An adaptive impedance control algorithm; application in exoskeleton robot

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Abstract. Exoskeleton is a well-known example of an unconstrained robot for which the desired path is not predefined. Regarding these two effective features, a formulation for impedance control algorithm is presented and its prominence is demonstrated both mathematically and through simulation. Moreover it is essential to control this robot by an adaptive method because at least dynamic characteristics of the load are unknown. Unfortunately the existing methods do not address aforementioned traits or become unstable as inertia matrix becomes singular. Here an adaptive algorithm is generated based on the logic of Least Squares identification method rather than the Lyapunov stability criterion to tackle those limitations.

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1. Introduction

Exoskeleton is worn like a cover and enhances physical might via powerful actuators. To accomplish this, it constitutes of anthropomorphic parts which follow movements of body simultaneously. A review on the major projects in this field shows that researchers have chosen various control methods [1]. BLEEX project by Kazerooni is among the premier robots of this kind (Figure 1). Its control method is based on measuring forces exerted on the robot’s links and producing a control law to reduce those forces [2]. Disability to distinguish between voluntary and involuntary movements is a major disadvantage of that method. Sankai developed HAL robot and utilized EMG sensors to detect intention of the user [3]. These sensors need to be attached on the skin and cause discomfort to the user. XOS is another exoskeleton manufactured by Sarcos Company which uses Impedance Control (IC) system. This method does not cause any of the mentioned insufficiencies.

IC introduced by Hogan in 1985, was specially created to control the interaction dynamics of robot with human or environment. But exoskeleton application has three important features which affect impedance control formulation:

• This robot does not have a specific end-effector and interacts with body at several points;
• Exoskeleton cannot be treated as a constrained robot;
• The desired path or equilibrium points are not predetermined.

In a few articles IC is applied to exoskeleton, some of which use an admittance model to calculate desired movement and impose position control to reach that goal [4,5]. In this way tracking quality will be dependent on the working point that is not desirable. Caldwell et al. [6] employed IC for a rehabilitation robot and assumed negligible veloc-
2. Impedance control method

2.1. The algorithm

From 1985 that Hogan explained impedance control in [10], supplementary issues about it were also discussed till around 2000. Adaptation is among these issues. According to the viewpoint of the mentioned article that aims industrial robots, the desired impedance relation is considered as:

\[ M \ddot{e} + C_d \dot{e} + K \ddot{e} = -f, \quad e = x - x_v, \quad (1) \]

where \( f \) represents interface force and \( x \), \( x_v \) are true and virtual positions of the end-effector in its task space. The coefficients of desired impedance are shown with index \( d \) and correspond to mass, viscous damping and stiffness. In situations that end-effector is constrained to move in contact with a specific solid surface, the desired path is known beforehand. Accordingly \( x_v \) is determined by designer such that desired force is produced at the interface. In some other applications \( x_v \) may represent an equilibrium trajectory around which the dynamics of the end-effector would obey Eq. (1). However our application is none of the above. \( x_v \) is replaced by desired position in [11], which is a well-known reference for robot dynamics and control topics. Therefore if we have the dynamic equation of the robot as:

\[ M(q \ddot{y} + C(q, \dot{q}) \dot{q} + G(q) = \tau + J^T(q) f. \quad (2) \]

where \( J \) is the Jacobian matrix and transfers local coordinates \( q \) to the task space, the control law and closed-loop dynamics are respectively given as:

\[ \tau = -J^T f + G + C \dot{q} \]

\[ + M(q \ddot{y}_d - M_d^{-1} (C_d \dot{q} + K \ddot{e} + J^T f)), \quad (3) \]

\[ M \ddot{e} + C_d \dot{e} + K \ddot{e} = -f, \quad e = q - q_d. \quad (4) \]

Here \( q_d \) represents desired trajectory. So this method needs both \( f \) and \( \ddot{q}_d \) to produce the control law. But in an application that the desired trajectory is not predetermined, there is no means to obtain desired path except calculation through desired impedance relation. By a little change in representation of IC concept, it is possible to attain the desired impedance dynamics without the mentioned calculation. More important point is that the final consequence of the above formulation, as in Eq. (4), does not guaranty convergence of error and its first and second time derivatives to zero, because the interface force on the right hand side of this equation acts as a disturbance itself. Only when the interface force becomes zero the error between the desired and actual states converges to zero.
In an application like exoskeleton that the desired path is not predetermined, \( q_d \) should be calculated from Eq. (4) and this needs \( q \) and its derivatives to be measured via sensors. So for example the \( \ddot{q} \) measured to calculate \( q_d \) and consequently to produce control law, is different from the \( \ddot{q} \) obtained from dynamic Eq. (2) which itself is dependent to the control law. To remove this ambiguity we name the measured position signal by \( q_0 \), and the position signal derived from dynamic equation using control law is depicted by \( q \). Therefore the desired impedance relation is rewritten as:

\[
M \ddot{\delta} + C \dot{\delta} + K \delta = f,
\]

