Territory concept to improve transmission expansion planning problem solution algorithms

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Abstract. This paper is devoted to solving Transmission Expansion Planning (TEP) problems, via a Constructive Heuristic Algorithm (CHA) that can be employed as a subroutine in a meta-heuristic procedure. In such a strategic methodology, CHA may improve the quality of trial solutions that speed up the convergence of the main algorithm. By introducing a “territory concept” for each derived local optimum, this paper proposes an approach, forcing CHA sub-procedures to explore new areas in the problem sub-space. Such modification is enforced on the Villasana-Garver-Salón (VGS) algorithm, as a well-known kind of CHA, to improve its performance. The improved VGS is called Territory-Based VGS (TBVGS). In order to evaluate the performance of TBVGS, it is implemented on a meta-heuristic algorithm in which the performance of the obtained meta-heuristic algorithm is examined with different standard test systems, as well as practical cases. Simulation studies and result analysis show a promising improvement in the computational efficiency of the algorithm and, even more importantly, in finding a higher quality set of TEP local optima.

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1. Introduction

Transmission Expansion Planning (TEP) is one of the most important parts of power system development that offers an optimal decision for transmission line construction with less investment cost in a predefined planning horizon. An optimal expansion of transmission systems should be capable of facilitating different load patterns, as well as future generation [1,2]. The desired solution of the TEP problem is an adequate network with minimum construction cost, subject to operating constraints [3]. Due to the high investment cost of electric power networks, finding the optimal solution of such a complicated problem is of great interest.

Simplified mathematical models are very common in solving TEP problems, such as the DC model [4]. The DC model of TEP represents the electrical aspect of the problem, neglecting voltage variations and reactive power flow. The solutions derived, based on such simplified models, may require modification and more accurate evaluation to handle real world objectives and constraints [5-7].

A solution that is just a local optimum of the DC model may lead to the global optimum of a more accurate model of the TEP problem. Therefore, instead of searching for just the global optimum of the DC model, studies should focus on finding a high-quality set of the best local optimum solutions.

Mathematically, TEP is a mixed integer, nonlinear and non-convex optimization problem [8]. Various algorithms have been addressed to solve this problem, which have been actually focused on finding
the global optimum solution. Traditionally, these algorithms are classified into three groups: (a) classical optimization algorithms, (b) heuristic algorithms (including constructive heuristics), and (c) meta-heuristics [9-12]. A Constructive Heuristic Algorithm (CHA) can be employed to improve the performance of a meta-heuristic [13-17]. In this type of application, CHA is employed as a sub-procedure to improve the quality of trial solutions. For instance, CHA may take an offspring solution generated by genetic algorithm operators as an initial solution. By adding and removing some transmission lines, CHA will converge to a local optimum. In fact, CHA may converge to a repeated local optimum that is already obtained in the meta-heuristic procedure. This phenomenon wastes computational effort and may cause the meta-heuristic procedure to trap in some local optimums. This may prevent discovery of some good quality local optimum solutions.

In this paper, the territory concept is introduced to avoid such repeated convergences. A modified CHA, the so-called Territory-Based VGS (TBVGS), is also proposed in this study. TBVGS can be derived by applying the territory concept to the Villasana-Garver-Salon (VGS) algorithm. TBVGS tries to find a new local optimal solution as it avoids repeated convergence. In order to examine its performance, TBVGS is employed in a meta-heuristic algorithm reported in [18]. Comparison of the computational efforts of the proposed algorithm with that of the original work in [18] is reported to illustrate the impact of the proposed modification.

The rest of this paper is organized as follows: Different types of CHA and, particularly, VGS algorithm, are discussed in Section 2. The territory concept and the proposed TBVGS algorithm are introduced in Section 3. A brief explanation of TBVGS usage in a sample meta-heuristic procedure is provided in Section 4. Simulation studies and result analyses are provided in Section 5. Finally, concluding remarks are presented in Section 6.

