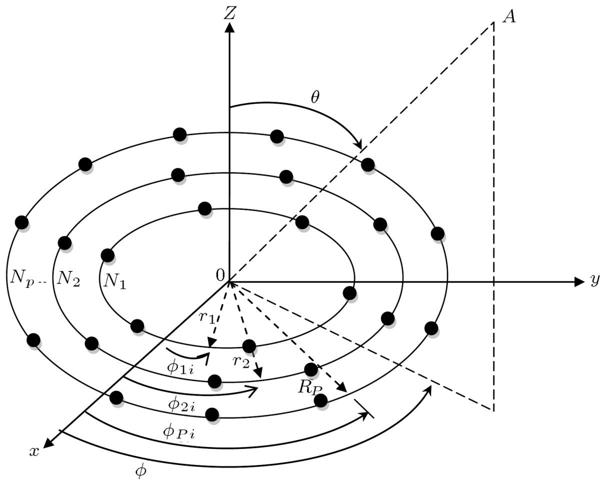

Figure 2. Concentric Circular Antenna Array (CCAA).

θ and ϕ symbolize the zenith angle from the positive z-axis and the azimuth angle from the positive x-axis to the orthogonal projection of the observation point, respectively. The angle ϕ_{pi} is the element to element angular separation measured from the positive x-axis. The elements in each ring are assumed to be uniformly distributed [27-30].

$$\phi_{pi} = 2\pi\left(\frac{i}{N_p}\right); \qquad p = 1, ..., P; \quad i = 1, ..., N_p. \quad (12)$$

The term α_{pi} is the phase difference between the individual elements in the array, which is a function of angular separation ϕ_{pi} and ring radii r_p.

$$\alpha_{pi} = -Kr_p \sin\theta_0 \cos(\phi_0 - \phi_{pi});$$
$$p = 1, ..., P; \quad i = 1, ..., N_p, \quad (13)$$

where θ_0 and ϕ_0 are the values of θ and ϕ, respectively, where the peak of the main lobe is obtained. In this paper, $\theta_0 = 90°$ and $\phi_0 = 90°$.

3. Fitness function evolution

After defining the array factor, the next step in design process is to formulate the cost function which is to be minimized to achieve the design goal. The cost function (*fit*) for improving the Side Lobe Level (SLL) of radiation patterns of non-uniform circular antenna array is given by:

$$fit = W_1 \times |AF(\varphi_{msl1}, I_n) + AF(\varphi_{msl2}, I_n)|$$
$$/\,|AF(\varphi_0, I_n)| + W_2$$
$$\times \frac{\left|\Pi_{\theta=0}^{\theta_{msl1}} AF(\varphi_{\text{local max } ima}, I_n)\right|}{|AF_{\max}|} + W_3$$
$$\times [FNBW_{\text{computed}} - FNBW(I_n = 1)]$$
$$+W_4 * 1/D_{\max}, \quad (14)$$

where $FNBW$ is an abbreviated form of first null beamwidth or, in simple terms, angular width between the first nulls on either side of the main beam. Thus, $FNBW_{\text{computed}}$ and $FNBW(I_n = 1)$ basically refer to the computed first null beamwidths in radian for the non-uniform excitation case and for the uniform excitation case, respectively [27-30]. The third term in Eq. (14) is computed only if $FNBW_{\text{computed}} > FNBW(I_n = 1)$ [27-30]. $D_{\max}$ is the maximum directivity. W_1, W_2, W_3, and W_4 are the weighting factors. ϕ_{msl1} is the angle where maximum SLL $AF(\phi_{msl1}, I_{mi})$ is attained in the lower band. ϕ_{msl2} is the angle where the maximum SLL $AF(\phi_{msl2}, I_n)$ is attained in the upper band. The second term in the cost function is used to reduce SLL by imposing the nulls at all local maxima of array factor $AF(\phi_{\text{local max } ima}, I_n)$ values within $\phi \in [-\pi : \phi_{msl1}, \phi_{msl2} : -\pi]$. $AF_{\max}$ is the maximum of AF. In cost function, both the numerator and denominator of the first and second terms are absolute. The weights W_1, W_2, and W_3 are chosen in such a way that optimization of SLL remains more dominant than optimization of $FNBW$. Minimization of (fit) means maximum reduction of SLL and reduction of $FNBW_{\text{computed}}$ possible maximum reduction. The OGSA technique employed for optimizing current excitation weights and inter-element spacing resulting in the minimization of (fit) and hence both SLL and $FNBW$, is described in the next section.

4. Evoluationary optimization employed

A meta-heuristic optimization algorithm based on the metaphor of gravitational interaction between masses is proposed by Rashedi et al. in Gravitational Search Algorithm (GSA) in [45]. GSA is inspired by the Newton theory that postulates every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them [45-52]. Promising results were reported in [45] for benchmark function optimization problems by adopting GSA.

Tizhoosh introduced the concept of Opposition-Based Learning (OBL) in [53]. This notion has been applied to accelerate reinforcement learning [53-63] and back propagation learning [62] in neural networks. The main idea behind OBL is the simultaneous consideration of an estimate and its corresponding opposite estimate (i.e., guess and opposite guess) in order to achieve a better approximation for the current candidate solution [53-62]. In this paper, OBL has been utilized to accelerate the convergence rate of GSA. Hence, our proposed approach has been called Opposition-based Gravitational Search Algorithm (OGSA). OGSA uses opposite numbers during population initialization and also for generating new populations during the evolutionary process of GSA [45-52]. The concept of opposite numbers has been utilized to speed up the convergence rate of an optimization algorithm.

4.1. Proposed opposition-based gravitational search algorithm

4.1.1. A brief description of gravitational search algorithm

GSA [40-47] is navigated by properly adjusting the gravitational and the inertial masses. In physics, gravitation is the tendency of objects with masses to accelerate towards each other. In the Newton gravitational law, each particle attracts every other particle with a force which is the gravitational force [45-52]. GSA is one of the newest heuristic algorithms that has been inspired by the Newtonian laws of gravity and motion [45-52].

In GSA [45-52], agents are considered as objects and their performances are measured by their masses. All these objects attract each other by the gravity forces and these forces cause a global movement of all the objects towards the objects with heavier masses. Hence, masses cooperate using a direct form of communication through gravitational forces. The heavy masses (which correspond to the good solutions) move more slowly than the lighter ones. This guarantees the exploitation step of the algorithm [45-52].

To describe the GSA, consider a system with S masses in which position of the ith mass is defined as in Eq. (15):

$$X_i = \left(x_i^1, ..., x_i^d, ..., x_i^n\right) \quad \text{for} \quad i = 1, 2, ..., S, \tag{15}$$

where x_i^d is position of the ith mass in the dth dimension and n is the dimension of the search space. Based on GSA, mass of each agent is calculated after computing current population's fitness given in Eqs. (16) and (17) [45-52]:

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}, \tag{16}$$

$$M_i(t) = \frac{m_i(t)}{\sum_1^N m_i(t)}, \tag{17}$$

where $M_i(t)$ and $fit_i(t)$ represent the mass and the fitness value of the agent i at iteration cycle (t), respectively, and for a minimization problem $worst(t)$ and $best(t)$ are defined in Eqs. (18) and (19) [45-52]:

$$best(t) = \min_{j \in \{1,...,n\}} fit_j(t), \tag{18}$$

$$worst(t) = \max_{j \in \{1,...,n\}} fit_j(t). \tag{19}$$

Total forces applied on an agent from a set of heavier masses should be considered based on the law of gravity as stated in Eq. (20), which is followed by the calculation of the acceleration using the law of motion as presented in Eq. (21) [46-47]. Afterwards, next velocity of an agent, as given in Eq. (22), is calculated as a fraction of its current velocity added to its acceleration. Then, its next position may be calculated by using Eq. (23) [45-52].

