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A heuristic approach for the sport equipment allocation and member recruiting management

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Abstract. Some practices arising in sport club industry include: (1) Most sport clubs run their business by taking membership programs, and divide their memberships into several distinct classes by applying restrictions; (2) They operate in chain business mode and allow their members to enjoy their services at any affiliate; and (3) They usually recruit members as many as possible to increase their revenues. Due to the limitless acceptance of customers' subscriptions, users usually face the situation that their preferred equipments are occupied. Thus, in order to improve revenues and enhance corporate image, sport clubs can not only pay attention to the supply management of fitness equipment allocation, but also attempt on the demand management in terms of member recruiting limit. This study develops a mathematical model for determining the resource allocation of fitness equipments and the member recruiting limit decisions for a chain fitness club. The model is a constrained nonlinear integer problem, and in this paper a computational approach is proposed to solve the considered problem. Numerical results show that the proposed heuristic approach can efficiently obtain compromised solutions.

1. Introduction

Sport club industry has many characteristics. First, most sport clubs adopt membership programs and divide their members into several membership classes under distinct restrictions. That is, users need to join their clubs as members. When joining a club, subscribers will get membership cards which indicates the validity periods and restrictions of their jointed membership classes. For example, in terms of validity periods, membership cards include annual cards, semi-annual cards and quarterly cards. In terms of restrictions on usable sport equipments, general members can enjoy full access to all sport equipments at their clubs. A freedom member card can use all sport equipments except for spa facility, swimming pool and thermal experience rooms. The second characteristic is that sport clubs usually have several affiliates and operates in chain business mode. Under chain business structure, they also allow their members to enjoy their services at any affiliate.

To serve members, sport clubs should consider the problem of what types of sport equipments and how many of them should be allocated to each affiliate. These decision variables will lead to situations of overprovision or underprovision for sport equipments. For the former situation, some sport equipments will become idle. For the latter situation, some of members' requests will be denied since their requested resources are occupied. Practically, most sport clubs attempt to avoid overprovision. In addition, since not all of the sport clubs' members would use resources at the same time, there is a tendency that most sport clubs accept

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as many members as possible to improve the utilization of sport equipments and increase their revenues. However, in common, the frequency of requests at any time is directly affected by the number of members. Thus, this strategy of accepting as many members as possible usually leads to underprovision and results in high occupation ratio, which is the proportion of the potential users to the allocated sport equipments. Especially at peak time periods, most members will be forced to wait since the allocated sport equipments may be insufficient to support members' requests. This result does not only sacrifice the members' rights, but also damages the sport clubs' corporate image.

Thus, to avoid such a situation, sport clubs can pay attention not only to their sport equipment allocation problem, but also to the problem of their member recruiting limit. The problem of member recruiting limit focuses on how many members should be accepted so as to reduce penalty costs of over-recruiting members.

The proposed sport equipment allocation is a nonlinear resource allocation problem which aims at optimizing the resource allocation for maximizing a nonlinear objective over the given resource constraint. Relevant literature has focused on the use of mathematical programming approaches, meta-heuristic algorithms or evaluation methods to solve the problems. For example, Ramanathan and Ganesh [1] applied the AHP approach to deal with resource allocation problems. Blake and Carter [2] developed an approach for allocating resources in hospitals. Their methodology uses two linear goal programming models which allow decision makers to set case mix and case costs such that the institution is able to break even while preserving physician income and minimizing disturbance to practice. Klein et al. [3] dealt with a multi-period resource allocation problem in which excess resources can be used in subsequent periods, and certain substitutions among the resources are feasible. Their objective was to find the maximal flow in a related multi-period network. Patriksson [4] surveyed the history and applications of the continuous nonlinear resource allocation problem, as well as algorithmic approaches to its solution.

Darmann et al. [5] investigated a Resource Allocation Problem (RAP) where each task requires a certain amount of a limited resource for a certain time interval. Lykina [6] investigated an infinite horizon resource allocation problem. He showed that there is no optimal solution for this problem. However, after modifying the problem as an adapted resource allocation problem by adding an additional state constraint of weight function, he showed that the problem has an optimal solution which was identified by means of the duality concept of Klotzler.

In addition, Medernach and Sanlaville [7] dealt with the allocation of a limited set of identical resources to a set of users. The requests of each user arrive at successive time. They are increasing and unknown in advance, but follow some anticipated pattern. Such a problem occurs in many application domains of resource allocation. Consider that a set of users share a common fixed pool of resources controlled by a central manager or fair regulation authority. The limited resources are water supply, [8] satellite orbit resources [9], or IP addresses pool shared by many teams in some organization.

Vissers [10] dealt with inpatient resource allocation problems for hospitals. The inpatient resources are such as beds, operating theatres and nursing staff. A procedure was developed to update resource allocations on a regular basis. Yang et al. [11] dealt with a resource allocation problem for time-reservation systems in which customers arrive at a service affiliate and receive service in two steps. In the first step, information is gathered from the customer, which is then sent to a pool of computing resources, and in the second step, the information is processed after the customer leaves the system. The purpose is to decide how many processors to be allocated for the second processing step such that reservation and holding costs are minimized.