2. CHA and VGS algorithms in brief

In this section, a brief description of Constructive Heuristic Algorithms (CHA) is presented, in which the Villasana-Garver-Salon (VGS) algorithm is described as a popular type of CHA associated with the DC model. Starting from an initial solution and performing a systematic procedure, CHA will converge to a local optimum. A solution is an array that represents the number of new lines in each candidate path, \( i = j \). A “local optimum” is defined here as an adequate solution (does not need a new line addition) with no unnecessary added circuits. An unnecessary circuit is an added circuit that can be simply removed without making the solution non-adequate. On the other hand, a CHA is based upon a stepwise procedure. In early stages, this procedure obtains an adequate solution by addition of new circuits to the initial solution. Then, the construction costs of the new solution will be reduced by removing unnecessary added circuits. Choosing a circuit to be added is based upon a “Sensitivity Index” (SI). The main difference between diverse CHAs is about the definition of SI. In the VGS algorithm, this sensitivity index is calculated via Eq. (1), where \( n_{ij} \) is the output of an Hybrid Linear Model (HLM) [19] and \( f_{ij} \) denotes the maximum active power flow limit of each circuit in the right-of-way, \( i - j \). A circuit with maximum value of SI is considered the most attractive:

\[
SI_{ij} = n_{ij}f_{ij}
\]  

The hierarchical of the VGS algorithm is depicted in Figure 1, while more detailed explanation is provided in the following steps:

Step 1. Add the initial solution to the base topology, where it is assumed to be a current topology, and solve HLM based on the current topology.

Step 2. Solve Linear Programming (LP) for HLM based on the current topology [19]. If the obtained solution indicates that the system requires no additional circuit, then go to Step 4. Otherwise, continue to Step 3.

Step 3. Calculate SI using Eq. (1) to identify the most attractive circuit, then, update the current topology by addition of the chosen circuit and return to Step 2.

Step 4. Sort the added circuits in cost decreasing order. Remove the circuit having maximum cost and check the adequate operation conditions via the DC operation model described in [4]. If such removal keeps the system in adequate operating condition, remove that circuit; otherwise, keep the circuit. Repeat the
process for simulating circuit removal until all added circuits have been tested.

All added circuits that were not removed represent the final solution of CHA. It is noteworthy that although the VGS algorithm uses a hybrid linear model to identify the most attractive circuit, the final solution is also feasible in a DC model as well [20].

3. Territory concept and TBVGS algorithm
CHA can be employed in the improvement stage of meta-heuristic algorithms. For example, a CHA can improve an offspring created by genetic algorithm operators to a high-quality local optimum trial solution [18]. In this type of use, CHA may converge to a repeated local optimum that is already found in a meta-heuristic procedure. Repeated convergence wastes computational effort without achieving any improvement in the overall solution procedure. These repeated convergences also may cause the meta-heuristic algorithm to be trapped in a local optimum solution. By introducing the “territory of local optimum solutions” as a new concept, this section presents the same method to prevent repeated convergence of CHA. Applying this modification to the VGS algorithm, a new algorithm, the so-called Territory-Based VGS (TBVGS), is proposed.

3.1. Concept of territory for local optimum solutions
In this work, two territories are considered for each local optimum: A “front territory” and a “behind territory.” It is worth remembering that a local optimum is an adequate solution with no unnecessary added circuit.

Definition 1. The behind territory of a local optimum: Solution “B” is in the behind territory of solution “A”, if it is possible to reach solution “A” by adding some circuits to solution “B”. It is possible if, and only if:

\[ B \prec A \Leftrightarrow n_{ij}^B \leq n_{ij}^A, \forall (i, j) \in \Omega, \]  

where \( n_{ij}^A \) and \( n_{ij}^B \) denote the number of added circuits in solutions “A” and “B”, respectively. The operator, “\( \prec \)”, denotes being in behind territory.