$$F_i^d(t) = \sum_{j \in Kbest, j \neq i}^{N} rand_j \times G(t) \times \frac{M_i(t) \times M_j(t)}{R_{ij}(t) + \varepsilon} \times \left(x_j^d(t) - x_i^d(t)\right), \tag{20}$$

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} = \sum_{j \in Kbest, j \neq i}^{N} rand_j \times G(t) \times \frac{M_j(t)}{R_{ij}(t) + \varepsilon} \times \left(x_j^d(t) - x_i^d(t)\right), \tag{21}$$

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t), \tag{22}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1). \tag{23}$$

In Eqs. (20)-(22), $rand_i$ and $rand_j$ are two uniformly distributed random numbers in the interval [0,1]. ε is a small value. $R_{ij}(t)$ is the Euclidean distance between two agents i and j (defined as $R_{ij}(t) = ||x_i(t), x_j(t)||_2$). $Kbest$ is the set of first K agents with the best fitness values and the biggest masses, which is a function of iteration cycle (t) (initialized to K_0 at the beginning and decreased with iteration cycle (t)). Here, K_0 is set to S (total number of agents) and is decreased linearly to 1. In GSA, the gravitational constant (G) will take an initial value (G_0), and it will be reduced with iteration cycle as given in Eq. (24) [45-52]:

$$G(t) = G_0 \times e^{-\tau\left(\frac{t}{t_{\max}}\right)}, \tag{24}$$

where G_0 is set to 100, τ is set to 20, t and $t_{\max}$ are the current iteration cycle and the total number of iteration cycles (the total age of the system), respectively. The steps of the GSA are as follows [45-52]:

Step 1. Population (S)-based initialization;

Step 2. Fitness evaluation of the agents;

Step 3. Updating $M_i(t)$ based on Eqs. (16) and (17), $best(t)$ based on Eq. (18), $worst(t)$ based on Eq. (19), and $G(t)$ based on Eq. (24) for $i = 1, 2, ..., S$;

Step 4. Calculation of the total forces in different directions by using Eq. (20);

Step 5. Calculation of the acceleration by Eq. (21) and the velocity by Eq. (22);

Step 6. Updating the agents' positions by Eq. (23);

Step 7. Checking for the constraints of the problem;

Step 8. Repeating Steps 2 to 7 until the stopping criterion is met.

4.1.2. Opposition-based learning: A concept

The process of searching terminates when some predefined criteria are satisfied. In the absence of a priori information about the solution, we usually start with random guesses [53-63]. The computation time, among others, is related to the distances of these initial guesses from the optimal solution [54]. We can improve on chance of starting with a closer (fitter) solution by simultaneously checking the opposite solution [53-63]. By doing this, the fitter one (guess or opposite guess) can be chosen as an initial solution. In fact, according to the theory of probability, for 50% of the time a guess is further from the solution than its opposite guess. Therefore, starting with the closer of the two guesses (as judged by its fitness values) has the potential to accelerate convergence. The same approach can be applied not only to the initial solutions but also continuously to each solution as iteration progresses [53-63].

Definition of opposite number. Let $x \in [a, b]$ be a real number. The opposite number is defined by Eq. (25):

$$\breve{x} = a + b - x. \tag{25}$$

Similarly, this definition can be extended to higher dimensions [53-63] as stated in the next sub-section.

Definition of opposite point. Let $P = (x_1^1, ..., x_1^d, ..., x_1^n)$ be a point in n-dimensional space, where $\{x_1^1, ..., x_1^d, ..., x_1^n\}$ and $x_i \in [a_i, b_i] \forall_i \in \{1, ..., d, ..., n\}$. The opposite point $\breve{P} = (\breve{x}_1^1, ..., \breve{x}_1^d, ..., \breve{x}_1^n)$ is completely defined by its components as stated in Eq. (26) [53-63]:

$$\breve{x}_i = a_i + b_i - x_i. \tag{26}$$

Now, by employing the opposite point definition, the opposition-based optimization is defined as follows.

Opposition-based optimization. Let $P = (x_1^1, ..., x_1^d, ..., x_1^n)$ be a point in n-dimensional space (i.e., a candidate solution). Assume $f = (.)$ is a fitness function which is used to measure the candidate's fitness. According to the definition of the opposite point, $\breve{P} = (\breve{x}_1^1, ..., \breve{x}_1^d, ..., \breve{x}_1^n)$ is the opposite of $P = (x_1^1, ..., x_1^d, ..., x_1^n)$ [53-63]. Now, if $f(\breve{P}) \leq f(P)$ (for a minimization problem), then the point P can be replaced with $\breve{P}$; otherwise, we continue with P. Hence, the point and its opposite point are evaluated simultaneously in order to continue with the fitter one [53-63].

4.1.3. Opposition-based gravitational search algorithm
Similar to all population-based optimization algorithms, two main steps are distinguishable for the OGSA, namely, (i) opposition-based population initialization and producing new generations by adopting the principles of the GSA, and then (ii) the opposition-based generation jumping [53-63]. This hybridization of GSA with opposition concept causes accelerated convergence to a better near global solution. Corresponding pseudo code of the proposed OGSA is as follows [53-63]:

Step 1. Opposition-based population initialization. Generate uniformly distributed initial population P_0

```
for (i = 0; i < S; i++)
        // S: Population size
   for (j = 0; j < n; j++)
        // n: Problem dimension
      OP_{0i,j} = a_j + b_j - P_{0_{i,j}}
        // OP_0: Opposite of initial population P_0
      // [a_j, b_j]: Range of the jth variable
   end for
end for

Select S fittest individuals from the set of
{P_0, OP_0} as initial population P_0.
```

End of opposition-based population initialization;

Step 2. Fitness evaluation of the agents;

Step 3. Updating $M_i(t)$ based on Eqs. (16) and (17), $best(t)$ based on Eq. (18), $worst(t)$ based on Eq. (19), and $G(t)$ based on Eq. (24) for $i = 1, 2, ..., S$;

Step 4. Calculation of the total forces in different directions by using Eq. (20);

Step 5. Calculation of the acceleration by Eq. (21) and the velocity by Eq. (22);

Step 6. Updating the agents' positions by Eq. (23);

Step 7. Checking for the constraints of the problem;

Step 8. Opposition-based generation jumping

```
if (rand(0,1) < J_r) // rand(0,1): Uniformly
 generated random number, J_r: Jumping rate
   for (i = 0; i < S; i++)
     for (j = 0; j < n; j++)
        OP_{i,j} = min_j^p + max_j^p - P_{i,j}
          // min_j^p: minimum value of the jth
          variable in the current population (p)
         // max_j^p: maximum value of the jth
          variable in the current population (p)
        end for
     end for
   end if
Selecting S fittest individuals from set of {P, OP}
 as current population P.
```

End of opposition-based generation jumping;

Step 9. Repeating Steps 2 to 8 until the stopping criterion of maximum generation cycle is met.

