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# Robust M-estimation of multivariate FIGARCH models for handling volatility transmission: A case study of Iran, United Arab Emirates and the global oil price index

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**Abstract.** The stochastic nature of price volatility, as an important issue in stock markets, significantly affects decision makers' decisions. In this paper, a new multivariate fractionally integrated generalized autoregressive conditional heteroscedasticity (MVFIGARCH) model is proposed. Being more comprehensive, in comparison with models in the literature, the proposed model considers the long term parameter, which is estimated simultaneously with other parameters. A well-known method of MVFIGARCH estimation is the Gaussian quasi-maximum likelihood method. The Gaussian quasi-maximum likelihood estimator of the MVFIGARCH model is known to be sensitive to data outliers. To correct this vulnerability, robust M-estimators are introduced for MVFIGARCH models. Volatility models with bounded innovation propagation properties are introduced to increase the robustness of the estimations. The applicability of the proposed model is justified by the volatility transmission between the Tehran stock index, the Dubai stock index and the global oil price index between December 5th, 2006 to January 30th, 2012, and is investigated using the MVFIGARCH model. The result of estimation in different models generally shows the volatility transmission from the global oil market to Tehran and Dubai markets. The volatility transmission from the Dubai to Tehran market was meaningfully observed as well. However, the effect of transmission was not observed in the reverse direction.



## 1. Introduction

Long-term memory is an important concept in many scientific areas. It has attracted much research into human behavior, hydrology, telecommunications and financial time series, and the study of such processes continues to be of interest [1]. Long-term memory in time series analyses occurs when the autocovariance for a stationary time series converges to zero so slowly that its sums diverge. Since autocovariance has a non-summability property, in theory, the autocorrelation function is usually used to characterize long-memory stationary processes, and, in practice, an estimator is utilized instead of its theoretical counterpart. Long memory in asset returns and volatilities is a new research area, both in theoretical and empirical modeling of high frequent financial time series. Several papers have addressed the long-run properties of stock returns, suggesting that absolute returns are sufficiently characterized by long-term memory processes. The most popular techniques of time series modeling with long-

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term memory is the ARFIMA-FIGARCH. The famous GARCH models [2,3] are known to be short memory; however, the sample autocorrelation function of these models has long range dependence type behavior, according to research by Mikosch and Starica [4].

Considering long-term memory processes as fractional models, the breaks can be regarded as non-linear to the data. In some cases, long memory models may generate data with nonlinear properties, as occasional structural breaks. These nonlinear properties are highly influenced by outliers in the data.

In this paper, we develop a model called MV-FIGARCH (FBEKK), which is more comprehensive than previous ones and which considers the long term parameter and its simultaneous estimation, along with others in the model.

The proposed model suggests that MVFIGARCH volatility models estimators that are more resistant to data outliers than classical MGARCH estimators.

For FIGARCH model estimation, usually, a Gaussian likelihood function is maximized. The aforementioned estimation procedure, known as the Gaussian Quasi-Maximum Likelihood estimation, results in consistent estimation of parameters, even in the case of non-Gaussian distribution. The merit of this property becomes bold when the distribution of the standardized return series is still fat tailed after correcting the returns for the dynamics in the conditional covariance matrix.

Robust estimators are proposed for dealing with the effect of outliers on the estimation of univariate GARCH models. One way to do this is iteratively pruning the outliers and using the remaining data for model fitting, so that no more outliers are detected [5]. A forward search algorithm was applied for GARCH model estimation by Grossi and Laurini [6]. Using an estimator which is robust to outliers is another approach. The robust measure of residual scales is minimized by some research in attempts to estimate GARCH models [7-10]. Mancini et al. [11] and Muler and Yohai [12,13] introduced a robust M-estimator with much smaller outlier weights in comparison with the Gaussian ML estimator. The collective opinion of all the research is focused on the inaccuracy of the Gaussian QML estimator in the presence of outliers and, thus, robust procedures are needed.