The member recruiting problem relates to the threshold setting problem. The value of threshold refers to the sales limit over which subscribers are no longer accepted. The threshold setting problems have been studied in airlines and hotel industries. According to the threshold values, airlines and hotels determine what overbooking levels are adapted for their distinct commodity classes [12].

The existing related models on the threshold setting problems are the perishable inventory models with cancelations. We refer the readers to the survey paper of McGilL and van Ryzin [13] and Weatherford and Bodily [14] for perishable inventory models. In addition, Zhao et al. [15] provided an integer programming model to analyze a downtown space reservation system. Serel et al. [16] proposed a capacity reservation decision for manufacturers in uncertain supply markets. Gutierrez et al. [17] examined the disability-accessibility of the online reservations systems of commercial US airline carriers, as well as those of foreign carriers that fly to/from the US. Wang et al. [18] employed airline reservation technology to improve a navy training quota management system. Meidan [19] investigated various hotel reservation methods. Haerian et al. [20] examined two nesting reservation systems and highlighted their differences. Boyar et al. [21] discussed a variant of the seat reservation problems in which passengers are allowed to change their seats during the trip. Morosan and Jeong [22] investigated the users' perceptions of two types of hotel reservation websites. You [23] proposed an algorithm to develop decisions to railway booking problems.

Many studies on airline and hotels have shown that setting sales threshold can improve revenues and service quality for airlines and hotels. However, it can not be directly applied to sport club industry since the available consuming time is in a continuous state for the service of sport clubs, while it is in a discrete state for an airline or a hotel. For example, a user of a sport club can enjoy their service at any time during his/her subscribing period. However, a customer of an airline can board and get off an airplane only at his departure time and arrival time, respectively. Thus, there exist some different factors among sport clubs, airline industry, and hotel industry.

Knop et al. [24] focused on the management practice in sports clubs, and they developed a method to introduce the principles of service and quality management in these particular organizations. The purpose of their approach is to help sports clubs and federations with the introduction of quality management in their respective sports.

Wicker and Breuer [25] provided an empirical evidence of the organizational capacity and the resource profile of non-profit sport clubs in Germany in 2007. Their study indicated that sport clubs seem to have organizational capacity, as they have many different types of resources at their disposal that can be attributed to four capacity dimensions, namely, human resources capacity, financial resources capacity, network resources capacity and infrastructure resources capacity. Their study showed that German sport clubs are indeed characterized by scarce resources, especially in the fields of human resources and infrastructure resources (public sport facilities).

To our knowledge, no work deals with the resource allocation and member threshold setting problems for sport clubs. This paper attempts to develop a mathematical model to deal with this problem. The proposed model is a constrained nonlinear integer problem and an NP problem. We will develop an efficient computational approach to acquire a compromising solution for the problem. The solutions obtained by the heuristic approach are compared with those found by the CPLEX software. Numerical results show that the proposed heuristic approach only requires a small amount of CPU time to attain confidential solutions.

2. Modeling assumption and formulation

Consider that a sport club has M club affiliates at M distinct locations and J types of sport equipments. The sport club wishes to make a member sale plan over a planning horizon which is divided into $T+1$ periods, and the periods are numbered in order sequence, i.e. $t=1$ refers to the first period, $t=2$ to the second period, and so on. A period step could be 1-day, 1-week, and so on.

The chain club has a number of membership classes indexed by ℓ, $\ell = 1,2,...,L$, and the prices for membership classes are $p_1, p_2, ..., p_L$, with $p_1 > p_2 > ... > p_L$. Thus, membership class-1 is the most expensive class, and is followed by class-2, and so on. The restrictions on distinct memberships are the usable equipments. We assume that sport equipment j is dedicated to users of membership classes 1 to k_j only. That is, a member whose membership class index is higher than k_j cannot use sport equipment j. The chain clubs have to allocate their available sport equipments among their club affiliates. The space needed by allocating one unit of sport equipment j is g_j space units. The available space capacity of club branch-m is G^m space units.

Since a subscription for a certain membership class may have different valid subscribed intervals, products are distinct in terms of membership classes and valid subscribed interval. We use symbol (ℓ, i, j) to denote a product with membership class ℓ and valid subscribed interval from period $i+1$ to period $j+1$. The sales price of product (ℓ, i, j) is assumed to be $p_{\ell,i,j}$ units of money. To simplify the modeling complexity, we assume that any subscription with valid subscription period including period t should be made no later than period $t-1$. Thus, the member sale plan for the club is the interval between periods 1 to T and the valid subscribed interval for subscribers is the interval between periods 2 to $T+1$.

