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Research Note

Comparing four ordering policies in a lost sales inventory model with Poisson demand and zero ordering cost

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Abstract. In this paper, we compare four ordering policies in a lost sales inventory model with zero ordering cost, constant lead time, and Poisson demand process. These ordering policies are 1) base stock policy, 2) full delay policy, 3) simple delay policy and 4) a recently developed ordering policy called $(1, T)$ *or one for one period policy*. Our work can be considered as an expansion of a previous research which compared the first three policies. We show that, for any fixed value of the ratio of unit lost sales cost over unit holding cost, there is a specific value of lead time demand beyond which the cost of $(1, T)$ policy is lower than the costs of the other three policies. Furthermore, the superiority of $(1, T)$ policy is more significant for low values of the above ratio and becomes more pronounced as the lead time demand increases.

1. Introduction

In this paper, we consider an inventory system with a Poisson demand process, constant lead time on replenishment and lost sales during stock out. There exist a holding cost per unit per unit time and a lost sale cost per unit but the ordering cost is zero or negligible. A common inventory policy for this system is the one-for-one ordering policy. In this policy which is also known as the $(S-1, S)$ or base stock policy, an order is placed whenever a demand occurs and is met. This policy is usually applicable in the control of low demand but important and possibly expensive items for which the replenishment lead time is relatively long.

An analytical approach to cost optimization in backorder assumption can be found in Hadley and Within [1]. Iglehard [2] shows that the base stock policy is optimal in the backorder situations. Karush [3] models a lost-sales base stock policy as a queuing system. He assumes that the demand process is Poisson and calculates the average cost per unit time. He also proves that the out-of-stock probability is convex in the base stock level. Convexity has been proven differently by Jagers and van Doorn [4]. An approximation for the base stock level is presented by Smith [5]. Hill [6] shows that the base stock policy can never be optimal in the lost sale case for $S \geq 2$. Haji and Haji [7] apply a new ordering policy in the inventory system with lost sales, and call it $(1, T)$ or one for one period policy. In this new ordering policy, an order of size one is placed at each fixed interval of time. They show that $(1, T)$ ordering policy is less costly than base stock policy for long lead times. They also presented the following advantages of $(1, T)$ policy:

1. The safety stock in suppliers is eliminated (cost reduction).

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2. Cost of the retailer is completely independent of the lead time.
3. Information exchange cost for the supplier due to elimination of uncertainty of its demand is eliminated.
4. Inventory control and production planning in suppliers are simplified.
5. It is very easy to apply. Just order 1 unit of product at each cycle time T.
6. Inventory review costs are eliminated.

Haji et al. [8] apply this new policy in the retailers of a two echelon inventory system with one warehouse and a number of retailers with Poisson demand and in lost sale situations. They compare the result of their work with the result of Anderson and Melchior [9] which had used $(S-1,S)$ policy in retailers and supplier. Haji et al. [10] consider a two echelon inventory system in which the retailers use $(1,T)$ policy with a common cycle.

Hill [11] introduces two new policies for the lost sale inventory systems with no ordering cost (derived from the base stock policy) which may have a lower cost than the base stock policy. He calls these polices: simple and full delayed inventory policies. He compares the cost of these new ordering policies with that of the base stock policy in a numerical example. In this paper, besides of re-introducing $(1,T)$ policy, we use a numerical example to compare $(1,T)$ policy with the other three policies, i.e. standard base stock policy, full delay policy, and simple delay policy. We show that, other than the known advantages of $(1,T)$ policy, for any fixed value of the ratio of unit lost sales cost over unit holding cost, there is a specific value of lead time demand were the cost of $(1,T)$ policy is lower than the costs of the other three policies. Furthermore, the superiority of $(1,T)$ policy is more significant for low values of the above ratio and becomes more pronounced as the lead time demand increases.

