Scientia Iranica E (2015) 22(3), 1171–1178

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
www.scientiairanica.com

Fractional order grey relational analysis and its application

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Received 11 December 2012; received in revised form 19 July 2013; accepted 17 June 2014

KEYWORDS
Grey relational analysis;
Fractional order;
R&D;
GDP;
High technology output value.

Abstract. The main idea behind this study is to introduce the fractional order grey relational degree to analyze the relationship between sci-tech input and the economic growth of China. Based on a fractional order difference operator, fractional order grey relational analysis (FGRA) is defined. The effect of different orders on grey relational analysis is discussed. Two examples show the process and efficiency of its application.

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1. Introduction

Grey Relational Analysis (GRA) proposed by Deng [1] is a very useful mathematical tool for dealing with the relationship between a given reference sequence and a given set of comparative sequences. Despite having been widely used in various fields [2-6], traditional GRA might not meet the needs of certain problems, so, numerous different methods are proposed to improve it. Yamaguchi et al. proposed a new grey relational analysis by expanding a range of treatable value [7], and developed a reliable topological-based grey relational analysis [8]. The absolute, relative and synthetic degrees of grey relations are defined to be the sum of the distances between two consecutive time moments [9]. A grey slope relational degree based on a first difference operator is proposed [10-12], and a grey convex relational degree based on a second difference operator is discussed [13]. However, the above GRA methods cannot deal with the memory property of a time series.

Fractional order systems with long or short memories are very common, such as energy systems [14-16], earthquakes [17], hydrological time series [18], a number of common variables used in political research [19], economics and financial time series data [20-23], transportation time-series [24], tourism time series [25], and signal processing [26]. So, in this paper, we put forward a fractional order grey relational analysis which can deal with fractional order systems with long or short memories.

The relationship between levels of Research and Development (R&D) investment and economic growth has become an increasingly significant issue. Mario found that the range of R&D investment in GDP between 2.3% and 2.6% maximizes the long-run impact on productivity growth [27]. Jeffery's empirical evidence shows that R&D investment gives rise to different short-term and medium impacts on real GDP among different industries [28]. A new method of estimating the impact of investment in R&D on long-term economic growth is provided using panel data [29]. Roman examines the effect of R&D on long-term economic growth using the Bayesian model averaging to deal rigorously with model uncertainty [30]. Chen et al. applied an artificial neural network to explore the relationships between the performance of R&D

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projects and relevant determinants [31], and so on. Because R&D has lagged effects on economic growth, the fractional order grey relational degree provides a powerful decision support and scientific basis for science and technology input and output evaluation.

An overview of the relevant literature on grey relational degree is presented in Section 2, and new fractional order grey relational degree is provided in Section 3. Two real examples are proposed in Section 4, and some conclusions are discussed in the final section.

2. Literature review

2.1. The traditional grey relational degree [1-4]

Given a system's behavioral sequence, $X_i = (x_i(1), x_i(2), \cdots, x_i(n)), i = 1, 2, \cdots, m$, the values of the original sequences must be normalized to be in the same order, because an inaccurate grey relational degree will be induced by the order variation of the data characterizing the factors. The original data are commonly normalized by (a) initial value or (b) mean value. The two normalized algorithms are as follows [13]:

(a) The data normalized by initial value. The normalized sequences, $X_i^{'}$, by initial value can be derived as:

$$X_i^{'} = \{x_i^{'}(1), x_i^{'}(2), \cdots, x_i^{'}(n)\}$$

$$= \{\frac{x_i(1)}{x_i(1)}, \frac{x_i(2)}{x_i(1)}, \cdots, \frac{x_i(n)}{x_i(1)}\},$$

$$i = 1, 2, \cdots, m,$$

where $x_i(1)$ is the first data of the corresponding X_i factor.

(b) The data normalized by mean value. The normalized sequences, $X_i^{'}$, by mean value can be derived as:

$$X_i^{'} = \{x_i^{'}(1), x_i^{'}(2), \cdots, x_i^{'}(n)\}$$

$$= \left\{ \frac{x_i(1)}{\overline{X_i}}, \frac{x_i(2)}{\overline{X_i}}, ..., \frac{x_i(n)}{\overline{X_i}} \right\},$$

$$i = 1, 2, ..., m,$$

where $\overline{X_i} = \frac{1}{n} \sum_{k=1}^{n} x_i(k)$.