5. Numerical simulation results

5.1. Simulation results of Circular Antenna Array (CAA)

Extensive MATLAB simulation studies have been performed for the proposed OGSA-based optimal designing of non-uniform circular antenna arrays. The control

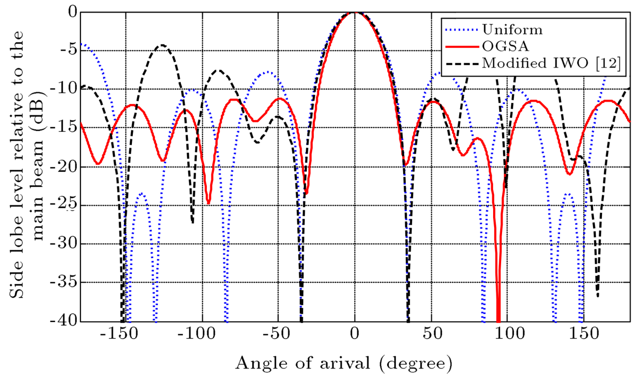

Figure 3. Comparative array patterns for the 8-element non-uniform circular array, OGSA showing much improved SLL.

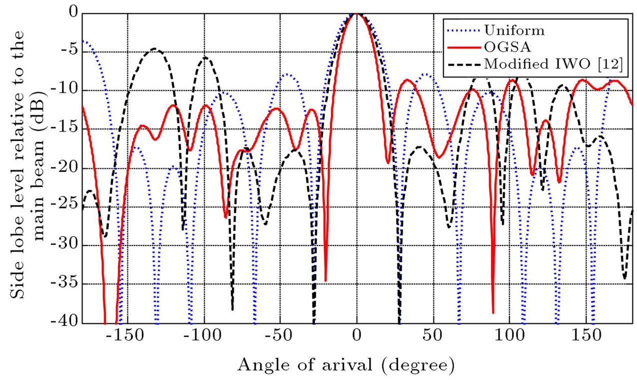

Figure 4. Comparative array patterns for the 10-element non-uniform circular array, OGSA showing much improved SLL.

parameters of OGSA are as follows: Population size (S) =120, Genration cycles = 300, Jr = 0.6; npCount = 0; twoNpCount = 1, alfa = 20, G0 = 10, rNorm = 2; rPower = 1, and elitistCheck = 1. Radiation patterns of the circular array with main lobe steered to $\phi_0 = 0$ degrees are considered. This section gives simulation results of the radiation patterns of three non-uniform circular antennas having 8, 10, and 12 elements, respectively. Figures 3-5 show a comparison among the radiation patterns for un-optimized uniform

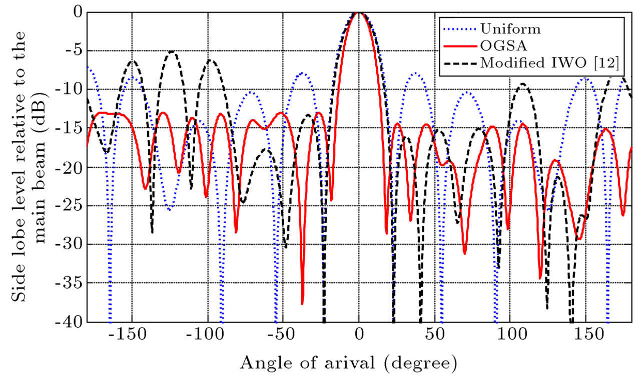

Figure 5. Comparative array patterns for the 12-element non-uniform circular array, OGSA showing much improved SLL.

circular antenna arrays ($d = \lambda/2$), and corresponding non-uniform circular antenna arrays ($\lambda/2 \leq d \leq \lambda$) optimized with the use of OGSA and modified IWO [12]. In case of uniform circular array, radius $a = N.\lambda/(4\pi)$ [26]. Table 1 shows optimal Design variable of current excitation weights and inter-element spacing, FNBW and aperture size obtained by OGSA for three arrays. Table 2 shows average SLLs, worst SLLs, and directivities for three antenna arrays obtained by OGSA. Average SLL is the average of all peaks of side lobes, and worst SLL is the maximum of all peaks of all side lobes. Directivities shown in Table 2 are calculated numerically by Simpson's 1/3 rule. Figure 3 illustrates the case for $N = 8$. For this value of N, average SLL is suppressed to a level of -12.2979 dB, worst SLL is reduced to -11.27 dB, directivity is improved to 11.17 dB, and BWFN is reduced to 65.16 degrees as a result of the optimization by OGSA, whereas modified IWO [12] shows average SLL of -10.75 dB, worst SLL of -2.283 dB, directivity of 11.17 dB, and BWFN of 70.27 degrees, respectively, as shown in Tables 2 and 1, respectively. Figure 4 illustrates the case for $N = 10$. For this value of N, OGSA provides a radiation pattern with average SLL of -14.0270 dB, worst SLL reduced to -8.659 dB, directivity improved to 12.27 dB, and BWFN reduced to 40.32 degrees as

Table 1. Design variables obtained with OGSA for three circular arrays.

No. of elements	FNBW	FNBW in [12]	$[I_1, I_2, I_3, I_4, ..., I_N]$	$[d_1, d_2, d_3, d_4, ..., d_N]$ in λ'_s	Aperture in (λ)
8	65.16	70.27	0.3575 0.3091 0.2807 0.8030 0.7008 0.3858 0.3064 0.8398	0.5150 0.5667 0.7001 0.7763 0.5351 0.8828 0.7736 0.9742	5.7238
10	40.32	55.85	1.0000 0.3102 0.2122 0.4407 1.0000 0.3701 0.4387 0.6342 0.5368 0.7088	0.9047 0.7843 0.7045 0.9976 0.5803 0.6445 0.9976 0.6805 0.9090 0.7639	7.9669
12	36.72	46.26	0.4021 0.5842 0.3538 0.3240 0.4750 0.8726 0.6292 0.3360 0.1304 0.5193 0.6874 0.9343	0.7744 0.9023 0.6236 0.6236 0.7997 0.9216 0.5377 0.9529 0.6633 0.8075 0.9565 0.7100	9.2731

Table 2. Average SLLs, worst SLLs, and directivities for different algorithms and different arrays.

Number of elements	Algorithm	Average SLL in dB	Worst SLL in dB	Directivity in dB
8	OGSA	-12.2979	-11.27	11.17
	Modified IWO [12]	-10.75	-2.283	9.14
	Classical IWO [12]	-9.488	NR*	8.27
	DE [12]	-10.34	NR*	8.87
	PSO [12]	-10.23	NR*	8.78
	GA [12]	-9.16	NR*	8.08
10	OGSA	-14.0270	-8.659	12.27
	Modified IWO [12]	-13.78	-4.636	11.75
	Classical IWO [12]	-11.35	NR*	9.93
	DE [12]	-12.46	NR*	10.79
	PSO [12]	-12.15	NR*	10.55
	GA [12]	-10.65	NR*	9.40
12	OGSA	-14.5239	-12.97	13.31
	Modified IWO [12]	-14.23	-5.105	12.25
	Classical IWO [12]	-10.99	NR*	9.86
	DE [12]	-11.66	NR*	10.39
	PSO [12]	-11.61	NR*	10.37
	GA [12]	-10.12	NR*	9.21

NR*: Not reported in the refereed literature/present work.

compared to average SLL of -13.78 dB, worst SLL of -4.636, directivity of 11.75 dB, and FNBW of 55.85 degrees, obtained, respectively, in modified IWO [12], as shown in Tables 2 and 1, respectively. Lastly, Figure 5 illustrates the case for $N = 12$. For this value of N, as shown in Tables 2 and 1, OGSA provides a radiation pattern having average SLL of -14.5239 dB, worst SLL reduced to -12.97 dB, directivity improved to 13.31 dB, and BWFN reduced to 36.72 degrees as compared to average SLL of -14.23 dB SLL, worst SLL of -5.105 dB, directivity of 12.25 dB, and BWFN of 36.72 degrees, respectively, in Modified IWO [12]. From Table 2 it is clear that as the number of antenna elements N increases, the SLL reduction for non-uniform circular antenna array increases.