(5)

Assuming equality of \( q_0 \) and \( q \), is mathematically possible but physically impossible as in [11]. However, ignoring this difference does not cause that formulation not to work, but will cause some errors which may usually be neglected. Making small changes in IC algorithm, we can reduce these errors to some extent and upgrade its performance.

Before calculating the closed loop dynamics, it should be noted that each link of this robot has interface with body and its state should be controlled. Therefore the task space comprised of links’ angles and robot dynamics is written as:

\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = \tau + B(q) f.
\]

(6)

In the above relation the matrix \( B \) is a function of link angles. Using the measured signals -and not \( q_d \)- the control law is given by:

\[
\tau = M(q_0 - M^{-1} q) \dot{q} + K\delta \delta + G(q) - B(q) f + G(q) \dot{q}.
\]

(7)

Inserting control law into the robot dynamics yields:

\[
M\delta \ddot{e} + C \delta \dot{e} + K \delta \delta = f.
\]

(8)

Combining Eqs. (5) and (8) gives the error dynamics as:

\[
M \ddot{e} + C \dot{e} + K \delta \delta = 0,
\]

(9)

In this formulation local coordinates \( \delta \) that are the angles of robot links are themselves considered as task space because all of the links have interfaces with body and their states should be controlled. Here \( f \) is of moment type. The final consequence of the proposed method in Eq. (9) represents error dynamics formed with desired impedance characteristics which guarantees stability and exponential convergence of error to zero. Also, state variables needed to construct control law are easily accessible.

**Remark.** In fact considering the difference of \( q \) and \( q_0 \) enables us to distinguish between two issues: One concept is user’s intended deviation from the present position \( (\sigma) \) that is desired, while the other concept is deviation of position that robot reaches to by obeying control law from the desired position, which is error and should converge to zero as fast as possible. Clarification of this separation in the theory makes it possible to design control law such that the right side of error dynamics becomes zero while interface force exists and is considered in the desired impedance relation. This makes error and its first and second derivatives to approach zero quickly. The effect of this is revealed in decreasing the interface forces exerted on the user.

Simulating the common IC formulation and the one presented in this article on a simple plant shows the advantage of the proposed method.

### 2.2. Simulation

A mass-spring-damper system equipped with a linear double acting actuator is considered (Figure 2). Two mentioned IC methods are simulated on this plant and the results are shown.

First, for applying common method it is assumed that user exerts interface force in the form \( f = 0.1 \sin(t) \). With common IC method the desired path is calculated and tracked. Second, the same desired path obtained from previous calculation is set to be tracked using the proposed method. This time, the interface force is not known. Consequently results of using the common and modified methods to track a specific path show that the modified method causes the person sustains less interface force.

Simulation parameters are:

\[
m = 2, \quad c = 15, \quad k = 100,
\]

\[
m_d = 1, \quad c_d = 10, \quad k_d = 100.
\]

(10)

To explain the above results (Figures 3 and 4) we can say, when \( f \) is positive, the user wants to increase the acceleration and velocity of the mass and move it forward. When \( f \) decreases or becomes negative, acceleration and velocity decrease and displacement gets close to change the direction. With common method larger magnitudes of interface force are needed and to slow down the movement.

![Figure 2. Actuated mass-spring-damper plant.](image-url)
keener changes are needed such that even the direction of force should be reversed. This verifies that the proposed method exhibits faster convergence; and as a result of which smaller magnitudes of interface force (user effort) with smoother slopes are needed.

3. Adaptive impedance control algorithm

According to the previous section, IC is a model-based method and the accuracy of modeling directly affects its performance. Although IC has an excellent robustness to any kind of uncertainties, in exoskeleton application due to close interaction between man and robot, it is essential to reduce errors caused by model uncertainties as much as possible. It is feasible to identify dynamic characteristics of all parts used in robot’s structure [12] but the load carried by robot is variable. Mass, moment of inertia and position of Center Of Mass (COM) are parameters of load needed in the dynamic model.