Given a reference sequence, $X_0 = (x_0(1), x_0(2), \cdots, x_0(n))$, normalized by initial value, and comparative sequences, $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$, normalized by initial value, $i = 1, 2, \cdots, m$, the grey relational coefficient between X_0 and X_i at point k was defined by Deng as follows:

$$\gamma(x_0(k), x(k)) = \frac{\Delta_{\min} + \xi\Delta_{\max}}{\Delta_{0i}(k) + \xi\Delta_{\max}},$$

where $\Delta_{0i}(k) = |x_0(k) - x_i(k)|, \Delta_{\min} = \Delta_{\min_{\forall i}\min_{\forall k}}\Delta_{0i}(k), \Delta_{\max} = \Delta_{\max_{\forall i}\max_{\forall k}}\Delta_{0i}(k)$. $\xi \in (0, 1)$ is a distinguishing coefficient used to control the level of differences of the relational coefficients.

$$\gamma_{0i} = \gamma(X_0, X_i) = \frac{1}{n}\sum_{k=1}^{n}\gamma(x_0(k), x_i(k)),$$

is named as the grey relation degree between X_0 and X_i. If $\gamma_{0i} \geq \gamma_{0j}$, we say that the factor X_i is superior to factor X_j, written $X_i \succ X_j$.

2.2. Grey slope relation degree [11-13]

Given a reference sequence, $X_0 = (x_0(1), x_0(2), \cdots, x_0(n))$, and comparative sequences, $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$, $i = 1, 2, \cdots, m$, the grey slope relational coefficient between X_0 and X_i at point k was defined as follows:

$$\gamma_{0i}(k) = \text{sgn}_k \frac{1}{1 + \left|\frac{|\triangle_i(k)|}{R_i} - \frac{|\triangle_0(k)|}{R_0}\right|},$$

$$k = 1, 2, \cdots, n, \qquad m = 1, 2, \cdots, n,$$

which is referred to as the grey slope relational coefficients between X_i and X_0, where:

$$\triangle_0(k) = x_0(k) - x_0(k-1),$$

$$\triangle_i(k) = x_i(k) - x_i(k-1),$$

$$R_0 = \max\{x_0(k)\} - \min\{x_0(k)\},$$

$$R_i = \max\{x_i(k)\} - \min\{x_i(k)\}, k = 2, 3, \cdots, n,$$

and:

$$\mathbf{sgn}_k = \begin{cases} -1 & \text{if} \triangle_0(k)\triangle_i(k) \geq 0 \\ 1 & \text{if} \triangle_0(k)\triangle_i(k) < 0 \end{cases}$$

$$\gamma_{0i} = \frac{1}{n-1}\sum_{k-2}^{n}\gamma_{0i}(k),$$

are said to be the grey slope relational degree between X_0 and X_i.

2.3. The grey convex relational degree [13]

Definition 1 [13]. Given a reference sequence, $X_0 = (x_0(1), x_0(2), \cdots, x_0(n))$, and comparative sequences, $X_i = (x_i(1), x_i(2), \cdots, x_i(n)), i = 1, 2, \cdots, m$, grey convex relational coefficient between X_0 and X_i at point k was defined as follows:

$$\gamma(x_0(k), x_i(k)) = \frac{1}{1 + |d_0(k) - d_i(k)|},$$

$$\gamma(X_0, X_i) = \frac{1}{n-2}\sum_{k=2}^{n-1}\gamma(x_0(k), x_i(k)),$$

which is said to be the grey convex relational degree between X_0 and X_i, where:

$$d_i(k) = \frac{x_i(k+1) + x_i(k-1)}{2x_i(k)}, \quad k = 2, 3, \cdots, n-1,$$

is the degree of convexity at point k of sequence $X_i (i = 0, 1, 2, \cdots, m)$. The grey convex relational degree does not need dimensionless transition.