Demand for product (ℓ, i, j) per period is assumed to follow a Poisson distribution with density function $f_{\ell ij}(x)$ and parameter $d_{\ell ij}$. At any time, only a portion of members may go to a club affiliate to do exercise. The potential users consist of a fraction, denoted by $r^m_{\ell j}$, of the members of membership class ℓ who will use equipment j at affiliate m in a period, where:

$$0 \leq r^m_{\ell} < 1 \text{ and } \sum_{m=1}^{M} \sum_{j=1}^{J} r^m_{\ell j} < 1.$$

Formally, we define:

$$\pi_\ell = 1 - \sum_{m=1}^{M} \sum_{j=1}^{J} r^m_{\ell j},$$

as the fraction of the members of membership class ℓ who never go to any affiliate to use equipment.

In addition, we assume that if the number of sport equipment j allocated to affiliate m is less than the number of units needed by expected users who will use this equipment, it incurs a waiting cost c_j per unit of unsatisfied demand for sport equipment j. The purpose of the chain club is to maximize the total profit by allocating all sport equipments among M club affiliates and determining the sales limits $b_{\ell ij}$s for all products. The value of sales limit refers to the sales limit over which subscribers are no longer accepted. The notations mentioned above are summarized as follows.

Notation:

J	The total number of sport equipment types;
L	The total number of membership classes in which the class level is in the natural order of sequence, that is, class-1 is the highest level and class-L is the lowest level;
M	The total number of service affiliates/locations;
T	The total number of planning periods;
(ℓ, i, j)	A product of membership class ℓ and its valid service interval is from period $i+1$ to period $j+1$;
c_j	The waiting cost per unit of unsatisfied demand of equipment j;
$d_{\ell ij}$	The mean demand for product (ℓ, i, j);
G^m	The space capacity for affiliate m;
g_j^m	The space capacity of equipment type j;
k_j	The lowest membership class that can use sport equipment j;
$p_{\ell ij}$	The unit sales price for product (ℓ, i, j);
$r_{\ell j}^m$	The fraction that the members of membership class ℓ will use equipment j at affiliate m at any time;
S_j	The total number of units of equipment type j;
a_j^m	The number of units of equipment j allocated to affiliate m;
$b_{\ell ij}$	The sales limit for product (ℓ, i, j).

The mathematical model is developed as follows.

Let $u_{\ell ij}(b_{\ell ij})$ be the expected sales number of product (ℓ, i, j) when the sales limit for that product is set at $b_{\ell ij}$. That is:

$$u_{\ell ij}(b_{\ell ij}) = \sum_{x=0}^{\infty} \min\{b_{\ell ij}, x\} f_{\ell ij}(x). \tag{1}$$

Let $R_{\ell,t}$ be the total revenue of membership class ℓ in period t when the sales limit for product (ℓ, i, j) is set at $b_{\ell ij}$. For the subscription made during period t, the starting period of the subscription must be $i \geq t+1$. Thus, products offered during period t is the set of $\{(\ell, i, j) | t \leq i \leq j \leq T\}$. Then, $R_{\ell,t}$ is given by:

$$R_{\ell,t} = \sum_{i=t}^{T} \sum_{j=i}^{T} u_{\ell ij}(b_{\ell ij}) p_{\ell ij}. \tag{2}$$

Let $V_{\ell,t}$ be the number of expected subscribers of membership class ℓ given that the valid interval of each subscription includes period $t+1$. Note that a subscription with a valid interval including period $t+1$ could only be sold over periods 1 to t. Thus, if a subscription with valid interval including period $t+1$ is made during period $k, 1 \leq k \leq t$, then the service start period i and the service expiration period j of the subscription must satisfy the conditions of $k+1 \leq i \leq t+1$ and $t+1 \leq j \leq T+1$. Accordingly, products with valid subscription interval including period $t+1$, offered during period $k, 1 \leq k \leq t$, are the set of $\{(\ell, i, j) | k \leq i \leq t, t \leq j \leq T\}$. Thus, we have:

$$V_{\ell t} = \sum_{k=1}^{t} \sum_{i=k}^{t} \sum_{j=t}^{T} u_{\ell ij}(b_{\ell ij}). \tag{3}$$

Therefore, the total expected number of users who use sport equipment j at affiliate m in period t is given by $\sum_{\ell=1}^{k_j} V_{\ell,t} r_{\ell,j}^m$. Suppose that a_j^m units of sport equipment j are located in branch club m. Then, the expected total waiting cost of sport equipment j at branch club m, $C_j^m(a_j^m)$, is given by:

$$C_j^m(a_j^m) = \max\{\sum_{\ell=1}^{k_j} V_{\ell,t} r_{\ell,j}^m - a_j^m, 0\} c_j, \forall\ m, j. \tag{4}$$

