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Introduction and optimization of a power split continuously variable transmission including several fixed ratio mechanisms

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Optimization.

Abstract. The goal of this paper is to introduce and optimize a Power Split Continuously Variable Transmission (PS-CVT) including several Fixed Ratio (FR) mechanisms. In this power train, FR mechanisms are embedded in all possible places. At first, the governing dynamics of the proposed power train is developed. A control algorithm on the base of minimizing the vehicle Fuel Consumption (FC) is introduced. Two vehicles in different classes are considered in which the proposed power train is used. Afterwards, two-stage optimization with the aim of minimizing the vehicles' FC is carried out. It is found that employing properly designed FR mechanisms improves the power train and decreases FC. Also, it is revealed that there is a specific power train for each of the vehicles that gives a remarkable fuel economy in each of the considered driving cycles.

1. Introduction

The finite supply of fossil fuel resources inspires researchers to explore ways of reducing consumers' Fuel Consumption (FC). Optimization of the vehicles, one of the major consumers, can play an important role in saving fuel resources. Several researches have focused on the improvement of the vehicles' different parts. One of the key elements of the vehicles that has a significant effect on their FC is the power transmission. Nowadays, some manufacturers use Continuously Variable Transmission (CVT) as the power train in order to decrease the vehicle FC. In these power trains, the Speed Ratio (SR) between the engine and wheels varies continuously. In the case of conventional power trains, the engine rpm is a function of the vehicle speed according to the power train SR. Despite the conventional power train, the engine rpm is almost independent of the vehicle speed through employing CVT. Therefore, CVT allows the engine to operate at the rpm in which the vehicle FC is minimal. However, CVT has two major shortcomings. The first, is its limited power transmission capacity, which is a result of limited friction coefficient in the V-belt CVT [1], and limited strength of contact surfaces in toroidal one [2]. The second weakness is its narrow SR range, because of the geometrical constraints. Moreover, in the case of CVT, another mechanism is necessary to create reverse gear in the transmission. Therefore, researchers attempted to combine CVT with a Planetary Gear train (PG) and a Fixed Ratio mechanism (FR) to extend the power train SR range. These mechanisms are called Power Split Continuously Variable Transmission (PS-CVT). Another advantage of PS-CVT over CVT is its greater capacity for power transmission. The reason behind it is that, in PS-CVT only a part of the input power flows through the CVT, while the rest flows through other parts [3]. It is notable that if PS-CVT be able to create zero speed ratio (output shaft fixed), it is named as Infinitely Variable Transmission (IVT).

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There has been some interesting research in the field of PS-CVT and IVT. Brockbank et al. [4] presented an IVT transmission including a full-toroidal traction drive used in tractors in the USA. Their introduced IVT had some advantages such as low weight and volume, and high efficiency. Mantriota [5] considered IVT including a V-belt CVT, and investigated the use of an automatically regulated speed ratio in different operating conditions. He calculated the power flows and efficiency of this power train. Mangialardi et al. [3] and Mantriota [6,7] introduced the parallel and series types of IVT including a V-belt CVT, planetary gear and a fixed ratio mechanism, and calculated their efficiencies and power flows, theoretically and experimentally. They showed that IVT efficiency would increase with the growth of its speed ratio. Delkhosh et al. [8] established the governing dynamics of a parallel PS-CVT equipped with full-toroidal CVT. They optimized the geometry of this power train to decrease the vehicle FC in NEDC driving cycle. Bottiglione et al. [9] studied the IVT efficiency and its control in the speed ratios close to zero (neutral gear), theoretically and experimentally. Fen Zhu et al. [10] examined the traction IVT (TIVT) and established a model for calculating its efficiency. They demonstrated that its efficiency can be almost 99% by proper designing. Pan et al. [11] studied the torque ratio of metal V-belt CVT to PS-CVT and investigated the toque ratio in series and parallel PS-CVT, and found the relation between the torque ratio and speed ratios of PS-CVT elements. Carbone et al. [12] developed a model for efficiency calculation of half-toroidal and half-toroidal CVT and calculated their efficiency as a function of their geometry and operating condition.