The effect of outliers on the Gaussian QML estimator of multivariate GARCH models is investigated by Boudt and Croux [14]. They proposed a robust alternative using the Mahalanobis Distance (MD). The MD is defined as the inner product of the return standardized by the corresponding MGARCH volatility prediction. Boudt et al. [15] proposed a multivariate volatility forecasting model that is accurate in the presence of large one-off events. This model, as an extension of the Dynamic Conditional Correlation (DCC) model, produced more precise out-of-sample covariance forecasts than the DCC model. Noureldin al. [16] introduced a new class of multivariate volatility model that utilizes high-frequency data. This paper discussed models dynamics and their covariance targeting specifications, and provides closed-form formulas for multi-step forecasts.

Outliers greatly influence the Gaussian QML estimates. We, therefore, have used M-estimators with loss functions in order to reduce the effects of outliers on the estimator. Moreover, the effect of outliers on regular observations is different from that of future volatility. According to research by Bauwens and Storti [17] and Hamilton and Susmel [18], usually, volatility growth is proportionally smaller after outlying shocks in comparison to that after moderate and small shocks. Nevertheless, in the majority of MGARCH models, the effect of outlying shocks on volatility is similar to that of moderate and small shocks. In this paper, MVFIGARCH models, with the effect of outlying returns on future volatility being bounded, are utilized to alleviate outlier effects on predicted volatility.

The rest of this paper is organized as follows. In Section 2, the proposed model is presented. Section 3 includes the Robust M-estimation of the parameters of the proposed multivariate FIGARCH model. Application of the proposed multivariate FIGARCH to the 3 stock markets is presented in Section 4. Finally, Section 5 concludes the paper.

## 2. Mathematical model

Let $(r_1, ..., r_T)$ be a sample of an $N$-dimensional return vector consisting of $T$ observations, and $I_{t-1}$ be the all available information up to time $t-1$. The random variable, $r_t$, has a mean equal to zero and a covariance matrix equal to $H_{t,\theta}$, where:

$$r_t = H_{t,\theta}^{1/2} u_t \text{ with } (E[u_t|I_{t-1}] = 0, \quad \mathrm{Cov}[u_t|I_{t-1}] = I_N). \tag{1}$$

Suppose an MVFIGARCH model, as a measurable function, denoted by $H_\theta(.)$, of past realizations, $r_t$, with $\theta$ as the unknown parameter vector, is utilized to parameterize $H_{t,\theta}$. Under these assumptions, we have Eq. (2) for $t > 2$:

$$H_{t,\theta} = H_\theta(r_1, ..., r_{t-1}). \tag{2}$$

In the above equation, a data-free method is used to initialize $H_{1,\theta}$ and the output of function $H_\theta(.)$ is a $N \times N$ positive definite symmetric matrix. For instance, one may refer to the parameterization of BEKK(p,q,K) for $H_{t,\theta}$ [19], then, we have:

$$H_{t,\theta} = C'C + \sum_{j=1}^{q}\sum_{k=1}^{K} A'_{kj} r'_{t-j} r_{t-j} A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{K} G'_{kj} H_{t-j,\theta} G_{kj}, \tag{3}$$

where $A_{kj}$, $G_{kj}$ and $C$ are $N \times N$ parameter matrices and is upper triangular. To develop a multivariate fractional BEKK (FBEKK), consider the BEKK (1,1) model introduced in the following equation:

$$H_{t,\theta} = C'C + A'r'_{t-1}r_{t-1}A + G'H_{t-1}G. \tag{4}$$

In Equation (4), $A$ and $G$ are arbitrary square matrices. To adapt BEKK (1,1) to fractional BEKK (1,d,1), the terms $A'r'_{t-1}r_{t-1}A$ must be replaced by the following expression (for further details in this regard, one may refer to [20-22].

$$r'_t r_t - G'(r'_{t-1}r_{t-1})G - (1-L)^d r'_t r_t + A'(1-L)^d r'_{t-1} r_{t-1} A. \tag{5}$$

In the above expression, $L$ is the lag operator, known also as the backshift operator, used to obtain a previous element given an arbitrary element:

$$H_{t,\theta} = C'C - G'(r'_{t-1}r_{t-1})G + \left[1-(1-L)^d\right](r'_t r_t) + (1-L)^d A'(r'_{t-1}r_{t-1})A + G'H_{t-1}G. \tag{6}$$

The multivariate model introduced in Eq. (6) is derived from the BEKK model considering long term parameter ($d$). The developed model considers the long-term memory parameter and estimates it through a modeling process. The long memory term, $(1-L)^d$, should be converted to Maclaurin expansion:

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \ldots \tag{7}$$

In case of having a trivariate model, Eq. (6) is elaborated according to the following expansion:

$$\begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} \\ h_{21,t} & h_{22,t} & h_{23,t} \\ h_{31,t} & h_{31,t} & h_{33,t} \end{bmatrix} =$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}^T \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}$$

$$- \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{31} & \beta_{33} \end{bmatrix}^T \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \\ r_{3,t-1} \end{bmatrix} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \\ r_{3,t-1} \end{bmatrix}^T$$

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{31} & \beta_{33} \end{bmatrix}$$

$$+ \left[dL - \frac{d(d-1)}{2!}L^2 + \frac{d(d-1)(d-2)}{3!}L^3\right]$$

$$\begin{bmatrix} r_{1,t} \\ r_{2,t} \\ r_{3,t} \end{bmatrix} \begin{bmatrix} r_{1,t} \\ r_{2,t} \\ r_{3,t} \end{bmatrix}^T$$

$$+ \left[1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3\right]$$

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{31} & \alpha_{33} \end{bmatrix}^T \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \\ r_{3,t-1} \end{bmatrix} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \\ r_{3,t-1} \end{bmatrix}^T$$

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{31} & \alpha_{33} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{31} & \beta_{33} \end{bmatrix}^T$$

$$\begin{bmatrix} h_{11,t-1} & h_{12,t-1} & h_{13,t-1} \\ h_{21,t-1} & h_{22,t-1} & h_{23,t-1} \\ h_{31,t-1} & h_{31,t-1} & h_{33,t-1} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{31} & \beta_{33} \end{bmatrix} \tag{8}$$

Here, we assume that $\{u_t, \forall t\}$ is a set of independent and identically distributed values of an elliptically symmetric distribution. In other words, there exists a function, $g*(.) : R^+ \rightarrow R^+$, such that the following equation is obtained for the $N$-dimensional density function of $r_t$:

$$p(r_t|I_{t-1};\theta) = (\det(H_{t,\theta}))^{-\frac{1}{2}}\, g*(d^2_{t,\theta}). \tag{9}$$

In the above equation, $d^2_{t,\theta}$ is the inner product of $r_t$, standardized by its MVFIGARCH volatility:

$$d^2_{t,\theta} = (H^{-1/2}_{t,\theta} r_t)'(H^{-1/2}_{t,\theta} r_t) = u'_t u_t. \tag{10}$$

One may name elliptic distributions with no finite second moment. However, here, only the distributions with a finite covariance matrix are considered. Usually, $d^2_{t,\theta}$ denotes the squared Mahalanobis Distance (MD) which lies between $r_t$ and zero. MD is measured by its conditional covariance matrix, $H_{t,\theta}$. It calculates the distance between a return and its MVFIGARCH volatility prediction; large values for MD indicate an extreme corresponding return. In other words, returns with large MD values are potential outliers. Considering the assumptions of the elliptical MVFIGARCH model explained by Eqs. (1), (2) and (9), the squared MD in Eq. (10) has a density function, written as:

$$f*(z) = \frac{\pi^{N/2}}{\Gamma(N/2)} Z^{\frac{N}{2}-1} g*(Z). \tag{11}$$

Consider a special case with $f*(.)$ being the chi-square density function with a freedom degree of $N$, and $g*(.)$ being the $N$-variate normal density function. In Eq. (9) function $g*(.)$ is assumed to be normalized, such that the conditional covariance matrix of $r_t$ is $H_{t,\theta}$. In case of a Gaussian distribution, $g*(.)$ is the standard multivariate Gaussian density function in which $\Gamma(.)$ is the gamma function.