2. Introducing four policies

In this section, we need the following notations:

μ	Customer arrival rate at retailer,
L	Lead time from supplier to retailer,
h	Holding cost rate at retailer,
π	Lost sale cost for a unit at retailer,
S	Base stock level,
I	Inventory average at the retailer,
K_B	Total cost of the base stock policy,
K_{BF}	Total cost of the full delay policy,
K_{BS}	Total cost of the simple delay policy,
K_T	Total cost of the $(1,T)$ policy,

2.1. $(S-1,S)$ policy

For the standard base stock policy, with base stock level, S, whenever a sale occurs, namely a demand is satisfied, an order for 1 unit is placed. Iglehard [2] shows that in the backorder situations and when the ordering cost is zero, the base stock policy is the optimal policy. But in the lost sales case the base stock policy can never be the optimal policy for $S \geq 2$ (see [6]). The total cost rate for $(S-1,S)$ policy with lost sales is as follows (e.g. see [5]):

$$K_B = h\left(S-\mu L\left(1-\frac{(\mu L)^S/S!}{\sum_{j=1}^{S}(\mu L)^j/j!}\right)\right) + \mu\pi\frac{(\mu L)^S/S!}{\sum_{j=1}^{S}(\mu L)^j/j!}. \quad (1)$$

It is obvious that K_B depends on the lead time L.

2.2. The base stock policy with delay

Hill [11] presents two new policies, which are derived from the standard base stock policy. These new policies can improve the base stock policy by imposing a delay, T', between the placements of successive orders. He calls these policies "Full delay policy" and "Simple delay policy". Whenever a demand is met, one must decide whether to place an order or delay it for some units of time. In the full delay policy T' is not a fixed value but depends on the state of the system and should be calculated every time the controller decides to impose a delay, but in the simple delay, policy T' is fixed and has a lower bound between two successive orders.

For applying both policies, the controller should consider the base stock level which is optimal for the standard base stock policy as the maximum level of the inventory position. In the full delay policy, time zero is considered the time in which a sale occurs. Hill defines a function PSO(t) as the probability that the system is out of stock at time $t+L$ if no further sales occur and no orders are placed during the interval $(0,t)$. He summarizes detailed operation of this policy in a procedure. In his procedure to decide whether to place an order immediately or delay it, one should consider three values which are PSO(t), the ratio of $h/\mu\pi$, and the inventory position whenever a sale occurs. As Hill mentions, the full delay policy is slightly complex in nature and requires a calculation whenever there is a sale. So he introduces the "simple delay policy". This policy only considers the times at which the last order is placed and ignores the values of stocks on hand and outstanding orders which may be in the system. Then he calculates a lower bound between two successive orders, below which it is not worth placing an order.

2.3. $(1, T)$ policy

In the $(1,T)$ policy, an order of one unit is placed in each fixed time interval, T. The $(1,T)$ policy can be interpreted as a $D/M/1$ queuing system (see [7]), i.e. a single channel queuing system in which the inter-arrival times are constant and equal to T, and the service times have exponential distribution with mean $1/\mu$. Thus, the arrival rate of units to the system is $\lambda = 1/T$, the service rate is μ, and the probability of stock out is:

$$P_0 = 1 - \rho, \tag{2}$$

where ρ is the ratio of the arrival rate to the service rate, i.e. the fill rate is:

$$\rho = \frac{1}{\mu T}, \tag{3}$$

and the total cost of the retailer is:

$$K_T = HI + \pi\mu(1-\rho), \tag{4}$$

where:

$$I = \frac{\rho}{1-\beta}, \tag{5}$$

and:

$$\beta = e^{-(1-\beta)/\rho} = e^{-1/I}. \tag{6}$$

From Eqs. (1), (4) and (5) we have:

$$\frac{1}{\mu T} = I(1 - e^{-1/I}). \tag{7}$$

Haji and Haji [7] proved the convexity of the total cost function. The optimal values of I and T, i.e. I^* and T^*, can be obtained from the following relations:

$$e^{-1/I^*} + \frac{1}{I^*}e^{1/I^*} = \frac{\mu\pi - h}{\mu\pi}, \tag{8}$$

and:

$$T^* = \frac{1}{\mu I(1 - e^{1/I^*})}. \tag{9}$$

It is important to note that based on the above relations the total cost of the $(1,T)$ policy is entirely independent of L.