One of the main kinds of PS-CVT is the series one in which CVT and the sun of PG are connected to the input shaft. As mentioned in [13], toroidal CVT efficiency is a function of its speed ratio, input speed and torque. In the simple type of series, where PS-CVT is including one FR, PG and CVT, CVT input torque and speed depend on the PS-CVT speed ratio and the engine exerted power and speed. Therefore, the value of CVT input torque, speed and speed ratio cannot be changed to increase its efficiency, and therefore PS-CVT efficiency is fixed. A remedy for this is to embed some FR mechanisms between simple PS-CVT's elements. In this paper, a number of FR mechanisms are embedded in all possible places in the PS-CVT. In this case, by properly designing the FRs, PS-CVT efficiency can be increased, and therefore, the vehicle FC will decrease. In the present study, at first, governing dynamics of the PS-CVT including one PG and full toroidal CVT and four FR mechanisms are established. Afterwards, the control strategy of the power train speed ratio is introduced. Two different vehicles are considered and the introduced PS-CVT is used as their power train. Finally, the optimization is accomplished with the aim of minimizing the vehicles' FC in various driving cycles, while the optimization parameters are the speed ratios of the FR mechanisms.

2. Method

In this section, the model of the proposed PS-CVT is presented and the control strategy of this power train is illustrated. Then the optimization process of the power train is explained.

2.1. Power split continuously variable transmission

Due to the fact that CVT creates a narrow SR range, it is used in combination with PG and FR mechanisms to extend this range. There are several kinds of PS-CVT. One of the major kinds of PS-CVT is the series one. A simple diagram of series PS-CVT including FR, PG and CVT elements is shown in Figure 1.

As demonstrated in [3,6], there are only two types for PS-CVT power flow, including types 1 and 2 for which the power flow diagram of this power train is shown in Figure 2.

Embedding various FR mechanisms in different places in the PS-CVT, allows the input torque and speed of CVT to be changed. Due to the fact that CVT and therefore PS-CVT efficiency is a function of input torque, speed ratio and rotational speed, proper design of FR mechanisms leads to improved power train efficiency by operating CVT in its most efficient region. Figure 3 shows the proposed model of PS-CVT

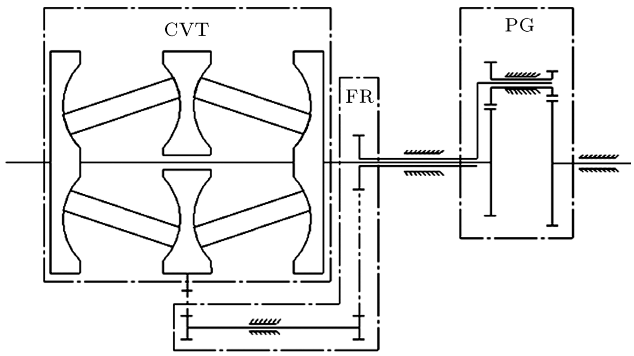


Figure 1. A simple diagram of PS-CVT including FR, PG and CVT elements.

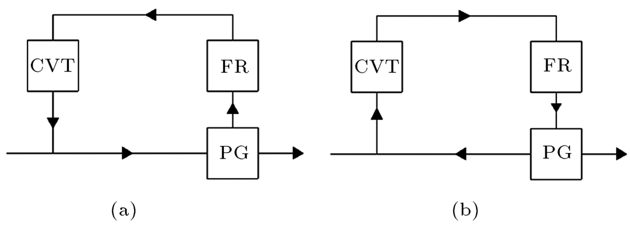


Figure 2. The power flow diagram of series PS-CVT for type 1 (a), and type 2 (b).

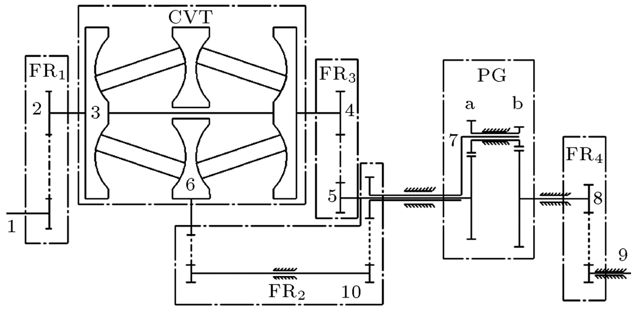


Figure 3. The proposed model of PS-CVT including one PG and full-toroidal CVT and four FR mechanisms.