Definition 2. The front territory of a local optimum: Solution “B” is in the front territory of solution “A”, if it is possible to reach solution “A” by removing some circuits from solution “B”. It is possible if, and only if:

\[ B \succ A \Leftrightarrow n_{ij}^B \geq n_{ij}^A, \forall (i, j) \in \Omega, \]  

where “\( \succ \)” denotes being in front territory.

Definition 3. Territories of a solution set: Consider \( \Gamma \) as a set of already obtained local optimums for the TEP problem. Solution “B” is out of the behind territory of set \( \Gamma \), if “B” is out of the behind territories of all members of the set. It means that:

\[ B \not\in \Gamma \Leftrightarrow \exists A \in \Gamma : B \prec A. \]  

Solution “C” is considered to be in the behind territory of set \( \Gamma \), if “C” is in the behind territory of at least one of the members of the \( \Gamma \) set. It means:

\[ C \prec \Gamma \Leftrightarrow \exists A \in \Gamma : C \prec A. \]  

The front territory of a solution set can be defined in a similar way. If solution “B” is out of the behind territory of solution “A”, it is not possible to reach solution “A” by adding new lines to solution “B”. In addition, if solution “B” is out of the front territory of solution “A”, it is not possible to reach solution “A” by removing some lines from solution “B”. The following section describes the usage of this concept in a CHA algorithm.

3.2. Territory based VGS algorithm
Suppose we have a \( \Gamma \) set of already found local optimums for the TEP problem. To guarantee the convergence of CHA to a non-repeated local optimum, one can start the line addition and line removal stages of CHA from solutions out of behind and front territories of \( \Gamma \), respectively. In fact, this is the main idea of this work, where, in this section, it is implemented in the VGS algorithm. In the proposed Territory Based VGS (TBVGS) algorithm, two stages are implemented to exit the initial solution from territories of \( \Gamma \). Therefore, it tries to find a new local optimum that has not been found before. The hierarchical of the proposed algorithm is depicted in Figure 2, while more detailed explanation is provided in the following steps:

Step 1. If the initial solution is a member of \( \Gamma \), stop the process, otherwise continue to Step 2.

Step 2. Check either the current solution is in the behind territory of \( \Gamma \). If it was, continue to Step 3 and try to exit the current solution from the behind territories, otherwise go to Step 4.

Step 3. Solve the LP of the Hybrid Linear Model (HLM), then calculate the sensitivity index as follows:

\[ SI_{ij,1} = m_{ij}^* T_{ij}, \]  

\[ SI_{ij,k} = SI_{ij,1}/k. \]  

Considering the action of simultaneous \( k \) circuit addition in the right-of-way, \( i - j \), \( SI_{ij,k} \) is the sensitivity index of this action. Apply
the most attractive action that can exit the current solution from all behind territories (i.e. the result must be out of $\Gamma$ behind territory). If no possible action is found to exit this solution from behind territories, then TBVGS cannot reach a non-repeated solution and, therefore, terminate the process. Otherwise, continue to Step 4.

Step 4. In a similar manner to the VGS algorithm, add circuits to reach a feasible solution.

Step 5. Check either the current solution is in the front territory of $\Gamma$. If it was, continue to Step 6 and try to exit the current solution from all front territories, otherwise, go to Step 7.

Step 6. In this step, try to exit the current solution from all front territories without loss of adequacy. Sort the right-of-ways in cost decreasing order. Start from the most expensive right-of-way and check if it is possible to do this job by elimination of one or more circuits in the current right-of-way. Go to Step 7, right after exiting the territory. Terminate the process if it was impossible to exit the territories without loss of adequacy.

Step 7. In a similar manner to VGS, try to reduce the solution costs by elimination of unnecessary added circuits. The remaining circuits form the final solution, and add the final solution to the set.