3. The fractional order grey relational degree

3.1. Fractional order difference operator and its properties

Let $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$ be an original sequence, $x^{(0)}(k)$ is the value at time k. If $x^{(0)}(k) - x^{(0)}(k-1)(k = 2, 3, \cdots, n)$ is first difference, then second difference is $x^{(0)}(k) - x^{(0)}(k-1) - (x^{(0)}(k-1) - x^{(0)}(k-2))(k = 3, 4, \cdots, n)$. In this section, we extend integer difference to fractional order difference.

Definition 2 [32]. Let the $\frac{p}{q}(0 \leqslant \frac{p}{q} \leqslant 1)$ order accumulated generating operator of the original non-negative sequence $X^{(0)}$ be $X^{(\frac{p}{q})}$, set $C^0_{\frac{p}{q}-1} = 1, C^{k-1}_{k} = 0$, then:

$$x^{(\frac{p}{q})}(k) = \sum_{i=1}^{k} C^{k-i}_{\frac{p}{q}+k-i-1} x^{(0)}(i), k = 1, 2, \cdots, n,$$

where:

$$C^{k-i}_{\frac{p}{q}+k-i-1} = \frac{(\frac{p}{q}+k-i-1)(\frac{p}{q}+k-i-2)\cdots(\frac{p}{q}]!+]!1)\frac{p}{q}}{(k-i)!}.$$

Definition 3. Let $X^{(0)} = \{x^{(0)}(1),\ x^{(0)}(2),\ \cdots,\ x^{(0)}(n)\}$ be an original sequence, first order inverse accumulated generating operator of $X^{(0)}$, as follows:

$$\alpha^{(1)} X^{(0)} = \{\alpha^{(1)} x^{(0)}(2), \cdots, \alpha^{(1)} x^{(0)}(n)\},$$

where $\alpha^{(1)} x^{(0)}(k) = x^{(0)}(k) - x^{(0)}(k-1); k = 2, \cdots, n$. Then, $\frac{p}{q}(0 \leqslant \frac{p}{q} \leqslant 1)$ order inverse accumulated generating operator of $X^{(0)}$ is:

$$\alpha^{(\frac{p}{q})} X^{(0)} = \alpha^{(1)} X^{(1-\frac{p}{q})}$$

$$= \{\alpha^{(1)} x^{(1-\frac{p}{q})}(1), \alpha^{(1)} x^{(1-\frac{p}{q})}(2), ..., \alpha^{(1)} x^{(1-\frac{p}{q})}(n)\}.$$

$\alpha^{(\frac{p}{q})} X^{(0)} = \alpha^{(1)} X^{(1-\frac{p}{q})}$ is called the $\frac{p}{q}$ difference operator.

Theorem 1. Assume that $X^{(0)} = \{x^{(0)}(1),\ x^{(0)}(2),\ \cdots,\ x^{(0)}(n)\}$ is a nonnegative sequence, for $0 \leqslant \frac{p}{q} \leqslant 1$:

$$\{\alpha^{(1)} x^{(1-\frac{p}{q})}(1), \alpha^{(1)} x^{(1-\frac{p}{q})}(2), ..., \alpha^{(1)} x^{(1-\frac{p}{q})}(n)\},$$

is the $\frac{p}{q}$ order difference sequence of $X^{(0)}$. When $\frac{p}{q}$ is certain, $C^0_{-\frac{p}{q}} = 1$; the larger $\frac{p}{q}$ is, the $\frac{p}{q}$ order difference sequence pays more attention to the recent information of a system; the smaller $\frac{p}{q}$ is, the $\frac{p}{q}$ order difference sequence pays more attention to the information of a system in the long-term.