In Section 1, the existing methods for adaptive IC were explained and the algorithm presented in [9] was declared to be the most efficient among them. The authors have implemented this method to control an exoskeleton in [13]. That method still has a limitation, which needs a non-singular inertia matrix.

In [14] the logic of a neural network is employed for adaptive impedance control of exoskeleton in which some assumptions, like approximating the dynamics of interaction between man and robot merely as a spring model, need to be revised. This approach is common for constrained robots but exoskeleton is unconstrained [15]. Besides, that method makes signals bounded and not asymptotically convergent. Neural network is also used in [16] to estimate desired path and then track it via IC. Elimination of dynamic model can be considered as the main purpose and advantage of such procedure, but for a robot with a very wide range of probable movements training a neural network and obtaining correct estimations of desired movement at every moment is not reliable; except for rehabilitative applications that usually involve repeated movements. For example in [17], ANFIS method is used to estimate joints’ torques with no need to calculate dynamic model of the intended rehabilitative exoskeleton.

Now based on IC algorithm developed in the previous section, a formulation is constructed to obtain adaptation law. Besides verifying the stability of the proposed algorithm mathematically, a simulation on dynamic model of a lower limb exoskeleton including all of its DOFs in sagittal plane is done which shows advantage of the proposed method comparing with the method of [9].

First we rewrite dynamic Eq. (6) as:

\[ M(q)\ddot{q} + G(q, \dot{q}) = \tau + B(q) f. \]  \hspace{1cm} (11)

Then considering uncertainties in dynamic model, the control law based on the IC method is written as:

\[ \tau = M(\ddot{q}_0 - M^{-1}d(C_d(\ddot{q} - \dot{q}_0) + K_d(q - \dot{q}_0) - f)) - Bf + \hat{G} + F_c, \hspace{0.5cm} F_c = K_c e, \]  \hspace{1cm} (12)

where symbol \( \hat{\cdot} \) shows the estimated value of the main parameter, \( e \) is design parameter \( F_c = K_c e \) which is used to improve the behavior of the controlled system. Matrix \( B \) is just dependent on robot kinematics and is known due to certainty of structure. Using this control law, the closed loop dynamics calculations result in:

\[ M\ddot{\hat{q}} + \hat{G} + \hat{F}_c + \hat{M}(\ddot{\hat{q}}_0 - M^{-1}_d(C_d(\ddot{q} - \dot{q}_0) + K_d(q - \dot{q}_0) - f)) = \]  \hspace{1cm} (13)

\[ \Rightarrow M_d\ddot{\hat{q}}_0 = M_d\ddot{\hat{q}} - C_d(\ddot{q} - \dot{q}_0) - K_d(q - \dot{q}_0) + f, \hspace{1cm} (14) \]

\[ \Rightarrow M_d\ddot{\hat{q}}_0 = (\hat{G} - G) + \hat{F}_c + C_d(\ddot{q} - \dot{q}_0) + K_d(q - \dot{q}_0) + f. \hspace{1cm} (15) \]
A parameter with superscript ~/~ equals to subtraction of its true value from the estimated value. Representing the unknown parameters of the system by p as a vector, factorizing them in dynamic equation as W.p = M.q + G and substituting for f from Eq. (5) the last equation can be rewritten as:

\[
W(\dot{p} - p) + F_c = \tilde{M}(\dot{c} + C_n\dot{c} + K_n\epsilon),
\]
\[
W(\dot{p} - p) = \tilde{M}\dot{q} + \tilde{G}, \quad \epsilon = q - q_d. \tag{16}
\]

Actually we have assumed the system parameters appear in equations linearly. This is not wrong because even when a nonlinear combination of some parameters appears in the equations, one can consider the whole combination as an uncertain parameter. Now the following parameters and notations are defined as:

\[
C_n = M_d^{-1}C_d, \quad K_n = M_d^{-1}K_d \Rightarrow
\]
\[
\text{if} \quad M_d = m_dI \rightarrow C_n = C_d/m_d. \tag{17}
\]

So, Eq. (16) can be rewritten as:

\[
\tilde{M}(\dot{c} + C_n\dot{c} + K_n\epsilon) - K_n\epsilon = W(\dot{p} - p). \tag{18}
\]