In the next section, we will compare the four ordering policies numerically. We will also establish the notable result that for a fix value of the ratio of unit lost sales cost over unit holding cost (π/h), the cost of the $(1,T)$ policy is lower than the costs of the other three policies beyond some specific value of lead time demand. Furthermore, the superiority of the $(1,T)$ policy is more meaningful for low values of π/h and becomes more significant as the lead time demand increases.

Table 1. The optimal cost and optimal ordering cycle of $(1,T)$ policy.

	$\mu\frac{\pi}{h}$				
	2	**4**	**6**	**8**	**10**
T^*	2.0636	1.5565	1.3679	1.3354	1.8804
K_T	1.6266	2.4704	3.1116	3.6505	4.1246

3. Comparing four policies

In this section, we numerically compare the $(1,T)$ policy with the three other policies (standard base stock policy, simple delay policy and full delay policy). Following Hill [11], without loss of generality, we assume that $h = 1$ and time is normalized so that $\mu = 1$. It reduces the parameters to L and π. We consider 5 cases for $\mu\pi/h$ and seven cases for μL. These values are:

$$\mu\frac{\pi}{h} = 2, 4, 6, 8, \text{ and } 10,$$

$$\mu L = 1, 2, 4, 6, 8, 10, 12, 14, 16, 18, \text{ and } 20.$$

In Table 1, we have obtained the optimal ordering cycle and optimal cost of the $(1,T)$ policy according to Eqs. (3) and (8). As can be seen the $(1,T)$ policy is entirely independent of the lead time.

In Table 2, the optimal base stock level of the $(S-1,S)$ policy, S^*, and the optimal costs of the three other policies, i.e., base stock policy, K_B, full delay policy, K_{BF}, and simple delay policy, K_{BS}, are shown. While S^* and K_B are determined analytically, as Hill [11] mentions, K_{BF} and K_{BS} can only be evaluated by simulation. In the first example, i.e. $\mu\pi/h = 2$ and $\mu L = 1$, the optimal base stock level is 1; therefore, the standard base stock policy is the optimal policy (see [6]).

Table 3 shows the percentage of cost reduction from using alternative policies instead of standard base stock policy. We denote the percentage of cost reduction from using the full delay policy, simple delay policy, and the $(1,T)$ policy by ΔK_{BF}, ΔK_{BS}, and ΔK_T, respectively. Delay policies always give a lower cost than the standard $(S-1,S)$ policy with the same base stock level, assuming that base stock is greater than 1 (see [11]). As we can see in Table 3, the $(1,T)$ policy is more costly than the other policies for low lead time demands (negative percentage), but except for low lead time demands, its cost is lower than the costs of the other three policies. Furthermore, for larger values of the lead time demands, the cost reduction when using the $(1,T)$ policy is much more significant than the cost reduction when using the delay policies. For example for $\mu\pi/h = 4$ and $\mu L = 10$ the cost reduction from using $(1,T)$ policy is equal to 9.40%, which is much higher than the savings from implementing the delay policies (2.08% and 0.97%).

Table 2. The optimal base stock level of standard base stock policy, its cost, the cost of full delay policy, and the cost of simple delay policy.