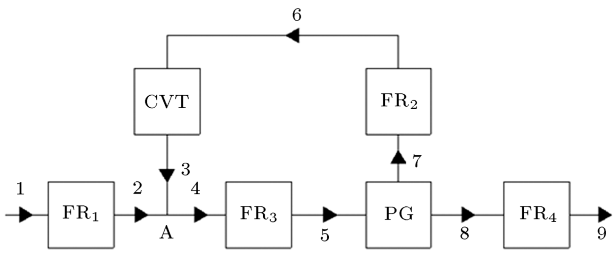


Figure 4. Power flow direction between PS-CVT elements for type 1 of power flow.

including one PG and full-toroidal CVT and four FR mechanisms.

As can be seen, the FR mechanisms are embedded in all possible places in the PS-CVT. The power flow diagram of this model for type 1 is shown in Figure 4.

2.1.1. Governing dynamics of series PS-CVT including four FR mechanisms

In order to calculate PS-CVT efficiency, it is necessary to study the value of power flows through its elements. According to Figure 4, speed ratios of PS-CVT elements are defined as:

$$\begin{aligned}
&\tau_{\mathrm{FR1}}=\frac{\omega_2}{\omega_1}, \quad \tau_{\mathrm{CVT}}=\frac{\omega_6}{\omega_3},\\
&\tau_{\mathrm{FR2}}=\frac{\omega_7}{\omega_6}, \quad \tau_{\mathrm{FR3}}=\frac{\omega_5}{\omega_4},\\
&\tau_{\mathrm{FR4}}=\frac{\omega_9}{\omega_8}, \quad \tau_{\mathrm{PG}}=\frac{\omega_8-\omega_7}{\omega_5-\omega_7}, \qquad (1)\\
&\omega_2=\omega_3=\omega_4.
\end{aligned}$$

Consequently, the speed ratio of the PS-CVT can be achieved by:

$$\tau_{\text{PS-CVT}}=\frac{\omega_9}{\omega_1}=\tau_{\mathrm{FR1}}\tau_{\mathrm{FR4}}\left[\tau_{\mathrm{PG}}\tau_{\mathrm{FR3}}+(1-\tau_{\mathrm{PG}})\tau_{\mathrm{FR2}}\tau_{\mathrm{CVT}}\right]. \qquad (2)$$

According to Eq. (2), $\tau_{\mathrm{FR1}}\tau_{\mathrm{FR4}}\tau_{\mathrm{FR3}}$ and $\tau_{\mathrm{FR1}}\tau_{\mathrm{FR4}}\tau_{\mathrm{FR2}}$ can be substituted with two other parameters in this equation, and utilization of different FRs do not have a significant effect in the kinematic behavior of PS-CVT. However, employing FR mechanisms in the input and output of the PS-CVT affects input torque, rotational speed and the speed ratio of CVT and PG. Due to the fact that CVT and PG efficiencies are a function of these parameters, the embedded FRs can change their efficiencies and therefore PS-CVT efficiency. Therefore, these FRs may have a significant effect on the power train efficiency and therefore, the vehicle FC.

Referring to Eq. (2), PS-CVT speed ratio is a linear function of CVT speed ratio. Therefore, knowing the maximum and minimum of CVT and PS-CVT speed ratio, two constraint equations between speed ratios of different elements can be derived. These equations are discussed in the next paragraph.

In order to determine PS-CVT efficiency, a dynamic analysis should be carried out. As mentioned above, there are two types of power flow for series PS-CVT. Each of these types has individual advantages. As shown in [3], type 2 of power flow has more efficiency in low speed ratios, while type 1 is more efficient in higher speed ratios. In this paper, we study type 1 and attempt to use it in its high efficiency region. As demonstrated in [3], for the case of employing toroidal CVT, if the speed ratio of PS-CVT is positive, and τ_{CVT} multiplicand in Eq. (2) (in this case, $(1-\tau_{\mathrm{PG}})\tau_{\mathrm{FR1}}\tau_{\mathrm{FR2}}\tau_{\mathrm{FR4}}$) is also positive, type 1 of power flow is obtained, and we will have:

$$\tau_{\text{PS-CVT(max)}}=\tau_{\mathrm{FR1}}\tau_{\mathrm{FR4}}\left[\tau_{\mathrm{PG}}\tau_{\mathrm{FR3}}+(1-\tau_{\mathrm{Pg}})\tau_{\mathrm{FR2}}\tau_{\mathrm{CVT(max)}}\right], \qquad (3)$$

$$\tau_{\text{PS-CVT(min)}}=\tau_{\mathrm{FR1}}\tau_{\mathrm{FR4}}\left[\tau_{\mathrm{PG}}\tau_{\mathrm{FR3}}+(1-\tau_{\mathrm{Pg}})\tau_{\mathrm{FR2}}\tau_{\mathrm{CVT(min)}}\right]. \qquad (4)$$