Until now we have obtained a form of equations that enables us to apply the logic of Least Squares (LS) method. In this method an objective function of error is considered so that minimizing it produces adaptation law and causes error to converge to zero. First the objective function is defined as:

\[
J = \int_0^T \epsilon^T \dot{\epsilon} = \int_0^T \epsilon^T (\dot{M} \dot{c} + C_n\ddot{c} + K_n\epsilon) - K_n\epsilon = W(\dot{p} - p). \tag{19}
\]

Uncertain parameters should be corrected in a way to minimize the function J, so:

\[
\frac{\partial J}{\partial \hat{p}} = \frac{\partial}{\partial \hat{p}} \left( \int_0^T (\dot{p}^T W^T \dot{W}) d\tau \right) = \int_0^T (2W^T \Delta p) d\tau = 0, \tag{20}
\]

\[
\Rightarrow 2 \int_0^T W^T (W \dot{p} - W p) d\tau = 0
\]

\[
\Rightarrow \dot{p} = \left[ \int_0^T (W^T W) d\tau \right]^{-1} \int_0^T (W^T W) d\tau.
\]

Defining the first term in Eq. (21) as H and differentiating it with respect to time yields:

\[
H = \left[ \int_0^T (W^T W) d\tau \right]^{-1} \Rightarrow \frac{dH^{-1}}{dt} = W^T W, \tag{22}
\]

\[
\Rightarrow \frac{dH}{dt} = -H \frac{dH^{-1}}{dt} H = -HW^T WH. \tag{23}
\]

So, we have:

\[
\dot{p} = H \int_0^T (W^T W) d\tau \Rightarrow \frac{d\dot{p}}{dt} = \frac{dH}{dt} \int_0^T (W^T W) d\tau + HW^T W p. \tag{24}
\]

And also we have:

\[
\frac{dH}{dt} \int_0^T (W^T W) d\tau = -HW^T W H \frac{dH}{dt} \left( \int_0^T (W^T W) d\tau \right), \tag{25}
\]

\[
= \frac{d\dot{p}}{dt} = -HW^T W \dot{p} + HW^T W p = -HW^T \epsilon. \tag{26}
\]

Based on the presented calculations, adaptation laws are obtained as:

\[
\begin{aligned}
\frac{d\dot{p}}{dt} &= -HW^T \epsilon \\
\frac{dt}{dt} &= -HW^T WH
\end{aligned} \tag{27}
\]

One can conclude that implicating this method makes function J converge to zero and due to stability of the impedance relation in Eq. (18) which is accomplished by choosing appropriate coefficients, the error converges to zero, too. By the way, we can investigate stability of this system through the Lyapunov stability theorem as well. Defining a Lyapunov function as below and taking time derivative of it we have:

\[
V = \delta^T H^{-1} \delta, \quad \delta = \dot{p} - p, \tag{28}
\]

\[
\dot{V} = 2\delta^T H^{-1} \dot{\delta} + \delta^T H^{-1} \dot{\delta}, \quad \dot{H}^{-1} = W^T W. \tag{29}
\]

Values of \( \dot{\delta} \) and \( \dot{H}^{-1} \) in Eq. (29) are substituted from Eqs. (27) and (22) respectively to obtain:

\[
\dot{V} = -2\delta^T H^{-1} \delta W^T \epsilon \delta^T + \delta^T W^T W \epsilon \delta^T \epsilon + \delta^T \epsilon \epsilon^T \epsilon \delta^T \epsilon \delta = -\epsilon^T \epsilon \delta \leq 0. \tag{30}
\]

So the stability of the system is proved. Till now it is verified that all components (signals) taking part in the system remain bounded. Also we have:

\[
- \int_0^\infty \dot{V} dt = -V(0) + V(0) < \infty
\]

\[
\Rightarrow \int_0^\infty (\epsilon^T \epsilon) dt < \infty \Rightarrow \epsilon \in L_2. \tag{32}
\]

Eq. (32) means that every component of \( \epsilon \) is bounded of type \( L_2 \), so according to Barbualt’s lemma [18], due to differentiability of \( \epsilon \), we have:

\[
\lim_{t \to \infty} \epsilon = 0. \tag{33}
\]
Again one can say choosing appropriate coefficients in Eq. (18), guarantees exponential convergence of $\varepsilon$ to zero. This means that on the right side of Eq. (18) the product of $W$ and $\ddot{p}$ converges to zero which does not necessarily yield that true values of uncertain parameters are attained. We know this matter depends on the richness of input.