μL		$\mu\frac{\pi}{h}$ 2	4	6	8	10
1	S^*	1	2	2	3	3
	K_B	1.5000	2.0000	2.4000	2.5625	2.6875
	K_{BF}	–	1.9895	2.3975	2.5413	2.6692
	K_{BS}	–	1.9878	2.3849	2.5594	2.6842
2	S^*	2	2	2	3	3
	K_B	1.6000	2.2632	2.6842	2.9524	3.1429
	K_{BF}	1.5771	2.2248	2.6368	2.9179	3.1092
	K_{BS}	1.5785	2.2538	2.6724	2.9478	3.1381
4	S^*	2	4	5	5	6
	K_B	1.6923	2.4854	2.9907	3.3888	3.6403
	K_{BF}	1.6744	2.4555	2.9582	3.3430	3.5819
	K_{BS}	1.6838	2.4644	2.9718	3.3832	3.6343
6	S^*	3	5	6	7	8
	K_B	1.7213	2.6040	3.1791	3.5908	3.9500
	K_{BF}	1.6823	2.5523	3.1215	3.5245	3.8941
	K_{BS}	1.7033	2.5910	3.1668	3.5810	3.9458
8	S^*	4	6	8	8	9
	K_B	1.7464	2.6770	3.2980	3.7691	4.1165
	K_{BF}	1.6898	2.6306	3.2527	3.7014	4.0273
	K_{BS}	1.7204	2.6623	3.2879	3.7645	4.1135
10	S^*	4	7	9	10	11
	K_B	1.7600	2.7266	3.3713	3.8625	4.2646
	K_{BF}	1.7164	2.6699	3.3068	3.7802	4.1712
	K_{BS}	1.7442	2.7000	3.3628	3.8534	4.2562
12	S^*	5	8	10	11	12
	K_B	1.7689	2.7625	3.4347	3.9553	4.3685
	K_{BF}	1.7133	2.7090	3.3740	3.8405	4.2862
	K_{BS}	1.7503	2.7342	3.4151	3.9486	4.3673
14	S^*	6	9	11	13	14
	K_B	1.7789	2.7898	3.4881	4.0205	4.4593
	K_{BF}	1.7249	2.7507	3.4036	3.9245	4.3704
	K_{BS}	1.7580	2.7656	3.4667	4.0147	4.4590
16	S^*	6	11	13	14	15
	K_B	1.7824	2.8109	3.5228	4.0741	4.5269
	K_{BF}	1.7479	2.7264	3.4221	3.9591	4.4499
	K_{BS}	1.7588	2.7966	3.5101	4.0695	4.5236
18	S^*	7	12	14	15	17
	K_B	1.7876	2.8274	3.5525	4.1270	4.5921
	K_{BF}	1.7288	2.7367	3.4176	4.0084	4.4628
	K_{BS}	1.7691	2.7964	3.5223	4.1208	4.5896
20	S^*	7	13	15	17	18
	K_B	1.7912	2.8412	3.5799	4.1600	4.6378
	K_{BF}	1.7353	2.7407	3.4963	4.0334	4.5040
	K_{BS}	1.777	2.8152	3.5518	4.1534	4.6321

Table 3. The percentage of cost reduction when using alternative policies instead of standard base stock policy.

μL		$\mu\frac{\pi}{h}$ 2	4	6	8	10
1	ΔK_{BF}	-	0.52%	0.10%	0.83%	0.68%
	ΔK_{BS}	-	0.61%	0.63%	0.12%	0.12%
	$\Delta K_{T\bullet}$	-	-23.52%	-29.65%	-42.46%	-53.47%
2	ΔK_{BF}	1.43%	1.69%	1.77%	1.17%	1.07%
	ΔK_{BS}	1.34%	0.41%	0.44%	0.16%	0.15%
	$\Delta K_{T\bullet}$	-1.66%	-9.16%	-15.92%	-23.65%	-31.24%
4	ΔK_{BF}	1.06%	1.20%	1.09%	1.35%	1.60%
	ΔK_{BS}	0.50%	0.85%	0.63%	0.17%	0.16%
	$\Delta K_{T\bullet}$	**3.88%**	0.61%	-4.04%	-7.72%	-13.30%
6	ΔK_{BF}	2.27%	1.99%	1.81%	1.85%	1.42%
	ΔK_{BS}	1.05%	0.50%	0.39%	0.27%	0.11%
	$\Delta K_{T\bullet}$	**5.50%**	**5.13%**	**2.12%**	-1.66%	-4.42%
8	ΔK_{BF}	3.24%	1.73%	1.37%	1.80%	2.17%
	ΔK_{BS}	1.49%	0.55%	0.31%	0.12%	0.07%
	$\Delta K_{T\bullet}$	**6.86%**	**7.72%**	**5.65%**	**3.15%**	-0.20%
10	ΔK_{BF}	2.47%	2.08%	1.91%	2.13%	2.19%
	ΔK_{BS}	0.90%	0.97%	0.25%	0.24%	0.20%
	$\Delta K_{T\bullet}$	**7.58%**	**9.40%**	**7.70%**	**5.49%**	**3.28%**
12	ΔK_{BF}	3.14%	1.94%	1.77%	2.90%	1.88%
	ΔK_{BS}	1.05%	1.02%	0.57%	0.17%	0.03%
	$\Delta K_{T\bullet}$	**8.04%**	**10.57%**	**9.41%**	**7.71%**	**5.58%**
14	ΔK_{BF}	3.04%	1.40%	2.42%	2.39%	1.99%
	ΔK_{BS}	1.18%	0.87%	0.61%	0.14%	0.01%
	$\Delta K_{T\bullet}$	**8.56%**	**11.45%**	**10.80%**	**9.20%**	**7.51%**
16	ΔK_{BF}	1.93%	3.01%	2.86%	2.82%	1.70%
	ΔK_{BS}	1.32%	0.51%	0.36%	0.11%	0.07%
	$\Delta K_{T\bullet}$	**8.74%**	**12.12%**	**11.67%**	**10.40%**	**8.89%**
18	ΔK_{BF}	3.29%	3.21%	3.80%	2.87%	2.82%
	ΔK_{BS}	1.03%	1.10%	0.85%	0.15%	0.05%
	$\Delta K_{T\bullet}$	**9.00%**	**12.63%**	**12.41%**	**11.55%**	**10.18%**
20	ΔK_{BF}	3.12%	3.54%	2.34%	3.04%	2.89%
	ΔK_{BS}	0.92%	0.92%	0.79%	0.16%	0.12%
	$\Delta K_{T\bullet}$	**09.19%**	**13.05%**	**13.08%**	**12.25%**	**11.07%**