Using these equations, the values of $\tau_{\mathrm{PG}}\tau_{\mathrm{FR3}}\tau_{\mathrm{FR1}}\tau_{\mathrm{FR4}}$ and $\tau_{\mathrm{FR1}}\tau_{\mathrm{FR2}}\tau_{\mathrm{FR4}}$ can be calculated. Considering Figure 4, the sum of input and output power at point A is zero. In addition, regarding the equality of rotational speeds of elements 2, 3 and 4 in Figure 3, the sum of torques is zero, as:

$$P_2+P_3+P_4=0, \qquad (5)$$

$$T_2+T_3+T_4=0. \qquad (6)$$

According to the static equilibrium and power flow direction in PG place, we obtain:

$$\eta_{\mathrm{PG}}P_5+P_7+P_8=0, \qquad (7)$$

$$T_5+T_7+T_8=0. \qquad (8)$$

Using the presented equations, PS-CVT efficiency for positive speed ratios can be calculated by Eq. (9) as shown in Box I.

$$\eta_{\text{PS-CVT}} = \left|\frac{P_9}{P_1}\right| = \frac{\left[\tau_{\text{CVT}}\tau_{\text{FR2}}(\eta_{\text{PG}}\tau_{\text{FR3}} - \tau_{\text{PS-CVT}}/\tau_{\text{FR1}}) + \eta_{\text{PG}}\tau_{\text{FR3}}\tau_{\text{PG}}(\tau_{\text{FR3}} - \tau_{\text{CVT}}\tau_{\text{FR2}})\right]\eta_{\text{FR1}}\eta_{\text{FR4}}}{\tau_{\text{CVT}}\tau_{\text{FR2}}\eta_{\text{CVT}}\eta_{\text{FR2}}\eta_{\text{FR3}}(\eta_{\text{PG}}\tau_{\text{FR3}} - \tau_{\text{PS-CVT}}/\tau_{\text{FR1}}) + \tau_{\text{FR3}}\tau_{\text{PG}}(\tau_{\text{FR3}} - \tau_{\text{CVT}}\tau_{\text{FR2}})}. \quad (9)$$

Box I

According to this equation, the efficiencies of FR, PG and CVT elements should be determined to calculate PS-CVT efficiency. Since the efficiencies of the FR mechanisms is almost fixed compared to the efficiencies of PG and CVT, they are assumed a fixed value (98%) [14]. Similarly, the efficiencies of gear pairs in PG ($\eta_{P\text{Gel}}$) are assumed to be 98%. The efficiency of PG is achievable as a function of the connected elements' SR and gear pairs' efficiency ($\eta_{P\text{Gel}}$) as follows [15,16]:

$$\eta_{\text{PG}} = 1 - \frac{1 - \eta_{P\text{Gel}}}{1 + \frac{\eta_{P\text{Gel}}\tau_{\text{FR2}}\tau_{\text{CVT}}}{\tau_{\text{PG}}(\tau_{\text{FR3}} - \tau_{\text{FR2}}\tau_{\text{CVT}})}}. \quad (10)$$

To calculate full-toroidal CVT efficiency, a model presented in [17] is used. In this model, its efficiency is achieved as a function of its geometry, lubricating oil condition, and operating conditions, such as its speed ratio, input torque and rotational speed.

Figure 5 shows the efficiencies of PS-CVT and its elements versus its SR, for a specific geometry of its elements.

As can be seen, PS-CVT efficiency increases with the growth of its speed ratio. Moreover, CVT efficiency will increase, too. The growth rate of CVT efficiency is lower than its value for PS-CVT. The reason behind this is that increasing PS-CVT speed ratio decreases the value of power flows through CVT. Since CVT has the lowest efficiency between PS-CVT elements, decreasing the power flow through CVT reduces the overall power loss, and therefore, PS-CVT efficiency increases [6]. Consequently, PS-CVT efficiency is severely dependent on its speed ratio.