4. Employing a meta-heuristic algorithm

As mentioned in the introduction, the popular application of CHA is like a sub-procedure in a meta-heuristic algorithm to improve the quality of trial solutions. Therefore, the proposed TBVGS is applied to a Scatter Search Algorithm (SSA) presented in [18]. In fact, this SSA is modified as the ordinary VGS is replaced by the proposed TBVGS. The obtained algorithm is called “Territory-based SSA” (TSSA). A flowchart of TSSA is provided in Figure 3.

In the next section, the performance of the proposed meta-heuristic algorithm (called TSSA) is studied to measure the improvements caused by this modification.

5. Case studies and results analysis

In this section, the performance of the proposed TSSA is compared with the original version of [18] to study the impact of the proposed territory concept. The ability to reach the best known solutions of the problem, the computational effort needed to find the global optimum and the ability to find high-quality local optimum solutions are considered three measures of performance.

Both algorithms are applied to Garver, 24-bus IEEE and the 46-bus Brazilian systems, and the obtained results are compared. The simulation studies are based on DC/TEP without generation rescheduling. To show the effectiveness of the proposed algorithm, the following studies are carried out.

5.1. Garver system

This system includes six transmission lines and six buses with 700 MW demand for the base topology that is shown in Figure 4. The solid lines represent existing circuits in the base case topology and the dotted lines represent the possible candidate right-of-ways. Black
circles and small arrows represent generators and loads
in buses, respectively.

The number of candidate lines is 15 circuits. The
system data can be found in [19]. In the numerical case
studies, the optimal solution of 200 M$ is obtained by
both algorithms; that is the best known solution for the
Garver system [21]. The added circuits in the obtained
solution are as follows: $n_{2-6} = 4$, $n_{3-5} = 1$, $n_{4-6} = 2$.

In Table 1, the number of LPs solved to obtain
the optimal solution is shown. It can be noticed that
TSSA shows a better performance than [18] because it
prevents repeated convergences. The computational
effort can be measured by the number of solved LPs.

In addition, as shown in Figure 5, the proposed
improvement leads to finding a higher number of high-
quality local optimums. In this case, local optimum
solutions cheaper than 230 M$ are considered high-
quality solutions.

5.2. IEEE 24-bus system

An IEEE 24-bus system consists of 24 buses, 41 right-
of-ways for the addition of new circuits, with 8550 MW
demand for the base topology, which is shown in
Figure 6. The data is available in [22]. Also [22]
provides the information of four generation patterns
to solve TEP without generation rescheduling.

Similar to [18], the proposed TSSA was successful
in reaching the optimal solution in all four generation
patterns, which indicates the capability of this algo-
rithm. Tables 2 and 3 compare the computational effort
of both algorithms. As noticed again, the TSSA shows a
better performance.

Figure 7 illustrates a promising improvement in
the number of high quality local optimums found.
This figure is illustrated, based on the IEEE 24-bus
test system with the generation pattern #1. In this
case, local optimum solutions cheaper than 430 M$

### Table 1. Computational effort in both algorithms for
Garver system.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of LPs</th>
<th>CPU time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18]</td>
<td>61-130</td>
<td>7.46-14.59</td>
</tr>
<tr>
<td>The proposed TSSA</td>
<td>55-61</td>
<td>5.61-6.54</td>
</tr>
</tbody>
</table>

### Table 2. Number of solved LPs in both algorithms for
IEEE 24-bus system.

<table>
<thead>
<tr>
<th>Number of generation pattern</th>
<th>Number of solved LPs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSSA</td>
<td>[18]</td>
</tr>
<tr>
<td>1</td>
<td>681-1785</td>
</tr>
<tr>
<td>2</td>
<td>854-1319</td>
</tr>
<tr>
<td>3</td>
<td>283-622</td>
</tr>
<tr>
<td>4</td>
<td>332-1135</td>
</tr>
</tbody>
</table>
Table 3. Comparison of CPU times in both algorithms for an IEEE 24-bus system.