Before going through simulation an important point should be mentioned. Because of heavy numerical computation needed in a case like exoskeleton dynamics, employing a suitable filter is essential and can reduce the derivative noisy fluctuations from the numerical calculation. For this purpose it is offered to implicate a filter as

$$\frac{a}{\Gamma(D)} \left( \dot{M} (\dot{\theta} + C_0 \dot{\theta}) - K_0 \varepsilon \right) = aW \frac{\Gamma(D)}{\Gamma(D)} (\dot{\theta} - \theta).$$

(34)

In the above relation $a/\Gamma(D)$ is the filter where D indicates time derivative operator. We can tune the filter by determining the coefficients of the polynomial:

$$\Gamma(D) = D^2 + \lambda_1 D + \lambda_2.$$  

(35)

4. Simulation

4.1. Modeling

A lower limb exoskeleton is considered with a structure similar to that of BLEEX project. Physical characteristics of robot links are determined in accordance with those of an average person weighing 75 kg and 175 cm tall derived from article [19] along with a load of 50 kilograms. Two linear double acting hydraulic actuators are used for knee and hip joints of each foot. For ankle joints two torque actuators are contemplated. The structure is shown schematically in Figure 5. Dynamic model is derived in sagittal plane. Hydraulic cylinders are modeled as two links joined by a prismatic joint. A thorough description about this

![Figure 5. Schematic of robot structure [2].](image)

plant is presented in [20]. Figure 6 represents chosen generalized coordinates.

To validate the model, the SimMechanics software is used. Also in order to run simulation and do verification, kinematic data of gate cycle is measured via Xsens system at Gait laboratory of Sharif University of Technology. Solving forward dynamics needs active control system because inverted pendulum-like equations of the model make it highly unstable. The obtained results make clear that the model is verified and IC method presented in this article works efficiently.

4.2. Model verification and control results

Solving inverse dynamics does not need control system, and results in forces or torques of actuators. The robot is modeled in SimMechanics as well, and running it in inverse dynamics mode gives results which can be compared with the model outputs. The control actions obtained from solving forward dynamics for the same desired kinematics, are expected to be in good agreement with the outputs of inverse dynamics analysis. These are all done on the entire robot including all of its DOFs. But regarding briefness of this article, randomly two parts of the robot are selected to present the results.

It should be said that initial conditions applied in forward dynamics simulation are equal to the desired values for position and velocity and zero for acceleration. Due to the fact that robot is worn by user and its links are attached to his body the above assumption is rational. The desired impedance coefficients are selected as:

$$M_d = 0.1I_t, \quad C_d = 10I_t, \quad K_d = 100I_t,$$

(36)

where $I$ is the identity matrix. In Figure 7, forces of two actuators obtained from the mentioned methods of calculation are demonstrated together that verifies the mathematical model. Similar results are obtained for level walking in [21].

![Figure 6. Designation of DOFs.](image)

Next, efficiency of the presented control system is investigated by checking tracking quality in Figure 8 in
addition to magnitudes of torques sustained by user during taking a step carrying a 50 kilograms load (Figure 9).

4.3. Adaptive impedance control results
Finally performance of the proposed adaptive IC algorithm is checked. For this purpose desired kinematic data to be tracked by the robot is designed such that insensitiveness of the proposed method to singularity of inertia matrix as well as good tracking quality is testified. Also the method of article [9] that has the least defects compared to the other existing methods is implicated for the same situation so as to illustrate the mentioned advantage of our method. Initial conditions are set to zero like those of desired kinematics. Other parameters used in simulation are set as below:

control coefficients:

\[ M_d = 0.1I_f, \quad C_d = 10I_f, \quad K_d = 100I_f, \]

\[ H(0) = 200I_4, \quad k_c = 50, \]

uncertain parameters:

\[ P = [m_l, j_l, b_2m_l, h_g^2m_l]^T = [50, 1, 16.5, 12]^T, \]

\[ \hat{P}(0) = [60, 0.6, 10, 5.1]^T, \]

filter:

\[ \lambda_1 = 7, \quad \lambda_2 = 12, \quad a = 5. \]  (37)

The uncertain parameters correspond to the load are mass, mass moment of inertia, product of mass and distance of load’s COG from hip joint and product of mass and square of the mentioned distance. As it was stated before when there are nonlinear combinations of parameters in dynamic model, one can take the whole combination as an uncertain parameter.