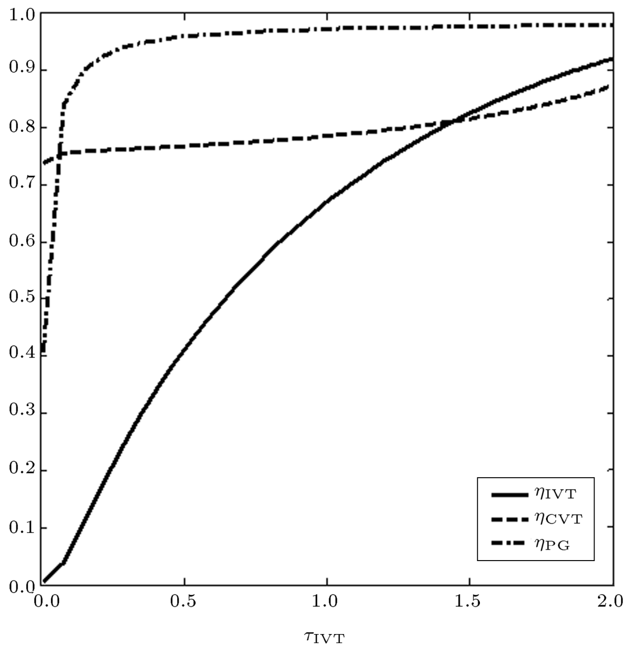


Figure 5. Efficiencies of PS-CVT and its elements for a specific geometry of its elements.

In this section, the efficiency of PS-CVT was derived as a function of its elements' efficiency and SR.

2.2. Simulation and control of power train

The model of the power train comprises a control unit which determines power flow through the power train elements including the engine, PS-CVT and clutch. The block diagram of the model is shown in Figure 6. According to this figure, the requested power of the vehicle at any point of the driving pattern can be calculated considering the vehicle speed and acceleration. Since the transmission efficiency is not 100%, the engine required power is different from the vehicle required power. This power is achievable as:

$$P_{\text{engine}} = \frac{P_{\text{required}}}{\eta_{\text{PS-CVT}}}. \quad (11)$$

In the first stage of the engine power calculation, PS-CVT efficiency is unknown. Therefore, a rough estimate for its efficiency is considered and the engine required power is determined. Subsequently, according to the control algorithm which will be illustrated in the following section, the engine operating point is determined, and regarding the vehicle speed, PS-CVT speed ratio is given by:

$$\tau_{\text{PS-CVT}} = \text{SR} = \frac{\frac{v}{R_d} n_d}{\omega_e}. \quad (12)$$

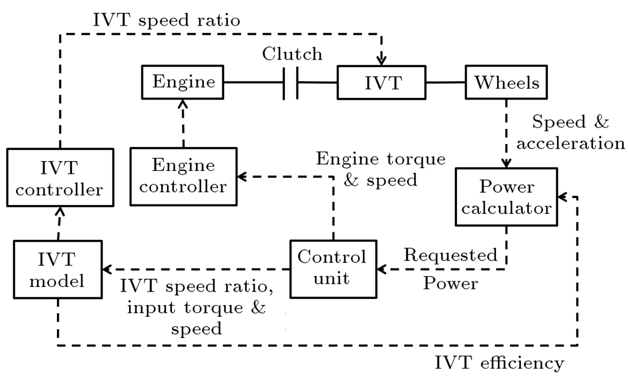
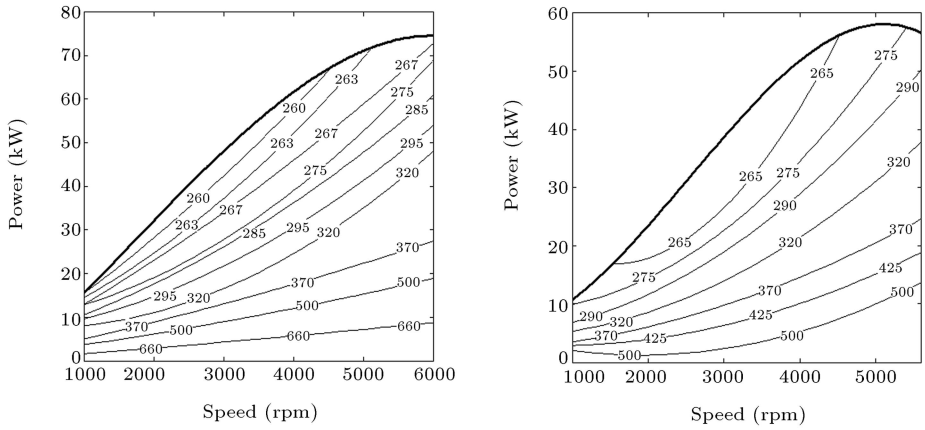


Figure 6. The block diagram of the powertrain simulation model.