Proof. By definition 3,

$$\alpha^{(1)} x^{(1-\frac{p}{q})}(k) = x^{(1-\frac{p}{q})}(k) - x^{(1-\frac{p}{q})}(k-1)$$

$$= \sum_{i=1}^{k} C^{k-i}_{1-\frac{p}{q}+k-i-1} x^{(0)}(i)$$

$$- \sum_{i=1}^{k-1} C^{k-1-i}_{1-\frac{p}{q}+k-1-i-1} x^{(0)}(i)$$

$$= \sum_{i=1}^{k} C^{k-i}_{k-i-\frac{p}{q}} x^{(0)}(i) - \sum_{i=1}^{k-1} C^{k-1-i}_{k-i-1-\frac{p}{q}} x^{(0)}(i)$$

$$= C^0_{-\frac{p}{q}} x^{(0)}(k) + \sum_{i=1}^{k-1} C^{k-i}_{k-i-\frac{p}{q}} x^{(0)}(i)$$

$$- \sum_{i=1}^{k-1} C^{k-1-i}_{k-i-1-\frac{p}{q}} x^{(0)}(i) = x^{(0)}(k)$$

$$+ \sum_{i=1}^{k-1} (C^{k-i}_{k-i-\frac{p}{q}} - C^{k-1-i}_{k-i-1-\frac{p}{q}}) x^{(0)}(i)$$

$$= x^{(0)}(k) + \sum_{i=1}^{k-1} C^{k-i}_{k-i-\frac{p}{q}-1} x^{(0)}(i).$$

$C^{k-i}_{k-i-\frac{p}{q}-1}$ is the weight coefficient of $x^{(0)}(i)$ for $\alpha^{(1)} x^{(\frac{p}{q})}(k)$ and is a decreasing function of i. Then, the difference between the weight of $x^{(0)}(i)$ and $x^{(0)}(i-1)$ is:

$$C^{k-i}_{k-i-\frac{p}{q}-1} - C^{k-(i-1)}_{k-(i-1)-\frac{p}{q}-1} = C^{k-i}_{k-i-\frac{p}{q}-1}$$

$$-C^{k-i+1}_{k-i-\frac{p}{q}} = -C^{k-i+1}_{k-i-\frac{p}{q}-1}.$$

If:

$$0 \leqslant \frac{p_1}{q_1} < \frac{p_2}{q_2} \leqslant 1,$$

then:

$$-C^{k-i+1}_{k-i-\frac{p_1}{q_1}-1} < -C^{k-i+1}_{k-i-\frac{p_2}{q_2}-1}.$$

We can see that the larger $\frac{p}{q}$ is, the faster is the decreased rate, and the larger $\frac{p}{q}$ is, the more attention the $\frac{p}{q}$ order difference sequence pays to the recent information of a system; the smaller $\frac{p}{q}$ is, the more attention the $\frac{p}{q}$ order difference sequence pays to the old information of a system in the long-term.

3.2. Fractional order grey relational degree

Given a reference sequence, $X_0 = (x_0(1),\ x_0(2),\ \cdots,\ x_0(n))$, normalized by mean value, and comparative sequences, $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$, normalized by mean value, $i = 1, 2, \cdots, m$.

$$\gamma_{0i}^{\frac{p}{q}} = \frac{1}{n-1}\sum_{k=2}^{n}\frac{1}{1+|\alpha^{(\frac{p}{q})}x_0^{(0)}(k) - \alpha^{(\frac{p}{q})}x_i^{(0)}(k)|},$$

is said to be the $\frac{p}{q}$ order grey relational degree between X_0 and $X_i, i = 0, 1, 2, \cdots, m$. If $\gamma_{0i}^{\frac{p}{q}} \geq \gamma_{0j}^{\frac{p}{q}}$, we say that factor X_i is superior to the factor X_j, written $X_i \succ X_j$. The order relation, $'\succ'$, is referred to as the $\frac{p}{q}$ order grey relational order.

Theorem 2. Given a reference sequence, $X_0 = (x_0(1),\ x_0(2),\ \cdots,\ x_0(n))$, normalized by mean value and comparative sequences, $X_i = (x_i(1),\ x_i(2),\ \cdots,\ x_i(n))(i = 1, 2, \cdots, m.)$, normalized by mean value, $\gamma_{0i}^{\frac{p}{q}} = \frac{1}{n-1}\sum\limits_{k=2}^{n}\frac{1}{1+|\alpha^{(\frac{p}{q})}x_0^{(0)}(k)-\alpha^{(\frac{p}{q})}x_i^{(0)}(k)|}$ satisfies:

1. Normality: $0 < \gamma_{0i}^{\frac{p}{q}} \leq 1, \gamma_{0i}^{\frac{p}{q}} = 1 \Longleftrightarrow X_0 = X_i$;
2. Duality symmetric: $\gamma_{0i}^{\frac{p}{q}} = \gamma_{i0}^{\frac{p}{q}}$;
3. Closeness: the smaller the $|\alpha^{(\frac{p}{q})}x_0^{(0)}(k) - \alpha^{(\frac{p}{q})}x_i^{(0)}(k)|$ is, the greater is the $\gamma_{0i}^{\frac{p}{q}}$.