Let $R(\mathbf{a}, \mathbf{b})$ represent the total expected profit when the sale limits are set at $\mathbf{b}$ where $\mathbf{b}$ is a $0.5LT(T+1)$-dimensional vector with element $b_{\ell ij}$ at $0.5(\ell-1)T(T+1) + 0.5j(j-1) + i$-th entry, and the allocation of the sport equipments is set at $\mathbf{a}$. Then, the total expected revenue, $R(\mathbf{a}, \mathbf{b})$, can be expressed as follows:

$$R(\mathbf{a}, \mathbf{b}) = \sum_{\ell=1}^{L} \sum_{t=1}^{T} R_{\ell,t} - \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{t=1}^{T} C_{jt}^m(a_j^m). \tag{5}$$

Since we assume that the number of resource units allocated to all affiliates is equal to their available resource units and the consumed space by those allocated resource cannot exceed the facility's space capacity, the capacity constraints can be expressed as follows:

$$\sum_{m=1}^{M} a_j^m = N_j,\ \forall\ \ell, j, \tag{6}$$

$$\sum_{j=1}^{J} a_j^m g_j \leq G^m,\ \forall\ m. \tag{7}$$

Accordingly to the above equations, we can formulate the problem as follows:

$$\max_{\mathbf{a},\mathbf{b}} R = \sum_{\ell=1}^{L}\sum_{t=1}^{T} R_{\ell,t} - \sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{t=1}^{T} C_{jt}^{m}(a_j^m), \tag{8}$$

subject to the constraints of Eqs. (6) and (7). In this paper, we refer to the above formulation as Problem Q_0.

3. Heuristic approach

The mathematical formulation of the considered problem is a constrained mixed-integer nonlinear minimization problem. In this problem, there are two kinds of variables **a** and **b**. Due to the computational complexity of the model, it seems impossible to derive the exact closed forms of the problem. To obtain a compromised solution within a reasonable CPU time, this study presents a hybrid genetic-based algorithm to solve the problem. The approach is divided into three steps. In the first step, the algorithm determines the equipment allocation decision. In the second step, the algorithm determines the sales limit through the determination of the expected sales number.

To develop the heuristic approach, we need to analyze the characteristics of $u_{\ell ij}(b_{\ell ij})$. Since the value of $u_{\ell ij}(b_{\ell ij})$ can be considered as the expected sales of product (ℓ, i, j), when the sales limit is set at $b_{\ell ij}$, the value of $u_{\ell ij}(b_{\ell ij})$ has the following properties.

Lemma 1. The function of $u_{\ell ij}(b)$ is increasing and concave in b, and can take an integer value close to every nonnegative number between $[0, d_{\ell ij}]$.

Proof. First, we observe that $u_{\ell ij}(b_{\ell ij})$ is increasing in $b_{\ell ij}$ since:

$$u_{\ell ij}(b_{\ell ij}+1) - u_{\ell ij}(b_{\ell ij}) = 1 - \sum_{x=0}^{b_{\ell ij}} f_{\ell ij}(x) > 0.$$

Second, let:

$$\Delta u_{\ell ij}(b_{\ell ij}) = u_{\ell ij}(b_{\ell ij}) - u_{\ell ij}(b_{\ell ij}-1).$$

We see that $u_{\ell ij}(b_{\ell ij})$ is concave in $b_{\ell ij}$ since:

$$u_{\ell ij}(b_{\ell ij}+2) - 2u_{\ell ij}(b_{\ell ij}+1) + u_{\ell ij}(b_{\ell ij})$$
$$= -f_{\ell ij}(1+b_{\ell ij}) < 0.$$

Moreover:

$$lim_{b\to\infty} u_{\ell ij}(b) = \sum_{x=0}^{\infty} x f_{\ell ij}(x) = d_{\ell ij},$$

and it implies that the upper bound of $u_{\ell ij}(b_{\ell ij})$ is not larger than $d_{\ell ij}$. Here, the proof is completed.□

Next, we develop our heuristic approach. The problem is composed of two integer variables, namely, the allocation and the threshold decisions. The mathematical model is still a nonlinear problem, even though the integer constraints on these two variables are relaxed. According to Lemma 1, we can find the value of threshold, $b_{\ell ij}$, when the value of $u_{\ell ij}(b_{\ell ij})$ is determined. Thus, instead of directly finding the values of thresholds, we can find them by determining the values of $u_{\ell ij}(b_{\ell ij})$s. In the first step, we find the allocation decision. However, since the constraints on thresholds cannot be ignored, we replace the function of $u_{\ell ij}(b_{\ell ij})$ in Problem Q_0 with a real variable $\tilde{u}_{\ell ij}$ to overcome this problem. Thus, solving Problem Q_1 helps us to find the allocation decision without violating the constraints on thresholds.