<table>
<thead>
<tr>
<th>Number of generation pattern</th>
<th>Number of solved LPs</th>
<th>TSSA</th>
<th>[18]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>78.26-165.18</td>
<td>96.93-204.46</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>82.17-119.71</td>
<td>150.67-314.42</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>28.18-58.37</td>
<td>52.09-317.27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>31.99-110.21</td>
<td>50.88-115.73</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7. Number of high-quality local optimums in IEEE 24-bus with generation pattern #1.

are considered high-quality solutions. Note that the optimal construction cost for this system is 390 M$.

5.3. Southern Brazilian system of 46 buses
The Southern Brazilian System has 46 buses, 79 right-of-ways for the addition of new circuits and 6880 MW of demand. The system data is available in [4]. The base topology of this system is shown in Figure 8. There is no limit for circuit additions in each right-of-way.

In this case, both algorithms reach the optimal solution of 154420M$. The added circuits are as follows:

\[ n_{20-21} = 1, \quad n_{42-43} = 2, \quad n_{46-46} = 1, \]

\[ n_{19-25} = 1, \quad n_{31-32} = 1, \quad n_{28-30} = 1, \quad n_{26-20} = 3, \]

\[ n_{24-25} = 2, \quad n_{20-30} = 2, \quad n_{5-4} = 2. \]

In Table 4, the number of LPs solved to obtain the optimal solution is shown. TSSA shows a better performance than in [18].

It can be observed through Figure 9 that the proposed improvement enables the algorithm to find more high-quality local optimums at the end. It indicates that the proposed improvement prevents the algorithm being trapped in local optimums and, therefore, improves its searching ability. In this case, local optimum solutions cheaper than 165000M$ are considered high-quality solutions.

Table 4. Computational effort in both algorithms for a 46-bus system.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of LPs</th>
<th>CPU time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18]</td>
<td>5079-17951</td>
<td>483.91-1623.73</td>
</tr>
<tr>
<td>The proposed TSSA</td>
<td>1332-3072</td>
<td>146.38-347.92</td>
</tr>
</tbody>
</table>

Figure 8. The base topology of southern Brazilian system of 46 buses.

Figure 9. Number of high-quality local optimums in case of Brazilian 46-bus system.

6. Concluding remarks
This paper presents a new method to optimize metaheuristic TEP solution algorithms that are integrated with CHAs. The TBVGS algorithm is proposed by introducing the territory concept and forcing the VGS algorithm to converge to a non-explored local optimum.

The proposed TBVGS algorithm has been inte-
grated in a meta-heuristic procedure and applied to test systems, including Garver, IEEE 24-bus and the Southern Brazilian system of 46 buses. The obtained results for these test systems show that considering the territories of found local optimums can lead to significant computational performance and to finding a higher number of high quality local optimum solutions. Finding a higher number of local optimums indicates that the proposed territory concept improves the searching ability of the algorithm. It prevents the algorithm being trapped in local optimums and makes it more reliable in finding the global optimum solution. Moreover, deriving a set of high-quality local optimums is an advantage, based on the need to apply modifications and more accurate evaluations when handling real world objectives and constraints.

The proposed algorithm is designed for a single-stage DC TEP problem that is a simplified model of TEP. This model does not completely consider operation, reliability, security, and quality costs. Also, it does not exploit alternative options, like redesigning, rearranging, and upgrading, etc. Therefore, modification of the proposed concept for more complete models of TEP can be suggested for future work. New theories may be developed to identify problem categories that the territory concept is suitable for. In addition, the authors speculate if it is possible to use the territory concept as an alternative to “tabulist” in tabu search algorithms.

References


**Biographies**

Mohammad R. Habibi was born in Baft, Iran, in 1988. He received his MS degree in Electrical Engineering, in 2011, from Kerman Graduate University of Technology (KGUT), Kerman, Iran, where he is currently working towards his PhD degree. His research interests include power system planning, power system optimization and power system operation.

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