## 3. Robust $M$-estimation

Let $\theta*$ be the true, unknown parameter vector in the parameter space, $\Phi$. $\theta*$ is usually estimated by the $\theta \in \Phi$, such that the log-likelihood of the observations $(r_1, ..., r_T)$ is maximized, assuming a case in which the innovations density function may, in fact, be different from the true function $g*(.)$. This approach for estimation results in the Quasi-Maximum Likelihood (QML) estimator, expressed as:

$$\hat{\theta}_{\text{QML}} = \arg\max_{\theta\in\Phi} -\frac{1}{T}\sum_{t=1}^{T}\Big[\log(\det H_{t,\theta}) - 2\log(r_t' H_{t,\theta}^{-1} r_t)\Big]. \tag{12}$$

In the special case of $g(.) = g*(.)$, the QML estimator is known as the ML estimator. The QML and ML estimators are members of the larger class of $M$-estimators defined as the value of $\theta \in \Phi$, in which, the $M$-function:

$$M(S_T;\theta,\rho) = \frac{1}{T}\sum_{t=1}^{T}\left[\log(\det H_{t,\theta}) + \sigma\rho(r_t' H_{t,\theta}^{-1} r_t)\right], \tag{13}$$

is minimized for sample $S_T$ of observations $(r_1, ..., r_T)$. In the above $M$-function, the scalar, $\sigma$, ensures the consistency of the $M$-estimator regarding the distribution, $g*(.)$, and $\rho(.)$ is called the loss function associated with the $M$-estimator. The standardized Student, $t_v$, and Gaussian $\varphi$ density functions can be regarded as the most popular elliptical density functions for describing financial return series. The loss functions of Student $t_v(v > 2)$ and Gaussian $(\varphi)$ are given by $\rho_{t_v}(z) = (N+v)\log(1+\frac{Z}{v-2})$ and $\rho_\varphi(z) = z$, respectively.

The goal here is to estimate the parameters of a MVFIGARCH model in the presence of outliers in the data. The outliers are identified by their squared Mahalanobis Distance (MD) values being unusually larger in comparison to others. To make the analysis invulnerable to outliers, an MGARCH model with bounded innovation propagation, i.e. an MVFIGARCH model with a bounded innovations effect on future volatility, is proposed.

Empirical studies indicate that huge shocks, e.g. the October 1987 crisis, have a proportionally weaker effect on subsequent volatility in comparison to small and moderate shocks [17,18]. In MVFIGARCH models, the past returns effect on future volatility is considered quadratic, resulting in the neglect of the difference in volatility reactivity to very strong shocks. Therefore, volatility in periods subsequent to extreme return realizations is predicted very pessimistically. Since the MVFIGARCH process is persistent, this overestimation of volatility fades slowly. To defuse this effect, MVFIGARCH models are equipped with a Bounded Innovation Propagation (BIP) property, in order to limit past shock effects on future volatility [12,13]. Although many MGARCH models do not have this property, they can be augmented to have the BIP property by specifying $H_\theta(.)$ introduced in Eq. (2), as the conditional covariance matrix, as a function of the returns, which are weighted in the function of their squared Mahalanobis distance:

$$\tilde{H}_\theta(.) = H_\theta(\tilde{r}_{1,\theta}, \tilde{r}_{2,\theta}, ..., \tilde{r}_{t-1,\theta}), \quad \text{with}$$

$$\tilde{r}_{t,\theta} = r_t\sqrt{w(r_t'\tilde{H}_{t,\theta}^{-1} r)}. \tag{14}$$

The effect of $r_t$ on $\tilde{H}_\theta(.)$ should be bounded by using the weight function. Here, our focus is on a case in which $H_\theta(.)$ has the FBEKK specification (Eq. (6)). The resulted BIP_FBEKK model has the conditional covariance matrix expressed as:

$$\tilde{H}_{t,\theta} = C'C - G'(\tilde{r}_{t-1}'\tilde{r}_{t-1})G + \left[1-(1-L)^d\right](\tilde{r}_t'\tilde{r}_t) + (1-L)^d A'(\tilde{r}_{t-1}'\tilde{r}_{t-1})A + G'\tilde{H}_{t-1}G. \tag{15}$$

From another point of view, by filtering out the outliers from a FBEKK model, a BIP_FBEKK model is obtained. Using a robust $M$-estimator of the BIP_FBEKK model for estimating the parameter $\theta_*$ of the FBEKK model shows its merit when there is an existence possibility of outliers in the data. This is motivated by the BIP_FBEKK model, which may produce more accurate volatility predictions and parameter estimates in the presence of outliers in comparison to the BEKK model, even if the observations are generated by the latter model. The obtained estimator is known as the BIP $M$-estimator, minimizing the $M$-function:

$$\tilde{M}(S_T;\theta,\rho) = \frac{1}{T}\sum_{t=1}^{T}\left[\log(\det\tilde{H}_{t,\theta}) + \sigma\rho(r_t'\tilde{H}_{t,\theta}^{-1} r_t)\right]. \tag{16}$$