Like before two parts of robot are selected randomly to show the results. In Figure 10 tracking quality and in Figure 11 proceeding of adaptation of uncertain parameters are shown. Two samples of actuator forces and interface forces are inserted in Figures 12 and 13 respectively. Graphs of Figure 14 show how reaching to a position in which inertia matrix becomes singular, causes a controller using adaptive method of [9] brings the system to instability despite its fine performance when that matrix is nonsingular.
5. Discussion

The first simulation results relate to verification of mathematical model. Curves of Figure 7 are representatives of good agreement between model and SimMechanics results. As it is expected, control signals applied to the controlled system and outputs of solving the inverse dynamics are very close. However, there exist some differences between model and software outputs which can be interpreted by various numerical calculations errors. Figure 8 indicates excellent tracking quality of the proposed IC method accompanied by little torques exerted on user’s body despite the 50 kg load which is shown in Figure 9.

After that, results of the proposed adaptive method performance are illustrated. In Figure 10 it can be seen that tracking quality is very good while Figure 11 shows uncertain parameters are converging to values which are not necessarily the true ones. It is clear that the input used here is not rich at all. The parameter $h_2 m_1$ is much more prominent in equations than other ones. At the same time it can be seen that this parameter approaches to its true value given in Eq. (16) better than others. Especially $h_2^2 m_1$ has negligible effect in the complicated and bulky dynamic equations of this robot. Forces of actuators have normal values and little torques are exerted on user’s body according to Figures 12 and 13.

Finally two curves of Figure 14 show that adaptive impedance control of article [9] exhibits very good performance but it is limited to cases in which inertia matrix does not become singular. It is seen that as the robot links reach such a position and remain there for a small time, the system gets destabilized. In contrast, our proposed method does not show any sensitivity to singularity of inertia matrix.

6. Conclusion

In this article an adaptive impedance control algorithm is presented which is applicable for any robot no matter is it linear or nonlinear, constrained or unconstrained and how many points are considered to control their impedances. The main feature is that adaptation law does not need inverse of inertia matrix so it can be used in conditions that this matrix becomes singular. The basis of this method is impedance control so before developing adaptive algorithm, we concentrated...
Figure 11. Adaptation of uncertain parameters.

Figure 12. Actuator forces of two legs.

Figure 13. Torques exchanged at interfaces.
on this basis. It was figured out that the existing representations of IC, face some delicate problems especially when they are to be used in an application like exoskeleton. Desired path is not predetermined in such an application. In the proposed method difference between desired deviation from current position and tracking error -at any moment- is taken into account theoretically. Major variation in formulation of IC is that the measured signals -instead of desired path data- are used directly to generate control law. As it was expected, results showed faster convergence of tracking error and its first and second derivatives to zero. In reality, tangible consequence is reduction of forces exerted on user’s body.

After specifying IC formulation, an adaptive algorithm was developed. This was achieved by writing the error dynamics relation of IC method in a form suitable for implementation of the LS method logic. In this way adaptation law was obtained with no need to calculate the inverse of the inertia matrix. Stability of the method was proved using the Lyapunov stability theory. In addition several simulations were carried out through which the dynamic model was verified and efficiency of the control system as well as the performance of the adaptive method was indicated especially where the inertia matrix became singular.

References


Figure 14. Tracking desired trajectory in a system using adaptive method of [9].


Biographies

Mohammad Mahdi Ataei was born in 1988. After finishing high school at the exceptional talents center in Isfahan he entered Sharif University of Technology, Tehran, Iran, to receive BS and MS degrees in Mechanical Engineering in 2010 and 2012 respectively. Adaptive control, robotics and mathematical modeling are among his research interests.

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Aria Alasty received his BSc and MSc degrees in Mechanical engineering from Sharif University of Technology, Tehran, Iran in 1987 and 1989. He also received his Ph. D. degree in Mechanical engineering from Carleton University, Ottawa, Canada, in 1996. At present, he is a professor of mechanical engineering in Sharif University of Technology. He has been a member of Center of Excellence in Design, Robotics, and Automation (CEDRA) since 2001. His fields of research are mainly in Nonlinear and Chaotic systems control, Computational Nano/Micro mechanics and control, special purpose robotics, robotic swarm control, and fuzzy system control.