(a) (b)

Figure 7. bsfc contours of the selected vehicles' engine: a) Vehicle A; and b) vehicle B.

Subsequently, knowing the power train input torque, speed and SR, its efficiency can be derived using the presented model in the previous section. The engine required power is computed again. This procedure continues until PS-CVT efficiency converges.

As mentioned, through application of PS-CVT, the speed ratio between the engine and the wheels is changed continuously, and there are an infinite number of gears. Therefore, its SR control strategy is more complex than the conventional one, where there are only five or six gears. One of the major concerns about PS-CVT power trains is their SR control strategy. One of the main control strategies is on the base of minimizing the vehicle FC. In this method, the experimental data of the engine Brake Specific Fuel Consumption (BSFC) is essential. In the present study, two vehicles in different classes are considered and the proposed power train is used in them. Figure 7(a) and (b) show BSFC contours of the selected vehicles' engine.

Vehicle A has an engine delivers power up to 100 hp, while the maximum power of the vehicle B's engine is 80 hp. As stated above, the amount of required power at any moment of the driving cycle can be calculated regarding the vehicle speed and its acceleration. For any value of the engine required power, there is a specific rpm, where the engine BSFC is minimal. For the selected vehicles' engine, these points are approximately on the curve of the engine maximum power. Having the vehicle speed and the engine rpm, the required SR of the power train can be determined.

In addition to the BSFC map of the engines, the characteristics of the selected vehicles are necessary to calculate their FC in the driving cycles. These data are presented in Table 1.

The algorithm of FC calculation when employing the presented control strategy is introduced in [13], thoroughly. This method has been validated by comparing the experimental data.

Table 1. The characteristics of the considered vehicles.

	Vehicle A	**Vehicle B**
f_r	0.013	0.013
C_D	0.325	0.355
A (m^2)	2.26	2.09
m (kg)	1220	1100
R_d (m)	0.3108	0.279
n_d	4.529	3.895

The SR range of the power train is determined according to the vehicle grade ability, its cruise speed and the engine fuel-optimal rpm [18]. As illustrated, by using the presented control strategy, the engine rpm is almost independent of the vehicle speed. Therefore, it is possible that the chosen SR is not in its determined domain. If it is larger than the upper bound of SR range (SR$_U$), the PS-CVT speed ratio will be adjusted to be equal to SR$_U$, and the engine rpm will change to create the demanded vehicle speed. Moreover, if the SR is smaller than the lower bound of the allowable range (SR$_L$), similar to the conventional transmissions, the power train creates the desired SR with the assistance of the clutch. In summary, if SR $>$ SR$_U$ then SR $=$ SR$_U$ and $\omega_e = \frac{v n_d}{\mathrm{SR}_U R_d}$, but if SR $<$ SR$_L$, then SR $=$ SR$_L$, $\omega_e = \omega_{\mathrm{idle}}$, and the clutch is engaged.

In this section, the control algorithm of PS-CVT speed ratio and its input torque and speed was presented. The optimization will be accomplished through controlling PS-CVT by this algorithm.

2.3. Optimization of the power train

According to Eqs. (1)-(10), speed ratios of FR mechanisms and PG can be changed to increase the overall power train efficiency and therefore decrease the vehicle FC in the driving cycle. In the present study, for the full-toroidal CVT part, the optimum geometry proposed in [17] is used. Its SR range is [-0.9,-

Table 2. The allowable range of the optimization parameters.

$\tau_{\text{FR1}}, \tau_{\text{FR2}}, \tau_{\text{FR3}}, \tau_{\text{FR4}}$	[1/8-8]
τ_{PG}	[0.1-10]

0.3]. Therefore, the optimization parameters are the speed ratios of FR_1, FR_2, FR_3, FR_4 and PG. Their allowable ranges in the optimization process are shown in Table 2. These ranges are determined according to the geometrical constraints.