Problem $\mathbf{Q}_1$:

$$\max_{\tilde{\mathbf{u}},\tilde{\mathbf{a}}} \tilde{R} = \sum_{\ell=1}^{L}\sum_{t=1}^{T}\sum_{i=t}^{T}\sum_{j=i}^{T} \tilde{u}_{\ell ij} p_{\ell ij}, \tag{9}$$

subject to:

$$\sum_{m=1}^{M} \tilde{a}_j^m = N_j, \ \forall\, j, \tag{10}$$

$$\sum_{j=1}^{J} \tilde{a}_j^m g_j \le G^m, \ \forall\, m, \tag{11}$$

$$\sum_{\ell=1}^{k_j}\sum_{k=1}^{t}\sum_{i=k}^{t}\sum_{j=t}^{T} \tilde{u}_{\ell ij} r_{\ell,j}^m \le \tilde{a}_j^m, \ \forall\, m, j, t, \tag{12}$$

$$\tilde{u}_{\ell ij} \le d_{lij}, \ \forall\, \ell, i, j. \tag{13}$$

Problem Q_1 is a linear programming model. By Lemma 1, $u_{\ell ij}(b_{\ell ij})$ is never larger than $d_{\ell ij}$. Thus, we include Eq. (13) to avoid the situation of $u_{\ell ij}(b_{\ell ij}) > d_{\ell ij}$. Let $\tilde{a}_j^{m*}$ be the optimal values of the variables $\tilde{a}_j^m$ in Problem Q_1. Then, we firstly assign $\bar{a}_j^m = \lfloor \tilde{a}_j^{m*} \rfloor$ units of equipment type j to affiliate m. After that, $n_j = N_j - \sum_{m=1}^{M} \bar{a}_j^m$ units of equipment type j remain. We further assign the remaining resource, one by one, to each affiliate, according to the order sequence of values of $\tilde{a}_j^{m*} - \bar{a}_j^m$. Then, we assign the sport equipments to all affiliates. Next, we determine the values of $b_{\ell ij}$. First, we introduce the symbol y_{mjt} which is determined by the following formula:

$$y_{mjt} = \begin{cases} 1, & \sum\limits_{\ell=1}^{k_j}\sum\limits_{k=1}^{t}\sum\limits_{i=k}^{t}\sum\limits_{j=t}^{T} d_{\ell ij} r_{\ell,j}^m > a_j^{m*}, \\ 0, & \sum\limits_{\ell=1}^{k_j}\sum\limits_{k=1}^{t}\sum\limits_{i=k}^{t}\sum\limits_{j=t}^{T} d_{\ell ij} r_{\ell,j}^m \le a_j^{m*}, \end{cases} \tag{14}$$

where a_j^{m*} is the amount of equipment j allocated into affiliate m. Next, we will derive the values of $b_{\ell ij}$ with the help of Problem Q_2.

Problem $\mathbf{Q_2}$:

$$\max_{\tilde{u}} \tilde{R} = \sum_{\ell=1}^{L}\sum_{t=1}^{T}\sum_{j=t}^{T}\sum_{j=i}^{T} \tilde{u}_{\ell ij} p_{\ell ij} - \sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{t=1}^{T} y_{mjt}$$

$$\left(\sum_{\ell=1}^{k_j}\sum_{k=1}^{t}\sum_{i=k}^{t}\sum_{j=t}^{T} \tilde{u}_{\ell ij} r_{\ell,j}^{m} - a_j^{m*}\right) c_j, \tag{15}$$

subject to:

$$\sum_{\ell=1}^{k_j}\sum_{k=1}^{t}\sum_{i=k}^{t}\sum_{j=t}^{T} \tilde{u}_{\ell ij} r_{\ell,j}^{m} - a_j^{m*} > (y_{mjt} - 1)Big,$$

$$\forall\ m, j, t, \tag{16}$$

$$\sum_{\ell=1}^{k_j}\sum_{k=1}^{t}\sum_{i=k}^{t}\sum_{j=t}^{T} \tilde{u}_{\ell ij} r_{\ell,j}^{m} - a_j^{m*} \leq y_{mjt} Big,$$

$$\forall\ m, j, t, \tag{17}$$

$$\tilde{u}_{\ell ij} \leq d_{lij},\ \forall\ \ell, i, j. \tag{18}$$

Problem Q_2 is used to find the threshold variables when allocation variables are determined. Problem Q_2 can be optimally solved since it is a linear programming problem. The optimal values of $\tilde{u}_{\ell ij}$ are used to derived the values of $ub_{\ell ij}$.

Note that, the allocation decision variables are first obtained by solving Problem Q_1. However, after rounding allocation decision variables, there remain some resources that should be further distributed among branches. Genetic Algorithm (GA) is an evolution approach and has the ability to avoid being trapped in local optimal. Thus, in this paper, we adopt it to find better solution to distribute the remaining resources. More clearly, we use genetic algorithm to expand the solution space of $u_{\ell ij}(b_{\ell ij})$ in Problem Q_0 and then the values of $b_{\ell ij}$. The genetic algorithm can be expressed as follows. In the first step, an initial population of size Pop_{size} is randomly generated. For each iteration, a generic chromosome V consists of the expanding factor vector $\mathbf{g}$ where $g_{\ell ij}$ is coded with LTT distinct numbers within the range of $[g_s, g_e]$. Let $q_{\ell ij} = g_{\ell ij}\tilde{u}_{\ell ij}$. By using the following formula, we can determine the values of $b_{\ell ij}$.