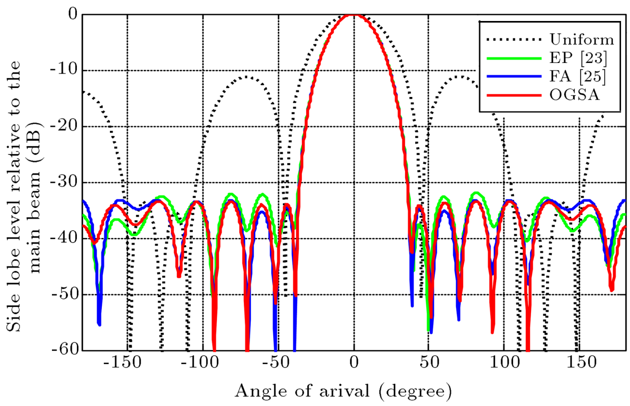

Figure 6. Radiation pattern of CCAA for 4, 6 and 8 elements without center element.

5.2. Simulation results of Concentric Circular Antenna Array (CCAA)

This segment gives the experimental results for various CCAA designs obtained by OGSA techniques. For each optimization technique, two three-ring ($M = 3$) CCAA structures are assumed, each maintaining a non-uniform excitation, the inter-element spacing is assumed to be constant being 0.55λ, 0.606λ, and 0.75λ for the first, second, and third rings with a design goal of maximizing SLL reduction [27-30]. For obtaining the non-uniform excitations sets in each ring, 30 trial generalized optimization runs are used for each structure. For all sets of experiments, the number of elements of the innermost circle is N_1 and for outermost circle is N_3, whereas the middle circle consist of N_2 number of elements [27-30].

Tables 3 and 4 show the optimal excitation coefficient of each element, Side Lobe Level (SLL), and FNBW obtained by the array set ($N_1 = 4$, $N_2 = 6$, and $N_3 = 8$ elements) with and without center element. Figures 6 and 7 show radiation pattern obtained by OGSA for the same array set with and without center element. From Tables 3 and 4 and Figures 6, and 7, it is clear that SLL obtained by OGSA is -33.44 dB, and -43.25 dB, for without center and with center element, respectively, which is better than EP [23], FA [25],

Table 3. Optimal excitation coefficient of each element, SLL, and FNBW, with $N_1 = 4$, $N_2 = 6$ and $N_3 = 8$ without center element.

Algorithms	$I_{1,1}$ $I_{1,2}$ $I_{1,3}$ $I_{1,4}$ $I_{2,1}$ $I_{2,2}$ $I_{2,3}$ $I_{2,4}$ $I_{2,5}$ $I_{2,6}$ $I_{3,1}$ $I_{3,2}$ $I_{3,3}$ $I_{3,4}$ $I_{3,5}$ $I_{3,6}$ $I_{3,7}$ $I_{3,8}$	**SLL (dB)**
OGSA	0.0990 0.5395 0.0643 0.5372; 0.3466 0.3406 0.8060 0.3228 0.3186 0.8120; 0.3901 0.5984 0.3864 0.1992 0.3923 0.5390 0.3971 0.1851	-33.44
FA [25]	0.7025 0.1410 0.6770 0.1215; 0.9999 0.4349 0.4084 0.9999 0.4076 0.4305; 0.2352 0.4789 0.7366 0.4831 0.2542 0.4790 0.7172 0.4730	-33.20
EP [23]	0.3416 0.0496 0.3242 0.0283; 0.5321 0.2114 0.1923 0.4901 0.1876 0.1994; 0.1204 0.2555 0.3527 0.2450 0.1229 0.2294 0.3449 0.240	-31.84
Uniform	1111;111111;11111111	-11.23

Table 4. Optimal excitation coefficient of each element, SLL, and FNBW, with $N_1 = 4$, $N_2 = 6$ and $N_3 = 8$ with center element.

Algorithms	I_{center}; $I_{1,1}$ $I_{1,2}$ $I_{1,3}$ $I_{1,4}$; $I_{2,1}$ $I_{2,2}$ $I_{2,3}$ $I_{2,4}$ $I_{2,5}$ $I_{2,6}$; $I_{3,1}$ $I_{3,2}$ $I_{3,3}$ $I_{3,4}$ $I_{3,5}$ $I_{3,6}$ $I_{3,7}$ $I_{3,8}$	**SLL (dB)**
OGSA	0.5432; 0.5527 0.5471 0.5506 0.3997; 0.4076 0.1523 0.4014 0.4096 0.1427 0.1855; 0.2653 0.1898 0.0510 0.1950 0.2761 0.1926 0.0510 0.3662	-43.25
FA [25]	0.5142; 0.9943 0.8029 0.9508 0.8087; 0.5727 0.7228 0.7056 0.6025 0.7020 0.7262; 0.1516 0.4732 0.6145 0.4837 0.1627 0.4748 0.6159 0.4648	-40.43
EP [23]	0.3770; 0.5502 0.5477 0.5530 0.5890; 0.0976 0.3830 0.3972 0.0999 0.4152 0.4051; 0.0417 0.1730 0.2290 0.1734 0.0401 0.1750 0.2755 0.1717	-39.73
Uniform	1; 1111; 111111; 11111111	-12.31

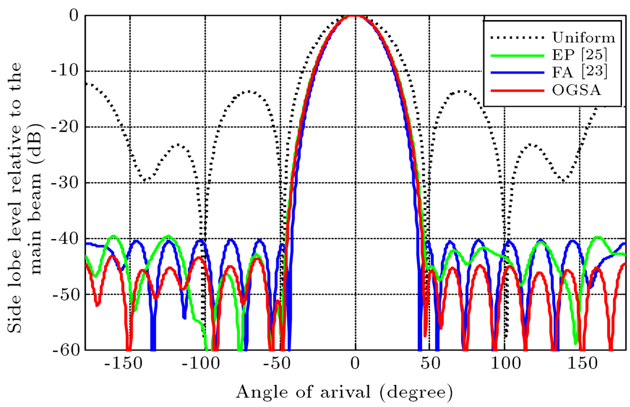

Figure 7. Radiation pattern of CCAA for 4, 6 and 8 elements with center element.

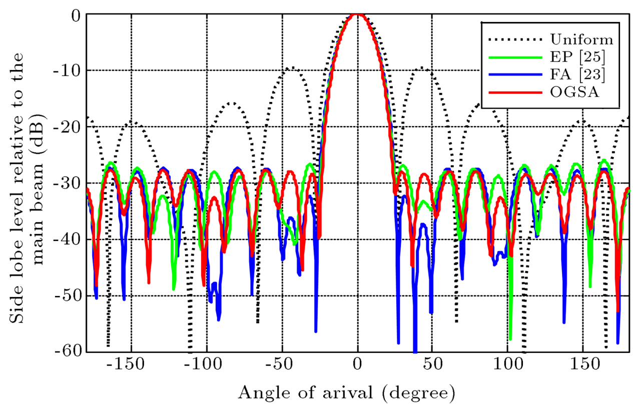

Figure 8. Radiation pattern of CCAA for 8, 10 and 12 elements without center element.

and uniform circular antenna array for corresponding without center and with center element, respectively. Tables 5 and 6 shows the optimal excitation coefficient of each element, Side Lobe Level (SLL), and FNBW obtained by the array set ($N_1 = 8$, $N_2 = 10$ and $N_3 = 12$ elements) with and without center element. Figures 8 and 9, shows radiation pattern obtained by OGSA for the same elements with and without center element. From Tables 5 and 6 and Figures 8, and 9, it is clear that SLL obtained by OGSA is -27.76 dB, and

Table 5. Optimal excitation coefficient of each element, SLL, and FNBW, with $N_1 = 8$, $N_2 = 10$ and $N_3 = 12$ without center element.