The aim of the optimization is to minimize the selected vehicles' FC in different driving cycles including ECE, EUDC, FTP and Urban Dynamometer. One of the chief concerns about optimization is that if the values of optimized parameters will be different in various conditions. In order to resolve this concern, a specific driving cycle which is the collection of the mentioned driving cycles, is considered. In this cycle, the vehicles' motion is considered in ECE, EUDC, FTP and Urban Dynamometer, subsequently. Since the travelled distance in these cycles are different, the value of FC in each of them is normalized with respect to the distance travelled in it. The sum of these values is chosen as the objective function. First, the optimization is implemented on the introduced parameters by considering the vehicles' motion in this cycle. Then, the vehicles' FC is calculated in each of the mentioned driving cycles, through using the optimized power train. Afterwards, the second stage, introduced in the following section, is applied. As mentioned, in the first optimization, the optimization objective function is the FC of the vehicles in the combined driving cycle (collection of mentioned cycles). The optimization is carried out using Particle Swarm Optimization (PSO). In this method, each set of solutions is considered as a particle which has specific velocity and position. At each iteration, a new velocity for each particle is computed considering its distance from the global best position, the distance from its previous best position and its present velocity. Then, the new velocity is used to compute the resulting position of the particle in the search zone. This process is then repeated for a set number of times or until a minimum convergence rate is achieved [19]. This method is described in [20] comprehensively.

There are two constraints during the optimization process. The first is on the SR range created by the optimized power train. As stated, this range is definite and is achieved considering the vehicle desired grade ability, cruise speed, and the engine fuel-optimal rpm. This range is [0.31-1.34] and [0.3-1.28] for vehicles A and B, respectively. During the optimization process, SR of the power train elements must be changed to create this range.

The second constraint is on the relation between maximum and minimum of CVT and PS-CVT speed ratios. As stated, the necessary condition to achieve type 1 power flow is presented in Eqs. (3) and (4). Therefore, the optimization parameters must satisfy these equations.

In the case of violating the introduced constraints, an infinite value is assigned to the objective function, and the optimization will start again.

In summary, the optimization problem can be described as follows:

Minimize FC subject to:

$$\text{for vehicle A} \begin{cases} g_1 = \tau_{\text{PS-CVT(min)}} - 0.31 = 0 \\ g_2 = \tau_{\text{PS-CVT(max)}} - 1.34 = 0 \end{cases}$$

$$\text{for vehicle B} \begin{cases} g_1 = \tau_{\text{PS-CVT(min)}} - 0.3 = 0 \\ g_2 = \tau_{\text{PS-CVT(max)}} - 1.28 = 0 \end{cases}$$

$$\begin{aligned} g_3 =& \tau_{\text{PS-CVT(max)}} - \tau_{\text{FR1}}\tau_{\text{FR4}} \\ & \left[\tau_{\text{PG}}\tau_{\text{FR3}} + (1-\tau_{\text{PG}})\tau_{\text{FR2}}\tau_{\text{CVT(max)}}\right] = 0, \end{aligned}$$

$$\begin{aligned} g_4 =& \tau_{\text{PS-CVT(min)}} - \tau_{\text{FR1}}\tau_{\text{FR4}} \\ & \left[\tau_{\text{PG}}\tau_{\text{FR3}} + (1-\tau_{\text{PG}})\tau_{\text{FR2}}\tau_{\text{CVT(min)}}\right] = 0, \end{aligned}$$

where:

$$X = \begin{Bmatrix} \tau_{\text{FR1}} \\ \tau_{\text{FR2}} \\ \tau_{\text{FR3}} \\ \tau_{\text{FR4}} \\ \tau_{\text{PG}} \end{Bmatrix}.$$

2.3.1. Optimization results

After several runs, the optimized parameters for each of the vehicles are achieved. These values are shown in Table 3. As shown in this table, none of FR's speed ratios is close to 1, and it seems that embedding them in the exhibited places has significant effects on the amount of optimization.

In order to precisely evaluate the effect of employing FRs, the value of FC through utilization of a simple PS-CVT (Figure 1) for both vehicles is calculated in different driving cycles. As illustrated, for the specific geometry of CVT, there is no flexible parameter in

Table 3. The optimized parameters in the combined driving cycle for vehicles A and B.