$$b_{\ell ij} = \max\{b_{\ell ij} | u_{\ell,ij}(b_{\ell ij}) \geq q_{\ell,ij} - \xi\}. \tag{19}$$

In our GA, four genetic operators are repeatedly performed until the maximum number of iterations, $K_{\max}$, is reached. The details are presented as follows:

1. Cloning operator: In our GA, an elitism strategy is used to retain good chromosomes. We select the best N_{best} individuals as elitist set, and directly copy them to the next generation, and then produce the remaining individuals by steps 2-4.
2. Parent selection: Roulette-wheel selection is used to produce the mating pool to produce the remaining $Pop_{\text{size}} - N_{\text{best}}$ individuals.
3. Crossover operator: The single point crossover is used to perform crossover.
4. Mutation operator: For each crossed individual, we generate a random number r within the range $[0, 1]$. If $r < r_m$, we generate random integer numbers, k_1, within $[1, LTT]$. Then, the heuristic algorithm interchanges the numbers at k_1-th and replace the number at k_1-th bit with the result of one minus that number.
5. Evaluating fitness: Use Eq. (19) to derive $b_{\ell ij}^{*}$ and substitute $\mathbf{a}^*$ and $\mathbf{b}^*$ in Eq. (8) to compute $\tilde{R}$.
6. The objective value is then the evaluated fitness of a chromosome, and we update the best values of $\mathbf{a}^*$ and $\mathbf{b}^*$ if a better solution is found. Go to the next iteration, if the stopping criterion is not satisfied.

The proposed heuristic approach can be summarized as follows.

Outline of the solution procedure

1. Solve Problem Q_1 to obtain $\tilde{\mathbf{a}}$ and set $\bar{n}_j = N_j - \sum_{m=1}^{M} \bar{a}_{mj}$ where $\bar{a}_{mj}$ is the rounded numbers of $\tilde{a}_{mj}$.
2. Sequentially assign the remaining resource, one, by one to each affiliate according to the order sequences of $\tilde{a}_{mj} - \bar{a}_{mj}$.
3. Solve Problem Q_2 to obtain $\tilde{\mathbf{u}}$.
4. Apply genetic algorithm to generate the value of $\mathbf{u}$ by varying the value of $\tilde{\mathbf{u}}$ and find $\mathbf{b}^*$ according to Lemma 1.
5. Output $\mathbf{a}^*$, $\mathbf{b}^*$ and R^*.

4. Numerical examples

4.1. Test problems and results

In order to evaluate the performance of the proposed solution procedure, four different problem sizes in terms of the number of membership classes L and planning periods T are considered. The combination of (J, L, M, T) in problem sizes 1, 2, 3 and 4 are fixed at $(J = 10, L = 3, M = 5, T = 30)$, $(J = 15, L = 3, M = 5, T = 60)$, $(J = 20, L = 3, M = 5, T = 80)$ and $(J = 25, L = 3, M = 10, T = 120)$, respectively. We refer these four problems as problems P1, P2, P3 and

Table 1. The values of k_j.

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
P1	1	1	1	2	2	2	2	3	3	3	–	–	–	–	–	–	–	–	–	–
P2	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	–	–	–	–	–
P3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3
P4	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3

P4, respectively. For each problem size, ten problem instances were generated and solved.

The required parameters for all problem instances in all problem sizes can be randomly generated, but this will largely increase the page length of this paper since each problem instance requires a large number of parameters. To shorten this paper, we assume that the parameters of all instances in all problem types were generated as follows. $c_j = 0.01 + 0.002j$, $d_{\ell ij} = 20 + L + |j - i| + 2n$ where n is the instance number of computational experience. The available units of equipment for these four problem sizes were set as:

$$S_j = 4M + 0.5j, \quad g_j = 1 + 0.4(J - j),$$

$$G_m = (\sum_{j=1} S_j g_j + \sum_{j=1} S_j g_j (m - 0.5(M - m)/M) + 20m).$$

Sale prices are assumed to be $p_{\ell ij} = 5\ell + |j - i| + 1.0$. The values of $\mathbf{k}$ are assumed and shown in Table 1. The values of $r_{\ell ij}$s was generated according to the following rules:

$$r_{\ell ij} = \hat{r}_{\ell ij} / \sum_{m=1}^{M} \sum_{j=1}^{J} \hat{r}_{\ell ij} Prob_\ell,$$

where:

$$\hat{r}_{\ell ij} = ((mod(mL, M + L + J) + 1) + jJL)$$
$$/(mod(mL, M + L + J) + 1),$$

and:

$$Prob_\ell = 0.3 + 0.05(\ell - 1).$$

The Poisson random variable should ranges from 0 to infinity. However, to speed up the computation, we ignored the trivial items. For each run, the Poisson probability of $f(y_\ell ij)$ for $x > m$ was viewed as zero, where Q is an integer up to which the cumulative Poisson distribution of $F(Q)$ is not less than 0.9995.