Algorithms	$I_{1,2}...I_{1,8}$; $I_{2,1}...I_{2,10}$; $I_{3,1}...I_{3,12}$	SLL (dB)
OGSA	0.6659 0.2415 0.5651 0.4619 0.5178 0.2829 0.5636 0.7579; 0.5410 0.0833 0.0038 0.4313 0.2713 0.2719 0.0385 0.1265 0.5840 0.4137; 0.2463 0.3035 0.7675 0.3238 0.2634 0.3247 0.3170 0.1510 0.7052 0.2780 0.2808 0.2586	-27.76
FA [25]	0.9354 0.7716 0.3013 0.7299 0.8924 0.7641 0.3044 0.7999; 0.5444 0.5686 0.2124 0.1958 0.5901 0.5647 0.6322 0.1498 0.1660 0.6379; 0.5044 0.4125 0.2457 0.9673 0.2516 0.3827 0.4854 0.3444 0.3209 0.9734 0.3290 0.365	-27.49
EP [23]	0.2242 0.2886 0.1891 0.3336 0.5458 0.3895 0.1000 0.2866; 0.1595 0.1378 0.1036 0.1000 0.4048 0.2686 0.3090 0.1000 0.1000 0.1696; 0.2419 0.1183 0.1144 0.4708 0.1685 0.2090 0.2566 0.2200 0.1000 0.4229 0.1273 0.1020	-26.12
Uniform	1 1 1 1 1 1 1 1; 1 1 1 1 1 1 1 1 1 1; 1 1 1 1 1 1 1 1 1 1 1 1	-9.56

Table 6. Optimal excitation coefficient of each element, SLL, and FNBW, $N_1 = 8$, $N_2 = 10$, and $N_3 = 12$ with center element.

Algorithms	I_{center}; $I_{1,2}...I_{1,8}$; $I_{2,1}...I_{2,10}$; $I_{3,1}...I_{3,12}$	SLL (dB)
OGSA	0.3951; 0.7232 0.4874 0.9148 0.4535 0.6670 0.3698 0.5583 0.4712; 0.0273 0.0240 0.7137 0.3321 0.6351 0.0638 0.0766 0.3463 0.2054 0.1856; 0.2865 0.6559 0.3471 0.2897 0.2662 0.3578 0.2577 0.5727 0.1477 0.2712 0.2229 0.5705	-31.85
FA [25]	0.5199; 0.9967 0.5631 0.7591 0.4567 0.6972 0.5239 0.6171 0.6279; 0.3579 0.7551 0.0321 0.1155 0.5299 0.4027 0.4977 0.1285 0.0531 0.7114; 0.3466 0.3824 0.3453 0.8170 0.2995 0.2844 0.3716 0.2866 0.2705 0.7644 0.3284 0.3990	-30.54
EP [23]	0.2750; 0.2989 0.4102 0.3979 0.7325 0.3989 0.3813 0.2785 0.2628; 0.2300 0.0187 0.0464 0.5620 0.2875 0.5240 0.0855 0.0166 0.1763 0.1283; 0.1225 0.1932 0.5081 0.2903 0.2285 0.2227 0.2858 0.2278 0.4828 0.0957 0.1756 0.2082	-28.92
Uniform	1; 11111111; 1111111111; 111111111111	-10.77

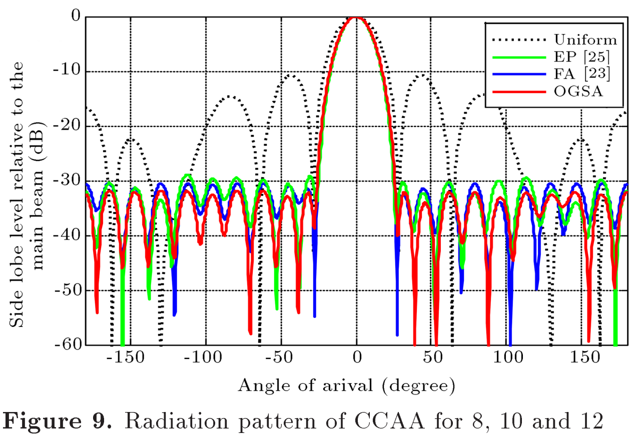

Figure 9. Radiation pattern of CCAA for 8, 10 and 12 elements with center element.

-31.85 dB for without center and with center element, respectively, which is better than EP [23], FA [25] and uniform circular antenna array for corresponding without center and with center element, respectively.

5.3. Comparisons of accuracy

Test for independent samples is conducted at the 5% significance level in order to judge whether the results obtained with the best performing algorithm differ from the results of the other algorithms in a statistically significant way [64]. Two-sample t-test is a hypothesis testing method for determining the statistical significance of the difference between two independent samples of an equal sample size [65]. Therefore, if the t-test result is positive, then it means that the results obtained by different algorithms are different. If the t-test result is negative, the difference between them are not significant. The t-value is defined as given in the following equation. Assume unequal population variances/standard deviations ($\sigma_1^2 \neq \sigma_2^2$), as we have taken two-sample t-test with equal sample size; in this paper sample size is 50 [64].

$$t - \text{test} = \frac{\bar{\alpha}_{22} - \bar{\alpha}_{11}}{\sqrt{\left(\frac{\sigma_2^2}{n_A}\right) + \left(\frac{\sigma_1^2}{n_B}\right)}}, \tag{27}$$

$$\text{Degree of freedom } (\beta) = \min\left(n_A - 1, n_B - 1\right), \tag{28}$$

where $\bar{\alpha}_{11}$ and $\bar{\alpha}_{22}$ are the mean values of the first and second methods, respectively; σ_1 and σ_2 are the standard deviations of the first and second methods, respectively; n_A and n_B are the numbers of sample sizes in the first and second methods, respectively. The degree of freedom for unequal variance and unequal sample size is given by Eq. (28) [64]. In this paper, we have taken n_A equal to n_B. Table 7 shows t-test values obtained between the OGSA and the other optimization methods. When the t-test value given in Table 7 is higher than 1.645, 1.960, 2.326, 2.576, and 3.090 ($\beta \geq 100$), there is a significant difference between the two algorithms with 95%, 97.5%, 99%, 99.5%, and 99.9% confidence level, respectively. The t-test values between the OGSA and the other optimization methods are presented in Table 7. The t-test values of all approaches are larger than 2.15 in case of OGSA/modified IWO [12] and larger than 3.090 in rest of the cases (degree of freedom = 49), meaning that there is a significant difference between the OGSA and other methods with a 98% confidence level in case of OGSA/modified IWO [12] and 99.9% in rest of the cases. Thus, from statistical analysis, it is clear that the OGSA-based optimization technique offers robust and promising results. Table 8 shows mean and slandered deviation of the final objective function for the 8-, 10-, and 12-element arrays. From the same table it is clear that mean and standard deviation values of objective function, as obtained in OGSA, are lower than those of other algorithms, classical IWO and modified IWO, presented in [12]. So, from all the above results, it is established that OGSA results in the best SLL, the lowest FNBW, and much improved directivity for each non uniform circular antenna array.