Proof. By $\frac{1}{n-1}\sum\limits_{k=2}^{n}\frac{1}{1+|\alpha^{(\frac{p}{q})}x_0^{(0)}(k)-\alpha^{(\frac{p}{q})}x_i^{(0)}(k)|}$, we can see that the smaller $|\alpha^{(\frac{p}{q})}x_0^{(0)}(k) - \alpha^{(\frac{p}{q})}x_i^{(0)}(k)|$ is, the greater $\gamma_{0i}^{\frac{p}{q}}$; $0 < \gamma_{0i}^{\frac{p}{q}} \leq 1, \gamma_{0i}^{\frac{p}{q}} = 1 \Longleftrightarrow X_0 = X_i$ is, and $\gamma_{0i}^{\frac{p}{q}} = \gamma_{i0}^{\frac{p}{q}}$.

Because the wholeness of the grey relational analysis is incompatible with the Duality Symmetric [33], it is reasonable that the $\frac{p}{q}$ order grey relational degree does not obey the wholeness axiom.

The larger $\frac{p}{q}$ is, the more attention the $\frac{p}{q}$ order difference sequence pays to the recent information of a system. So, the $\frac{p}{q}$ order grey relational degree pays more attention to the short-term relation when $\frac{p}{q}$ is relatively large; the smaller $\frac{p}{q}$ is, the more attention the $\frac{p}{q}$ order difference sequence pays to the old information of a system. So, the $\frac{p}{q}$ order grey relational degree pays more attention to the long-term relation when $\frac{p}{q}$ is relatively small.

The calculating method of the fractional order grey relational degree can be summarized as follows:

Step 1. The raw data are normalized by the mean value for dimensionless.

Step 2. Set $\frac{p}{q}$, calculate the $\alpha^{(\frac{p}{q})}x_i^{(0)}(k), i = 0, 1, 2, \cdots, m, k = 2, 3, \cdots, n$.

Step 3. Calculate the fractional order grey relational degree, $\gamma_{0i}^{\frac{p}{q}}, i = 1, 2, \cdots, m$.

Step 4. Arrange the factors into order.

4. Numerical experiments

In this section, the fractional order grey relational degree is used to analyze the effects of science and technology input on long-term economic growth in a country or a region.

Case 1: The relational analysis of the relationship between Science and Technology (ST) input and output in China [34]. The dynamics and performance of innovation systems in China have been dramatically altered over the last 30 years. But, regional markets are still not functioning well. These difficulties have been highlighted in many recent studies on Chinese national and regional innovation systems [35-36]. Jiang and Zhao used a grey T correlation analysis to study the relationship between the ST input and economic growth from 2003 to 2008 in China [34]. In this paper, we used a fractional order grey relational degree to discuss the relationship between the ST input and economic growth. Let the Gross Domestic Product (GDP), the GDP expenditure on R&D, the value of contracts imported and the personnel engaged in ST activities be $X_0^{(0)}$, $X_1^{(0)}$, $X_2^{(0)}$ and $X_3^{(0)}$, respectively. The raw values of $X_0^{(0)}$, $X_1^{(0)}$, $X_2^{(0)}$ and $X_3^{(0)}$ from paper [34] are presented in Table 1. Set $\frac{p}{q} = 0.9, 0.7, 0.5, 0.2, 0.1$, respectively.

$$\alpha^{(0.5)}X_0^{(0)} = \{0.392, 0.371, 0.419, 0.446, 0.503, 0.597\},$$

$$\alpha^{(0.5)}X_1^{(0)} = \{0.358, 0.358, 0.453, 0.538, 0.633, 0.776\},$$

then:

$$\gamma_{01}^{0.5} = \frac{1}{n-1}\sum_{k=2}^{n}\frac{1}{1+|\alpha^{(0.5)}x_0^{(0)}(k) - \alpha^{(0.5)}x_1^{(0)}(k)|}$$

$$= \frac{0.9669+0.9869+0.9673+0.9153+0.8848+0.8482}{6}$$

$$= 0.928.$$

Similarly, we can obtain grey relational degree with different orders as shown in Table 2.