The proposed heuristic procedure was coded in Visual C++ programming language and the GAMS/CPLEX models were implemented on an Intel(R) Core2 Quad CPU 2.4 GHz personal computer with 2.99 GB RAM. We compared the solutions obtained by the proposed heuristic approach with those found by the CPLEX solver. First, to explain the application of the equipment allocation and member recruiting limits decisions, we show the equipment allocation and member recruiting threshold decisions for the first instance of problem P1.

Table 2. The values of a_{mj} in Case 1 of problem P1.

	j									
m	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	3	5	8	5	6	8	8	12	9	10
2	14	10	4	11	9	8	5	4	5	6
3	1	2	4	3	2	3	4	3	5	4
4	1	2	2	1	3	2	3	3	2	3
5	1	2	3	2	2	2	3	2	3	2

Tables 2 and 3 show the decisions found by the proposed heuristic approach, respectively. To shorten the page length, we only show the values of b_{2ij}s. We use the data shown in Tables 2 and 3 to explain their applications. Table 2 reveals that, for example, the units of equipment 1 allocated to the five club affiliates are 3, 14, 1, 1 and 1, respectively. Table 3 indicates the member recruiting thresholds. From this table, we obtain the acceptance rule for any arriving customers. For example, during period 10, we see that an arriving customer who expects to join the club for product (2,10,22) should be accepted if and only if the number of members of product type (2,10,22) does not reach the level of $b_{2,10,22} = 77$. Similarly, if the number of members for subscription type (2,10,22) has been up to the level of $b_{2,10,22} = 77$, then any request for subscription type (2,10,22) is no longer accepted.

The criteria of performances of algorithms were measured by solution quality and computational efficiency. We refer to the proposed heuristic approach as HGA. The percent points, defined as 100 (feasible solution obtained by CPLEX solver / feasible solution obtained by HGA) is used to evaluate the solution quality of HGA. The quotient, defined as (the CPU time used by CPLEX solver) /(the CPU time used by heuristic) is used to evaluate the efficiency of the proposed heuristic approach. The computational experiences show that no feasible solution was found after running the CPLEX solver for all instances in problem P4. However, the proposed heuristic can obtain feasible solution for all problem instances within reasonable CPU time. The computational results for problems P1-P3 were reported in Tables 4-6 in which

Table 3. The values of b_{lij} in instance 1 of problem P1.

	j																													
i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94	97	101	101	101	101	101
2	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94	97	101	101	101	101
3	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94	97	101	101	101
4	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94	97	101	101
5	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94	97	101
6	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94	97
7	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92	94
8	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91	92
9	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89	91
10	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88	89
11	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86	88
12	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85	86
13	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83	85
14	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82	83
15	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80	82
16	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79	80
17	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77	79
18	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76	77
19	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74	76
20	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73	74
21	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71	73
22	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69	71
23	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68	69
24	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66	68
25	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65	66
26	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63	65
27	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61	63
28	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60	61
29	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58	60
30	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	58

the problem instance that cannot be solved is expressed by the symbol "N/A".

In terms of solution quality, from the last columns of Tables 4-6, we observe that the proposed heuristic approach obtain the same solutions as those of CPLEX solver in all problem instances. In terms of computational time, from the third and sixth columns of Tables 4 and 5, we observe that the proposed heuristic approach is slightly inferior to CPLEX solver for problems P1 and P2. However, from Table 6, we find that the CPU time used in finding feasible solutions for problem P4 by the CPLEX solver was nearly 2 times slower than the propose heuristic approach. This implies that the proposed heuristic is superior to CPLEX solver for problem P3. Tables 4-7 also reveal that as the problem size increases, the computational time consumed to obtain solutions by the proposed

Table 4. Computational results for problem P1.

	CPLEX		HGA		
No	**Sol**	**Time**	**Sol**	**Time**	**GAP**
1	8108084	24	8108068	23	0.00%
2	8590451	17	8590451	22	0.00%
3	9072817	14	9072817	23	0.00%
4	9555184	20	9555184	23	0.00%
5	10037550	14	10037550	24	0.00%
6	10519916	16	10519916	23	0.00%
7	11002282	16	11002282	25	0.00%
8	11484644	14	11484644	24	0.00%
9	11966998	15	11966998	25	0.00%
10	12449328	15	12449328	25	0.00%
Average		16		24	0.00%

Table 5. Computational results for problem P2.

No	CPLEX		HGA		GAP
	Sol	Time	Sol	Time	
1	119998598	118	119998598	134	0.00%
2	125365388	134	125365388	137	0.00%
3	130732173	134	130732173	139	0.00%
4	136099014	147	136099014	143	0.00%
5	141465800	134	141465800	144	0.00%
6	146832583	130	146832583	145	0.00%
7	152199429	118	152199429	149	0.00%
8	157566213	125	157566213	150	0.00%
9	162932993	134	162932993	153	0.00%
10	168299843	160	168299843	156	0.00%
Average		133		145	0.00%

Table 6. Computational results for problem P3.