Originally, univariate GARCH model BIP $M$-

estimators were introduced by Muler and Yohai (2002, 2008). In a study by Boudt and Croux, it is shown that, when there are additive outliers, $\theta_*$ BIP $M$-estimator acts more accurately than the $M$-estimator obtained from the correctly set BEKK model [14]. The following weight function is used throughout the rest of the paper:

$$w(Z) = \begin{cases} 1 & \text{if } z \leq c_1 \\ 1 - (1 - c_1/Z)^3 & \text{if } c_1 < Z \leq c_2 \\ (c_2/Z)(1 - (1 - c_1/c_2)^3) & \text{else.} \end{cases} \tag{17}$$

There is a tradeoff between robustness and efficiency in the choice of $c_1$ and $c_2$. In the case of all return observations following the FBEKK model, the use of the BIP_FBEKK model leads to a bias and larger mean squared errors for the estimated parameters. As the values of $c_1$ and $c_2$ increase, the BIP_FBEKK model gets closer to the BEKK model, and, as a result, the bias of misspecification is reduced. However, large values of $c_1$ and $c_2$ also imply that the outlying returns effect future volatility, with a squared Mahalanobis distance below $c_1$ being larger. Following Boudt and Croux, parameters $c_1$ and $c_2$ are set to the 99% and 99.9% quantile of the distribution in Eq.(11) of the squared Mahalanobis distances. The corresponding weight function is plotted in Figure 1.

It is noticeable that only observations which have an extraordinary large Mahalanobis distance are down-weighted. In addition, the distributional assumption affects the weighting. Also displayed in Figure 1 is the function, $w(z)z$, which should be focused on, especially when $z$ is the squared MD of $r_t$. This is because, in such a case, $w(z)z$ is the squared MD of $\tilde{r}_t$. It should be pointed out that downweighting is such that $w(z)z$ is non-decreasing and bounded by $w(c_2)c_2$. In order to stay away from numerical problems arising from parameter estimation, the smoothness of the weight function is needed.

## 4. Case study

From 2025, only five Middle Eastern countries; Iran, Saudi Arabia, Kuwait, Iraq and the United Arab Emirates (UAE), as the owners of 70 percent global oil and gas resources, will be producers and major role players in fossil fuel markets. Among those, Iran and the UAE are major oil producers and, therefore, their markets are influenced by oil price fluctuations. In addition, Iranian residents of UAE own a total of ten thousand different businesses in the UAE. Furthermore, economic cooperation between these two countries has grown drastically from 2005 to 2009 [23].

Nowadays, the UAE is regarded as an easy and short detour for Iranian businesses to evade limitations enforced by the sanctions on Iran. International sanctions against Iran, including restrictions on Iranian bank transactions and legislations to confine Iranian international trade, are the motives for Iranians to invest in businesses in the UAE. It should be noted that Tehran and Dubai stock markets are completely different from those of other developing countries, because these two markets, due to the efficiency level and market liberation, are independent of markets in developed countries. Hence, international investors regard these countries as an opportunity for investment risk distribution; however, the effect of oil on these markets should be noticed. In what follows, first, the statistical properties of these indexes are discussed and then the results of volatility transmission for international oil prices (WTI) and Tehran and Dubai Stock Markets are investigated using the robust estimator of the FBEKK model.

### 4.1. Data

In this study, daily data on the Iranian (Tehran) stock price index (TEPIX), the UAE (Dubai) stock price index (DFM), and the daily crude oil price index are used in modeling, estimating and testing in the period December 5th, 2006 to January 30th, 2012. By the crude oil price index, we mean the spot price for West Texas Intermediate (WTI), traded on the domestic spot market at the Cushing, Oklahoma center, and obtained from Reuters. Tehran Stock Exchange index data was extracted from www.tse.ir, the UAE Exchange Index from www.btflive.net and the daily crude oil price index from www.opec.org.