From the relational order, $\gamma_{02}^{0.9} \succ \gamma_{01}^{0.9} \succ \gamma_{03}^{0.9}$, it can be concluded that the value of contracts imported has the greatest influence on economic growth among the $X_1^{(0)}, X_2^{(0)}$ and $X_3^{(0)}$ in the short-term, and personnel engaged in ST activities have the least influence on economic growth in the short-term. When $\frac{p}{q} = 0.7, 0.5, 0.2, 0.1$, the relational order is $\gamma_{01}^{\frac{p}{q}} \succ \gamma_{03}^{\frac{p}{q}} \succ \gamma_{02}^{\frac{p}{q}}$.

Table 1. The GDP and science and technology input in China.

Year	$X_0^{(0)}$ (100 million RMB)	$X_1^{(0)}$ (100 million RMB)	$X_2^{(0)}$ (billion US dollar)	$X_3^{(0)}$ (person)
2001	109655	1042.49	90.09	3141085
2002	120333	1287.64	173.89	3221822
2003	135823	1539.63	134.51	3284005
2004	159878	1966.33	138.6	3481417
2005	183218	2449.97	190.43	3814654
2006	211924	3003.1	220.23	4131542
2007	249530	3710.24	254.15	4543868

Table 2. Fractional order grey relational degree with different orders.

Order	γ_{01}	γ_{02}	γ_{03}
0.9 order	0.909	0.921	0.902
0.7 order	0.919	0.906	0.911
0.5 order	0.928	0.891	0.917
0.2 order	0.937	0.873	0.930
0.1 order	0.939	0.868	0.931

Let the grey relational degree be reviewed from a vertical orientation. With a decrease in $\frac{p}{q}$, $\gamma_{02}^{\frac{p}{q}}$ becomes gradually small. This reveals that, at present, technology development is in a transition phase from technical use to technical transformation. Technology has a great influence on economic growth in the short-term, and has little influence on economic growth in the long run. With the decrease of $\frac{p}{q}$, $\gamma_{01}^{\frac{p}{q}}$ and $\gamma_{03}^{\frac{p}{q}}$ become gradually larger. These results indicate that GDP expenditure on R&D and personnel engaged in ST activities have a long-term influence on economic growth. The above results are quite close to the actual situation in China. It shows that a fractional order grey relational degree can reflect the correlation situation of practical systems.

This study calculated the Pearson product-moment correlation coefficient (r) to study the correlation between both values, and the statistics are summarized in Table 3.

The Pearson product-moment correlation coefficient is static analysis. The fractional order grey relational analysis shows that the Chinese government needs to strengthen the ability of digestion and absorption when importing advanced and applicable technology.

Table 3. Correlation coefficient of different input.

Correlation coefficient	Value
$r_{X_0^{(0)}X_1^{(0)}}$,	1.00
$r_{X_0^{(0)}X_2^{(0)}}$,	0.90
$r_{X_0^{(0)}X_3^{(0)}}$,	0.99

Table 4. The R&D investment and output in Anhui province (Unit: billion RMB).