	τ_{FR1}	τ_{FR2}	τ_{FR3}	τ_{FR4}	τ_{PG}
Vehicle A	1.19	-4.32	1.14	0.97	1.4
Vehicle B	1.44	1.46	0.72	1.16	-3.1

Table 4. The vehicles' FC in different driving cycles for the case of using simple series PS-CVT, the optimized power train, and their differences.

		ECE	**EUDC**	**FTP**	**Urban dynamometer**
Vehicle A	FC for optimized power train (L/100 km)	8.96	6.89	7.37	7.38
	FC for simple series PS-CVT (L/100 km)	10.2	7.69	7.92	7.9
	Fuel economy	12%	10%	7%	7%
Vehicle B	FC for optimized power train (L/100 km)	6.97	5.95	6.23	6.24
	FC for simple series PS-CVT (L/100 km)	7.7	6.8	7.3	7.4
	Fuel economy	10%	13%	15%	16%

simple series PS-CVT to optimize. Therefore, this PS-CVT does not need to be optimized in order to compare it with the proposed PS-CVT. The vehicles' FC for the case of using simple series PS-CVT and the optimized power train besides their differences are shown in Table 4.

As clearly shown, a remarkable decrease in the vehicles' FC is observed when substituting the optimized PS-CVT for the simple one in all the driving cycles. Thus embedding FR mechanisms in the exhibited places and optimizing them has a significant effect on the improvement of the vehicles' FC. Moreover, although the aim of optimization was to minimize the vehicles' FC in the combined driving cycle, there is also a considerable fuel economy in each of the driving cycles.

In the second stage of the optimization, the optimization is accomplished in case of considering the vehicles' motion in each of ECE, EUDC, FTP and Urban Dynamometer cycles, and the minimal FC values are obtained. Finally, these values are compared with the values achieved in the first stage of optimization. If the difference between these values is negligible, it can be concluded that there is a specific power train that gives a remarkable fuel economy in all the mentioned driving cycles. The values of the vehicles' FC equipped with the optimized power train, and the amounts of FC reduction compared to the application of simple series PS-CVT are shown in Table 5.

The results shown in Tables 4 and 5, reveal that there is no significant difference in the amounts of the fuel economies between the first and second optimization results, for each of the vehicles and for any of the driving cycles. For example, in the FTP driving cycle, the values of fuel economies of vehicle A for stages 1 and 2 of optimization are 7% and 8%, respectively. Therefore, the vehicles power train can be designed according to the parameters shown in Table 4.

3. Conclusion

The present study aimed to introduce the series PS-CVT in which four additional fixed ratio mechanisms are embedded as the vehicle power train, and then to optimize the added fixed ratio mechanisms. After presenting the governing dynamics of this power train, the proposed power train was optimized for two considered vehicles. A combined driving cycle which was a collection of ECE, EUDC, FTP and Urban Dynamometer was introduced and an optimization was accomplished in this driving cycle for both considered vehicles. FC values of the vehicles equipped with the optimized power trains were calculated. Then, the optimization was repeated for each of the mentioned driving cycles. Finally, it was investigated whether there is a remarkable difference in the vehicles' FC between the first and second optimization results. It was shown that, the optimized power train achieved

Table 5. The optimization results for vehicles A and B.

		ECE	**EUDC**	**FTP**	**Urban dynamometer**
Vehicle A	FC for optimized power train (L/100 km)	8.87	6.87	7.32	7.31
	Fuel economy	13%	11%	8%	8%
Vehicle B	FC for optimized power train (L/100 km)	6.83	5.67	6.22	6.16
	Fuel economy	11%	17%	15%	17%

from the first optimization gives a fuel economy near to the case of using the second optimization results in each of driving cycles and for each of the considered vehicles. Therefore, it seems that there is a specific power train for each of the vehicles that gives a remarkable fuel economy in each of the mentioned driving cycles.

Nomenclature

τ	Speed ratio
ω_e	Engine rpm
A	Vehicle frontal area
C_D	Aerodynamic drag coefficient
R_d	Wheel's radius
m	Vehicle mass
n_d	Speed ratio of final drive
P	Power
T	Torque
η	Mechanical efficiency
f_r	Rolling resistance coefficient of the vehicle
$\eta_{P\mathrm{Gel}}$	Gear pairs efficiency in planetary gear train
v	Vehicle speed

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