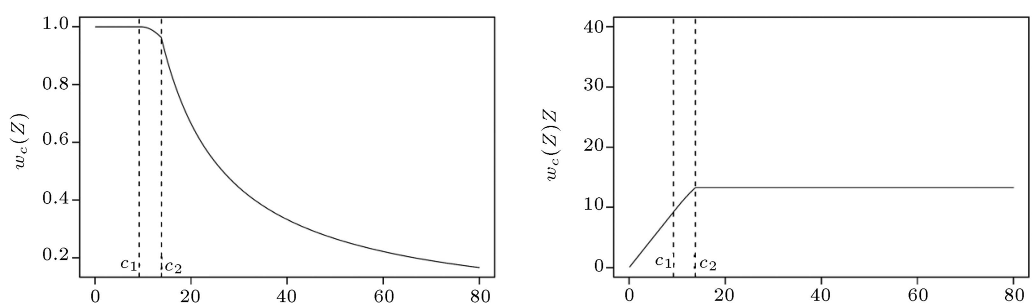


**Figure 1.** Plot for the functions $w(z)$ and $w(z)z$ of BIP_BEKK model. $c_1$ and $c_2$ are set to the 99% and 99.9% quantiles of the squared MD under Gaussian innovations [14].

**Table 1.** Characteristics of the returns of index's distribution.

| | **Iran stock returns** | **The UAE stock returns** | **Oil** |
|---|---|---|---|
| Mean | 0.000424 | -0.001027 | 0.000582 |
| Standard deviation | 0.005658 | 0.022017 | 0.012544 |
| Skewness | 0.553995 | -0.011712 | -0.136782 |
| Kurtosis | 28.55958 | 6.679280 | 6.067216 |
| Jarque-Bera | 342.67 (00.0) | 18.5(00.0) | 92(00.0) |
| Q(16) | 48.56(00.0) | 31.63(00.0) | 32.46(00.0) |

Notes: The daily returns from December 5, 2006 to January 30, 2012 constist of the samples. There are 1124 usable observations. Normality test for the series is performed through Jarque-Bera statistic. Ljung-box statistic for serial correlation is denoted by Q(16) and the actual probability values are presented in parentheses.

The indices analysis using long term memory assessment methods indicates a long term memory effect in all three time series. The method of calculation for the Tehran and Dubai stock exchange indices are the same; hence, it is possible to include these two indices in a multivariate model. Because of the differences in working days between the Tehran stock exchange and international markets, the data are adapted in such a way that the maximum overlap between indices return and weekdays are ensured. Statistical characteristics of the returns of index distributions are given in Table 1.

As reported in Table 1, the daily return mean of the Iranian stock index in the period under study was 0.000424, and the standard deviation was 0.005658. The distribution has a skewness of 0.553995, which means skewness to the right. The kurtosis is 28.55958, which is much more than the kurtosis of normal density functions. Hence, it has a high-peak and fat-tail curve. The test for normal distribution of returns shows that the distributions are not normal. The Jarque-bera statistics used for the test of normality indicates the same result. Jarque-bera statistics for logarithmic returns of indices is 342.67 for Iran (Tehran) stock,18.5 for the UAE (Dubai) stock, and 92 for the oil price. The Jarque-Bera statistic rejects the null hypothesis of normality for all return series. The analysis shows that all the return series are leptokurtic, meaning they have fat tails, suggesting that the existence of autoregressive conditional heteroscedasticity (ARCH) should be tested in each of the mean equations. In each case, ARCH effects are observed in the mean equation for the return series, and, therefore, a GARCH estimation model is appropriate. It is noticeable that the Ljung-Box Statistic (Q-Statistic) utilized for autocorrelation detection is significant in all cases, indicating that the past market behavior may be more relevant.