Since the cost of the base stock policy is strictly increasing by the lead time demand and the cost of the $(1,T)$ policy is independent of the lead time demand (note that time is normalized so that $\mu = 1$), for each fixed value of $\mu\pi/h$ there is a specific value of lead time demand, after which the $(1,T)$ policy has a lower cost than the standard base stock policy. Table 4 shows this specific value for each value of $\mu\pi/h$ in our numerical example. We denote these values of the lead time demands by $\mu\hat{L}_B$. For each fixed value of $\mu\pi/h$, we have also obtained the specific value of

Table 4. Specific values of lead time demands after which $(1, T)$ policy has a lower cost than the standard base stock policy, simple delay policy, and full delay policy.

	$\mu\frac{\pi}{h}$				
	2	**4**	**6**	**8**	**10**
$\mu\hat{L}_B$	2.59	3.81	5.23	6.67	8.12
$\mu\hat{L}_{BS}$	2.74	4.02	5.45	7.11	8.14
$\mu\hat{L}_{BF}$	2.78	4.16	5.86	7.73	8.68

the lead time demand after which the $(1, T)$ policy is less costly than the simple delay policy and the full delay policy, and denoted them by $\mu\hat{L}_{BS}$ and $\mu\hat{L}_{BF}$, respectively. Notice that, because there are no mathematical formulas for calculating the total costs of the delay politics, we resorted to simulation for obtaining $\mu\hat{L}_{BS}$ and $\mu\hat{L}_{BF}$.

Table 4 also shows that when $\mu\pi/h$ takes small values, the superiority of the $(1, T)$ policy occurs in the lower range of the lead time demands.

4. Conclusion

In this paper we considered an inventory system with a Poisson demand process, constant lead time and lost sales during stock out. Numerically we compared the cost of four ordering policies for the case of zero or negligible ordering cost: 1) the base stock policy, 2) full delay policy, 3) simple delay policy, and 4) $(1, T)$ policy. Hill [11] compared the cost of delay policies with that of the base stock policy and numerically showed that when the inventory position S is greater than or equal to 2 ($S \geq 2$) the two delay policies always result in a lower cost than the base stock policy. We showed that for fixed value of the ratio of unit lost sales cost over unit holding cost, the cost of the $(1, T)$ policy is lower than the costs of the other three policies after some specific values of lead time demand. Furthermore, the superiority of the $(1, T)$ policy is more significant for low values of the above ratio and becomes more notable as the lead time demand increases.

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