Table 7. t-test values obtained for comparison of OGSA algorithm with different algorithms.

Number of elements	Algorithm	t-test value
8	OGSA/modified IWO [12]	2.3627
	OGSA/classical IWO [12]	30.8512
	OGSA/DE [12]	36.3721
	OGSA/PSO [12]	12.2462
	OGSA/GA [12]	8.6639
10	OGSA/modified IWO [12]	52.1555
	OGSA/classical IWO [12]	15.2529
	OGSA/DE [12]	25.1210
	OGSA/PSO [12]	12.9509
	OGSA/GA [12]	9.7242
12	OGSA/modified IWO [12]	54.6665
	OGSA/classical IWO [12]	6.9307
	OGSA/DE [12]	23.5768
	OGSA/PSO [12]	6.2444
	OGSA/GA [12]	7.8848

6. Convergence profile of OGSA

The minimum objective function (fit) values against number of generation cycles are recorded to get the convergence profile of OGSA. Figure 10(a) and (b)

Table 8. Means and standard deviations of the final objective function values for different algorithms.

Number of elements	Algorithm	Mean objective function value	Standard deviation for the objective function
8	OGSA	0.4434	0.01419
	Modified IWO [12]	1.0687	0.0122
	Classical IWO [12]	1.1990	0.1726
	DE [12]	1.1355	0.1338
	PSO [12]	1.2079	0.4412
	GA [12]	1.4011	0.7815
10	OGSA	0.4341	0.01569
	Modified IWO [12]	0.6799	0.0294
	Classical IWO [12]	0.8705	0.2017
	DE [12]	0.7895	0.0988
	PSO [12]	0.8461	0.2244
	GA [12]	1.1215	0.4996
12	OGSA	0.01835	0.01207
	Modified IWO [12]	0.7288	0.0911
	Classical IWO [12]	0.9873	0.9885
	DE [12]	0.8046	0.2355
	PSO [12]	0.8918	0.9890
	GA [12]	1.1256	0.9929

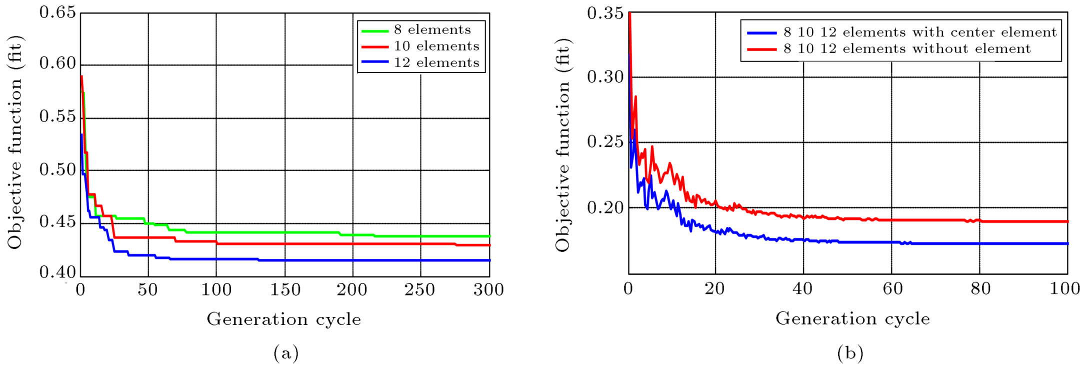

Figure 10. Convergence profile OGSA for (a) CAA, and (b) CCAA.

Table 9. Comparison of execution times, means, and standard deviations of the final (fit) values for different algorithms.

Array type	**No. of elements**	**Execution times (in sec) of OGSA**
CAA	8	358.9920
	10	480.2500
	12	630.2300
CCAA	4, 6, 8 without centre element	925.4566
	4, 6, 8 with centre element	984.2541
	8, 10, 12 without centre element	1205.6556
	8, 10, 12 with centre element	1254.1548

portrays the convergence profile of minimum (fit) for the circular arrays having 8, 10, and 12 elements, and concentric circular antenna arrays having sets of $N_1 = 4$, $N_2 = 6$, $N_3 = 8$ and $N_1 = 8$, $N_2 = 10$, $N_3 = 12$ with and without centre element, respectively. Table 9 shows OGSA's execution times, mean values, and standard deviations for the cost function (fit). The simulation programming was written in MATLAB language using MATLAB 7.5 on dual core (TM) processor, 2.88 GHz with 1 GB RAM.

7. Conclusion

This paper illustrates how to model the optimal design of non-uniform circular and non-uniform concentric circular antenna array for maximal SLL reduction as well as improvement in the FNBW for three non-uniform circular arrays antenna arrays. The new algorithm OGSA efficiently determines the optimal design of non-uniform circular antenna arrays generating radiation pattern with the best optimal SLL reduction and lesser FNBW than other algorithms. Array patterns obtained by OGSA are generally better than those presented in modified IWO [12]. The same algorithm determines optimal designs of non-uniform concentric circular antenna array having elements sets of $N_1 = 4$, $N_2 = 6$, $N_3 = 8$ and $N_1 = 8$, $N_2 = 10$, $N_3 = 12$ with and without centre element, respectively, which generate radiation pattern with the best optimal SLL reduction, as compared to the cases of uniform excitation and uniform inter-element spacing and those of some published works like EP [23], and FA [25]. Thus, OGSA algorithm has proved comparatively more suitable for the present work and thus seems to be a good evolutionary technique to deal with complex nonlinear problem of array antenna design. Thus, OGSA seems to be a suitable algorithm for future research with other array geometries and constraints.

References

1. Ballanis, C.A., *Antenna Theory Analysis and Design*, 2nd Edition, John Willey and Son's Inc., New York (1997).
2. Ram, G., Kar, R., Mandal, D. and Ghoshal, S.P. "Optimal design of non-uniform circular antenna arrays using PSO with wavelet mutation", *Int. J. Bio-Inspired Computation*, **6**(6), pp. 424-433 (2014).
3. Blank, S.J. and Hutt, M.F. "On the empirical optimization of antenna arrays", *IEEE Antennas and Propagation Magazine*, **47**(2), pp. 58-67 (2005).
4. Cengiz, Y. and Tokat, H. "Linear antenna array design with use of genetic, memetic and tabu search optimization algorithms", *Progress in Electromagnetics Research C*, **1**, pp. 63-72 (2008).
5. Mandal, D., Chandra, A., Ghoshal, S.P. and Bhattacharjee, A.K. "Side lobe reduction of a concentric circular antenna array using genetic algorithm", *Ser-*

bian Journal of Electrical Engineering, **7**(2), pp. 141-148 (2010).