### *4.2. Result*

To estimate the multivariate FIGARCH (FBEKK) model, we used the robust $M$-estimator quasi-maximum likelihood method introduced in Section 3. Due to the nonlinear structure of the memory function $(1-L)^d$, the Mac-Lauren extension was used in the program structure and the likelihood function. The model is developed in such a way to imply a long-term memory effect in the estimation. This procedure has not been considered in previous studies. In the FBEKK model, $\alpha_{ii}$ notifies the ARCH effects of each variable, and $\alpha_{ij}$ notifies the volatility spillover of variable $i$ for prior periods to the volatility of variable $j$ for the current period. This transmission effect is measured by the squared residuals obtained from the models of return forecasting. $\beta_{ii}$ notifies the GARCH effects and signifies the persistence of volatility in each series. $\beta_{ij}$, which is based on the recent forecast of the variance, notifies the volatility spillover effect of the variance of prior period of variable $i$ to the current variance of variable $j$. Notably, both $\alpha_{ij}$ and $\beta_{ij}$ may indicate the spillover between indices, and the volatility spillover is determined by the non-diagonal values of the matrix. Table 2 reports the estimation results of BEKK and FBEKK models.

According to the results of estimates in Table 2, in the multivariate FIGARCH model, $\alpha_{ii}$ and $\beta_{ii}$ coefficients are significant, and indicate the volatility spillover rate and the persistence of conditional volatility in each of the mentioned indices. Empirical analysis and the evaluation process in terms of price indices indicate that the UAE stock markets and global oil markets are turbulent.

The effect was significant for Iran's stock return $((\alpha_{11} = 0.13441)$ and $(\beta_{11} = 0.86055))$; for the UAE stock return $((\alpha_{22} = 0.09032)$ and $(\beta_{22} = 0.9020))$; and for crude oil $((\alpha_{33} = 0.06531)$ and $(\beta_{33} = 0.92082))$.

$(\alpha_{21} = 0.08532)$ and $(\beta_{21} = 0.91471)$ coefficients are significant and show the volatility spillover from the UAE (Dubai) stock exchange to the Iranian (Tehran) stock exchange. There was no significant adverse effect from Iran to the UAE in the FBEKK model. In the BEKK model, the $\beta_{21} = 0.5836$ coefficient was

**Table 2.** Estimation of MVFIGARCH (FBEKK) coefficient model; comparison of estimated parameters of BEKK and FBEKK models.

| | **BEKK (without consideration of long memory effect)** | | **FBEKK** | | **Status of estimated parameters** |
|---|---|---|---|---|---|
| **Coef.** | **Value** | $\boldsymbol{Pr(>\|t\|)}$ | **Value** | $\boldsymbol{Pr(>\|t\|)}$ | |
| d | - | - | -0.30451 | 0.0000 | |
| $\alpha_{11}$ | 0.2015 | 0.0000 | 0.13441 | 0.0000 | Significant in both models |
| $\alpha_{12}$ | 0.8531 | 0.6338 | 0.6392 | 0.542 | Insignificant in both models |
| $\alpha_{13}$ | 0.3419 | 0.4380 | 0.0468 | 0.762 | Insignificant in both models |
| $\alpha_{21}$ | 0.0739 | 0.0004 | 0.08532 | 0.0000 | Significant in both models |
| $\alpha_{22}$ | 0.1034 | 0.0000 | 0.09032 | 0.0000 | Significant in both models |
| $\alpha_{23}$ | 0.0159 | 0.148 | 0.00345 | 0.6302 | Insignificant in both models |
| $\alpha_{31}$ | 0.0743 | 0.504 | 0.00235 | 0.0000 | **Significant in FBEKK model** |
| $\alpha_{32}$ | 0.0294 | 0.0000 | 0.02531 | 0.0000 | Significant in both models |
| $\alpha_{33}$ | 0.0578 | 0.0000 | 0.06531 | 0.0000 | Significant in both models |
| $\beta_{11}$ | 0.7013 | 0.0000 | 0.86055 | 0.0000 | Significant in both models |
| $\beta_{12}$ | 0.03201 | 0.6483 | 0.1161 | 0.8302 | Insignificant in both models |
| $\beta_{13}$ | 0.0005 | 0.5283 | 0.0070 | 0.5704 | Insignificant in both models |
| $\beta_{21}$ | 0.5836 | 0.0749 | 0.91471 | 0.0000 | **Significant in FBEKK model** |
| $\beta_{22}$ | 0.628 | 0.0000 | 0.90920 | 0.0000 | Significant in both models |
| $\beta_{23}$ | 0.8706 | 0.5192 | -0.0320 | 0.7301 | Insignificant in both models |
| $\beta_{31}$ | 0.5537 | 0.6839 | 0.0004 | 0.439 | Insignificant in both models |
| $\beta_{32}$ | 0.7881 | 0.0000 | 0.97472 | 0.0000 | Significant in both models |
| $\beta_{33}$ | 0.8527 | 0.0000 | 0.92082 | 0.0000 | Significant in both models |
| | AIC=-25.02 | BIC=-28.56 | AIC=-21.64 | BIC=-22.09 | Optimal lag selection criteria |

Notes: The parameter $d$ is a long memory parameter and other estimation parameters ($\alpha_{ij}, \beta_{ij}$) indicate the volatility structure between time series.