No	CPLEX		HGA		GAP
	Sol	Time	Sol	Time	
1	397985840	393	397985840	340	0.00%
2	413182473	398	413182473	344	0.00%
3	428379208	414	428379208	224	0.00%
4	443575843	415	443575843	224	0.00%
5	458772472	402	458772472	225	0.00%
6	473969214	430	473969214	227	0.00%
7	489165846	437	489165846	229	0.00%
8	504362473	442	504362473	229	0.00%
9	519559221	446	519559221	231	0.00%
10	534755851	451	534755851	219	0.00%
Average		423		249	0.00%

Table 7. Computational results for problem P4.

No	CPLEX		HGA	
	Sol	Time	Sol	Time
1	N/A[a]	N/A	231818366	1083
2	N/A	N/A	438007839	3377
3	N/A	N/A	454278454	2428
4	N/A	N/A	470549308	2915
5	N/A	N/A	486819931	2783
6	N/A	N/A	502808013	3108
7	N/A	N/A	519038295	2808
8	N/A	N/A	534892104	2233
9	N/A	N/A	550993935	3588
10	N/A	N/A	567181256	2604
Average				2693

[a] N/A means that no feasible solution is found in 4 hours.

approach increases slowly. However, the CPLEX solver increases quickly. It further indicates that the proposed heuristic approach can solve larger scale problems while the CPLEX solver cannot (Table 7).

4.2. Sensitivity analysis

To understand the impact of the model parameter on the solution, we performed a sensitivity analysis on the values of $r_{\ell ij}$ and c_j in problems P5 and P6, respectively. In these two examples, the data is the same as that of experimental case 1 of problem type P2 except that the values of $Prob_\ell$ in problem P5 and c_j in problem P6 were varied according to the formula of $Prob_\ell = 0.30 + 0.05(\ell - 1) + 0.02(case_{no} - 1)$ and $c_j = 0.010 + 0.002j + 0.02case_{no}$, respectively. The symbol $case_{no}$ is used to express the experimental case number. Since $r_{\ell ij}$ increases in the values of $Prob_\ell$, the values of utilization rate, $r_{\ell ij}$, increases as the case number increases. In addition, the value of waiting cost, c_j, also increases with the increase of the case number. The computational results are shown in Table 8.

As shown in Table 8, the objective values tend to decrease as the values of $r_{\ell ij}$ and c_j increase. For example, Table 8 indicates that the best objective value decreased from 119,707,629 to 119,707,629 for problem P5. In addition, we also observe that the best objective value decreased from 119,567,976 to 115,692,380 for problem P6.

5. Conclusion

To satisfy the various requirements of sport clubs' customers, sport clubs usually divide their membership into several classes with distinct restrictions. A sport club with more members will need more sport equipments than a sport club with fewer members. This phenomenon implies that a member may wait a long time to enjoy the service once members are

Table 8. Sensitivity analysis.

No	Problem P5	Problem P6
	Sol	Sol
1	119998598	119567976
2	119966980	119137355
3	119934119	118706733
4	119905002	118276111
5	119873209	117845489
6	119836371	117414868
7	119806392	116984246
8	119772233	116553624
9	119745205	116123002
10	119707629	115692380

over recruited. To avoid such a situation, this paper has developed a mathematical model to determine the equipment allocation for a chain club with the consideration of the sales limits on member recruitment. In order to reduce the waiting cost, sales limits can be considered as a complementary tool to avoid the situation of overloaded members. The proposed model aims to maximize total expected profit over a fixed period of time. A hybrid heuristic approach based on linear programming was developed to acquire a feasible solution for this problem.

To evaluate the performance of the proposed approach, this study has solved forty test problems, i.e. ten instances for P1, P2, P3 and P4, respectively, by using the proposed approach and the well-known commercial solver, CPLEX solver. The computational results have shown that the performance of the proposed approach is as good as that of the CPLEX solver on both solution quality and CPU time for small scale problems (problems P1, P2 and P3). Compared with the CPLEX solver, the computational time required to obtain solutions by using the presented approach increases slowly. Numerical experiences have also shown that the proposed algorithm can obtain a comprised feasible solution for larger scale problem instances (problem P4), which could not be solved by CPLEX. In the light of these numerical experiences, the proposed algorithm can be considered as an efficient tool for dealing with large scale problems. Moreover, sensitivity analysis of the optimal decisions with respect to the system parameter has also been conducted to illustrate the optimal decision characteristics in this study.

In addition, the required parameters were assumed to be known and arbitrarily determined. For the parameters to be more realistic, it is necessary to collect the information on the pricing setting from a sport club, and it needs to adopt an appropriate method to estimate the demand parameters. These complicated topics can be explored in future research.

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