6. Elliott, R.S. *Antenna Theory and Design*, Revised Edition, John Wiley, New Jersey (2003).
7. Guney, K. and Onay, M. "Amplitude-only pattern nulling of linear antenna arrays with the use of bees algorithm", *Progress in Electromagnetics Research*, **70**, pp. 21-36 (2007).
8. Pathak, N., Mahanti, G.K., Singh, S.K., Mishra, J.K. and Chakraborty, A. "Synthesis of thinned planar circular array, antennas using modified particle swarm optimization", *Progress in Electromagnetics Research Letters*, **12**, pp. 87-97 (2009).
9. Das, S., Bhattacharjee, S. and Mandal, D. "Improvement of far field radiation pattern of linear array antenna using genetic algorithm", *ICTACT Journal of Communication Technology*, **1**, pp. 23-31 (2010).
10. Ram, G., Mandal, D., Kar, R. and Ghoshal, S.P. "Design of non-uniform circular antenna arrays using firefly algorithm for side lobe level reduction", *World Academy of Science, Engineering and Technology, International Journal of Electrical, Computer, Electronics and Communication Engineering*, **8**(1), pp. 36-41 (2014).
11. Deen, Z., Mady, S., Awadalla, E. and Harsher, K. "Controlled radiation pattern of circular antenna array", *IEEE Anennas and Propagation Symp.*, pp. 3399-3402 (2006).
12. Roy, G.G., Das, S., Chakraborty, P. and Suganthan, P.N. "Design of non-uniform circular antenna arrays using a modified invasive weed optimization algorithm", *IEEE Transactions on Antennas and Propagation*, **59**(1), pp. 110-118 (2011).
13. Panduro, M., Mendez, A., Dominguez, R. and Romero, G. "Design of non-uniform antenna arrays for side lobe reduction using the method of genetic algorithm", *Int. J. Electron. Communication (AEU)*, **60**(10), pp. 713-717 (2006).
14. Sahib, M., Najjar, Y., Dib, N. and Khodier, M. "Design of non-uniform circular antenna arrays using the particle swarm optimization", *Journal of Electrical Engineering*, **59**(9), pp. 216-220 (2008).
15. Stearns, C. and Stewart, A. "An investigation of concentric ring antennas with low sidelobes", *IEEE Trans. Antennas Propag.*, **13**(6), pp. 856-863 (1965).
16. Das, R. "Concentric ring array", *IEEE Trans. Antennas Propag.*, **14**(3), pp. 398-400 (1966).
17. Goto, N. and Cheng, D.K. "On the synthesis of concentric-ring arrays", *IEEE Proc.*, **58**(5), pp. 839-840 (1970).
18. Biller, L. and Friedman, G. "Optimization of radiation patterns for an array of concentric ring sources", *IEEE Trans. Audio Electroacoust.*, **21**(1), pp. 57-61 (1973).
19. Huebner, M.D.A. "Design and optimization of small concentric ring arrays", in *Proc. IEEE AP-S Symp.*, pp. 455-458 (1978).
20. Haupt, R.L. "Optimized element spacing for low sidelobe concentric ring arrays", *IEEE Trans. Antennas Propag.*, **56**(1), pp. 266-268 (2008).
21. Dessouky, M., Sharshar, H. and Albagory, Y. "Efficient sidelobe reduction technique for small-sized concentric circular arrays", *Progress in Electromagnetics Research*, **65**, pp. 187-200 (2006).
22. Haupt, R.L. "Thinned concentric ring arrays", *Antennas and Propagation Society International Symposium*, pp. 1-4 (2008).
23. Mandal, D., Ghoshal, S. and Bhattacharjee, A. "Design of concentric circular antenna array with central element feeding using particle swarm optimization with constriction factor and inertia weight approach and evolutionary programing technique", *J. Infrared Millimeter Terahertz Waves*, **31**(6), pp. 667-680 (2010).
24. Elsaidy, S., Dessouky, M., Khamis, S. and Albagory, Y. "Concentric circular antenna array synthesis using comprehensive learning particle swarm optimizer", *Progress in Electromagnetics Research Letters*, **29**, pp. 1-13 (2012).
25. Sharaqa, A. and Dib, N. "Circular antenna array synthesis using firefly algorithm", *International Journal of RF and Microwave Computer-Aided Engineering*, **24**(2), pp. 139-146 (2014).
26. Ram, G., Mandal, D., Kar, R. and Ghoshal, S.P. "Circular and concentric circular antenna array synthesis using CSO", *IETE Technical Review*, Taylor & Francis, **32**(3), pp. 204-217 (2015).
27. Mandal, D. and Ghoshal, S.P. "Wavelet mutation based novel PSO for optimal design of concentric circular array antennas", *Journal of Electrical and Control Engineering*, **3**(1), pp. 35-46 (2013).
28. Mandal, D., Ghoshal, S.P. and Bhattacharjee, A.K. "Determination of the optimal design of three-ring concentric circular antenna array using evolutionary optimization techniques", *International Journal of Recent Trends in Engineering*, **2**(5), pp. 110-115 (2009).
29. Mandal, D., Ghoshal, S.P. and Bhattacharjee, A.K. "Optimal design of concentric circular antenna array using particle swarm optimization with constriction factor approach", *International Journal of Computer Applications*, **1**(17), pp. 94-98 (2010).
30. Mandal, D., Goswami, B., Kar, R. and Ghoshal, S.P. "Particle swarm optimization with constriction factor and inertia weight approach based synthesis of concentric circular antenna array with non-isotropic elements", *ACEEE Int. J. on Communications*, **3**(1), pp. 5-9 (2012).
31. Das, S., Mandal, D., Kar, R. and Ghoshal, S.P. "Comparative performance of RGA and BAT algorithms in location only null synthesis of circular antenna arrays", *International Journal of Signal Processing Systems*, **1**(2), pp. 159-163 (2013).

32. Reyna, A., Panduro, M.A., Covarrubias, D.H. and Mendez, A. "Design of steerable concentric rings array for low side lobe level", *Scientia Iranica, D*, **19**(3), pp. 727-732 (2012).

33. Basu, B. and Mahanti, G.K. "Thinning of concentric two-ring circular array antenna using firefly algorithm", *Scientia Iranica, D*, **19**(6), pp. 1802-1809 (2012).

34. Wu, H., Liu, C. and Xie, X. "Thinning of concentric circular autenna arrays using improved binary invasive weed optimization algorithm", *Hindawi Publishing Corporation Mathematical Problems in Engineering*, **2015**, pp. 1-8 (2015) http://dx.doi.org/10.1155/2015/365280.

35. Wang, Y., He, X., Wang, J., Berezin, S. and Mathis, W. "Antenna array pattern synthesis via coordinate descent method", *Journal of Electromagnetic Analysis and Applications*, **7**, pp. 168-177 (2015).

36. Civicioglu, P. "Circular antenna array design by using evolutionary search algorithms", *Progress in Electromagnetics Research B*, **54**, pp. 265-284 (2013).

37. Mandal, D. and Ghoshal, S.P. "Wavelet mutation based novel PSO for optimal design of concentric circular array antennas", *Journal of Electrical and Control Engineering*, **3**(1), pp. 35-46 (2013).

38. Mandal, S., Kar, R., Mandal, D. and Ghoshal, S.P. "Design of optimal linear phase FIR high pass filter using craziness based particle swarm optimization technique", *Journal of King Saud University - Computer and Information Sciences*, **24**(1), pp. 83-92 (2012).

39. Ram, G., Mandal, D., Kar, R. and Ghoshal, S.P. "Optimized hyper beamforming of linear antenna srrays using collective animal behaviour", *The Scientific World Journal*, Hindwai, pp. 1-13 (2013).

40. Haupt, R.L. and Werner, D.H., *Genetic Algorithms in Electromagnetics*, IEEE Press Wiley-Interscience (2007).

41. Nadjafi, B.A., Rahimi, A. and Karimi, H. "A genetic algorithm for mode identity and the resource constrained project scheduling problem", *Scientia Iranica, E*, **20**(3), pp. 824-831 (2013).

42. Bidabadi, M., Sadaghiani, A.K. and Azad, A.V. "Spiral heat exchanger optimization using genetic algorithm", *Scientia Iranica, B*, **20**(5), pp. 1445-1454 (2013).