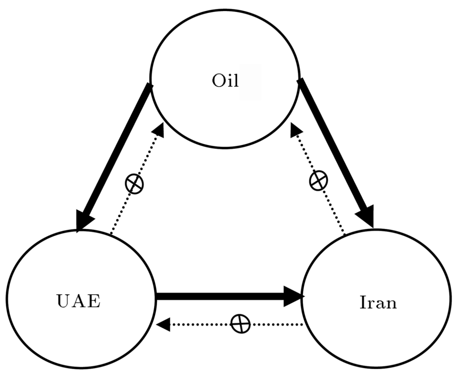


**Figure 2.** Volatility transmission conceptual model.

not significant and the model could not identify the volatility spillover.

Figure 2 depicts the relationship based on volatility transmissions between the examined indices in the form of a conceptual model. The black arrows indicate the spillover between indices and the $\otimes$ symbol indicates that there is no spillover in the examined direction.

Actually, due to the volume of trade between Iran and the UAE, which is more than 15 billion dollars, and due to the fact that more than 35 percent of Iran's imports are from the UAE, it is logical that the volatility transmission is from the UAE stock market to the Iranian stock market. A weak transmission was observed from the oil market to the Iranian stock exchange and the ($\alpha_{31} = 0.00235$) coefficient was significant, but there was no significant adverse effect. In the BEKK basic model, the $\alpha_{31} = 0.0743$ coefficient was not significant, indicating that according to economic theory, the basic model was less explanatory. ($\alpha_{32} = 0.02531$) and ($\beta_{32} = 0.97472$) coefficients are significant and this shows the volatility spillover from the oil market to the UAE market, but there was no significant adverse effect. The long-term memory parameter was estimated as $d = -0.30451$ and this indicates that the time series are stationary and have mid-term memory. By mid-term memory, we mean the condition in which the longevity of volatility effects is reduced. In the literature, they are classified in the field of long-term memory.

In order to justify the applicability of the proposed model, we investigated the volatility transmission between the Tehran stock index, the Dubai stock index and the global oil price index using the MVFI-

GARCH model within the time span from December 5th, 2006 to January 30th, 2012. The result of estimation in different models generally shows volatility transmission from the global oil market to Tehran and Dubai markets. Volatility transmission from the Dubai market to Tehran was meaningfully observed as well. However, the effect of transmission was not observed in the reverse direction.

Empirical analysis and the evaluation process, in terms of price indices, indicate that the UAE stock markets and global oil markets are turbulent. Due to the volume of trade between Iran and the UAE, which is more than 15 billion dollars, and due to the fact that more than 35 percent of Iran's imports are from the UAE, it is logical that the volatility transmission is from the UAE stock market to the Iranian stock market. Also, a weak transmission was observed from the oil market to the Iranian market, but no significant adverse effect was observed. However, there was a transmission from the oil market to the UAE market.

## 5. Conclusion

An important issue in stock markets, which significantly affects the decisions of decision makers, is the stochastic nature of price volatility. In this paper, by introducing a class of robust $M$-estimators for multivariate FIGARCH time series models, a new multivariate fractionally integrated generalized autoregressive conditional heteroscedasticity (MVFIGARCH) model was proposed. Actually, we aimed to consider the effect of bounding the innovation propagation in MVFIGARCH models on the robustness of the $M$-estimator to outliers. The application of a volatility model that has the property of bounded innovation propagation is highly recommended if the data is suspected to be contaminated by outliers. The model developed in this study enables us to observe the market volatility spillover effects. The $d$ parameter of long memory is estimated throughout the modeling process and this leads to more adaptability and more precision.

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