Year	High technology output value	R& DRI	R& DHE	R& DIB
2003	758.6	9.31	7.97	14.93
2004	928.3	7.59	6.97	22.35
2002	1236.5	9.66	7.68	27.87
2006	1851.8	11.70	10.40	36.50
2007	2518.3	14.50	10.70	47.20
2008	3212.3	18.03	9.57	71.49

Case 2: The time correlation analysis of the relationship between the ST input and output in Anhui province [37]. Zhou and Zhang used the grey T correlation analysis to discuss the relationship between ST input and output in Anhui province. In this paper, we used the fractional order grey relational degree to discuss the relationship between R&D investments and high technology output value in Anhui province. Let the high technology output value, R&D investment in research institutes (R&DRI), R&D investment in higher education (R&DHE) and R&D investment in business (R&DIB) be $X_0^{(0)}$, $X_1^{(0)}$, $X_2^{(0)}$ and $X_3^{(0)}$, respectively. The raw values of $X_0^{(0)}$, $X_1^{(0)}$, $X_2^{(0)}$ and $X_3^{(0)}$ from paper [37] are presented in Table 4.

Set $\frac{p}{q} = 0.9, 0.7, 0.5, \frac{1}{3}, 0.1$ respectively. The fractional order grey relational degree is calculated and the fractional order grey relational degree is shown in Table 5.

For the fractional order grey relational degree with different orders, the relational orders are all $\gamma_{03}^{\frac{p}{q}} \succ \gamma_{01}^{\frac{p}{q}} \succ \gamma_{02}^{\frac{p}{q}}$.So, it was concluded that R&DIB and R&DRI play a more significant role in promoting high technology output in Anhui province than R&DHE. This is because R&DHE was less from 2003 to 2008.

From a vertical orientation, $\gamma_{03}^{\frac{p}{q}}$ becomes gradually smaller with a decrease in $\frac{p}{q}$. This is because the business policymakers like to pursue maximal short-term benefits when the R&D projects, are chosen [38].

Table 5. Fractional order grey relational degree with different orders.

Order	γ_{01}	γ_{02}	γ_{03}
0.9 order	0.872	0.739	0.914
0.7 order	0.890	0.750	0.910
0.5 order	0.888	0.763	0.907
$\frac{1}{3}$ order	0.886	0.776	0.906
0.1 order	0.885	0.796	0.899

Table 6. Correlation coefficient of different input.

Correlation coefficient	Value
$r_{X_0^{(0)}X_1^{(0)}}$,	0.98
$r_{X_0^{(0)}X_2^{(0)}}$,	0.76
$r_{X_0^{(0)}X_3^{(0)}}$,	0.98

With the decrease in $\frac{p}{q}$, $\gamma_{01}^{\frac{p}{q}}$ and $\gamma_{02}^{\frac{p}{q}}$ become gradually larger. These results indicate that R&DRI and R&DHE have a long-term influence on economic growth. The above results are quite close to the actual situation in Anhui province, which shows that the fractional order grey relational degree can reflect the correlation situation of practical systems.

This study calculated the Pearson product-moment correlation coefficient (r) to study the correlation between both values, and the statistics are summarized in Table 6.

The Pearson product-moment correlation coefficient is static analysis. The above analysis shows that the R&DHE is too small to fulfill its role, which is worth emphasizing. R&DRI and R&DHE are too great to be ignored by the Anhui government, in order to realize the sustainable growth of high technology output values.

5. Conclusion

It is well known that a fractional order system itself is an infinite dimensional filter, due to the fractional order in the differentiator or integrator, while the integer order system memory is limited. The significance of the fractional order theory is a generalization of the classical integral order theory. Similarly, in this paper, the fractional order grey relational degree is a generalization of the grey slope relation degree and grey convex relational degree. The results reveal that the fractional order grey relational degree is effective and feasible, thus, providing an excellent modeling tool for describing many actual dynamic processes. Future studies could investigate other actual dynamic processes in order to confirm the generality of the fractional order grey relational degree. The difference between the grey relational degree of different order is not large, and how to expand the difference is a problem which deserves further consideration.

Acknowledgments

The authors are grateful to the anonymous referees for their helpful and constructive comments regarding this paper. The relevant research undertaken in this paper is supported by a Marie Curie International Incoming Fellowship within the 7th European Community Framework Programme (No. FP7-PIIF-GA-2013-629051), the National Natural Science Foundation of China (No. 71401051), the Natural Science Foundation of Jiangsu Province (No. BK20130786), a project of Social Science Foundation of China (No. 12AZD102), and the funding for talent introduction in NUAA (1009-YAH14003).

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