43. Holland, J.H., *Adaptation in Natural and Artificial Systems*, Ann Arbor, MI: Univ. Michigan Press (1975).

44. Kennedy, J. and Eberhart, R. "Particle swarm optimization", in *Proc. IEEE Int. Conf. on Neural Network*, **4**, pp. 1942-1948 (1995).

45. Rashedi, E., Pour, H.N. and Saryazdi, S. "GSA: A gravitational search algorithm", *Information Sciences*, **179**(13), pp. 2232-2248 (2009).

46. Saha, S.K., Ghoshal, S.P., Kar, R. and Mandal, D. "Design and simulation of FIR band pass and band stop filters using gravitational search algorithm", *Journal of Memetic Computing*, Springer, **5**(4), pp. 311-321 (2013).

47. Islam, R., Kar, R., Mandal, D. and Ghoshal, S.P. "Swarm intelligence based design and simulation of FIR high pass filter using gravitational search algorithm", *Swarm, Evolutionary, and Memetic Computing, LNCS*, Springer, **8297**, pp. 547-557 (2013).

48. Saha, S.K., Kar, R., Mandal, D. and Ghoshal, S.P. "Optimal IIR filter design using gravitational search algorithm with wavelet mutation", *Journal of King Saud University - Computer and Information Sciences*, Elsevier, **27**(1), pp. 25-39 (2015).

49. Saha, S.K., Ghoshal, S.P., Kar, R. and Mandal, D. "Gravitational search algorithm in digital FIR low pass filter design", *Proc. IEEE EAIT*, pp. 52-55 (2012).

50. Saha, S.K., Kar, R., Mandal, D. and Ghoshal, S.P. "Gravitational search algorithm with wavelet mutation applied for optimal IIR band pass filter design", *Proc. IEEE ICCSP*, pp. 14-18 (2013).

51. Ram, G., Chakravorty, P., Mandal, D., Kar, R. and Ghoshal, S.P. "GSAWM for beamforming and directivity of linear antenna arrays", *IEEE ICECS'15*, pp. 1541-1545 (2015).

52. Kumar, Y. and Sahoo, G. "A review on gravitational search algorithm and its applications to data clustering & classification", *I.J. Intelligent Systems and Applications*, **06**, pp. 79-93 (2014).

53. Saha, S.K., Kar, R., Mandal, D., Mukherjee, V. and Ghoshal, S.P. "IIR system identification using opposition-based BAT", *International Journal on Bio-Inspired Computing, Inderscience*, **5**(2), pp. 99-132 (2013).

54. Saha, S.K., Datta, R., Chowdhury, R., Kar, R., Mandal, D. and Ghoshal, S.P. "Efficient and accurate optimal linear phase FIR filter design using opposition based harmony search lagorithm", *The Scientific World Journal, Hindwai*, Article ID 320489, pp. 1-16 (2013).

55. Upadhyay, P., Kar, R., Mandal, D., Mukherjee, V. and Ghoshal, S.P. "IIR system identification using opposition-based harmony search algorithm", *Journal of the Franklin Institute*, Elsevier, **351**(5), pp. 2454-2488 (2014).

56. Upadhyay, P., Kar, R., Mandal, D. and Ghoshal, S.P. "Opposition based differential evolution for IIR system identification problem", *International Journal of Modeling, Simulation, and Scientific Computing, World Scientific*, **5**(4), Paper 1450016 (2014).

57. Mallick, S., Kar, R., Mandal, D. and Ghoshal, S.P. "Optimal sizing of CMOS analog circuits using gravitational search algorithm with particle swarm", *International Journal of Machine Learning and Cybernetics*, Springer, pp. 1-23, 07 (Jan. 2015). DOI 10.1007/s13042-014-0324-3.

58. Tizhoosh, H.R. "Opposition-based learning: A new scheme for machine intelligence", In *Proc. Int. Conf. on Computational Intelligence for Modelling Control and Automation*, **1**, pp. 695-701 (2005).

59. Tizhoosh, H.R. "Reinforcement learning based on actions and opposite actions", In *Proc. ICGST Int. Conf. Artif. Intell. Mach. Learn. Egypt*, pp. 94-98 (2005).

60. Tizhoosh, H.R. "Opposition-based reinforcement learning", *Journal of Advanced Computational Intelligence and Intelligent Informatics*, **10**(4), pp. 578-585 (2006).

61. Shokri, M., Tizhoosh, H.R. and Kamel, M. "Opposition-based $Q(\lambda)$ algorithm", *IEEE Int. Joint Conf. on Neural Networks*, Vancouver, BC, pp. 254-261 (2006).

62. Ventresca, M. and Tizhoosh, H.R. "Improving the convergence of back propagation by opposite transfer functions", *IEEE Int. Joint Conf. on Neural Networks*, Vancouver, BC, pp. 4777-4784 (2006).

63. Rahnamayan, S., Tizhoosh, H.R. and Salama, M.M.A. "Opposition-based differential evolution", *IEEE Trans. on Evolutionary Computation*, **12**(1), pp. 64-79 (2008).

64. Ram, G., Mandal, D., Kar R. and Ghoshal S.P. "Improvement in various radiation characteristics of time modulated linear antenna arrays using evolutionary algorithms", *Journal of Experimental & Theoretical Artificial Intelligence*, (2015) DOI: 10.1080/0952813X.2015.1020522.

65. Walpole, R.E. and Myer, R.H., *Probability and Statistics for Engineers and Scientists*, New York: Macmillan (1978).

Biographies

Gopi Ram received the BE degree in Electronics and Telecommunication Engineering from Government Engineering College, Jagdalpur, Chhattisgarh, India, in 2007. He received the MTech degree from National Institute of Technology, Durgapur, West Bengal, India, in 2011. Presently, he is attached to National Institute of Technology, Durgapur, West Bengal, India, as Institute PhD Research Scholar at the Department of Electronics and Communication Engineering. His research interest includes array antenna design via evolutionary computing techniques.

Durbadal Mandal received the BE degree in Electronics and Communication Engineering from Regional Engineering College, Durgapur, West Bengal, India, in 1996. He received the MTech and PhD degrees from National Institute of Technology, Durgapur, West Bengal, India, in 2008 and 2011, respectively. Presently, he is attached to National Institute of Technology, Durgapur, West Bengal, India, as Assistant Professor at the Department of Electronics and Communication Engineering. His research interest includes array antenna design and filter optimization via evolutionary computing techniques. He has published more than 220 research papers in International Journals and Conferences.

Rajib Kar received the BE degree in Electronics and Communication Engineering from Regional Engineering College, Durgapur, West Bengal, India, in 2001. He received the MTech and PhD degrees from National Institute of Technology, Durgapur, West Bengal, India, in 2008 and 2011, respectively. Presently, he is attached to National Institute of Technology, Durgapur, West Bengal, India, as Assistant Professor at the Department of Electronics and Communication Engineering. His research interest includes VLSI signal processing, and filter optimization via evolutionary computing techniques. He has published more than 250 research papers in International Journals and Conferences.

Sakti Prasad Ghoshal received the BSc and BTech degrees in 1973 and 1977, respectively, from Calcutta University, West Bengal, India. He received MTech degree from I.I.T (Kharagpur), in 1979. He received PhD degree from Jadavpur University, Kolkata, West Bengal, India, in 1992. Presently, he is acting as Professor at Electrical Engineering Department in N.I.T. Durgapur, West Bengal, India. His research interest areas are: Application of evolutionary computing techniques to electrical power systems, digital signal processing, array antenna optimization, and VLSI. He has published more than 270 research papers in International Journals and Conferences.